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ON THE THEORETICAL FOUNDATIONS OF THE PERMANENT INCOME HYPOTHESIS

by

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ABSTRACT

In its certainty equivalence form, consumption is proportional to the sum of human and non-human wealth. With labor income uncertainty the proportionality takes the form of homogeneity of consumption with respect to the two components of wealth. In this paper we analyze the stochastic properties of labor income which yield the homogeneity property as the utility maximizing solution. A sufficient condition is derived on the way in which a certain income shift (in time series analysis) or difference (in cross section comparison) preserves the homogeneity result. Application of this condition to some geometric processes and normal distribution of income is made. For other income processes the response of consumption to a certain income movement may be larger, which appears as excess sensitivity.

1. Introduction

The certainty equivalence version of the permanent income hypothesis (PIH) can be summarized as follows. Consumption is proportional to permanent income, which is the sum of human and non-human wealth, where the factor of proportionality is independent of the wealth. Although this form of the PIH is simple and widely used in empirical work, its theoretical basis with uncertainty is extremely narrow; in particular, the utility function must be of the quadratic form.

Except for the special case of quadratic utility, consumption cannot be proportional to the sum of human and non-human wealth with labor income uncertainty. Since human wealth, but not financial wealth, is uncertain, the mixture of the two does matter. As noted by Hayashi (1982), the proportionality property takes in this case the form of homogeneity of degree one with respect to the vector of human and non-human wealth.

In this paper we study conditions which guarantee the validity of the proportionality result in Hayashi's sense, the homogeneity property. Instead of looking for utility functions which yield this property, we adopt an isoelastic utility and study the stochastic properties of labor income which preserve the homogeneity of consumption to wealth. For example, if financial and human wealth (which is the expected value of lifetime labor earnings) are both 10% higher for person A relative to person B, what are the income processes which guarantee that person A will consume 10% more than person B? Similarly, in time series analysis, we look for income innovations which yield the homogeneity result.

We find that, in general, forward looking, optimizing consumers take every moment of labor income in each period into consideration in making their consumption decision. However, when the income distribution in each period is different (in cross section analysis) or shifts (in time series analysis) by some constant factor, the homogeneity result still holds.

In this case, when the m^{th} moment of labor income changes by a factor of α^m, $\alpha > 0$, and when non-human wealth changes by α, both consumption and permanent income will change by α, as implied by the poroportionality of consumption to permanent income. For instance, when labor income is distributed normally, then if the mean and standard deviation of labor income and nmon-human wealth all change by the same factor, then consumption and permanent income will change by this factor as well. But when the mean of future income increases without a similar change in the standard deviation, then rises in permanent income signal not only additional expected income, but also lower relative uncertainty about lifetime resources. For an isoelastic utility this will lead to an increase in consumption above that implied by the standard PIH form, a result which is sometimes termed "excess sensitivity".

We conclude that the empirical question of the validity of the PIH might hinge upon the stochastic properties of labor income. When the mean and standard deviation of future labor income are fully correlated, as in some geometric processes, changes in human wealth will not change the relative uncertainty about future income and therefore the homogeneity property is still valid. However, when the stochastic process for labor income is such that human wealth changes without similar changes in the uncertainty about lifetime wealth, then we expect to observe a large consumption response. Hence the question of the correlation of the mean and standard deviation of lifetime resources might have strong implications on the empirical validity of the homogeneity result.

The structure of the paper is as follows. Section 2 presents the optimization problem of the consumer. Some of the theoretical results are presented in section 3. Section 4 studies implications of these results, and section 5 draws the main conclusions.

2. The Consumer's Optimization Problem

Consider the problem of a consumer who lives for T periods and chooses optimal current consumption and contingency plan for future consumption.[1] His objective is to maximize the expected value of lifetime utility subject to a series of budget constraints. The time separable isoelastic utility function U(C) exhibits constant relative degree of risk aversion; that is, $U(C) = (1/\beta)C^{\beta}$ $(\beta < 1)$, where C is consumption. The only source of uncertainty that I consider here is uncertainty about exogenous labor income. Hence, labor income is a

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[1]The exposition of the consumer's problem follows closely that of Zeldes (1986).

general stochastic process, but expected real rates of return on (non-human) assets follow known, but possibly time-dependent, sequence.

