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COMPARATIVE ADVANTAGE AND LONG-RUN GROWTH*

by

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Comparative Advantage and Long-Run Growth

Abstract

We construct a dynamic, two-country model of trade and growth in which endogenous technological progress results from the profit-maximizing behavior of entrepreneurs. We study the role that the external trading environment and that trade and industrial policies play in the determination of long-run growth rates. We find that cross-country differences in efficiency at R&D versus manufacturing (i.e., comparative advantage) bear importantly on the growth effects of economic structure and commercial policies. Our analysis allows for both natural and acquired comparative advantage, and we discuss the primitive determinants of the latter.

Keywords: Comparative advantage, long-run growth, commercial policy, technological change, R&D

JEL Classification: 411, 111, 621, 422

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I. Introduction

What role do the external trading environment and commercial policy play in the determination of long-run economic performance? This central question of international economics has received surprisingly little attention in the theoretical literature over the years.

Previous research on trade and growth has adopted the neoclassical framework to focus on factor accumulation in the open economy. (See the surveys by Findlay (1984) and Smith (1984)). This research largely neglects the effects of trade structure on rates of growth, however, addressing instead the reverse causation from growth and accumulation to trade patterns.[1] The direction that the research followed almost surely can be ascribed to the well-known property of the standard neoclassical growth model with diminishing returns to capital that (endogenous) growth in per capita income dissipates in the long run. For this reason, the familiar models which incorporate investment only in capital equipment seem ill-suited for analysis of long-run growth.

The available evidence collected since the seminal work of Solow (1957) also leads one to look beyond capital accumulation for an explanation of growth. Exercises in growth-accounting for a variety of countries generally find that increases in the capital to labor ratio account for considerably less than half of the last century's growth in per capita incomes.[2] Although econometric efforts to explain the residual have been somewhat disappointing, (see e.g. Griliches (1979)) professional opinion and common sense continue to

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[1] An exception is Corden (1971), who studies how the opening up of trade affects the speed of transition to the steady state in a two-factor neoclassical growth model with fixed savings propensities.

[2] See Maddison (1987) for a recent, careful exercise in growth accounting.

impute much of this residual to improvements in technology.[3] We share the view, expressed by Romer (1986, 1988), that a full understanding of growth in the long run requires appreciation of the economic determinants of the accumulation of knowledge.

In this paper, we draw on the pioneering work by Romer to construct a model that highlights the roles of economies of scale and technological progress in the growth process. As in Romer's work, our model implies an endogenous rate of long-run growth in per capita income, and we study its economic determinants. Our primary contribution lies in casting the growth process in a two-country setting. We provide, for the first time, a rigorous analysis linking long-run growth rates to trade policies and other international economic conditions. Moreover, we find that recognition of cross-country differences in economic structure impinges upon conclusions about the long-run effects of domestic shocks and policies.

Our model incorporates the essential insights from Romer (1988), although we introduce some differences in detail. The building blocks are an R&D sector that produces designs or blueprints for new products using primary resources and previously accumulated knowledge, an intermediate-goods sector consisting of oligopolistic producers of differentiated products, and a consumer goods sector in each country that produces a country-specific final output using labor and intermediate inputs. As in Ethier (1982), total factor productivity in final production increases when the number of available varieties of differentiated inputs grows. Thus, resources devoted to R&D

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[3] The benefits of education and experience undoubtedly contribute part of the explanation for the growth residual. See, for example, Lucas (1988) and Becker and Murphy (1988) for growth models that highlight the role of human capital accumulation as a source of growth.

contribute over time to productivity in the production of final goods, as well as to the stock of scientific and engineering knowledge.

The new elements in our analysis stem from the assumed presence of cross-country differences in the effectiveness with which primary resources can perform different activities; i.e. comparative advantage. For simplicity we specify a one-primary-factor model, and allow the productivity of this factor in the three activities to vary internationally. We suspect that similar results could be derived from a multi-factor model with inter-industry differences in factor intensities. In any event, we find that many comparative dynamic results hinge on a comparison across countries of efficiency in R&D relative to efficiency in manufacturing the goods that make use of the knowledge generated by R&D, namely middle products. The effects of policy in a single country, of accumulation of primary resources in a single country, and of a shift in world tastes toward the final output of one of the countries all depend upon the identity of the country in which the change originates in relation to the international pattern of comparative advantage.

We provide a more complete verbal description of the economic setting, followed by a formal statement of the model, in Section II immediately below. Then, in Section III, we derive the dynamic equilibrium of the world economy, discuss conditions under which there exists a steady-state equilibrium with positive growth of per capita income, and calculate two reduced-form equations that determine the steady-state growth rate. In Section IV we investigate the structural determinants of long-run growth. There, the implications for growth of variations in consumer preferences, primary-input coefficients in one or both countries, and stocks of available primary resources are considered. Section V contains policy analysis. We study barriers and inducements to trade in consumer goods and subsidies to research and development. Then, in Section VI, we introduce an important elaboration of the model. There we extend the analysis to incorporate lags in the dissemination of knowledge and asymmetries in the speed of diffusion within and between countries. We use the extended model to reconsider the effects of trade policies on the steady-state rate of growth. Finally, Section VII provides a brief summary of our findings.

Before proceeding, a brief disclaimer may be in order. Our results in this paper concern steady-state rates of growth. Because we perform steady state comparisons and also because growth rates have no immediate implications for discounted utility, we do not mean to confer upon our findings any normative interpretation. We do hope to report on the welfare properties of our model in a future article.

II. The Model

A. General Description

In Figure 1 we provide a schematic representation of our model. We study a world economy comprising two countries. Each country engages in three productive activities: the production of a final good, the production of varieties of differentiated middle products (i.e., intermediate inputs), and research and development (R&D). The two final goods are imperfect substitutes, and both are demanded by consumers worldwide. A single primary factor is used in production, and is taken to be in fixed and constant supply in each country. Although we refer to this factor as "labor", we have in mind an aggregate of irreproducible resources that for any given level of technical know-how limits aggregate output.

We follow Romer (1988) in assuming that R&D generates two distinct outputs. First, as in our earlier paper (Grossman and Helpman, 1988; see also Judd, 1985), research effort produces "blueprints" for new products. The returns to this component of R&D output, coming in the form of an infinite stream of monopoly profits, are assumed to be perfectly appropriable by the originator due either to perfect and indefinite patent protection or technical barriers to imitation. Blueprints are not tradable, so the manufacture of each middle product takes place in the country in which it was developed.

Second, R&D contributes to the stock of disembodied knowledge. Knowledge here includes all general scientific information, as well as some forms of engineering data with more widespread applicability, generated in the course of developing marketable products. Knowledge contributes to the productivity of further research efforts, by reducing the amount of labor needed for an inventor to develop a new product. Due to the more general and non-patentable nature of this product of the R&D effort, appropriation of the resulting returns by the creator becomes problematic. We assume to begin with that general knowledge disseminates immediately and costlessly throughout the world. This approximates a situation in which information spreads through technical journals, professional organizations, and interpersonal commercial contacts, and where literature, scientists, and businessmen move freely across international borders (see Pasinetti, 1981, ch.11). We relax this assumption by introducing lags in the dissemination of knowledge in Section VI.

Once developed, middle products are manufactured with labor alone under conditions of constant returns to scale. These products are freely traded between the countries. The middle products, along with labor, serve as inputs into the production of the final goods in each country. Given the number of available varieties, the production function for each of the final goods exhibits constant returns to scale. But an increase in the number of varieties of middle products used as inputs raises total factor productivity. This specification, which we borrow from Ethier (1982), captures the notion that an increasing degree of specialization generates technical efficiency gains. In effect, the economy's potential for augmenting the degree of specialization by developing new middle products implies the existence of dynamic scale economies at the industry level that are external to the individual final-good-producing firms.

At each point in time, competitive producers of final goods earn zero profits. Patent holders for middle products engage in oligopolistic competition, earning monopoly rents. Forward-looking entrepreneurs in each country elect to devote resources to R&D if the present discounted value of future profits exceeds the current cost of development. Free entry into R&D ensures that this activity earns at most a competitive return. Finally, consumers maximize intertemporal utility, with savings devoted to the acquisition of corporate bonds or ownership claims in input-producing firms. We assume that financial capital is internationally mobile, although many of our results also hold in the absence of international borrowing and lending.

We shall study the dynamic evolution of this world economy. Over time, the number of available varieties of middle products grows, affecting both profitability in the intermediate-goods sector and productivity in the final goods sector. The stock of technical knowledge also expands, reducing the resource cost of inventive activity. Under certain conditions, the world economy approaches a steady-state rate of growth of per-capita income, the determinants of which are the focus of our attention in Sections IV and V.

We turn now to the formal specification of the model.

B. Consumers

Consumers worldwide share identical, homothetic preferences. We represent these preferences by a time-separable intertemporal utility function

$$(1) \qquad U_t = \int_t^\infty e^{-\rho(\tau-t)} \log u[y_1(\tau), y_2(\tau)] d\tau$$

where ρ is the subjective discount rate and $y_i(\tau)$ is consumption of final goods from country i in period τ. The instantaneous sub-utility function $u(\cdot)$ is non-decreasing, strictly quasi-concave and positively linearly homogeneous.

A typical consumer maximizes (1) subject to an intertemporal budget constraint, which requires that the present value of all future expenditures not exceed the present value of factor income plus the market value of current asset holdings. With $u(\cdot)$ linearly homogenous, this problem can be solved in two stages. First, the consumer maximizes static utility for a given level of expenditure at time τ, $E(\tau)$. The solution to this sub-problem generates an indirect utility function, $v[p_{Y1}(\tau), p_{Y2}(\tau)]E(\tau)$, where p_{Yi} is the price of y_i. In the absence of barriers to trade in final goods, these prices are common to consumers in the two countries. The second-stage problem now can be formulated as one of choosing the time pattern of expenditures to maximize

$$(2) \qquad V_t = \int_t^\infty e^{-\rho(\tau-t)} \{\log v[p_{Y1}(\tau), p_{Y2}(\tau)] + \log E(\tau)\} \, d\tau$$

subject to

$$(3) \qquad \int_t^\infty e^{-[R(\tau)-R(t)]} E(\tau) \le \int_t^\infty e^{-[R(\tau)-R(t)]} w(\tau) L d\tau + Z(t),$$

where R(t) is the cumulative interest factor from time 0 to time t (R(0)=1), $w(\tau)$ is the consumer's wage rate at time τ, L is his labor supply, and Z(t) is the value of his time t asset holdings. The interest factor in (3) is common to all individuals as a result of trade on the integrated world capital market, but the wage rate may vary across countries, with residents of country i receiving $w_i(\tau)$.

From the first-order conditions to this problem, we see that the optimal path for expenditure obeys

$$(4) \qquad \frac{\dot{E}}{E} = \dot{R} - \rho \ .$$

Savings are used to accumulate either ownership claims in input-producing firms or riskless bonds issued by these same firms.[4] Arbitrage ensures that the rates of return on these two assets are equal, and in equilibrium consumers are indifferent as to the composition of their portfolios.

C. <u>Producers</u>

At a point in time, output of final goods in country i is given by

$$Y_i = BA_i L_{Yi}^{1-\beta}\left[\int_0^n x_i(\omega)^\alpha d\omega\right]^{\frac{\beta}{\alpha}}, \qquad 0 < \alpha,\beta < 1,$$

where L_{Yi} represents employment in the final goods sector, $x_i(\omega)$ denotes the input of middle product ω, and n is (the measure of) the number of varieties

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[4] Firms that produce final goods earn zero profits, hence their stock market value is nil. Input-producing firms command a market value equal to the discounted value of their future operating profits.

of middle products available at that time.[5] Notice that the production functions are the same for both countries except for the productivity parameter A_i. This productivity parameter may represent differences in technology or in the endowments of sector-specific inputs.

Competition in this sector ensures marginal-cost pricing. Hence, by appropriate choice of the constant B, producer prices satisfy

$$(5) \qquad p_{Yi} = \left(\frac{w_i}{A_i}\right)^{1-\beta}\left[\int_0^n p_X(\omega)^{1-\epsilon}d\omega\right]^{\frac{\beta}{1-\epsilon}}, \qquad \epsilon = \frac{1}{1-\alpha} > 1,$$

where $p_X(\omega)$ is the price of variety ω. Final-good producers worldwide pay the same prices for (freely traded) middle products.

At every moment in time the existing producers of middle products engage in oligopolistic competition. Each producer takes as given the prices of his rivals, as well as the outputs and prices of final goods. The producer of a variety ω in country i chooses $p_X(\omega)$ to maximize profits,

$$\pi_i(\omega) = [p_X(\omega) - w_i a_{LXi}] \frac{p_X(\omega)^{-\epsilon}}{\int_0^n p_X(\omega)^{1-\epsilon}d\omega} \beta\Sigma_i p_{Yi} Y_i \ ,$$

where a_{LXi} is the unit labor requirement for production of intermediates in country i. This expression for profits comprises the product of profits per unit (in square brackets) and derived demand for variety ω, where the latter incorporates the assumption that neither prices nor volumes of final production vary with $p_X(\omega)$. The first-order condition for a profit maximum

[5] Here, and henceforth, we omit time arguments when no confusion is caused by doing so.

implies the usual fixed-markup pricing rule,

(6) $$\alpha p_X(\omega) = w_i a_{LXi} \ .$$

It is clear from (6) that varieties originating from the same country bear the same price. Letting p_{Xi} represent the price of a variety produced in country i and n_i be the number of intermediate inputs produced there, equations (5) and (6) imply

(7) $$p_{Yi} = \left(\frac{w_i}{A_i}\right)^{1-\beta} \left(\sum_j n_j p_{Xj}{}^{1-\epsilon}\right)^{\frac{\beta}{1-\epsilon}} ,$$

(8) $$\alpha p_{Xi} = w_i a_{LXi} \ .$$

With these prices, profits per firm can be expressed as

(9) $$\pi_i = (1-\alpha) p_{Xi} X_i / n_i \ ,$$

where X_i is aggregate output of intermediates in country i (n_i times per-firm output) and is given by

(10) $$X_i = \frac{n_i p_{Xi}^{-\epsilon}}{\Sigma_j \ n_j p_{Xj}^{1-\epsilon}} \beta(\Sigma_j \ p_{Yj} Y_j) \ .$$

The number of intermediates produced in country i evolves over time according to the amount of R&D that takes place there. If resources are devoted to R&D in country i at time t, then the present value of future operating profits -- discounted to time t -- must be equal to the current cost of R&D, denoted by $c_{ni}(t)$. We write this analog to the zero-profit condition of static, monopolistic-competition models as

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]}\pi_i(\tau)d\tau = c_{ni}(t) .$$

Differentiating this condition with respect to t, we find

$$\text{(11)} \qquad \frac{\pi_i + \dot{c}_{ni}}{c_{ni}} = \dot{R} .$$

Equation (11) expresses a standard no-arbitrage condition. Recognizing that $c_{ni}(t)$ represents the value of an input-producing firm in country i at time t, (11) equates the instantaneous rate of return on shares in such a firm (the sum of dividends and capital gains) to the rate of interest.

