AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

TEL-AVIV

35-88

THE FOERDER INSTITUTE FOR ECONOMIC RESEARCH
TEL-AVIV UNIVERSITY
RAMAT AVIV ISRAEL

GIANNINI FOUNDATION OF
AGRICULTURAL ECONOMICS
LIBRARY

WITHDRAWN
FEB 15 1989

מכון למחקר כלכלי ע"ש ד"ר ישעיהו פורדר ז"ל
ע"י אוניברסיטת תל-אביב

THE DYNAMICS OF INFLATION WITH CONSTANT DEFICIT
UNDER EXPECTED REGIME CHANGE

by

Benjamin Bental* and Zvi Eckstein **

Working Paper No.35-88

November, 1988

* Technion, Israel Institute of Technology

** Tel-Aviv University

We would like to thank Ben Eden, Dan Peled and Neil Wallace for very helpful comments on previous versions of this paper. We alone are responsible for the contents of the paper.

Financial assistance from the Foerder Institute of Economic Research is gratefully acknowledged

FOERDER INSTITUTE FOR ECONOMIC RESEARCH
Faculty of Social Sciences,
Tel-Aviv University, Ramat Aviv, Israel.

THE DYNAMICS OF INFLATION WITH CONSTANT DEFICIT
UNDER EXPECTED REGIME CHANGE

by

Benjamin Bental* and Zvi Eckstein **

Abstract

A model that describes the dynamics of inflation with a fully anticipated stabilization program is developed. It is shown that with a constant deficit, an economy tends to stay at the neighborhood of the lower of the two state inflation rates that are compatible with the deficit. Further, increasing inflation rates prior to the stabilization are compatible only with deficits that are lower than the maximal feasible steady-state deficits, and with a stabilization program that reduces the demand for real balances. If these conditions are not met, inflation decreases towards the stabilization date. It is argued that the data generated by actual inflationary episodes fit the conditions described above and corroborate the model.

1. Introduction

The economic theory of high or hyper-inflation was motivated by observations on major historical inflationary episodes in different countries. These episodes tend to display some important common features. Monthly inflation rates have a clear upward trend and then drop abruptly to zero at the stabilization date. Real balances have a downward trend until stabilization, then increase for a while and stabilize a few months after the stabilization date. In contrast, real government revenues from expanding the money supply display no clear trend.¹ Moreover, these revenues seem to be quite small. Cagan (1956), Barro (1972), and Sargent (1977) conclude that in many of the post World War I inflationary episodes the seignorage collected by the government fell short of the maximal amount sustainable in the steady state.

The behavior of prices, real balances and the inflation tax revenue during the inflationary period, is well suited to a multi-steady-state theory (for example, Sargent and Wallace (1987), Quah (1987)). In this theory there are two steady-state inflation rates that correspond to the same government deficit. The lower of the two is dynamically unstable while the higher is stable. Accordingly, an economy in which the government runs a constant deficit is likely to be on a path of increasing inflation rates that converge to the higher steady-state inflation. However, this theory does not explain the end of the inflationary process and does not incorporate the stabilization policy.²

The goal of this paper is to provide a framework that is compatible with all the common characteristics of the inflationary episodes under the discipline of rational expectations. Specifically, using a

conventional monetary model we argue that the rationally expected stabilization program rules out the explanations offered by the multi-steady-state approach. With expected stabilization, economies have a unique equilibrium which is associated with the low steady-state inflation.

As we concentrate on the analysis of the trends of the different variables, we choose to construct a deterministic model. We assume the existence of a stable money demand that increases with disposable income and decreases with expected inflation. Agents have perfect foresight and know the whole future path of government policies. In particular, they know the date and the composition of any stabilization package.³ After the stabilization date the government's deficit is eliminated and the price level remains fixed.⁴

As a result of these assumptions the equilibrium pre-stabilization inflation path is uniquely determined. Its properties depend on the size of the (constant) pre-stabilization deficit as well as on the particulars of the stabilization program. Specifically, we show that high deficits (higher than the deficit that is sustainable in steady state) require a future stabilization, and lead to a sequence of inflation rates that are decreasing towards such stabilization. Increasing inflation rates are therefore compatible only with low pre-stabilization deficits. However, this condition is not sufficient to generate increasing inflation rates. We show that an additional requirement is a stabilization program that reduces the demand for real

balances. A reduction in disposable income due to increased post-stabilization taxes can create this effect.