In each period t, $t = 1,\ldots,T$, the consumer chooses the contingent plan $\{C_t\}$ to maximize expected utility as follows:

(1) $$\underset{\{c_t\}}{\text{Max}} \; E_t \sum_{i=0}^{T-t} b^i U(C_{t+i})$$

(2) $$\text{s.t.} \quad W_{t+i+1} = (W_{t+i} - C_{t+i})(1+r_{t+i}) + Y_{t+i+1} \qquad i = 0,\ldots,T-t-1$$

(3) $$C_t \geq 0$$

(4) $$W_T - C_T = 0$$

where:

E_t = the expectation operator, conditional on information known a time t.

b = utility discount factor; $b = 1/(1+\delta)$ where δ is the subjective rate of time preference.

W_t = financial wealth in period t.

C_t = consumption in period t. $\{C_t\}$ = consumption plan for periods t, $t+1,\ldots,T$.

r_t = real rate of return between t and $t+1$.

Y_t = real labor income in period t. $\{Y_t\}$ = labor income in periods $t+1,\ldots,T$.

The timing of the problem is as follows. Income is received at the beginning of the period such that W_t includes also Y_t, as in equation (2). Consumption is then chosen and the remaining wealth earns the rate of return r_t.

Recursive substituion in the sequence of budget constraints (2) and using the "no bankruptcy" condition (4) yield the following sequence of budget constraints:

$$(5) \qquad W_\tau + \sum_{j=1}^{T-\tau} R_{\tau,j} Y_{\tau+j} = C_\tau + \sum_{j=1}^{T-\tau} R_{\tau,j} C_{\tau+j} \quad \tau = t,\ t+1,\ldots,T-1$$

where

$R_{\tau,j}$, the discount factor from period $\tau+j$ to period τ, is defined as:

$$R_{\tau,j} = \Pi_{i=1}^{j}(1+r_{\tau+i-1})^{-1}$$

where equation (4) gives the budget constraint for the last period τ = T. Given the stochastic process $\{Y_t\}$ for periods $t+1,\ldots,T$, with a known distribution, the consumer chooses consumption $\{C_t\}$, which includes current consumption C_t and a plan for future consumption

C_{t+i}, $i = 1,\ldots,T-t$, which is contingent upon the realization of future income $Y_{t+1},\ldots,Y_{t+T}$.

It is, in general, impossible to derive a closed-form solution to the stochastic optimal control problem (1)-(4). However, it is widely assumed in empirical work that the solution can be approximated by the standard permanent income hypothesis form:[2]

$$(6) \qquad C_t = \alpha_t(W_t + H_t)$$

where H_t is real human wealth which is defined as the present discounted value of expected future real labor income:

$$(7) \qquad H_t = \sum_{i=1}^{T-t} R_{t,i}\ {}_tY_{t+i} \qquad t = 1,\ldots,T-1$$

where ${}_tY_{t+i}$ represents expectations as of t of Y_{t+i}. Following Flavin (1981),[3] define the expected value of lifetime resources, $W_t + H_t$ as permanent income.[4] This yields:

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[2]Hall and Mishkin (1982), Hayashi (1982) and Campbell (1986) are a few of the numerous examples.

[3]See also Hayashi (1982) and Stock and West (1987).

[4]Permanent income, which represents expected lifetime wealth, can be defined in terms of stock or flow, although the latter is more common.

$$(8) \qquad C_t = \alpha_t Y_t^P$$

where Y_t^P is permanent income as of period t. Equation (8) is the standard form of the PIH,[5] where the marginal propensity to consume out of permanent income, α_t, is usually assumed to be independent of permanent income.