As we have discussed above, R&D produces a joint output; new varieties of middle products and additions to the stock of knowledge. If L_{ni} units of labor engage in research in country i, they generate a flow of new products $\dot{n}_i$ given by

$$\text{(12)} \qquad \dot{n}_i = L_{ni}K/a_{Lni} ,$$

where K is the current stock of knowledge and a_{Lni} is a country-specific productivity parameter. We assume until Section VI that the by-product contribution to knowledge occurs instantaneously and that diffusion is worldwide. Also, we take the stock of knowledge to be proportional to cumulative experience in R&D; i.e., there are no diminishing returns to research in adding to scientific understanding. By choosing units for K so that the factor of proportionality is unity, we have K=n and

$$\dot{K} = \Sigma_i L_{ni}K/a_{Lni}. \tag{13}$$

Finally, since knowledge is a free input to each individual entrepreneur, the cost of product development in country i can be written as

$$c_{ni} = w_i a_{Lni}/n . \tag{14}$$

This completes our description of the model.

III. Equilibrium Dynamics

During the course of the development of our model in the previous section, we provided some of the equilibrium conditions. For example, we derived pricing equations for goods and a no-arbitrage condition relating equilibrium asset returns. In this section we complete the list of equilibrium requirements by adding conditions that stipulate market clearing in factor and final-goods markets. We then derive and discuss a reduced-form system that describes equilibrium dynamics.

Static equilibrium in the markets for the two final goods implies

$$p_{Yi}Y_i = s_iE , \tag{15}$$

where $s_i(p_{Y1},p_{Y2})$ is the share of world spending allocated to Y_i and E is world spending on consumer goods. The share function is, of course, homogenous of degree zero. We establish below that relative commodity prices are constant in the vicinity of a steady state with active R&D sectors in both countries. For this reason, we take s_i to be constant in our subsequent analysis, and omit its functional dependence on relative prices.[6]

The labor-market clearing conditions equate labor supply and labor demand in each country. Using (7) and Shephard's lemma, we see that final-goods producers demand $(1-\beta)p_{Yi}Y_i/w_i$ workers. The demand for labor by middle-products producers is $a_{LXi}X_i$, while (12) and the fact that K=n imply demand for labor by product developers of $(a_{Lni}/n)\dot{n}_i$. Hence,

$$(16) \qquad (a_{Lni}/n)\dot{n}_i + a_{LXi}X_i + (1-\beta)p_{Yi}Y_i/w_i = L_i \ ,$$

where L_i is the labor force available in country i.

Since we neglect here the monetary determinants of the price level, we may choose freely a time pattern for one nominal variable. It proves convenient to specify the numeraire as follows:

$$(17a) \qquad p_{X1} = n(a_{LX1}/a_{Ln1})^{1/\epsilon} \ .$$

We show in Appendix A via equations (8)-(11) and (14) that, with this normalization, a necessary condition for convergence to a steady state with positive R&D in both countries (i.e., non-specialization) is

$$(17b) \qquad p_{X2} = n(a_{LX2}/a_{Ln2})^{1/\epsilon} \ .$$

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[6] Of course, if u(·) takes a Cobb-Douglas form, then expenditure shares are independent of relative prices in any equilibrium.

Together, (17a) and (17b) imply that relative prices of middle products are constant along the convergent path, which further implies with (8) the constancy of relative wages, and with (7) the constancy of relative prices of final goods. This last fact justifies our treatment of expenditure shares as constants.

Let g denote the rate of growth of the number of products and the stock of knowledge; i.e., $g=\dot{n}/n=\dot{K}/K$. Then from (17) and (8) we see that prices of intermediates and wages grow at rate g, while from (14), product development costs are constant. Moreover, equations (9)-(11), (14), (15), and (17) imply

$$(18) \qquad X_i = \frac{n_i b_i^{1/\alpha}}{\Sigma_j \, n_j b_j} \frac{\beta E}{n}$$

and

$$(19) \qquad \dot{R} = \frac{1}{\epsilon - 1} \frac{\beta E}{\Sigma_j \, n_j b_j} \quad ,$$

where $b_i = (a_{Lni}/a_{LXi})^{\alpha}$. The coefficients b_i will serve as our measures of <u>comparative advantage</u>. Country 1 enjoys comparative advantage in conducting R&D if and only if $b_1 < b_2$.

Since wages grow at the same rate as n, it proves convenient to define $e=E/n$. Letting $\sigma_i=n_i/n$ be the share of products manufactured in country i and noting that $g=\Sigma_i \dot{n}_i/n$, (16), (15), (17), (8), and (18) imply

$$(20) \qquad \frac{\dot{n}}{n} = g = H - \frac{\beta e}{\sigma} - \frac{1-\beta}{\alpha} \, se \quad ,$$

where we have defined $H=\Sigma_i L_i/a_{Lni}$, the total <u>effective</u> labor force, $\sigma=\Sigma_i \sigma_i b_i$, a weighted average of the comparative advantage parameters with product shares as weights, and $s = \Sigma_i s_i / b_i$. Observe that the parameter σ, which provides a useful summary of the static intersectoral resource allocation, grows (shrinks) over time if and only if the growth rate of the number of differentiated middle products in the country with comparative <u>disadvantage</u> in R&D exceeds (falls short of) that of the other country.

We are now prepared to derive two equations that describe the dynamic evolution of the world economy. From the definition of e, we have $\dot{e}/e = \dot{E}/E - g$, or, substituting (4), (19), and (20),

$$(21) \qquad \frac{\dot{e}}{e} = \frac{\beta e}{\alpha \sigma} + \frac{1-\beta}{\alpha} se - H - \rho \, .$$

Hence, the rate of increase of spending per middle product is larger the greater is spending per product and the smaller is the share of the country with comparative disadvantage in R&D in the total number of varieties.

Now, from the definition of the product shares σ_i, their rates of change are given by $\dot{\sigma}_i/\sigma_i = \dot{n}_i/n_i - \dot{n}/n$. Using (16) together with (17), (8), (18), and (20), we obtain

$$(22) \qquad \dot{\sigma}_i = h_i - \frac{s_i}{b_i} \frac{1-\beta}{\alpha} e - \sigma_i \left[H - \frac{1-\beta}{\alpha} se \right] ,$$

where $h_i = L_i / a_{Lni}$ is effective labor in country i, and $\Sigma_i h_i = H$. Since the evolution of the two product shares are related by $\Sigma_i \dot{\sigma}_i = 0$, we can replace (22) by a single differential equation in σ. Making use of the fact that $\dot{\sigma} = \Sigma_i \dot{\sigma}_i b_i$, we find

$$(23) \qquad \dot{\sigma} = h - \frac{1-\beta}{\alpha} e - \sigma\left(H - \frac{1-\beta}{\alpha} se\right) ,$$

where $h \equiv \Sigma_i h_i b_i$.

Equations (21) and (23) constitute an autonomous system of differential equations in e and σ. The solution to this system, together with (3), (20), and the definition of σ, provide a complete description of the evolution of spending and the number of products in each country. From these, the paths for outputs, employments and final-goods prices are easily derived. Thus, we shall use this two-equation system to analyze equilibrium dynamics.

Equations (21) and (23) do not apply if the steady state falls outside the shaded region in Figure 2; i.e., σ must lie between the smaller and larger of the b_i's, while the growth rate must fall between zero and ρ. A negative growth rate is impossible, because the number of blueprints cannot decline. And a steady-state growth rate in excess of the subjective rate of discount would imply unbounded utility and thus invalidate our use of (4) in describing the evolution of spending. The shape of the iso-growth-rate curves (increasing and concave) follows from (20). We limit our attention to economies with parameter configurations that ensure a steady state in the interior of the shaded area.[7]

Contrary to the impression given by the figure, the likelihood of reaching a steady state in the indicated region does not depend directly on the spread between b_1 and b_2. This point is seen most clearly by considering

[7] A further restriction that we have not explicitly taken into account to this point is that product development must be non-negative in each country. This condition certainly is satisfied in the neighborhood of a steady-state equilibrium with positive growth, but it need not hold all along the convergent path to such an equilibrium. Thus, strictly speaking, the equilibrium dynamics that we describe below apply for sure only in the vicinity of a steady state.

the case in which neither country enjoys comparative advantage in R&D, a case that is of some interest in its own right. When $b_1=b_2=b$, the shaded area collapses to the vertical line segment between points 1 and 2 in Figure 3. However, in this case, $h=bH$ and $s=1/b$, which implies via (23) that $\dot{\sigma}=0$ for $\sigma=b$, irrespective of the value of e. When considerations of comparative advantage are absent, any intersection of the two curves inevitably falls within the horizontal "width" of the relevant region.

Consider next the shape and location of the curve depicting stationary points for e. We draw the $\dot{e}=0$ locus as increasing and concave (see (21)). To understand the positive slope of this curve, observe from (19) that the interest rate can be expressed as

$$(24) \qquad \dot{R} = \beta e/\sigma(\epsilon-1) \ .$$

Comparing (24) and (20), we see that an increase in spending per product increases the interest rate and reduces the rate of growth of n (the former because the profitability of R&D rises with derived demand, the latter because more spending means less savings and hence less investment). Since an increase in the interest rate raises the rate of growth of nominal spending, and the rate of growth of e is just the difference between the rates of growth of E and n, it follows that an increase in e raises the growth rate of e. To compensate for this acceleration in spending per product, if e is to be stationary, σ must rise. An increase in σ lowers the interest rate and raises the rate of growth of n, thereby reducing the rate of growth of e.

In the figure, the $\dot{e}=0$ curve intersects the $\dot{\sigma}=0$ line at point 3, which lies between points 1 and 2. Clearly, there are many constellations of parameters values that admit a steady state with g in the permissible range. In the figure, we also indicate with arrows the direction of the system's movement. Point 3 is seen to be unstable. The intertemporal budget constraint can only be satisfied with equality if the initial value of e corresponds to the ordinate of point 3. Hence, in this case of no comparative advantage, e jumps immediately to its long-run value, and the world economy remains always in a steady state.

Now let us reintroduce comparative advantage. We distinguish two sub-cases depending on the relative sizes of h/H and 1/s. It can be shown that $h/H > 1/s$ if and only if $(b_2-b_1)(h_2b_2/s_2 - h_1b_1/s_1) > 0$. If the shares of the two countries' final outputs are in proportion to their relative <u>effective</u> labor forces, then the second inequality will be satisfied. But a bias in size relative to budget share of final output can reverse the inequality and thus the relationship between h/H and 1/s. We consider the alternative cases in turn.

Figure 4 depicts equilibrium dynamics when $1/s > h/H$. Both 1/s and h/H must lie between b_{min} and b_{max}. The $\dot{\sigma}=0$ curve here is everywhere downward sloping, crosses the horizontal axis at h/H, and is discontinuous at $\sigma=1/s$. The slope of the curve is understood as follows. For $\dot{\sigma}=o$, we must have $\dot{\sigma}_1=0$, which requires that the resources available for R&D in each country be just sufficient to preserve the country's <u>share</u> in the world's number of varieties. Consider country 1 and suppose for concreteness that this country has comparative advantage in R&D. Then an increase in σ lowers σ_1, thereby reducing the resources needed for production of middle products. The fall in σ_1 also reduces the amount of R&D country 1 must perform to preserve its share in the number of products. <u>Ceterus paribus</u>, σ_1 would tend to rise. An increase in e, on the other hand, diverts resources away from R&D to production of middle and final products in country 1. But it also causes the world's rate of product growth to fall, thereby diminishing the amount of R&D country 1 must undertake to maintain its share of middle products. The relative magnitudes of these two effects depend upon country 1's relative size, and on the share of its final product in aggregate spending. In the case under consideration, the second effect dominates, and so the $\dot{\sigma}=0$ curve slopes downward.

In this case, there exists a unique steady-state point, labelled 1 in the figure. For initial values of σ not too different from that at point 1, a unique trajectory (saddle-path) converges to the steady state. This trajectory, labelled SS, fulfills all equilibrium requirements and satisfies the intertemporal budget constraint with equality. Along this trajectory (in the vicinity of the steady state), the interest rate and profit rate are declining (see (24)) and nominal expenditure E is rising. If the country with comparative advantage initially has a share of products that is smaller (larger) than its steady-state share, expenditure rises more slowly (rapidly) than the number of products.