We argue in the Appendix that the conditions that create increasing inflation in our model were present in the actual episodes including their end. Sargent (1982) has already described the stabilization programs in the post-World War I European episodes and has shown that all programs balanced the the government's budget mainly by increasing taxes and decreasing subsidies. The actual behavior of real balances also seems to be consistent with our model. Despite their recovery after the end of the inflation, real balances stay below their pre-inflation levels. Thus, we show that the end of the inflation if it was expected, is consistent with its beginning.

We discuss the properties of the model in section 2. In particular, we make some assumptions on the shape of the inflation-tax curve. In section 3, we show that these assumptions can be justified by several money-demand models. A money demand equation that is similar to that suggested by Cagan (1956), is one possible approach. In addition we show that money demand equations that are generated explicitly in an overlapping generations model or derived from cash in advance constraints as in Lucas and Stocky (1987), are also compatible with our specification. In section 4 we characterize the pre-stabilization equilibrium path and derive the conditions that yield increasing inflation rates. Section 5 summarizes our findings.

2. A General Class of Monetary Models

We analyze a discrete time, constant population, single good model in which the per-capita government consumption each period is g_t units of the good. The government finances its consumption by per-capita taxes, τ_t , and monet creation. Accordingly, the government's budget constraint is given by

$$(1) \quad d_t = g_t - \tau_t = \frac{M_t - M_{t-1}}{p_t}$$

where d_t is the per-capita deficit, M_t is the per-capita nominal balances at the "end" of the period t and p_t is the price level.

We postulate that the per-capita equilibrium demand for real balances, m_t , is given by

$$(2) \quad m_t = f(y - \tau_t, \pi_{t+1}^e)$$

where y represents per-capita income (constant over time) and π_{t+1}^e is the expected gross inflation rate between periods t and $t+1$, given by $\frac{p_{t+1}}{p_t}$. We assume that the money demand is increasing in disposable income so that $f_1 > 0$ and is decreasing in expected inflation, that is, $f_2 < 0$, where subscripts denote the appropriate partial derivatives. In the following section we discuss some models that yield a money demand equation of this form.⁵

An equilibrium consists of sequences $\{m_t, \pi_{t+1}\}_{t=1}^{\infty}$ such that for given policy sequences $\{g_t, r_t, M_t\}_{t=1}^{\infty}$ and an initial M_0 (i) the money market clears for every t , and (ii) perfect foresight prevails, that is

$$\pi_{t+1}^e = \frac{p_{t+1}}{p_t} = \pi_{t+1}.$$

Accordingly, in the money market we have

$$(3) \quad m_t = \frac{M_t}{p_t}.$$

The inflation process (from (3) and (11)) follows

$$(4) \quad \pi_t = \frac{m_{t-1}}{m_t - d_t}.$$

The equilibrium inflation sequence is determined from (4) and (2), and is given by

$$(5) \quad \pi_t = \frac{f(y - r_{t-1}, \pi_t)}{f(y - r_t, \pi_{t+1}) - d_t}.$$

Since f_2 is assumed to be negative everywhere, we can use the implicit function theorem to obtain

$$(6) \quad \pi_t = h(y - \tau_{t-1}, y - \tau_t, d_t, \pi_{t+1}) .$$

By differentiating the implicit function (5), we establish that $h_1 > 0$, $h_2 < 0$, $h_3 > 0$ and $h_4 > 0$.