In spite of the widespread use of the PIH form (8), it is the correct solution to the consumption problem (1)-(4) only in some very special cases. Campbell (1986) and Zeldes (1986) identify the following conditions for equations (6) and (7), or equation (8), to be the solution of the consumption problem (1)-(4):

(1) Labor income $\{Y_t\}$ is deterministic and the utility function $U(C_t)$ is of the constant relative aversion form. In this case the factor of proportionality α_t depends on the sequence of real rates of return $\{r_t\}$ and the subjective rate of time preference.

(2) When there is uncertainty about future labor income, then (8) is the solution for a quadratic utility function and when the (constant) real rate of return equals the subjective rate of time preference, and when consumption C_t is allowed to range from $-\infty$ to $+\infty$.

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[5] An error term, representing transitory consumption, is often added to equation (8).

The assumption that consumption with uncertainty can be identified with the certainty equivalence solution (6) is very convenient. It implies that in making their consumption decisions consumers look only at the expected value of lifetime wealth. As such, any innovation in labor income Y_t should affect consumption behavior only through its effect on human wealth H_t and wealth W_t. However, we have seen that this result extends to uncertainty only under extremely restrictive conditions. We will now explore the possibility that the consumption function under uncertainty can still take a simple form.

3. Some Theoretical Results

The certainty equivalence version of the PIH cannot, in general, hold with uncertainty. It is well known since the works of Leland (1968), Sibley (1975), and others, that increased uncertainty in labor income will affect the consumption level. In particular, consumers with utility function with positive third derivative reduce consumption with increased income uncertainty in some cases. Thus optimal consumption must depend, in the general case, upon more moments of the income distribution than the first moment only, as implied by the strict version of the PIH.

It is important to note that with uncertainty, the combination of human and non-human wealth does matter. Except for the special case of quadratic utility mentioned above, we cannot expect that consumption will be proportional to the sum of W_t and H_t, as in equation (6).

Instead, as Hayashi (1982) notes, with labor income uncertainty the proportionality property takes the form of homogeneity of degree one with respect to (W_t, H_t). In what follows we concentrate on the stochastic processes for income $\{Y_t\}$ which give rise to proportionality result in Hayashi's sense. To accomplish this we state the following theorem:

Theorem 1: Suppose that a consumer has constant relative risk aversion (CRRA) utility and that his labor income follows general stochastic process. Sufficient condition for the consumption plan $\{C_t\}$ to change to $\{\alpha C_t\}$, where $\alpha > 0$ is some constant factor, is that the income series $\{Y_t\}$ and nonhuman wealth W_t will change by the factor α.

Proof: The essence of the proof is to show that if $\{C_t\}$ is the contingent consumption plan for income $\{Y_t\}$ and financial wealth W_t then $\{\alpha C_t\}$ is the consumption plan for income and wealth $\{\alpha Y_t\}$ and αW_t, respectively.[6] This can be proved straightforwardly without a

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[6] Notice that when a stochastic variable Y has a probability density function (p.d.f.) $f(y)$ then the p.d.f. of $Z = \alpha Y$, $\alpha > 0$, is $g(z) = (1/\alpha)\ f(z/\alpha)$. This implies that the probability (abbreviated Pr) of Z being in the region $[\alpha a, \alpha b]$ equals that of Y being in $[a, b]$),

$$\Pr(\alpha a \le Z \le \alpha b) = \Pr(a \le Y \le b).$$

Hence, Z can be described as a shift of Y by α. The corresponding cumulative distribution functions (c.d.f.) of Z and Y are:

$$G(z) = F(z/\alpha).$$

closed-form solution, which is impossible to derive for CRRA utility. From the sequence of budget constraints (4) and (5) we get that if $\{C_t\}$ is a feasible consumption program with labor income $\{Y_t\}$ and wealth W_t then $\{\alpha C_t\}$ is feasible with income $\{\alpha Y_t\}$ and wealth αW_t. Denote by V_t the value function which is the maximum expected lifetime utility for the consumption problem (1)-(4). When the single-period utility function has the CRRA form $U(C) = (1/\beta)C^{\beta}$, $\beta < 1$, then