The second case arises when $h/H > 1/s$. Then the $\dot{\sigma}=0$ schedule slopes upward, as depicted in Figure 5. If the curve intersects the $\dot{e}=0$ locus in the positive orthant at all, it must intersect it twice, as at points 1 and 2.[8] The lower point (point 1) represents the steady state with the higher rate of growth (growth rates increase as we move down along the $\dot{e}=0$ schedule, as we demonstrate below) and indeed the growth rate corresponding to point 2 may be

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[8] The geometry supports this claim, once we recognize that the $\dot{\sigma}=0$ curve asymptotes to the horizontal line at $\alpha H/s(1-\beta)$, whereas the $\dot{e}=0$ curve asymptotes to the horizontal line at $\alpha(H+\rho)/s(1-\beta)$. The algebra provides confirmation, as simple manipulation reveals that the steady-state growth rate solves a quadratic equation.

negative. More importantly, as we show in Appendix B, the equilibrium at point 1 exhibits saddle-path stability, whereas that at point 2 is locally unstable.[9] To the right of point 1, the saddle-path leading to that point remains trapped in the area bounded by the $\dot{e}=0$ locus and the line segment joining points 1 and 2, and is everywhere upward sloping. Thus, the qualitative properties of the dynamic trajectory that leads to a stable, positive-growth equilibrium in Figure 5 mimic those of the stable saddle-path in Figure 4.

For the remainder of this paper, we shall restrict our discussion to stable steady-state equilibria with positive growth rates. That is, we focus our attention on equilibria such as those at the points labelled 1 in Figures 4 and 5. In the steady state there occurs intra-industry trade in middle products and inter-industry trade in consumer goods, with the long-run pattern of trade determined by comparative advantage, productivities in the two final-goods industries, and consumer preferences.

IV. Determinants of Long-Run Growth

Our model generates an endogenous rate of long-run growth. We now are prepared to explore how economic structure and economic policy affect this growth rate. In this section, we derive the implications of sectoral productivity levels, country sizes and demand composition for the steady-state

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[9] We strongly suspect, however, that whenever there exist two positive-growth, steady-state equilibria in the admissible range, there also exists a third (saddle-path stable) steady-state equilibrium with zero growth. We have established the existence of such an equilibrium for some parameter values, but so far have been unable to construct a general existence proof. Since the equilibrium at point 1 in Figure 5 can only be reached if the initial value of σ is less than that at point 2, we suspect that initial values of σ in excess of that at point 2 (and perhaps only these) imply convergence to a steady state with zero growth.

growth rate. The influence of trade policies and of subsides to R&D are treated in the next section.

We calculate the steady-state values, $\bar{e}$ and $\bar{\sigma}$, from the following pair of equations, derived form (21) and (23) with $\dot{e}$ and $\dot{\sigma}$ set to zero:

$$\frac{\beta\bar{e}}{\alpha\bar{\sigma}} + \frac{1-\beta}{\alpha} s\bar{e} = H + \rho \;; \tag{25}$$

$$\frac{1-\beta}{\alpha} \bar{e}(1-\bar{\sigma}s) + \bar{\sigma}H = h \;. \tag{26}$$

Whenever $1/s > h/H$, these equations provide at most one solution for $(\bar{e},\bar{\sigma})$ consistent with $\bar{g} > 0$. When $1/s < h/H$, there may be two such solutions, in which case we select the stable equilibrium; i.e., the one with the smaller values for $\bar{e}$ and $\bar{\sigma}$. Stability implies, in this latter case, that the $\dot{\sigma}=0$ curve intersects the $\dot{e}=0$ curve from below (see Appendix B). We make use of this condition, namely

$$\frac{\beta\bar{e}}{(H+\rho)\bar{\sigma}^2} > \frac{\alpha H - (1-\beta)s\bar{e}}{\bar{\sigma}H - h} \qquad \text{for} \quad 1/s < h/H \;, \tag{27}$$

in signing the comparative-dynamics derivatives below.

The growth rate of the number of varieties in the steady-state equilibrium can be derived from the solution to (25) and (26), together with (15). From this, we can easily calculate the growth rate of output. In the steady state nominal expenditure grows at rate $\bar{g}$, while (7) implies that p_{Yi} grows at rate $[1 - \beta/(1-\epsilon)]\bar{g}$. From these facts and (15), we deduce that final output grows at rate $\beta\bar{g}/(1-\epsilon)$.

It is worth noting at this point that the steady-state equations (25) and

(26), as well as the equation for $\bar{g}$, do not rely on our assumption of perfect capital mobility. In the absence of capital mobility, the steady state would be the same as long as consumers worldwide share identical preferences (and therefore common subjective discount rates).[10]

It is instructive to begin the discussion with the case in which neither country exhibits comparative advantage in conducting R&D; i.e., $b_1=b_2=b$. As we noted in Section III, this case has $h=bH$ and $s=1/b$. Then (25) and (26) provide a unique solution for $\bar{e}$ and $\bar{\sigma}$, which upon substitution into (20), yields the long-run growth rate

$$(28) \qquad \bar{g} = \frac{\beta(H+\rho)}{\epsilon} - \rho \ .$$

This equilibrium growth rate shares much in common with that derived by Romer (1988) for a closed economy. In particular, the growth rate rises with effective labor H and declines with the subjective discount rate. Our measure of effective labor adjusts raw labor for productivity in R&D (recall that $H=\Sigma_i L_i/a_{Lni}$), so greater effectiveness in research in either country, as well as a larger world labor force, necessarily mean faster growth. Long-run growth does not, however, depend upon coefficients that determine absolute productivity in the intermediate or final goods sectors (such as A_i or a_{LXi}). Nor do properties of the instantaneous utility function $u(\cdot)$, including the product composition of final demand, play any role in the determination of $\bar{g}$. As we shall see presently, all these features (except for the absence of an

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10 The cases of perfect and imperfect capital mobility do differ in their implications for the steady-state share of each country in aggregate spending E. However, as should be clear from (25) and (26), the cross-country composition of E does not matter for the issues taken up in the present section.

effect of A_1 on $\bar{g}$) are special to a world without any comparative advantage.

Consider next the case with $1/s > h/H$. The curves $\dot{e}=0$ and $\dot{\sigma}=0$ in Figure 6 describe the initial situation, with a unique initial steady state at point 1. Now suppose that preferences change so that s increases. This corresponds to a shift in tastes in favor of the final good produced by the country with comparative advantage in performing R&D. From (25), we see that the $\dot{e}=0$ curve shifts down, say to $\dot{e}'=0$ in the figure. Equation (26) implies that the $\dot{\sigma}=0$ schedule shifts out (in the positive orthant) to $\dot{\sigma}'=0$. The new steady state occurs at a point such as 2. But observe that all points on $\dot{e}'=0$ to the right of its intersection with ray OR are characterized by slower steady-state growth than at point 1. This claim follows from (20) and (25), whence

$$\text{(29)} \qquad \bar{g} = \frac{\beta\bar{e}}{(\epsilon-1)\bar{\sigma}} - \rho \ .$$

Since the intersection of $\dot{\sigma}'=0$ and $\dot{e}'=0$ necessarily lies to the right of the intersection of the latter curve with OR, we have established that an increase in s reduces steady-state growth.

When tastes shift unexpectedly toward the final good of the country with comparative advantage in R&D, resources there must be reallocated to satisfy the relatively higher consumer demand. A process begins whereby labor there shifts out of R&D and the manufacture of middle products. Products accumulate more slowly in this country than in the other, and over time its share of middle products falls (i.e., σ rises). Output per middle product changes by the same proportion in both countries (see (18)). So, in the new steady state, the country with comparative disadvantage in R&D is responsible for a relatively larger share of the world's innovation, with adverse consequences for the common steady-state growth rate. Of course, the opposite conclusion applies when s falls. Moreover, the same results obtain at stable equilibrium points when $1/s < h/H$.[11] We have thus proven:

Proposition 1: Stronger relative demand for the final good of the country with comparative advantage in R&D lowers the long-run share of this country in the number of middle products and slows long-run growth of the world economy. In the absence of comparative advantage in R&D, the long-run growth rate is independent of the relative demand for final goods.

Next we consider the dependence of growth on the sizes of the effective labor forces. Effective labor may grow without affecting cross-country comparative advantage either because the stock of irreproducible resources expands, or because the productivity of labor in all uses (or in R&D and intermediate-good production) rises equiproportionately. In the first experiment, suppose that both countries experience equiproportionate, once-and-for-all increases in the sizes of their effective labor forces. We have already seen that this change would augment world growth in the absence of comparative advantage. Now H and h both rise, with their ratio unchanged. We illustrate in Figure 7 the resulting impacts on the long-run equilibrium for the case in which $1/s > h/H$. The increase in H shifts the $\dot{e}=0$ curve up to $\dot{e}'=0$. Once again we draw the ray OR through point 1, along which e/σ is equal to its initial long-run level. Comparing points 1 and 2 (where the latter is the intersection of $\dot{e}'=0$ with OR), we see from (25) that expenditure

[11] In this case the $\dot{\sigma}=0$ curve rotates to the right in the relevant region.

per product differs by $de = \alpha dH/s(1-\beta)$. Since in the comparison of these two points, $d\sigma = \sigma de/e$, we find e higher in percentage terms at point 2 by

$$\frac{de}{e} = \frac{dH}{H}\frac{\alpha H}{(1-\beta)se} > \frac{dH}{H} .$$

The inequality stems from the fact that the $\dot{\sigma}=0$ curve intersects the vertical axis at $\alpha h/(1-\beta)$ and $h/H < 1/s$. The implication we wish to highlight is that in moving from point 1 to a point such as 3 (which has the same ordinate as point 2), the proportionate expansion of expenditure per product exceeds that of the world's effective labor force.

Now, from (26), the global expansion of the world's labor force also shifts up the $\dot{\sigma}=0$ curve. The vertical shift of this curve exceeds that of the $\dot{e}=0$ locus (compare (25)), and indeed is exactly equal to dH/H. So the intersection of the new $\dot{\sigma}=0$ schedule (not drawn) with the vertical line through point 1 must fall above point 4 but below point 3. This implies, finally, that the new steady-state point lies on $\dot{e}'=0$ between points 2 and 4. For all these points, σ is higher than at point 1, and -- since the new point is above OR -- so is the growth rate.

Figure 8 depicts the case in which $1/s < h/H$. Again in this case, an equiproportionate increase in the effective labor forces of the two countries accelerates growth, although here the share of products in the country with comparative advantage in R&D rises. To verify these claims, note that the $\dot{\sigma}'=0$ locus lies above the $\dot{\sigma}=0$ curve in proportion to the increase in H. The new $\dot{e}=0$ schedule (not drawn) intersects the vertical line through point 1 below point 1'. Consequently, the new steady-state equilibrium occurs at a point such as 2, to the left of point 1 and above the ray OR. This proves

Proposition 2: An equiproportionate, once-and-for-all increase in the effective labor forces of both countries accelerates long-run growth. The middle-product share of the country with comparative advantage in R&D rises if and only if $1/s < h/H$.

The interesting aspect of this proposition concerns the case where uniform expansion of effective labor reduces the share in world R&D of the country that is the relatively more efficient innovator. We have shown that, even in this case, where the market-share effect certainly is detrimental to world growth, the direct growth-augmenting effect of a greater resource base dominates. Greater resources generate higher growth rates in our model essentially because dynamic scale economies characterize long-run production.

We investigate next the effects of an increase in the effective labor force of a single country. Conceptually, it proves convenient to decompose this change into two elements. First, we increase h and H by the same percentage amount equal to the product of the share of the expanding country in the world's effective stock of labor and the percentage increase in effective labor force that this country experiences. This accounts for the total percentage change in H when H_i changes. Then we adjust h with H fixed to arrive at the appropriate change in h.

As an intermediate step, let us consider the effects of an increase in h alone. This corresponds to an increase in the effective labor of the country with comparative disadvantage in R&D, and a decrease in the effective labor of the other, so that the sum remains constant. This imaginary reshuffling of the world's resources shifts the $\dot{\sigma}=0$ schedule upward when $1/s > h/H$, and

These results suggest that findings reported by Krugman (1988) may be somewhat special. A country need not enjoy faster growth by joining the integrated world economy, if the country enjoys substantial comparative advantage in R&D. Moreover, growth in resources or improvements in the productivity of existing resources do not guarantee faster long-run growth in a world equilibrium with free trade. If resources expand or become more efficient in the country with comparative disadvantage in R&D, then the resulting intersectoral reallocation of resources worldwide might slow innovation and growth everywhere.

V. Economic Policy

In this section we discuss the effects of tariffs, export subsidies, and R&D subsidies on long-run growth. In order to do so, it is necessary for us to introduce the relevant policy parameters into the equations that describe instantaneous and steady-state equilibrium. To avoid repetition of the detailed arguments presented in Section II, we present here only the necessary modifications of the model, and then explain their implications for the steady-state conditions. We restrict attention to small taxes and subsidies; this restriction facilitates exposition, as the channels through which economic policies affect long-run growth can be seen more clearly. We also confine our analysis of trade policies to those that impede (or encourage) trade in final goods.

The introduction of taxes and subsidies to the model necessitates consideration of the government's budget. As usual, we assume that the government collects and redistributes net revenue by lump-sum taxes and subsidies. In a static framework, this specification suffices to determine completely the government's budgetary policy. But in a dynamic framework, the budget need not balance period by period, so budgetary policy in general must specify the intertemporal pattern of lump-sum collections and transfers. However, with perfectly-foresighted and infinitely-lived agents, our model exhibits the Barro-Ricardo neutrality property. Hence, we need not concern ourselves with the intertemporal structure of budget deficits so long as the present value of the government's net cash flow equals zero.

The presence of the aforementioned policies modifies the decision problem for consumers in country 1 in two ways. First, we replace the price of good i in (1) by $T_i p_{Yi}$, , where $T_1 = 1$. With this formulation, p_{Yi} remains the producer price of final good i, $T_2 > 1$ represents a tariff in country 1 on imports of consumer goods, and $T_2 < 1$ represents a subsidy by country 2 on exports of final output.[12] Second, we add the present value of net taxes to the right-hand-side of (3) as a lump-sum addition to consumer wealth. The amount of this collection or redistribution will differ across countries according to their policies.