In addition, we assume that in a steady state in which $\tau_t = \tau$, $d_t = d$, (6) satisfies the following requirements:

- (i) $1 = h(y - \tau, y - \tau, 0, 1)$, that is, at zero deficit an inflation rate of zero is a solution;
- (ii) Given τ , there exists a finite deficit $\bar{d}$ and a corresponding gross inflation rate $\bar{\pi}$ that satisfies $h(y - \tau, y - \tau, \bar{d}, \bar{\pi}) = \bar{\pi}$ such that for $d > \bar{d}$, $h(y - \tau, y - \tau, d, \pi) > \pi$ for all π . This condition implies that there is a limit to the amount of seignorage that can be raised by the government.

Conditions (i) and (ii) are not always imposed in macroeconomic models, but are implied by any reasonable monetary model. However, these conditions do not suffice to determine the dynamics of the inflation process for a given policy sequence. The crucial missing information is the sign of h_{44} (see figure 1). Clearly this sign depends on second derivatives of $f(\cdot)$. In particular, sufficiently small $|f_{22}|$ yields $h_{44} > 0$.

Standard macroeconomic theory provides no guidance related to the second derivative of the money demand function with respect to the inflation rate. Nevertheless, if we assume that $h_{44} > 0$, then property (ii) implies that for any d such that $0 < d < \bar{d}$ there exist

two values of π , π^1 and π^2 , that solve $h(y-\tau, y-\tau, d, \pi) = \pi$, with $\pi^1 < \pi^2$ (see figure 1).

These assumptions are compatible with particular models used in theoretical and empirical analyses of high inflation economies. The main feature of these models is the fact that for fixed levels of τ and $d < \bar{d}$, there are two dynamic solutions to equation (6). The first is a dynamically unstable path with $\pi = \pi^1$ for all t . The second is a path characterized by $\pi_t > \pi_{t-1}$ and $\lim_{t \rightarrow \infty} \pi_t = \pi^2$. The fact that the high inflation root is dynamically stable provided the basis for the analyses of runaway inflations in Sargent and Wallace (1986), Bruno and Fischer (1986) and Quah (1987).

The dynamic instability of the low inflation path applies also for the case in which the deficit is zero. Then, the unique monetary equilibrium in which the value of money remains fixed over time is unstable, and the stable steady-state solution is the one in which money has no value. This problem no longer exists if the value of money is bounded from below by a strictly positive number ϵ (see, for example, Wallace (1981) for a scheme which provides such a bound). Under such a constraint, the only equilibrium with a fixed money supply is the one that corresponds to $\pi = 1$. To show that this result is correct, suppose that it is not, so that $\pi_{t_0} > 1$ for some t_0 , then, ignoring the constraint, $\{m_t\}_{t=t_0}^{\infty}$ is a strictly decreasing sequence, tending to zero. Therefore, due to the constraint, there exists a period $T < \infty$,

such that $m_T = \epsilon$. Since $m_t \geq \epsilon$ for $t = T + 1$, we obtain from (4) that $\pi_{T+1} \leq 1$. Accordingly, from (6) we obtain $\pi_T \leq 1$. Applying (6) repeatedly, we get $\pi_t \leq 1$, a contradiction.

The analysis of the economy when deficits are positive (equation (5)) remain unaffected as long as $\epsilon < d_t$. Since ϵ can be arbitrarily small, the sole effect of the additional assumption is to guarantee the existence of a unique equilibrium which is monetary for a zero-deficit economy.

The next section provides three monetary models that fit the general framework discussed above. The reader may proceed directly to section 4 without a loss of continuity.

3. Three Underlying Models

To support our claim that the features of the general model are compatible with existing monetary theories, we discuss three examples of monetary environments that fit the framework of section 2.

Cagan demand for money

This classical demand for money specification is of particular importance in empirical studies. The demand for money is given by

$$(7) \quad m_t = A e^{-b(\pi_{t+1}-1)} (y - r_t)$$

with $b > 0$, which clearly satisfies the assumptions on $f(\cdot)$. Accordingly the $h(\cdot)$ function (equation (6)) satisfies all conditions concerning the first derivatives.