$$V_t\{\alpha C_t\} = \alpha^{\beta} V_t\{C_t\}. \tag{9}$$

Thus, if $V_t\{C_t\} \geq V_t\{\bar{C}_t\}$ for some consumption plan $\{\bar{C}_t\}$ which satisfies the sequence of budget constraints with $\{Y_t\}$ and W_t, then $V_t\{\alpha C_t\} \geq V_t\{\alpha\bar{C}_t\}$ where $\alpha\bar{C}_t$ is feasible with $\{\alpha Y_t\}$ and αW_t. Hence $\{\alpha C_t\}$ maximizes expected lifetime utility and is the new optimal consumption plan. This concludes the proof of Theorem 1.

The implication of this theorem is that ptimal consumption is proportional to the whole income process, and not necessarily to permanent income, which is the mean of lifetime resources. In fact, theorem 1 implies that the consumer takes into account, in general, all moments of lifetime wealth and not only the first moment. A conclusion in this spirit is stated in the following corollary:

In this case the m^{th} moment of Z equals α^{m} times the m^{th} moment of Y (a proof is given below, equation (12)).

Corollary 1: When the sufficient condition stated in theorem 1 for a change in consumption from $\{C_t\}$ to $\{\alpha C_t\}$ is satisfied, then the m^{th} moment of lifetime resources changed by α^m, $m = 1,2,\ldots$.

Proof: Denote the stochastic lifetime wealth (resources) by A_t

$$(10) \qquad A_t = W_t + \sum_{i=1}^{T-t} R_{t,i}\, Y_{t+i}$$

where the expected value of A_t, ${}_tA_t$, is permanent income:

$$(11) \qquad {}_tA_t = W_t + H_t = Y_t^P$$

where H_t, defined in equation (7), is human capital. When W_t and $\{Y_t\}$ are multiplied by α, the new level of A_t, denoted by B_t, will be

$$B_t = \alpha W_t + \sum_{i=1}^{T-t} R_{t,i}\, \alpha Y_{t+i} = \alpha A_t \qquad (\alpha > 0).$$

This implies that if the p.d.f. of A_t is $f(a_t)$ then the p.d.f. of the new level of lifetime wealth B_t is $g(b_t) = (1/\alpha)f(b_t/\alpha)$ (see footnote 7). Hence the m^{th} moment of B_t, $E^{(m)}(B_t)$, is:

$$(12) \qquad E^{(m)}(B_t) = \int_{-\infty}^{\infty} (b_t)^m g(b_t) db_t = \int_{-\infty}^{\infty} (\alpha a_t)^m (^1/\alpha) f(a_t) \alpha da_t =$$

$$= \alpha^m \int_{-\infty}^{\infty} (a_t)^m f(a_t) da_t = \alpha^m E^m(A_t)$$

as stated in corollary 1.

The conclusion from this corollary is that we can expect the consumption plan $\{C_t\}$ in general, and current consumption C_t in particular, to be proportional to permanent income when other moments of lifetime wealth change in the manner described in corollary 1. For example, 1% increase in permanent income might increase consumption by 1% when the standard deviation of lifetime resources increases also by 1%, the third moment of A_t increases by $(1.01)^3$ relative to its previous level, etc. In the case in which labor income, and therefore lifetime wealth, have normal distribution,[7] consumption can be homogenous of degree 1 with respect to permanent income and the standard deviation of A_t (denoted σ_t):

$$(13) \qquad C_t = f(Y_t^P, \sigma_t)$$

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[7]When labor income is distributed normally, the horizon T must satisfy $T \to \infty$. Otherwise it is impossible to guarantee the satisfaction of equation (4), the "no-bankruptcy" condition.

where $f(\cdot,\cdot)$ is homogenous of degree 1 in its arguments, the mean and standard deviation of A_t.[8] Equation (13) can also be written as:

$$(14) \qquad C_t = \alpha(I_t)Y_t^P$$

where different realizations of $I_t = \sigma_t/Y_t^P$, the coefficient of variation of lifetime wealth A_t, change the coefficient α.