These modifications do not affect (4), which continues to describe the optimal intertemporal pattern of expenditures for consumers worldwide as a function of the pattern of equilibrium interest rates. In a steady state with $\dot{e}=0$, (4) reduces to

$$(30) \qquad \dot{R} = \bar{g} + \rho \ .$$

[12] The effects of a country 2 import tariff and a country 1 export subsidy can be derived symmetrically, so we neglect these policies here and leave the maximand for consumers in country 2 as before.

Notice that (30) implies that in any steady state in which countries grow at the same rate, long-run equalization of interest rates obtains. This property of our model holds irrespective of the presence or absence of international capital mobility, and the presence or absence of tariffs or export subsidies on final goods and subsidies to research and development.

Turning to the production side, our policies do not alter equations (7)-(12) describing pricing and output relationships in the intermediate and final goods sectors and the technology for knowledge creation. However, R&D subsidies do change the private cost of R&D. We replace (14) by

$$(14') \qquad c_{ni} = w_i a_{Lni}/nS_i \ ,$$

where $S_i > 1$ represents subsidization of research costs in country i. It also proves convenient to redefine our numeraire to normalize for the effect of the R&D subsidy on the price of intermediate inputs in country 1. Our new normalization dictates a modified equation for the price of intermediates produced in country 2 as well. Together, these relationships, which replace (17a) and (17b) can be written as

$$(17') \qquad p_{Xi} = n(S_i a_{LXi}/a_{Lni})^{1/\epsilon}.$$

As for the market-clearing conditions, the factor-markets equation (16) is not affected, but we must replace (15) by

$$(15') \qquad p_{Yi} Y_i = \frac{s_{i1} E_1}{T_i} + s_{i2} E_2 \quad .$$

where E_i denotes aggregate spending by consumers in country i, and the shares of spending devoted to good i by residents of country 1 and country 2 are $s_{i1}=s_i(p_{Y1}, p_{Y2}T_2)$ and $s_{i2}=s_i(p_{Y1}, p_{Y2})$, respectively. Although import tariffs and export subsidies on final goods do not affect steady-state producer prices of final output in our model,[13] the direct response of spending shares in country 1 to changes in trade policy must now be treated explicitly for utility functions with an elasticity of substitution between the final goods other than unity. Moreover, R&D subsidies, if introduced at different rates in the two countries, will affect the steady-state value of p_{Y1}/p_{Y2}, and may influence, therefore, the long-run spending shares in both countries.

This completes the necessary modifications of the equilibrium relationships. We can now use the extended model to derive the equations describing steady-state equilibrium in the presence of policy intervention. In a steady state, employment in the R&D sector is given by $a_{Lni}\dot{n}_i/n = a_{Lni}\bar{g}\bar{\sigma}_i$. Making use of (8), (9), (11), (30), (14′) and (17′) (which together imply $\dot{c}_{ni}=0$ in a steady state), we find employment in the manufacture of middle products equal to $a_{Lni}\bar{\sigma}_i(\bar{g}+\rho)(\epsilon-1)/S_i$. Substitution of these terms into (16) yields the steady-state labor-market-clearing condition,

$$(31)\qquad \bar{g}\bar{\sigma}_i + \frac{(\epsilon-1)(\bar{g}+\rho)}{S_i}\bar{\sigma}_i + \frac{1-\beta}{\alpha b_i S_i^{1/\epsilon}}\bar{q}_i = h_i ,$$

where $q_i=p_{Yi}Y_i/n$. Next, from (8)-(11), (30), and (17′) we obtain

$$(32)\qquad (\epsilon-1)(\bar{g}+\rho)\left(\sum_i \frac{\bar{\sigma}_i b_i}{S_i^{\alpha}}\right) - \beta\,\Sigma_i\bar{q}_i = 0 .$$

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[13] This statement can be verified using equations (7), (8) and (17′).

Naturally, we also require

$$(33) \qquad \Sigma_i \, \bar{\sigma}_i = 1 \, .$$

Finally, (15′) implies

$$(34) \qquad \bar{q}_i = \frac{\bar{s}_{i1}\bar{e}_1}{T_i} + \bar{s}_{i2}\bar{e}_2 \, .$$

It is straightforward, now, to verify that (31)-(34) imply (25) and (26) when $T_i=S_i=1$ for $i=1,2$ (with $\bar{e}=\Sigma_i\bar{e}_i$). This provides a consistency check on the extended model with policy instruments.

A complete solution to the model requires specification of the determinants of the cross-country composition of world spending; i.e., e_i for $i=1,2$. For this, we need to distinguish between alternative cases based on the presence or absence of international capital mobility. When international capital flows are ruled out, steady-state spending per middle product by consumers in each country is proportional to the sum of that country's (per-product) labor income, operating profits, and net transfers from the government (including interest on internal debt). When capital flows do take place, on the other hand, spending per product is proportional to the sum of these components of income plus income on net foreign asset holdings. In this latter case, it is not possible to calculate the comparative dynamic response of $\bar{e}_i$ to a policy change without accounting for the effects of that change on foreign debt accumulation along the entire trajectory leading to the steady state. Fortunately, the long-run responses to "small" doses of policy do not depend on whether or not financial assets are tradable, so there is no need for us to deal in what follows with the entire equilibrium trajectory.

We consider trade policies first. From (34), the ratio q_1/q_2 satisfies

$$\frac{\bar{q}_1}{\bar{q}_2} = \frac{\bar{s}_{11}\bar{e}_1 + \bar{s}_{12}\bar{e}_2}{\bar{s}_{21}\bar{e}_1/T_2 + \bar{s}_{22}\bar{e}_2} . \tag{35}$$

Now, for given expenditure levels e_i, equations (31)-(33) and (35) -- which constitute a system of five equations -- provide a solution for $(\bar{g},\bar{\sigma}_1,\bar{\sigma}_2,\bar{q}_1,\bar{q}_2)$. In this system, the trade policy parameters appear only in (35). Therefore, the long-run effects of trade policy depend only on their effects on $\bar{q}_1/\bar{q}_2$, taking into account the induced adjustment in the spending levels $\bar{e}_1$ and $\bar{e}_2$. Moreover, for small trade policies (i.e, with an initial value of $T_2=1$), the spending shares are equal across countries ($\bar{s}_{11}=\bar{s}_{12}$), so the effect on $\bar{q}_1/\bar{q}_2$ of changes in the cross-country composition of aggregate spending "washes out".

Further inspection of (35) reveals that an increase in T_2 starting from free trade with $T_2=1$ (i.e., a small import tariff in country 1) unambiguously raises $\bar{q}_1/\bar{q}_2$.[14] A tariff shifts demand by residents of country 1 toward home consumer products, and since relative producer prices do not change in the long run, steady-state relative quantities must adjust. The effect of this

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[14] The easiest way to see this is to write the right-hand-side of (35) as

$$(\bar{p}_{Y1},\bar{p}_{Y2})[\phi_1(\bar{p}_{Y1},T_2\bar{p}_{Y2})\bar{e}_1 + \phi_1(\bar{p}_{Y1},\bar{p}_{Y2})\bar{e}_2]/[\phi_2(\bar{p}_{Y1},T_2\bar{p}_{Y2})\bar{e}_1 + \phi_2(\bar{p}_{Y1},\bar{p}_{Y2})\bar{e}_2]$$

where $\phi_i(\cdot)$ is minus the partial derivative of $v(\cdot)$ from (1) with respect to its i^{th} argument divided by $v(\cdot)$. Then an increase in T_2 with $\bar{p}_{Y1}/\bar{p}_{Y2}$ constant clearly raises demand for final good 1 in country 1 (the first component of the bracketed term in the numerator increases) and lowers the demand there for final good 2 (the first component of the bracketed term in the denominator falls).

change on the steady state is qualitatively the same as for an exogenous increase in world preference for final good 1, such as we studied in the previous section when we varied s_1. Similarly, a small export subsidy in country 2 (a reduction in T_2 to a value slightly below one) biases country 1 demand in favor of foreign final output. So we may apply directly our results from Proposition 1 to state:

Proposition 4: The imposition of a small tariff on imports of final goods reduces a country's steady-state share in middle products and R&D. It increases the rate of long-run growth in the world economy if and only if the country has comparative disadvantage in R&D.

Proposition 5: The provision of a small export subsidy for consumer products reduces a country's steady-state share in middle products and R&D. It increases the rate of long-run growth in the world economy if and only if the country has comparative disadvantage in R&D.

Commercial policies do affect long-run growth rates in our model. They do so by shifting resources in the policy-active country out of the growth-generating activity (R&D) and into production in the favored sector. At the same time, a resource shift of the opposite kind takes place abroad in the dynamic general equilibrium. The net effect on world growth hinges on the identity of the country that favors its consumer-good industry. If import protection or export promotion is undertaken by the country that is relatively less efficient in conducting R&D, then growth accelerates; otherwise, growth decelerates.

Next, we investigate the effects of small subsidies to R&D, introduced from an initial position of laissez faire. For these policy experiments, $T_2=1$ before and after the policy change, so the expenditure levels $\bar{e}_i$ cancel from (35). Suppose first that both countries apply subsidies at equal ad valorem rates; i.e., $S_1=S_2=S$. In this case, relative prices of final output do not change across steady states. Therefore, the spending shares $\bar{s}_{ij}$ do not change. In Appendix C we totally differentiate (31)-(33) and (35) with respect to S to prove:

Proposition 6: A small R&D subsidy by both countries at a common rate increase the rate of long-run growth in the world economy.

This proposition is not surprising, and corresponds to a similar result for the closed economy derived by Romer (1988). Since R&D represents the only source of gains in per capita income in our model, stimulation of this activity promotes growth.

What is more interesting, perhaps, is the effect of a small R&D subsidy in a single country. As for bilateral subsidies, a unilateral subsidy promotes growth by bringing more resources into product development in the policy-active country. But now, relative final-good prices change, so the spending shares in (35) must be allowed to vary unless the utility function has a Cobb-Douglas form. Depending on whether the elasticity of substitution between final products exceeds or falls short of one, this induced change in the pattern of spending can be conducive to or detrimental to growth. Moreover, an R&D subsidy in a single country will alter the relative shares of the two countries in product development. If the subsidy is introduced by the country that is relatively less efficient at performing R&D, this effect too can impede growth. In Appendix D we show, by means of a numerical example using a Cobb-Douglas utility function, that an R&D subsidy introduced by the country with comparative disadvantage in R&D might (but need not) reduce the world's growth rate. We also prove in Appendix C that, for the case of constant spending shares, an R&D subsidy must encourage growth if it is undertaken by the country with comparative advantage in R&D. Thus we have

Proposition 7: The provision of a subsidy to R&D in one country increases long-run growth if spending shares on the two final goods are constant and the policy is undertaken by the country with comparative advantage in R&D. Otherwise, the long-run growth rate may rise or fall.

VI. Lags in the Diffusion of Knowledge

We have assumed above that research and development creates as a by-product an addition to the stock of knowledge that facilitates subsequent R&D. Moreover, we supposed that the knowledge so created becomes available immediately to scientists and engineers worldwide. We now relax the latter assumption, in recognition of the fact that privately created knowledge, even if non-appropriable, may enter the public domain via an uneven and time-consuming process. Also, since legal and cultural barriers may inhibit the free movement of people and ideas across national borders, we shall allow here for the possibility that information generated in one country disseminates more rapidly to researchers in the same country than it does to researchers in the trade partner country. We shall use the extended model to reconsider the effects of trade policies on the steady-state rate of growth.

In place of our earlier assumption that world knowledge accumulates exactly at the rate of product innovation (eq. 13)), we suppose now that R&D expenditures contribute to country-specific stocks of knowledge according to

$$(13\dagger) \qquad K_i(t) = \lambda_h \int_{-\infty}^{t} e^{\lambda_h(\tau-t)} n_i(\tau) + \lambda_f \int_{-\infty}^{t} e^{\lambda_f(\tau-t)} n_j(\tau) d\tau \ ,$$

where $K_i(t)$ is the stock of knowledge capital at time t in country i. With this specification, the contribution of a particular R&D project to general knowledge is spread over time. At the moment after completion of the project, none of its findings have percolated through the scientific and professional community. After an infinite amount of time has passed, the R&D project makes, as before, a unit contribution to knowledge. After finite time, the contribution lies between these extremes of zero and one, as given by the exponential lag structure in (13†). The parameters λ_h and λ_f (with $\lambda_h \geq \lambda_f$) distinguish within-country and cross-country rates of diffusion.

The introduction of lags in the diffusion of knowledge alters two of the fundamental equations of the model. First, (12) becomes

$$(12\dagger) \qquad \dot{n}_i = L_{ni} K_i / a_{Lni} \ .$$

Second, we have in place of (14),

$$(14\dagger) \qquad c_{ni} = w_i a_{Lni} / K_i \ .$$

Since these two equations are the only ones in the development of the model in which the productivity parameter a_{Lni} appears, the change in specification compels us to substitute a_{Lni}/K_i for a_{Lni}/n in all equilibrium relationships where the latter term appeared formerly.

In a steady state with $\dot{n}_1=\dot{n}_2=g$, we have $n_i(\tau) = n_i(t)e^{g(\tau-t)}$, so that

$$K_i(t) = \frac{\lambda_h}{\lambda_h+\bar{g}} n_i(t) + \frac{\lambda_f}{\lambda_f+\bar{g}} n_j(t)$$

$$= \left[\frac{\lambda_h}{\lambda_h+\bar{g}} \bar{\sigma}_i + \frac{\lambda_f}{\lambda_f+\bar{g}} \bar{\sigma}_j \right] n(t) = \mu_i(\bar{\sigma}_1,\bar{\sigma}_2,\bar{g})n(t) \quad . \tag{36}$$

So in the steady state, knowledge in each country once again is proportional to the total number of middle products, but the factor of proportionality has become country-specific and endogenous. This means that the steady state labor-input coefficient for R&D in country i, a_{Lni}/μ_i, also is endogenous; i.e., relative productivity in R&D depends now not only on relative natural abilities in performing this activity, but also on relative cumulative experience in research, as summarized by the σ_i's. This consideration leads us to draw a distinction henceforth between <u>natural</u> and <u>acquired</u> comparative advantage in R&D.