For this model, it is convenient to express π_{t+1} as a function of π_t , $y - \tau_{t-1}$, $y - \tau_t$ and d_t . At constant values of τ_t and d_t the inverse function of equation (6) is given by

$$(8) \quad \pi_{t+1} = -\frac{1}{b} \ln \left(\frac{1}{\pi_t} e^{-b(\pi_t - 1)} + \frac{d}{A(y - \tau)} \right) + 1.$$

Obviously, when $d = 0$, $\pi_t = \pi_{t+1} = 1$ solves equation (8). Further, for sufficiently small values of b (in particular $0 < b < 1$), the r.h.s. of (8) is a concave function of π_t , so that the equivalent to $h_{44} > 0$ is satisfied. Hence, the Cagan equation satisfies the conditions of the general model.

An overlapping generations model

In this setting the economy is populated by two-period lived overlapping generations of equal size.

Agents are endowed with $y_1 > 0$ units of a nonstorable consumption good in the first period of their lives and with $y_2 \geq 0$ units of the good in the second period. Every member of the initial old generation is endowed with M_0 units of money (in addition to y_2). The young agents have identical, strictly concave utility functions defined over

their lifetime consumption bundle, given by $u(c_1(t), c_2(t))$, with $c_i(t)$ denoting the consumption of an agent born at period t at age i , $i = 1, 2$. The initial old want to maximize their consumption.

Assuming that only the young are taxed, demand for real balances is derived from the maximization of $u(c_1(t), c_2(t))$, subject to

$$(9) \quad c_1(t) + \frac{M_t}{p_t} = y_1 - \tau_t$$

and

$$(10) \quad c_2(t) = y_2 + \frac{M_t}{p_{t+1}}.$$

The demand for real balances m_t satisfies

$$(11) \quad \pi_{t+1} = \frac{u_2(y_1 - \tau_t - m_t, y_2 + \frac{1}{p_{t+1}} m_t)}{u_1(y_1 - \tau_t - m_t, y_2 + \frac{1}{\pi_{t+1}} m_t)}.$$

Under certain conditions on $u(\cdot)$, y_1 , y_2 , τ_t (see Wallace (1980)) equation (11) can be solved for m_t , yielding a money-demand equation that is similar in form to equation (2). The differences between (2) and the money demand equation resulting from (11) are due to the overlapping generation structure. For example, the real balances (m) are measures per member of the younger generation. Likewise, y in (2) corresponds to y_1 here, and y_2 could be viewed as a shift parameter

of the function $f(\cdot)$. If we assume that $c_1(t)$ and $c_2(t)$ are both normal goods and gross-substitutes, we obtain the desired signs for the derivatives of this money-demand function, namely $\frac{\partial m_t}{\partial (y_1 - \tau_t)} > 0$ and $\frac{\partial m_t}{\partial \pi_{t+1}} < 0$.⁶ Accordingly, the first derivatives of $h(\cdot)$ (equation (6)) have the correct signs. The sign of h_{44} depends on third derivatives of the utility function so that different examples may be established. For the popular example of $u(c_1, c_2) = \ln c_1 + \ln c_2$, equation (2) is linear, given by $m_t = \frac{1}{2}(y_1 - \tau_t - \pi_{t+1}y_2)$ and $f_{22} = 0$. The function $h(\cdot)$ takes the form

$$\pi_t = \frac{y_1 - \tau_{t-1}}{y_1 - \tau_t - 2d_t - (\pi_{t+1} - 1)y_2} \quad \text{so that} \quad \frac{\partial^2 \pi_t}{\partial \pi_{t+1}^2} > 0, \text{ i.e. } h_{44} > 0.$$

A cash in advance model

Here we use the framework suggested by Lucas and Stokey (1987). The economy is populated by an infinitely lived representative consumer. The consumption goods are classified into two subsets. First, there are goods that can be obtained by credit, called the "credit goods". The second subset consists of goods that have to be bought with cash, called the "cash goods". Each subset is represented by a single good. The momentary utility derived from consuming c_{1t} units of the cash good and c_{2t} units of the credit good at period t is given by $u(c_{1t}, c_{2t})$.