We conclude that when income is normally distributed we can expect consumption to be proportional to permanent income where the factor of proportionality depends on the coeffiient of variation of lifetime resources. When both the mean and standard deviation of A_t change by the same ratio then consumption might be a constant proportion of permanent income. Otherwise we expect to see the marginal propensity to consume out of permanent income varying with the change in the uncertainty about lifetime income.

4. Applications

Theorem 1 can be used to find stochastic processes for labor income such that the homogeneity of consumption to income holds.

(i) Geometric White Noise.[9]

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[8]When the mean and standard deviation of normal p.d.f. changes by a factor $\alpha > 0$, the m^{th} moment, $m = 3,4,\ldots$, changes by α^m.

[9]This case is discussed by Hayashi (1982).

$$(15) \qquad Y_{t+i} = \bar{Y}(1 + \epsilon_{t+i}) \qquad i = 1,\ldots,T-t$$

where $\bar{Y}$ is a constant parameter and ϵ_{t+1} is independent and identically distributed random variable with mean zero.

Suppose that W_t and $\bar{Y}$ are multiplied by the factor $\alpha > 0$. In this case the sufficient condition stated in theorem 1 is satisfied. Labor income in each period is shifted by the factor α, and therefore the m^{th} moment of Y_{t+i} changes by α^m. By theorem 1, consumption is homogenous of degree one in W_t and $\bar{Y}$, a result which Hayashi (1982) interprets as showing that "the proportionality postulate that consumption is proportional to total wealth carries over to the case of labor income uncertainty" (p.898).

Notice, however, that consumption is proportional to permanent income only when the ratio of W_t to $\bar{Y}$ does not change. When both W_t and $\bar{Y}$ change by the positive factor α, so will permanent income, consumption and also the standard deviation of lifetime resources. Hayashi's example is a special case of our analysis because a change in $\bar{Y}$ in the case of a geometric white noise changes the income distribution in the manner described in theorem 1.

(ii) White Noise.

$$(16) \qquad Y_{t+i} = \bar{Y} + \epsilon_{t+i} \qquad i = 1,\ldots,T-t$$

where $\bar{Y}$ and ϵ_{t+i} have the same interpretation as in the former case.

The expression for the permanent income for cases (i) and (ii) is identical.[10] A change of W_t and $\bar{Y}$ by the constant factor $\alpha > 0$ will thus change permanent income by the same factor. However, the sufficient condition of theorem 1 is not satisfied and therefore we cannot state that consumption is proportional to permanent income in Hayashi's sense. In fact, when W_t and $\bar{Y}$ change, the mean of Y_{t+i} changes with no change in any other central moment, including the variance. Similarly, the mean of lifetime resources A_t (which is Y_t^P) changes with no similar change in any central moment. Thus an increase in W_t and $\bar{Y}$ (which happens when $\alpha > 1$) will reduce the relative uncertainty about any future labor income and lifetime resources, as measured, for instance, by the coefficient of variation. For a CRRA utility this implies that consumption C_t will increase by more than the increase α in permanent income. We conclude that when labor income is i.i.d. with multiplicative error term, the homogeneity of consumption to permanent income, in general, holds; however, when the error term is additive, consumption is more sensitive to changes in permanent income.[11]

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[10]When $T \to \infty$ and the interest rate r is constant, then $Y_t^P = W_t + \bar{Y}/r$.