From (36) we see that, when $\lambda_h=\lambda_f \rightarrow \infty$ (i.e., when diffusion lags a very short), $\mu_1=\mu_2 \rightarrow 1$, and the extended model reverts to the earlier formulation. For $\lambda_h=\lambda_f$ finite, $\mu_1=\mu_2$, so that the ratio of the natural-plus-acquired productivity parameters for each country is the same as for the natural productivity parameters alone. In this case, the pattern of comparative advantage cannot be reversed by endogenous learning, and all results from before continue to apply. We concentrate here on cases in which the rates of diffusion are unequal but the difference between them is small.[15]

We derive the long-run effects of trade policy in the extended model using equations (31)-(33) and (35), but with $S_1=S_2=1$ (no R&D subsidies), with b_i replaced by b_i/μ_i^α (natural plus acquired comparative advantage in place of just natural comparative advantage), and with h_i replaced by $h_i\mu_i$ (natural plus acquired effective labor in place of natural effective labor). For clarity of exposition, we shall also assume for the remainder of this section that the spending shares s_i are constant. Recall that this assumption corresponds to taking static preferences as Cobb-Douglas.

The new elements that diffusion lags introduce to the analysis of policy stem from the effects of relative size and demand-side bias. Before considering these new aspects, let us suppose that labor forces are equal and demand for the two final goods is symmetric. By totally differentiating the system of steady-state equations (see Appendix E), we establish

Proposition 8: Suppose $L_1=L_2$, $s_1=s_2$, $a_{LX1}=a_{LX2}$ and $\lambda_h-\lambda_f$ small. Then a tariff on imports of final goods in country i raises the long-run growth rate if and only if $a_{Lni} > a_{Lnj}$.

In this case, the effects of acquired comparative advantage necessarily reinforce those of natural comparative advantage. The country that is

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15 A large difference between the within-country and across-country rates of diffusion may imply that, in the steady-state equilibrium, all R&D is carried out by one country. Such specialization, which is common in models with a national component to increasing returns to scale, necessarily occurs here if static preferences are Cobb-Douglas and $\lambda_f=0$ (i.e., all spillovers are internal). Then, the equations that we have developed to describe the steady-state equilibrium (which presume non-specialization in each country) would not be valid.

relatively more productive in creating new blueprints will attain, in the steady-state equilibrium prior to the introduction of policy, a majority share of the world's middle products. By its greater concentration in R&D, it will gain more experience in research and attain a higher steady-state stock of knowledge. Thus, the effects of learning will augment its initial comparative advantage in R&D. Then, when policy is introduced in one country or the other, the implications of the dynamic resource reallocation for the global efficiency of R&D will be all the more significant.

Now suppose that the two countries differ initially only in (effective) size, as measured by h_i. Recall that, with equal rates of diffusion, a small tariff in either country does not affect the long-run rate of growth. We find now, however,

Proposition 9: Suppose $b_1=b_2$, $s_1=s_2$ and $\lambda_h-\lambda_f$ small. Then a small tariff on imports of final goods raises the long-run growth rate if and only if the policy is introduced by the country with the relatively smaller effective labor force.

Here, the larger country will come to acquire comparative advantage in R&D, though it starts with none. The reason is as follows. With differential rates of diffusion, knowledge takes on the characteristics of a local public good. The larger country will have more (effective) scientists to benefit from this non-excludable good as its share in world R&D exceeds one half. So it acquires over time a relatively larger knowledge base and hence a relatively more productive corps of researchers. Trade policy that serves to divert resources away from the R&D sector in the larger country once comparative advantage has been established must be detrimental to growth.

The effects of demand-size bias are similar. We have

Proposition 10: Suppose $b_1=b_2$, $h_1=h_2$ and $\lambda_h-\lambda_\ell$ small. Then a small tariff on imports of final goods raises the long-run growth rate if and only if the policy is introduced in the country whose final good captures a majority share of world spending.

The argument should be apparent. The country whose good is in relatively greater demand must devote relatively more of its resources to final-goods production. Thus, its R&D sector initially will be smaller. This country develops over time a comparative disadvantage in R&D, as its learning lags that in its trade partner country. Protection in this country will improve world efficiency of R&D and thereby speed growth.

Once we allow for lags in the diffusion of scientific knowledge and differential speeds of diffusion within versus between countries, we find a richer set of possibilities for the long-run effects of trade policy. Comparative advantage continues to play a critical role in determining whether policy in one country will speed or decelerate growth. But comparative advantage now must be interpreted with care, since its measure combines natural ability and the (endogenous) benefits from cumulative experience.[16] Since steady-state productivity in R&D varies positively with the size of the R&D sector, all determinants of the equilibrium allocation of resources to

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[16] Endogenous comparative advantage also plays a central role in Krugman's (1987) analysis of commodity-specific learning-by-doing. There, as here, productivity increases with cumulative experience. But each good is produced in only one country in Krugman's model, so long-run comparative advantage is fully determined by the initial pattern of specialization.

this sector come to be important in the analysis of policy.

VII. Conclusions

In this paper, we have analyzed a dynamic, two-country model of trade and growth in which long-run productivity gains stem from the profit-maximizing behavior of entrepreneurs. We have studied the determinants of R&D, where research bears fruit in the form of designs for new intermediate products and in making further research less costly. New intermediate products permit greater specialization in the process of manufacturing consumer goods, thereby enhancing productivity in final production. In order to highlight the role of endogenous technological improvements as a source of growth, we have abstracted entirely from factor accumulation. But Romer (1988) has shown that capital accumulation can be introduced into a model such as the one we have studied without affecting the analysis in any significant way.

The interesting features of our analysis arise due to the assumed presence of cross-country differences in efficiency at R&D and manufacturing. Considerations of comparative advantage in research versus manufacturing of intermediate goods bear importantly on the implications of economic structure and economic policy for long-run patterns of specialization and long-run rates of growth. We find, for example, that growth in world resources or improvements in R&D efficiency need not speed the rate of steady-state growth, if those changes occur predominantly in the country with comparative disadvantage in R&D. Similarly, shifts in preferences in favor of the final good produced by the country with comparative advantage in R&D will reduce the long-run rate of world growth.

Concerning policy, we find for the first time a link between trade intervention and long-run growth. Any (small) trade policy that switches spending toward the consumer good produced by the country with comparative advantage in R&D will cause long-run growth rates to decline. Subsidies to R&D will accelerate growth when applied at equal rates in both countries, but need not do so if introduced only in the country with comparative disadvantage in R&D. When knowledge spillovers occur with a time lag and diffusion is faster within the country of origin than across national borders, comparative advantage becomes endogenous. Once we recognize that comparative advantage can be acquired as well as natural, we find a role for country size and demand-size bias in determining the long-run effects of policy.

Our emphasis on comparative advantage in research and development highlights only one channel through which trade structure and commercial policy might affect long-run growth. In other contexts, the trade environment might influence the rate of accumulation of human capital or the rate at which a technologically lagging (less developed) country adopts for local use the existing off-the-shelf techniques of production. Investigation of the links between trade regime and these other sources of growth seems to us a worthy topic for future research.

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Appendix A: Price of Intermediates in Country 2

We show in this appendix that (8)-(11), (14) and (17a) imply (17b). Observe first that (8), (14) and (17a) imply $\dot{c}_{n1} = 0$. That is, R&D costs are constant in country 1, so that the wage rate grows at the rate of new knowledge creation, g. This in turn implies that the price of intermediates in country 1 grows at rate g. Then (9) and (11) imply:

$$\dot{R} = \frac{1}{(\epsilon-1)} \frac{nX_1 a_{LX1}}{n_1 a_{Ln1}} . \tag{A.1}$$

We implicitly define a new variable, γ, by

$$p_{X2} = \gamma n(a_{LX2}/a_{Ln2})^{1/\epsilon} , \tag{A.2}$$

where γ may vary over time. Then (8), (14) and (A.2) give

$$\frac{\dot{c}_{n2}}{c_{n2}} = \frac{\dot{\gamma}}{\gamma} ,$$

which together with (9) and (11) yields

$$\dot{R} = \frac{1}{(\epsilon-1)} \frac{nX_2 a_{LX2}}{n_2 a_{Ln2}} + \frac{\dot{\gamma}}{\gamma} . \tag{A.3}$$

Now (A.1) and (A.3), together with (10), (17a) and (A.2) imply:

$$\frac{\dot{\gamma}}{\gamma} = \frac{n^{1-\epsilon}(1-\gamma^{-\epsilon})}{\epsilon-1} \frac{\beta(\Sigma_i p_{Yi} Y_i)}{\Sigma_i n_i p_{Xi}^{1-\epsilon}} . \tag{A.4}$$

Clearly, if $\gamma \neq 1$ at any point in time, then the price p_{X2} must explode or converge to zero. This is impossible along a convergent path, so $\gamma = 1$ at all times t, which together with (A.2) implies (17b).

Appendix B: Stability

Here we discuss local stability of the system of autonomous differential equations (21) and (23). In so doing, we assume that the necessary condition for stability embodied in equation (17b) and discussed in Appendix A is satisfied. We establish that, together with (17b), a necessary and sufficient condition for local stability is that either $\bar{\sigma} < h/H$ or else $\bar{\sigma} > h/H$ and the $\dot{\sigma}=0$ curve cuts the $\dot{e}=0$ schedule from below.

Taking a linear approximation around a steady state $(\bar{e},\bar{\sigma})$, we have

$$\begin{pmatrix} \dot{e} \\ \dot{\sigma} \end{pmatrix} = B \begin{pmatrix} e-\bar{e} \\ \sigma-\bar{\sigma} \end{pmatrix},$$

where

$$B = \begin{bmatrix} H+\rho & -\beta\bar{e}^2/\alpha\bar{\sigma}^2 \\ (\bar{\sigma}H-h)/\bar{e} & \frac{1-\beta}{\alpha}\, s\bar{e}-H \end{bmatrix}.$$

In calculating B, we have made use of (25) and (26). The trace of B is positive. So the system has at most one negative characteristic root. There exists such a negative characteristic root if and only if the determinant of B is negative. But, the determinant of B is negative for $\bar{\sigma} > h/H$ if and only if (27) is satisfied; that is, the $\dot{\sigma}=0$ curve must intersect the $\dot{e}=0$ schedule from below. When $\bar{\sigma} < h/H$ (which applies for $1/s > h/H$), the determinant of B is always negative. We conclude that points 1 in Figures 4 and 5 are saddle-path stable, whereas point 2 in Figure 4 is a source (i.e., is unstable).

Having established the local instability of point 2 in Figure 5, there remains to rule out limit cycles about this point. This we accomplish as follows. From (21) and (23) we have

$$\text{(A.5)} \qquad \frac{\dot{e}}{e} - \frac{\dot{\sigma}}{\sigma} = \frac{1}{\alpha}\frac{e}{\sigma} - \frac{h}{\sigma} - \rho .$$

Equation (A.5) implies that the points for which $\dot{e}/e = \dot{\sigma}/\sigma$ (i.e., e/σ is constant) lie on a line in e-σ space. This line passes through points 1 and 2 in Figure 5 since both $\dot{e}$ and $\dot{\sigma}$ vanish at these points. Above the line, e/σ is rising, while below it e/σ is falling. Thus, any trajectory which passes below the line passing through points 1 and 2 can never return to this line. This immediately rules out a limit cycle about point 2. It implies, moreover, that the saddle-path to point 1 and to the right of this point lies trapped in the area between the $\dot{\sigma}=0$ schedule and the line passing through points 1 and 2. For initial values of σ in excess of the ordinate of point 2, it is impossible to get into this area for any initial e, so the steady state at point 1 cannot be approached for these initial values of σ.

Appendix C: Comparative Dynamics of Policy Changes

In this appendix, we calculate the long-run comparative dynamic responses to small policy changes, using the steady-state equations (31)-(33) and (35). Letting "hats" represent proportional changes (i.e. $\hat{z}=dz/z$), and differentiating about the free trade point with $(T_2, S_1, S_2) = (1,1,1)$, we have

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ (g+\rho)b_1 & (g+\rho)b_2 & -bs_1e/\alpha\epsilon & -bs_2e/\alpha\epsilon & \sigma \\ \epsilon(g+\alpha\rho) & 0 & \frac{1-\beta}{\alpha}\frac{s_1e}{b_1} & 0 & \epsilon\sigma_1 \\ 0 & \epsilon(g+\alpha\rho) & 0 & \frac{1-\beta}{\alpha}\frac{s_2e}{b_2} & \epsilon\sigma_2 \end{bmatrix} \begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ \hat{q}_1 \\ \hat{q}_2 \\ dg \end{bmatrix} =$$

$$\begin{bmatrix} 0 \\ \Omega \\ \alpha(g+\rho)(\Sigma_i\sigma_i b_i \hat{S}_i) \\ \left[\alpha\epsilon\sigma_1(g+\rho) + \frac{1-\beta}{\alpha\epsilon}\frac{q_1}{b_1}\right]\hat{S}_1 \\ \left[\alpha\epsilon\sigma_2(g+\rho) + \frac{1-\beta}{\alpha\epsilon}\frac{q_2}{b_2}\right]\hat{S}_2 \end{bmatrix}$$

where Ω is the total derivative of the right-hand-side of (35) with respect to the policy parameters (T_2, S_1, S_2) evaluated at $(1,1,1)$, including the effect on relative demands of the induced change in the relative prices of final goods. Recall the argument in the text that at $T_2=S_1=S_2=1$: Ω has the same sign as $\hat{T}_2$

when $\hat{S}_1=\hat{S}_2=0$; $\Omega=0$ when $\hat{T}_2=0$ and $\hat{S}_1=\hat{S}_2=\hat{S}$; and Ω can have any sign when $\hat{T}_2=0$ and $\hat{S}_1\neq\hat{S}_2$.