Each period, agents are endowed with y units of a resource. Every unit of this resource can be costlessly converted to one unit of the cash good or the credit good. In addition, the government consumes g units per capita of the resource. Therefore, the per capita resource constraint of the economy is given by

$$(12) \quad c_{1t} + c_{2t} + g \leq y.$$

The existence of a credit good necessitates a modification of the government's budget constraint (1). The government supplies to the economy a one-period nominal credit instrument (bonds) in addition to money. The per-capita amount of bonds issued at t is B_t . The net nominal interest rate on bonds issued at $t-1$ that is paid at t is R_t . The amount of money issued at t is M_t . Accordingly, the government's budget constraint at period t is given by

$$(13) \quad g - \tau = \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - (1+R_t)B_{t-1}}{P_t}.$$

Individual budget constraints take the following form. First, the overall expenditure has to satisfy

$$(14) \quad p_t(c_{1t} + c_{2t}) + M_t + B_t \leq p_t(y - \tau) + (1+R_t)B_{t-1} + M_{t-1}.$$

In addition, the cash good at period t must be purchased with money accumulated at period $t-1$ so that

$$(15) \quad p_t c_{1t} \leq M_{t-1}.$$

Given M_0 and B_0 , the representative consumer maximizes

$\sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t})$ subject to (14) and (15), where $\beta < 1$ is the subjective discount factor. We assume that in equilibrium the cash in advance constraint (15) is binding, as well as the budget constraint (14). Using (12), the equilibrium first-order conditions for an optimum are

$$(16) \quad -\pi_{t+1} u_2(m_t, y-g-m_t) + \beta u_1(m_{t+1}, y-g-m_{t+1}) = 0$$

and

$$(17) \quad -\pi_{t+1} u_2(m_t, y-g-m_t) + (1+R_{t+1})\beta u_2(m_{t+1}, y-g-m_{t+1}) = 0$$

where $m_t = \frac{M_{t-1}}{p_t}$.

We now fix $\frac{B_{t-1}}{p_t} = b$ for all t , and rewrite (13) as

$$(18) \quad g - \tau = \pi_{t+1} m_{t+1} - m_t + b(\pi_{t+1} - (1+R_t)).$$

Equations (16), (17), and (18) describe the dynamics of the system. The equivalent of equation (2) can be obtained as follows. Solve for $(1+R_t)$ from (16) and (17) in terms of m_t , substitute the result in (18), obtain m_{t+1} in terms of m_t and π_{t+1} and use (16) again. Finally, the equivalent to equation (6) can be obtained by substituting the result of the procedure described so far in (18) both for m_t and m_{t+1} and solving for π_{t+1} in terms of π_{t+2} .

The complexity of the model precludes a meaningful general analysis. However, for a special case where $u(c_{1t}, c_{2t}) = \alpha c_{1t} + \ln c_{2t}$ we obtain

$$(19) \quad m_t = y - \tau - d - \frac{\pi_{t+1}}{\alpha\beta}$$

and

$$(20) \quad \pi_{t+1} = \frac{y - \tau}{y - \tau - d - \frac{\pi_{t+2}}{\alpha\beta} + \frac{1}{\alpha\beta} + b(1 - \frac{1}{\beta})}$$

where $d = g - \tau$. Equations (19) and (20) have the required properties (in particular $h_{44} > 0$). Notice, however, that in this model $\pi_{t+1} = 1$ for all t implies $1 + R = \frac{1}{\beta}$, and this is consistent only with $d = b(1 - \frac{1}{\beta}) < 0$, so that the government runs a surplus in order to finance the necessary interest payments.