[11]The issue of excess sensitivity of consumption typically focuses on the question of whether $\partial C_t/\partial Y_t = \alpha_t \, \partial H_t/\partial Y_t$, where α_t is the annuity value of a dollar of wealth, or whether it is greater than this amount. Hall and Mishkin (1982) focus on the response of consumption to transitory innovations in Y_t, while Flavin (1981) focuses on the

Up to now we did not restrict the p.d.f. of future labor income Y_{t+i} to any specific distribution. When Y_{t+i} is distributed normally (or uniformly) then Y_{t+i} changes by a factor α, as stated in theorem 1, if and only if the mean and standard deviation of Y_{t+i} changes by this factor. Hence, a straightforward corollary of theorem 1 is that for normally distributed labor income, consumption is homogenous of degree one in W_t and the vectors of the means and standard deviations of Y_{t+i}, $i = 1,\ldots,T-t$. We state this as a corollary.

Corollary 2: When the labor income series $\{Y_t\}$ is drawn from normal (or uniform) distribution, then consumption is homogenous of degree one in $(W_t, \mu_{t+i}, \sigma_{t+i})$, $i = 1,\ldots,T-t$, where μ_t and σ_t stand for the mean and standard deviation, respectively, of Y_t.

Application of corollary 2 to case (ii) of white noise labor income together with the normality assumption means that consumption is homogenous of degree one in $(W_t, \bar{Y}, \sigma)$ where $\bar{Y}$ and σ are the (common) mean and standard deviation of Y_{t+i}. As long as the coefficient of variation of Y_{t+i}, $i = 1,\ldots,T-t$, does not change, then consumption is proportional to permanent income; otherwise, the proportionality result does not hold.

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response to anticipated changes in Y_t. Given an income process, the question then boils down to how large the estimated α is, relative to the one predicted by the certainty version of the model. What we see here is that the sensitivity depends in a critical way on the nature of the stochastic process for income.

(iii) Geometric Autoregressive (1).

The stochastic process for labor income is:

$$(17) \qquad Y_{t+i} = \rho Y_{t+i-1}(1 + \epsilon_{t+i}) \quad i = 1,\ldots,T-t$$

where ρ is a constant parameter and ϵ_{t+i} is white noise error term. Consecutive substitution yields:

$$(18) \qquad Y_{t+i} = \rho^{i} Y_t (1 + \epsilon_{t+1})(1 + \epsilon_{t+2}),\ldots,(1 + \epsilon_{t+i}) \quad i = 1,\ldots,T-t.$$

A change of Y_t to αY_t will shift Y_{t+i} by α, in the way described in theorem 1. Hence, for a geometric AR(1), consumption is homogenous of degree one in (W_t, Y_t). In particular, when the ratio of W_t to Y_t is constant, consumption is proportional to permanent income.

(iv) Autoregressive (1).

$$(19) \qquad Y_{t+i} = \rho Y_{t+i-1} + \epsilon_{t+i} \quad i = 1,\ldots,T-t$$

where ρ and ϵ have the same interpretation as in equation (17). The expression of Y_{t+i} as a function of Y_t and the error terms is:

$$(20) \qquad Y_{t+i} = \rho^{i} Y_t + \rho^{i-1}\epsilon_{t+1} + \ldots + \rho\epsilon_{t+i-1} + \epsilon_{t+i} \quad i = 1,\ldots,T-t.$$

The conclusions from equation (20) are similar to those drawn in the case of white noise labor income. A change of Y_t shifts the p.d.f. of Y_{t+i} such that the mean changes proportionally, but no other central moment changes. Thus, even when W_t and Y_t are multiplied by a factor $\alpha > 1$, consumption will not change by this factor. Instead, consumption will increase by a larger proportion than that of permanent income. The difference between the two stochastic processes is that with multiplicative error term a change in the mean of any future labor income by α changes the m^{th} moment of labor income distribution by α^m; in particular, the coefficient of variation does not change. For an additive error term, a change in the mean of labor income does not necessarily imply a change in any other central moment. This implies that uncertainty about future income is relatively smaller, and optimizing consumers react by reducing what Leland (1968) calls "precautionary saving".[12]