Let Δ denote the determinant of the Jacobian matrix. Then

$$\Delta = -\frac{\beta e}{\alpha\sigma}(\sigma H-h) + \left(H - \frac{1-\beta}{\alpha}se\right)\sigma(H+\rho)$$

and $\Delta > 0$ for a stable steady-state equilibrium (see 27)).

Consider first the effects of trade policy. We find that at $T_2=S_1=S_2=1$

$$dg = \frac{\Omega}{\Delta}\,\frac{1-\beta}{\alpha(\epsilon-1)}[\epsilon g+(\epsilon-\beta)\rho]\,\frac{s_1 s_2}{b_1 b_2}\,e^2(b_1-b_2)\ ,$$

where $\Omega=e_1\hat{T}_2/e$ in this case. This expression has the sign of $(b_1-b_2)\hat{T}_2$. So a small tariff in country 1 raises the growth rate when $b_1 > b_2$, and a small export subsidy in country 2 raises the growth rate when $b_1 < b_2$.

Next consider equal-rate subsidies to R&D in each country, $\hat{S}_1=\hat{S}_2=\hat{S}$. We find

$$dg = \frac{\hat{S}}{\Delta}\left[\alpha(g+\rho)\Delta + (g+\alpha\rho)\frac{1-\beta}{\alpha(\epsilon-1)}\beta se^2\right] > 0\ .$$

Finally, if spending shares are constant (i.e., Cobb-Douglas utility) and country 1 alone introduces a small subsidy to R&D, the effect on the growth rate is given by

$$\Delta\frac{dg}{\hat{S}_1} = \epsilon(g+\rho)(g+\alpha\rho)(1-\beta)\sigma_1 b_1 se +$$

$$\left(\frac{\sigma_1(g+\rho)}{1-\alpha} + \frac{1-\beta}{\alpha(\epsilon-1)}\,\frac{s_1 e}{b_1}\right)\left[(g+\rho)(1-\beta)\,\frac{s_2 e}{b_2}(b_2-b_1) + (g+\alpha\rho)\beta e\right]$$

If $b_2 > b_1$, the expression on the right-hand side is positive, but otherwise it can have any sign. For a numerical example that establish this ambiguity, see Appendix D.

Appendix D: Numerical Example

We have solved the model numerically for several sets of parameter values under the assumption that the sub-utility function $u(\cdot)$ has a Cobb-Douglas form. In support of the statements made in the text, we report the following example:

Let $\alpha = 0.6$, $\beta = 0.5$, $\rho = 0.007$, $s_1 = 0.9$, $a_{Ln1}/a_{LX1} = 20$, $a_{Ln2}/a_{LX2} = 1$, $h_1 = .02$, and $h_2 = .03$. Then the unique steady-state equilibrium with positive growth is characterized by

$$e \simeq 0.3178, \ \sigma \simeq 14.2402, \ g \simeq 0.000439, \ \sigma_1 \simeq 0.697$$

Starting from this steady-state equilibrium an increase in the labor force of country 1 (the country with comparative disadvantage in R&D) reduces the steady-state growth rate. Also, the introduction of a small R&D subsidy in this country slows world growth.

Appendix E: Differential Rates of Diffusion and the Effects of Trade Policy

In this appendix, we provide proofs of Propositions 8-10. We first replace b_i with b_i/μ_i^α and h_i with $h_i\mu_i$ in the system comprising (31)-(33) and (35), and then calculate the response of the endogenous variables in this system to the introduction of a small tariff in country 1. Differentiating the revised system about the free trade point, we obtain:

$$\text{(A.6)} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ A_{12} & A_{21} & D_1 & D_2 & G_1 \\ B_1 & C_1 & F_1 & 0 & G_2 \\ C_2 & B_2 & 0 & F_2 & G_3 \end{bmatrix} \begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ \hat{q}_1 \\ \hat{q}_2 \\ dg \end{bmatrix} = \begin{bmatrix} 0 \\ \Omega \\ 0 \\ 0 \\ 0 \end{bmatrix} ,$$

where

$$A_{ij} \equiv (g+\rho)(b_i/\mu_i^\alpha - \alpha\sigma_i b_i \delta_H/\mu_i^{\alpha+1} - \alpha\sigma_j b_j \delta_F/\mu_j^{\alpha+1}), \qquad j\neq i,$$

$$B_i \equiv \epsilon(g+\alpha\rho) + (1-\beta)s_i e\mu_i^{\alpha-1}\delta_H/b_i - h_i\delta_H,$$

$$C_i \equiv (1-\beta)s_i e\mu_i^{\alpha-1}\delta_F/b_i - h_i\delta_F,$$

$$D_i \equiv -\beta s_i e/(\epsilon-1),$$

$$F_i \equiv (1-\beta)\mu_i^\alpha s_i e/\alpha b_i$$

and

$$\delta_i \equiv \lambda_i/(\lambda_i + g) , \qquad i = H,F .$$

As before, we use $\Omega > 0$ to denote the response of the relative demand for good 1 to the tariff, and $\hat{q}_i = dq_i/q_i$. We do not provide the details for G_i, because they are not needed for the calculations that follow.

We showed in Appendix C that the determinant of the matrix in (A.6) is positive at a stable equilibrium when $\lambda_H = \lambda_F \rightarrow +\infty$. We believe that a stability argument would also sign the determinant for the general case with diffusion rates λ_i, but here we simply assume that this determinant, Δ, is positive.

We first consider the case in which $s_1 = s_2 = 1/2$, $L_1 = L_2 = L$, and $a_{LX1} = a_{LX2} = a_{LX}$; i.e., countries differ only with respect to productivity in R&D. Then the modified version of (31) can be expressed as:

$$\text{(A.7)} \qquad \epsilon(g+\alpha\rho)\sigma_i b_i + \frac{1-\beta}{2\alpha} e\mu_i^{\alpha} = \mu_i L/a_{LX} ,$$

where by definition

$$\text{(A.8)} \qquad \mu_i \equiv \delta_F + \delta\sigma_i ,$$

and

$$\text{(A.9)} \qquad \delta \equiv \delta_H - \delta_F .$$

For $\delta = 0$ equations (A.7)-(A.9) imply $\sigma_1 > \sigma_2$ if and only if $b_1 < b_2$. That is, if the across-border diffusion rate does not differ from the within-border diffusion rate, then the country with comparative advantage in R&D has the larger steady-state share of R&D and middle products.

The solution to the system of linear equations (A.6) implies, for the case at hand,

$$(A.10)\quad -Gdg = \beta(g+\alpha\rho)(\mu_1^{\alpha}/b_1 - \mu_2^{\alpha}/b_2)$$
$$+ (g+\rho)(1-\beta)\mu_1^{\alpha}\mu_2^{\alpha}(b_2/\mu_2^{\alpha} - b_1/\mu_1^{\alpha})/b_1b_2$$
$$+ \beta La_{LX}(\delta_H-\delta_F)(\mu_2^{\alpha} - \mu_1^{\alpha})/a_{Ln1}a_{Ln2}\epsilon$$
$$+ e\beta(1-\beta)\mu_1^{\alpha}\mu_2^{\alpha}(\delta_H-\delta_F)(1/\mu_2 - 1/\mu_1)/2b_1b_2\epsilon$$
$$+ \alpha(g+\rho)(1-\beta)\mu_1^{\alpha}\mu_2^{\alpha}(\delta_H-\delta_F)(\sigma_1b_1/\mu_1^{\alpha+1} - \sigma_2b_2/\mu_2^{\alpha+1})/b_1b_2 ,$$

where $G = 4\alpha^2\Delta/\Omega(1-\beta)e^2 > 0$. In order to sign the change in the rate of growth, observe first that among the five terms on the right-hand side of (A.10), all but the first two approach zero as $\delta \to 0$. Therefore, for small differences in diffusion rates it suffices to sign the first two terms. But we have already established that $\sigma_1>\sigma_2$ if and only if $b_1<b_2$. Also, we have as a first-order approximation to the coefficient of acquired comparative advantage raised to the power θ, $\mu_i^{\theta} \simeq \delta_F^{\theta} + \theta\delta_F^{\theta-1}\delta\sigma_i$. Using these facts together with (A.8), we find that the first two terms are positive for δ close to zero if and only if $b_1<b_2$. This proves proposition 8.

In order to prove the remaining two propositions, we solve (A.6) after setting $b_1=b_2=b$, and obtain:

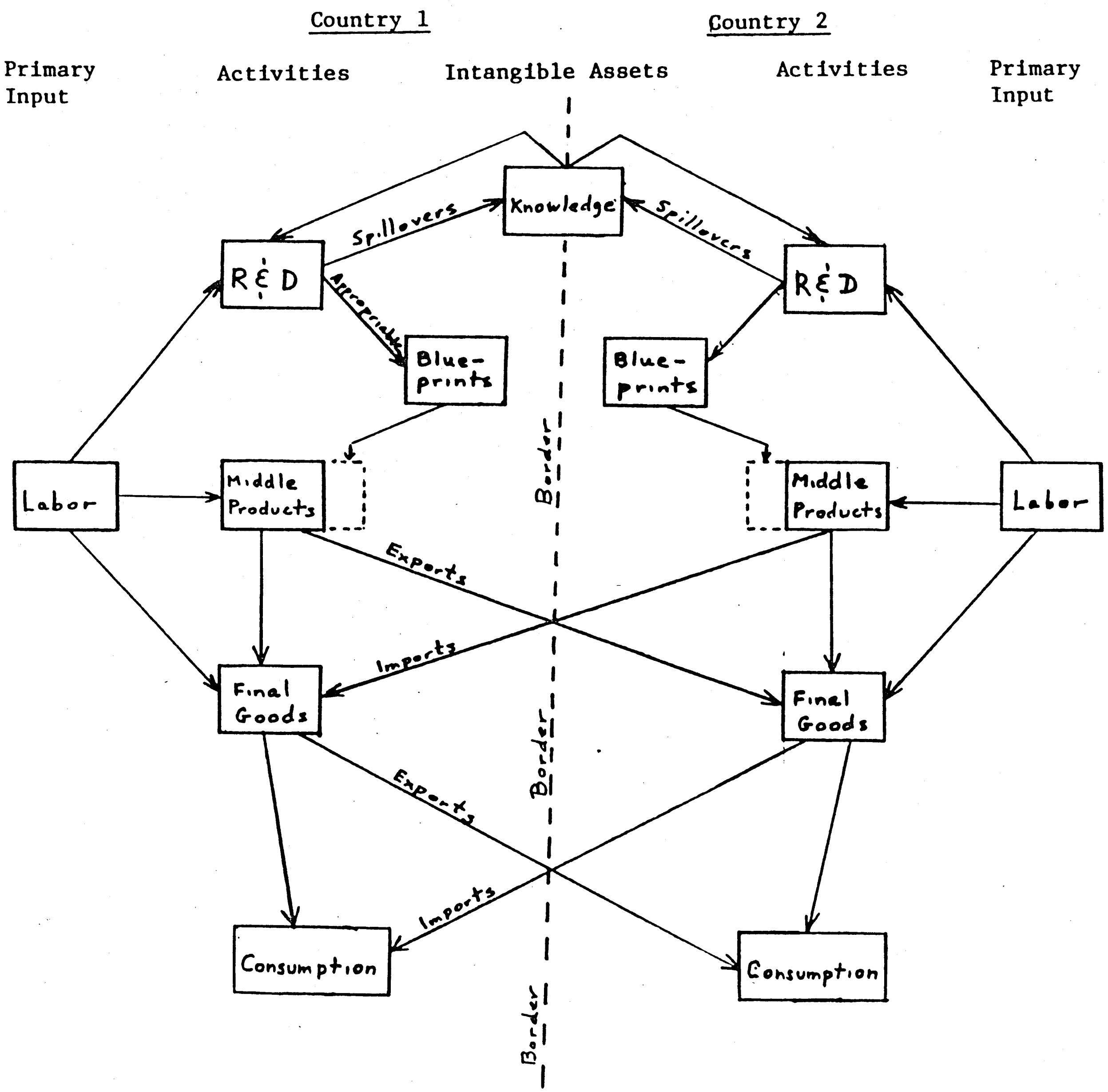
$$(A.11)\quad -G_bdg = [\beta(g+\alpha\rho) + (g+\rho)(1-\beta)](\mu_1^{\alpha} - \mu_2^{\alpha})$$
$$+ \beta\mu_1^{\alpha}\mu_2^{\alpha}(\delta_H-\delta_F)(h_1/\mu_1^{\alpha}-h_2/\mu_2^{\alpha})/\epsilon$$
$$+ \alpha(g+\rho)(1-\beta)(\delta_H-\delta_F)(\sigma_1\mu_2^{\alpha+1} - \sigma_2\mu_1^{\alpha+1})/\mu_1\mu_2$$
$$+ \beta\mu_1^{\alpha}\mu_2^{\alpha}(\delta_H-\delta_F)(s_2/\mu_2-s_1/\mu_1)(1-\beta)e/\epsilon b ,$$

where $G_b = \alpha^2b\Delta/\Omega s_1s_2e^2(1-\beta) > 0$. We note that the modified version of (31) now reads:

$$(A.12)\quad \epsilon(g+\alpha\rho)\sigma_ib + es_i\mu_i^{\alpha}(1-\beta)/\alpha = \mu_ih_ib .$$

Consider the case in which $s_1=s_2=1/2$. Then (A.8) and (A.12) imply that for δ close to zero $\sigma_1>\sigma_2$ if and only if $h_1>h_2$. Using this fact together with (A.8), (A.9) and the first-order approximation to $\mu_1{}^\theta$, it is easy to see that the terms on the right-hand side of (A.11) that are proportional to δ have the sign of (h_1-h_2). Since the remaining terms are proportional to δ^2, for sufficiently small values of δ the right-hand side has the sign of (h_1-h_2), which proves Proposition 9.

Finally, to prove Proposition 10, we consider the case in which $h_1=h_2$. In this case, for small values of δ, equations (A.8) and (A.12) imply $\sigma_1>\sigma_2$ if and only if $s_1< s_2$. Now we can repeat the argument from the previous paragraph. Equations (A.8), (A.9) and the approximation to $\mu_1{}^\theta$ imply that all the terms on the right-hand side of (A.11) that are proportional to δ have the sign of (s_2-s_1), while the remaining terms are proportional to δ^2. Hence, for sufficiently small values of δ, the right-hand side has the sign of (s_2-s_1), which proves the proposition.



Technology

$$\dot{K} = \dot{n}_1 + \dot{n}_2$$

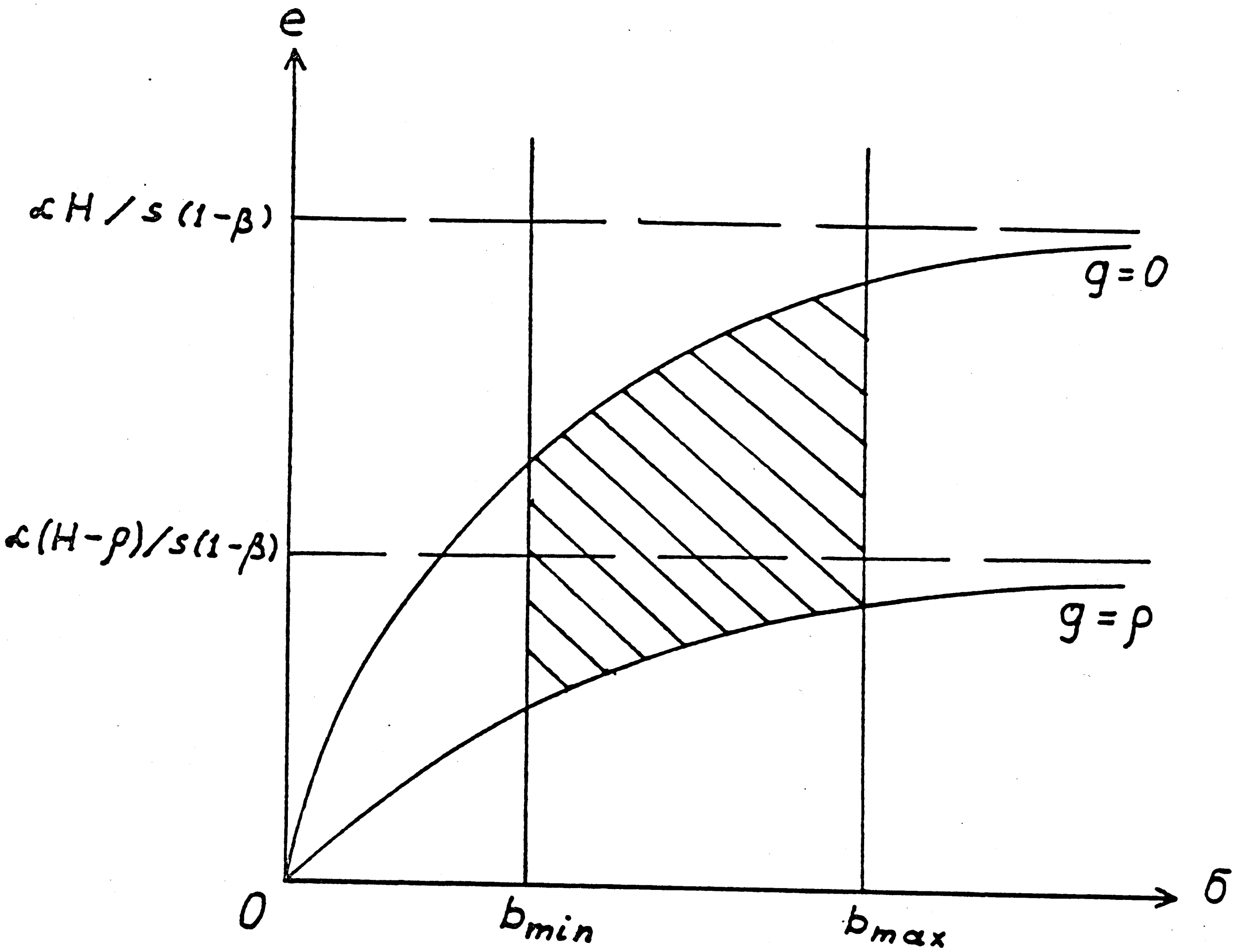
$$\dot{n}_i = KL_{ni}/a_{Lni}$$

$$X_i = L_{xi}/a_{Lxi}$$

$$Y_i = A_i L_{yi}^{1-\beta}\left[\int_0^n x(\omega)^\alpha d\omega\right]^{\beta/\alpha}$$

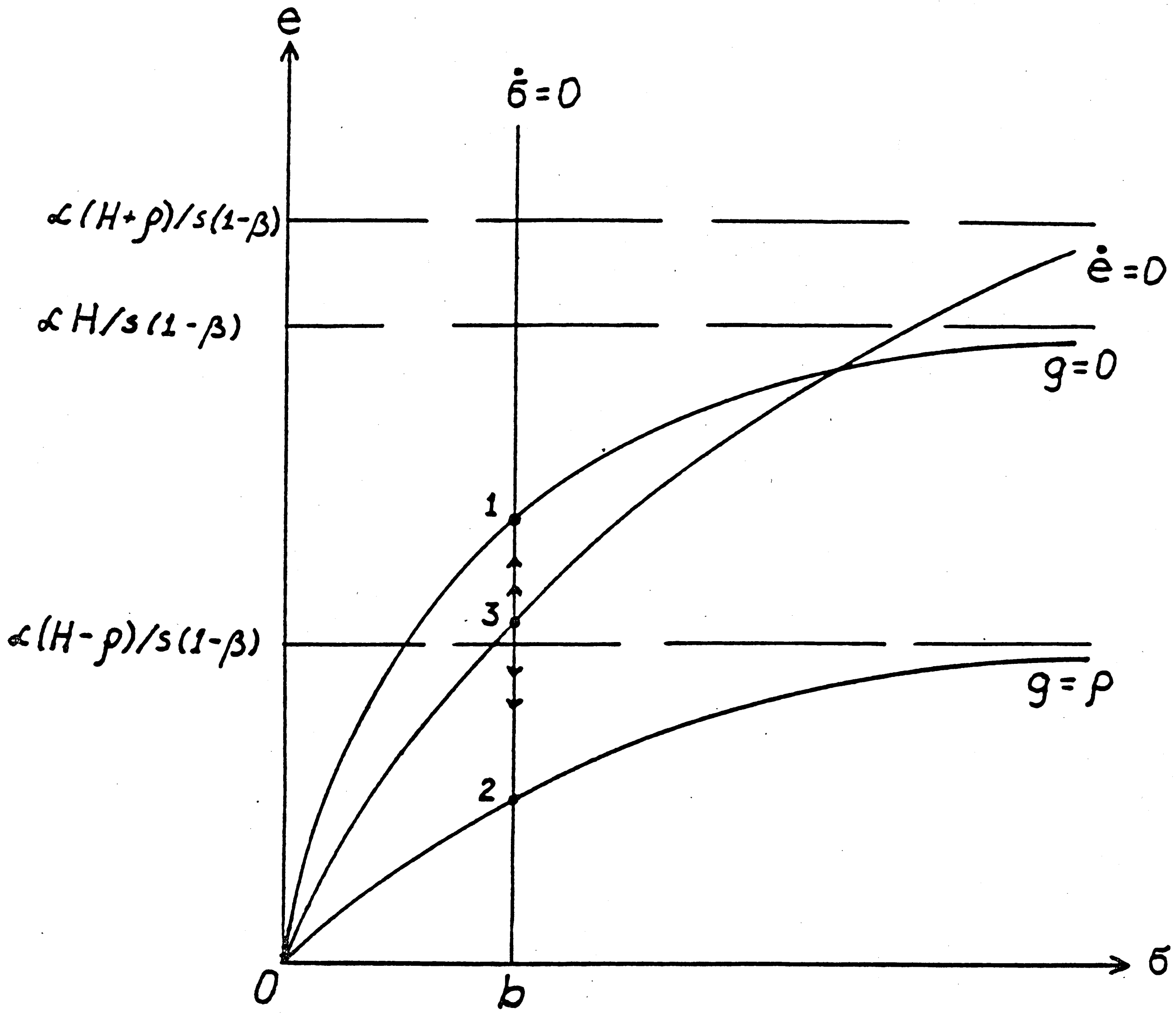
$$U = \int_t^\infty e^{-\rho(\tau-t)} \log u(y_1,y_2)\, dt$$