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[12] Blanchard and Mankiw (1988) make a similar point. Notice also that the distinction between geometric and nongeometric processes can be described as measuring labor income in logs or levels.
For example, if income is geometric AR(1), $y_{t+i} = \rho y_{t+i-1}(1+\epsilon_{t+i})$, then:

$$\log y_{t+i} = \log \rho + \log y_{t+i-1} + \log (1 + \epsilon_{t+i}).$$

Approximating $\log (1+\epsilon_{t+i})$ by ϵ_{t+i} yields

$$Y_{t+i} = R + Y_{t+i-1} + \epsilon_{t+i}$$

where capital letters stand for logs. Hence the geometric process can be estimated by modelling labor income in logs instead of levels. However, in estimating permanent income the values Y and R should be

At this stage it is implortant to note that there are two conceptual experiments which seem to be most relevant to the PIH. The first is a comparative statics or cross section exercise: compare two individuals who are identical except for their permanent income and ask whether they will both consume the same fraction of their permanent income. The second conceptual experiment is a time series one, and it involves examining the optimal response of consumption to a realization of a given income process. The question is whether an individual consumes the same fraction of his permanent income for different realizations of income.

The answer to these two questions is, in general, no, since the most we can expect is that consumption will be homogenous of degree one with respect to human and non-human wealth. Consumption is proportional to permanent income only when two conditions hold: All realizations of income are higher (in cross section experiment) or increase (in time series comparison) by some factor, and at the same time financial wealth is higher (increases) by the same factor. Otherwise, the proportionality hypothesis does not hold.

The proposition that consumption with uncertainty is homogenous with respect to W_t and H_t instead of being proportional to $W_t + H_t$ as in certainty equivalence makes a big difference. It implies that in cross section comparison the standard proportionality result can hold only when the two individuals chosen have the same mixture of human to non-human wealth. This is even more problematic for time series

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converted back to y and ρ.

experiments since financial wealth is an endogenous variable which evolves according to the dynamics of equation (2). In this case innovation in labor income will not be accompanied by a similar change in financial wealth and the standard proportionality hypothesis of the PIH will not hold.[13]

5. Summary and Conclusions

The goal of this paper is to make the microfoundations of the PIH more solid. The standard form of the PIH, which is widely used in empirical work, is that consumption depends linearly on the expected discounted value of total lifetime resources, or permanent income. For example, a 1% increase of permanent income should induce a 1% increase in consumption.

In spite of its widespread empirical applications, the PIH consumption function has a surprisingly narrow theoretical basis. The theory is valid in a world with no uncertainty and with CRRA utility function. However, even with a simple inclusion of uncertainty in labor income, consumption is proportional to permanent income only in the implausible case of quadratic utility function, when negative consumption is allowed and when the rate of time preference equals the real interest rate. The idea in this paper is to find the stochastsic

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[13] In order to preserve the proportionality result in time series experiments we have to add a term which represents windfall earnings to the r.h.s. of equation (2). This can make possible, but not very plausible, the change of human and non-human wealth by identical factors.

properties of the uncertain labor income which make consumption proportional to permanent income also for a CRRA utility function, for any value of interest rates and time preferences, and when negative consumption is excluded. Proportionality in a stochastic sense, due to Hayashi (1982), is homogeneity with respect to human and non-human wealth, the components of permanent income.

Optimizing consumers make consumption decisions based on the whole income distribution in each future period. In general, any moment of the distribution of labor income in every period is a relevant information. However, when the m^{th} moment of labor income in any future period is changing to α^m of its previous level, then the homogeneity property for consumption can hold. For example, when labor income is distributed normally, a change of the mean and standard deviation of future income and of financial wealth by a factor α will change consumption (and permanent income) by this factor.

In light of this paper we can pose the empirical question of the validity of the homogeneity result of the PIH in the following way. What is the correlation between the mean and the standard deviation of income? If the stochastic process of labor income is such that the two are fully correlated, like in some geometric processes, consumption is homogenous with respect to income. When the income process is such that mean income can change with no similar change in the standard deviation, such as in the case of homoskedastic income, we expect to observe "excess sensitivity" relative to the benchmark PIH result.