FIGURE 1



$b_{min} = min(b_1, b_2)$, $b_{max} = max(b_1, b_2)$

FIGURE 2



$b = b_1 = b_2$

FIGURE 3

$h/H < 1/s$

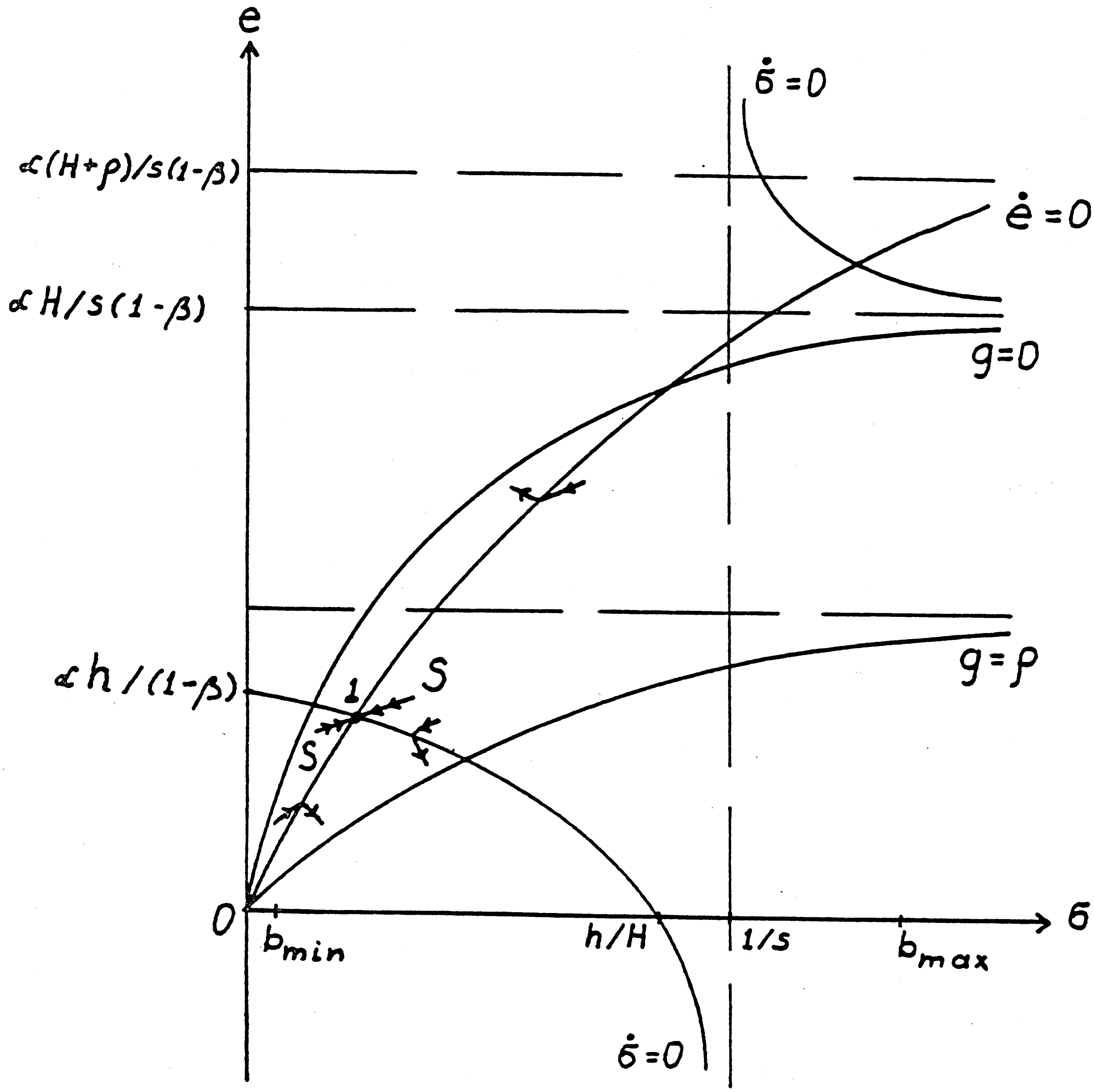


FIGURE 4

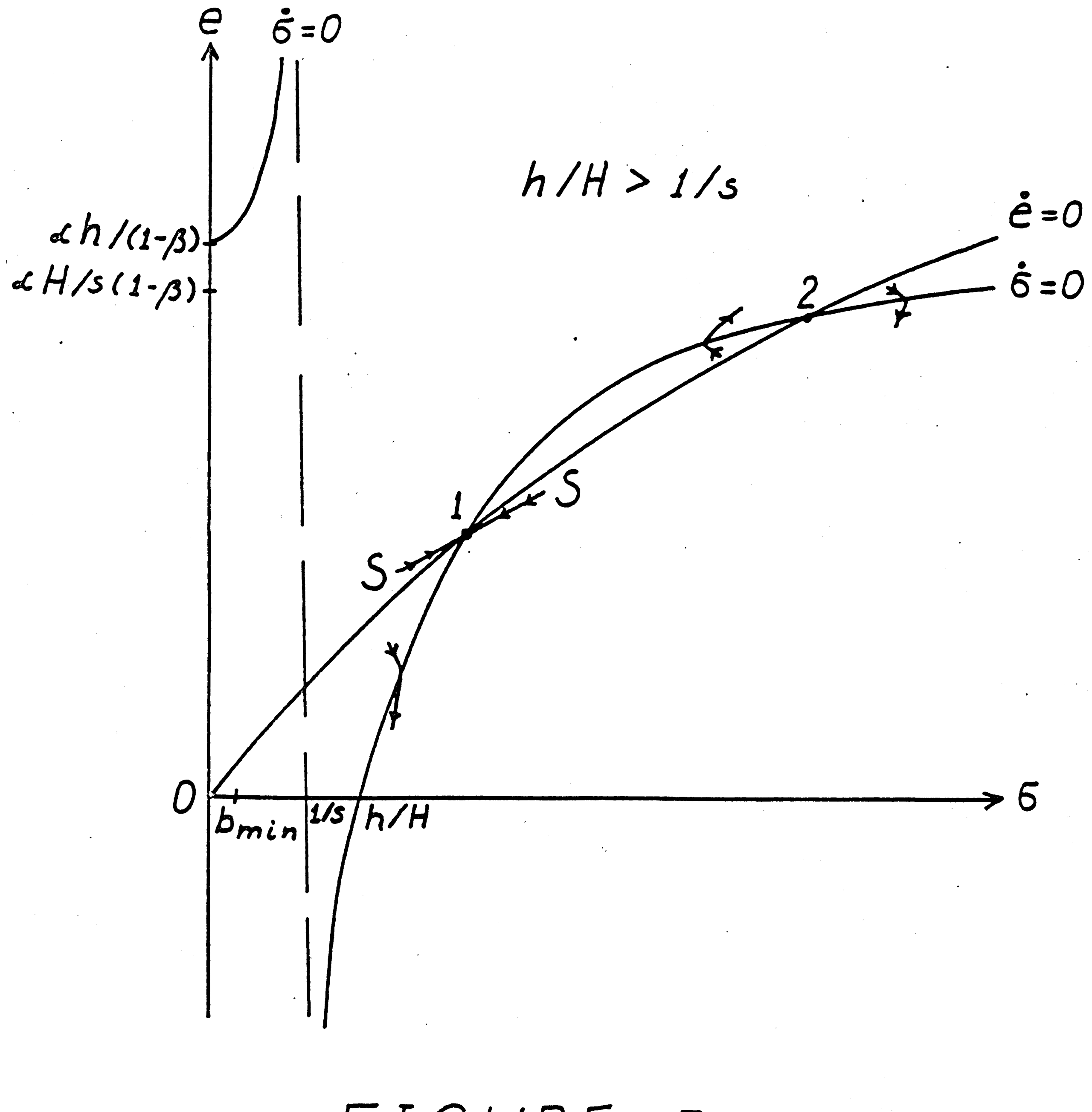


FIGURE 5

$h/H < 1/s$

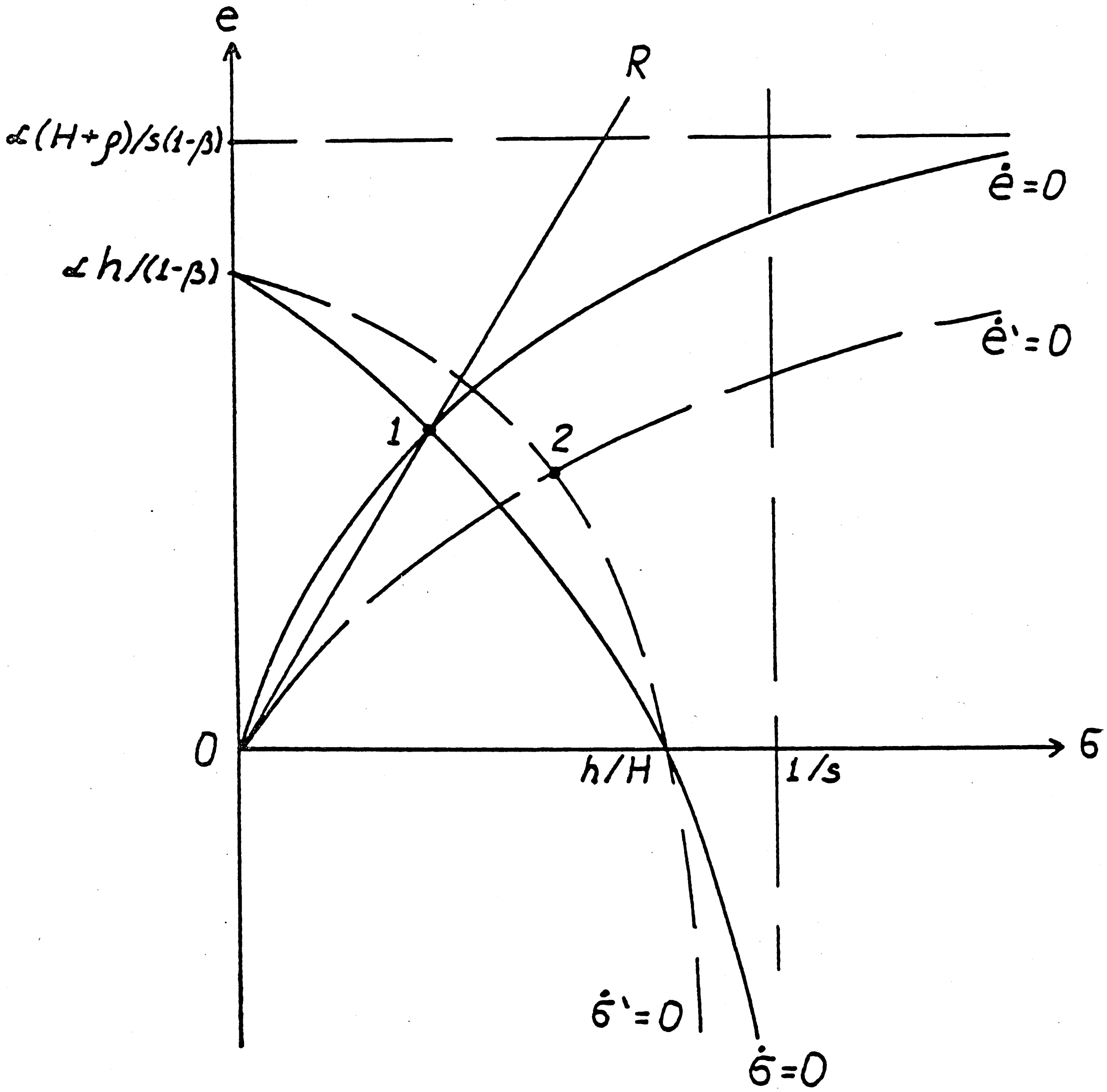


FIGURE 6

$h/H < 1/s$

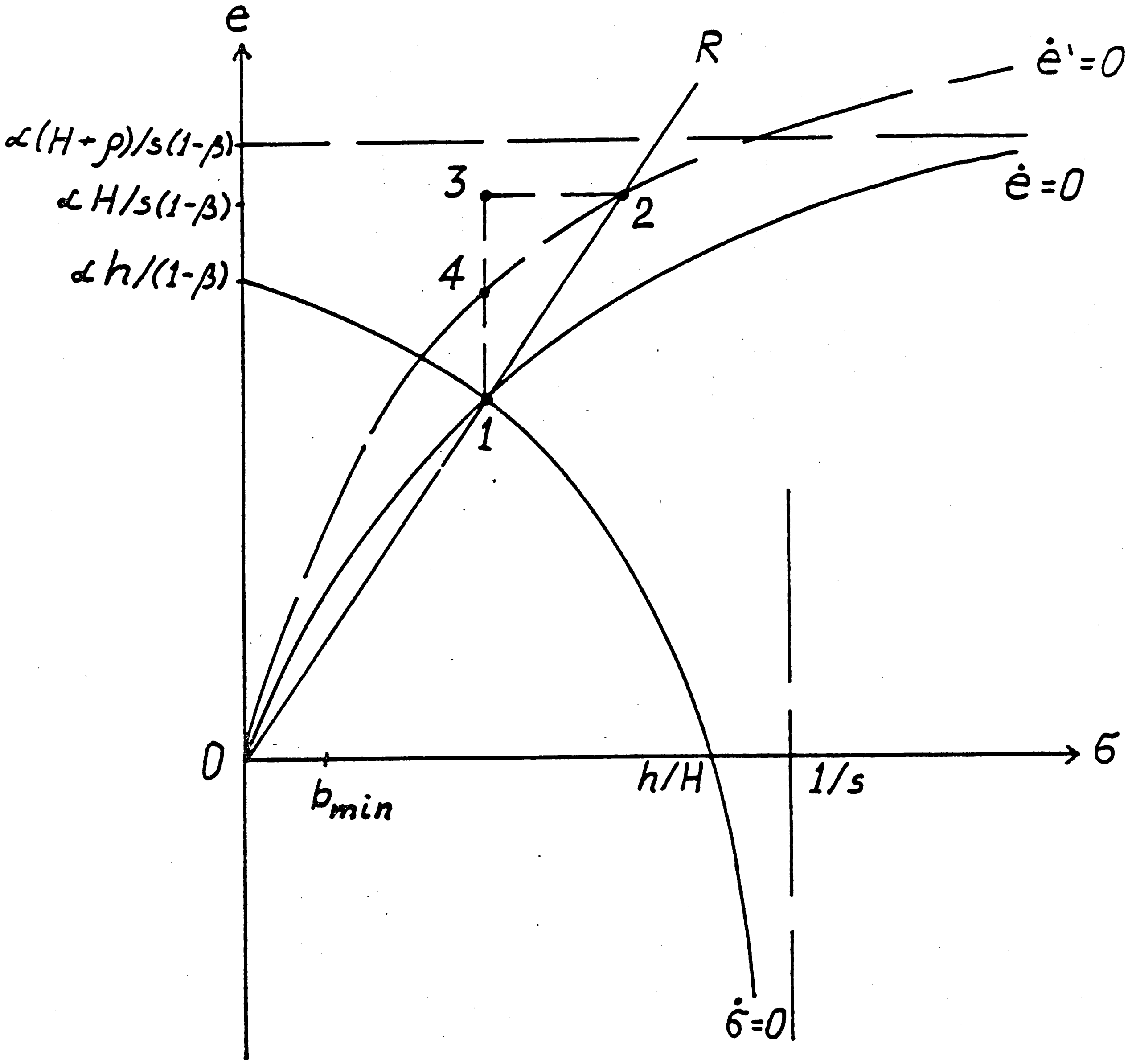


FIGURE 7

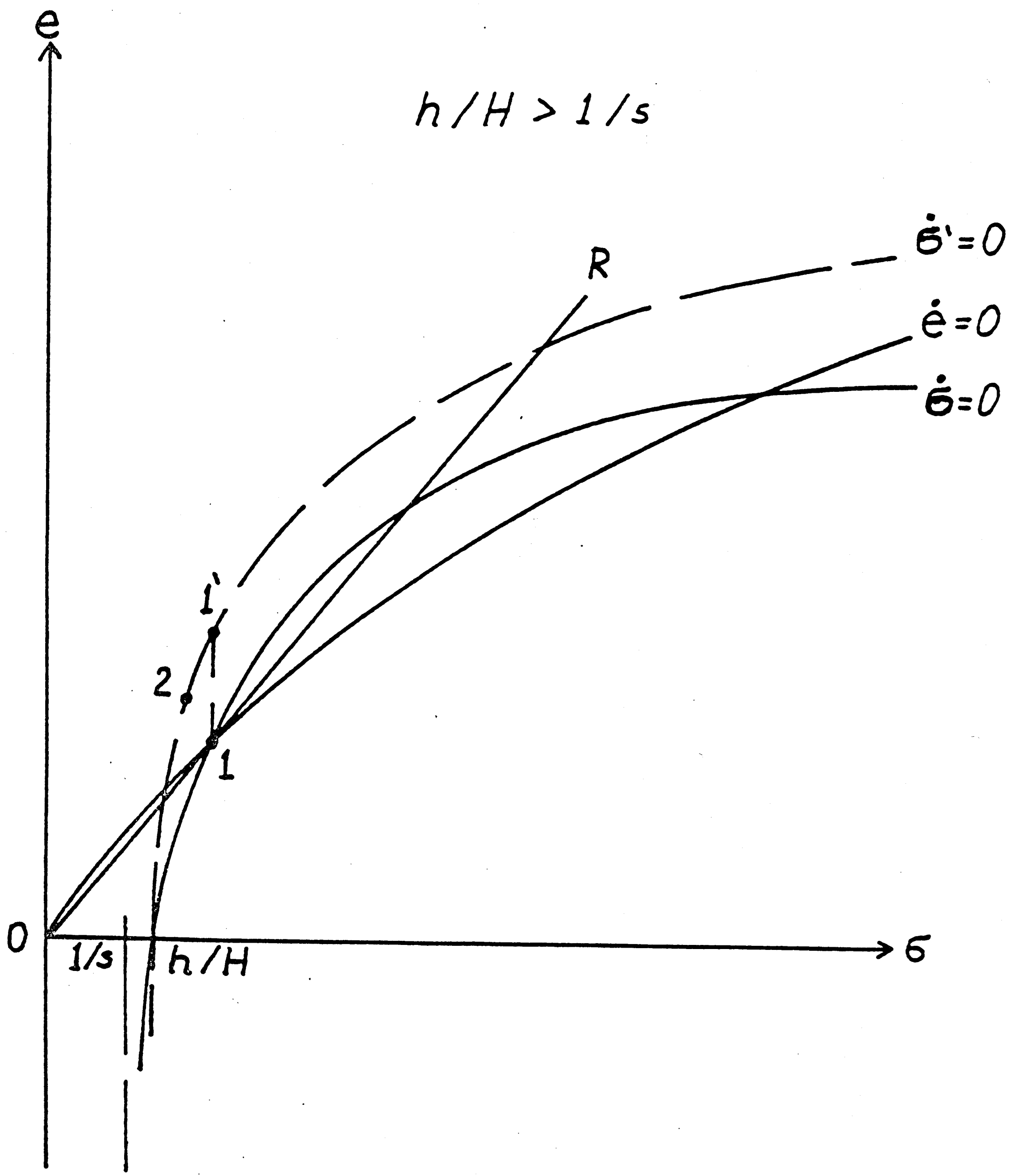


FIGURE 8