The insight of this paper can explain also some theoretical results. Zeldes (1986) solves the optimization problem (1)-(4) numerically, when the interest rate equals the rate of time preferences. His main conclusion is the following (p.20):

> "The standard consumption function posits a linear relationship between consumption and "permanent income," defined as the annuity value of the sum of non-human wealth and the present discounted value of expected future labor income. The results here indicate that such a consumption function is likely to be severely misspecified, especially at low levels of wealth."

Many of the theoretical experiments which led Zeldes to his "misspecifiation" conclusion are as follows. Given a certain distribution of future labor income, Zeldes looks for optimal consumption as a function of non-human wealth levels. He finds that for low levels of financial wealth, an increase in this wealth yields a large increase in consumption, considerably larger than implied by the change in permanent income. However, this "excess sensitivity" almost disappears for households with very high assets relative to expected future labor income.

In light of the analysis in the previous sections, Zeldes' (1986) results can be explained as follows. A rise in non-human wealth W_t, which is nonstochastic, when the distribution of the uncertain future labor income is given, implies not only a rise in permanent income, but also a decrease in the uncertainty about lifetime resources relative to the certain part. This reduction in uncertainty, which can be measured by a lower coefficient of variation of lifetime wealth, implies for a CRRA utility an increase in consumption above the increase implied by the rise in permanent income. In particular, the lower is the non-human wealth, the larger the decrease in relative uncertainty about lifetime resources when non-human wealth increases, and therefore the larger the "excess sensitivity".[14]

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[14] It is interesting to note that the two distributions of labor income which Zeldes (1986) used are geometric distributions. For example, distribution number 1A is a geometric white noise and can therefore satisfy the sufficient condition of theorem 1. Hence, a shift of both the non-human wealth and labor income by the same factor will just reproduce the results of the standard PIH. The "excess sensitivity" that Zeldes found result from changing the wealth level without a similar change in labor income.

Similar results which show that as financial wealth increases consumption approaches the standard PIH level can be found in Schectman (1976) and Bewley (1977).

REFERENCES

Bernanke, Ben S.,"Adjustment Costs, Durables, and Aggregate Consumption," **Journal of Monetary Economics**, Vol.15, No.1, Jan. 1985.

Bewley, Truman, "The Permanent Income Hypothesis: A Theoretical Formulation," **Journal of Economic Theory**, 176, 1977, 252-292.

Blanchard, O.J., and N.G.Mankiw, "Consumption: Beyond Certainty Equivalence," **American Economic Review**, 78, May 1988, 173-177.

Campbell, John Y., "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis," National Bureau of Economic Research Working Paper No.1805, 1986.

Flavin, Marjorie, A., "The Adjustment of Consumption to Changing Expectations about Future Income," **Journal of Political Economy**, 89, October 1981, 974-1009.

Hall, Robert E. and Frederic S.Mishkin, "The Sensitivity of Consumption to Transitory Inome: Estimates from Panel Data on Households," **Econometrica**, 50, March 1982, 461-481.

Hayashi, Fumio, "The Permanent Income Hypothesis: Estimation and Testing by Instrumental Variables," **Journal of Political Economy**, Vol.90, No.5, 895-916, October 1982.

Leland, Hayne, E., "Saving and Uncertainty: The Precautionary Demand for Saving," **Quarterly Journal of Economics**, 82, 1968, 465-473.

Schectman, Jack, "An Income Fluctuation Problem," **Journal of Economic Theory**, 12,1976, 218-241.

Sibley, David S., "Permanent and Transitory Income Effects in a Model of Optimal Consumption with Wage Income Uncertainty," **Journal of Economic Theory**, 11, 1975, 68-82.

Stock, James H. and Kenneth D.West, "Integrated Regressors and Tests of the Permanent Income Hypothesis," Princeton University, Discussion Paper No.127, April 1987.

Zeldes, Stephen P.,"Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence," Wharton School, Working Paper No.20-86, 1986.