4. Stabilization Policies

We consider equilibria that result from a specific class of government policies. In particular, we assume that there exists a date T , with $1 < T < \infty$, after which the government conducts a stabilization policy that generates a stable price path. Accordingly, $\pi_{t+1} = 1$ for $t \geq T$. The date T is called the stabilization date. There are several possible stabilization policies that would generate a constant price level, and proposition 1 below holds for any stabilization program. Here we are particularly interested in the zero deficit program. To guarantee that $\pi_{t+1} = 1$ for $t \geq T$, we assume that the stabilization policy, or even during the period prior to that policy, there exists a requirement that effectly binds the value money to be larger than $\epsilon > 0$.⁷ We can now establish the following result.

Proposition 1: If there exists a date T such that $\pi_{t+1} = 1$ for $t \geq T$, the equilibrium is unique.

Proof: Since $\pi_{T+1} = 1$ and $h_4 > 0$, equation (6) determines π_T uniquely. Repeated application of this procedure for $t < T$ yields the unique equilibrium path. $\square$

Notice that the uniqueness of the equilibrium depends only on the fact that $h_4 > 0$, which implies that the demand for money must be strictly decreasing with respect to expected inflation. Further, proposition 1 holds regardless of the particular nature of the policy

sequence after the stabilization date, as long as the policy generates price level stability in a credible way. Conditional on the assumption that stabilization is expected, the proposition rules out theories which explain hyperinflations as being a result of the multiplicity of rational expectations equilibria mentioned above.

We turn next to the analysis of the pre-stabilization inflation rates. In particular, we restrict the class of government policy sequences at the prestabilization date. To mimic our general observation that the deficits were approximately constant during the inflationary phase, we set $d_t = d$ and $r_t = r$ for $t \leq T-1$. Under these restrictions our aim is to identify conditions under which the pre-stabilization inflation rates are increasing towards the stabilization date.

Proposition 2: Let $d_t = d$ and $r_t = r$, for $t \leq T-1$. Then, a necessary condition for the inflation rates to increase is that the pre-stabilization deficit satisfies $d < \bar{d}$.

Proof: Using the assumption that $h_{44} > 0$ and that property (ii) of the function $h(\cdot)$ we have that for any $d > \bar{d}$,
 $\pi_t = h(y-r, y-r, d, \pi_{t+1}) > \pi_{t+1}$ for all $t \leq T-1$. Therefore, inflation must be decreasing towards stabilization. $\square$

Proposition 2 establishes the fact that if the pre-stabilization government deficit is bigger than the maximal steady state deficit, inflation cannot be increasing regardless of the tax/transfers related to the stabilization policy. Furthermore, it is not feasible to have such deficits unless the economy is to be stabilized. The properties of $h(\cdot)$ imply that if $d > \bar{d}$, there exists a t_0 such that for $t \geq t_0$, $\pi_t < 1$. Therefore, in the "short-run" the government can finance a deficit which is not maintainable in the "long-run" and inflation rates are decreasing. This result has the same flavor as that of Sargent and Wallace's (1981) "unpleasant monetarist arithmetic". There the government uses bonds to finance its deficit, a policy which is not feasible in the long run. Eventually the government reverts to money printing. As a result, inflation rates may be increasing during the initial phase in which no money is printed in anticipation of the high future steady-state inflation.

As a result of proposition 2, we further restrict the class of prestabilization policies that may be consistent with increasing inflation rates. Specifically, the deficit is now assumed to be at a level that could be maintained in steady state. We first show that the unique equilibrium path established in proposition 1 converges (backwards) to the lower steady-state inflation rate. This we do by delaying the stabilization date T indefinitely and evaluate the initial inflation rate, π_1 .

Proposition 3: Let $d_t = d$ and $r_t = r$ for $t = 0, 1, 2, \dots, T-1$, with $0 < d < \bar{d}$. Then, for any $\pi_T < \pi^2$, $\lim_{T \rightarrow \infty} \pi_1 = \pi^1$ (where π_1 is the inflation at the initial date, and π^1 and π^2 are the roots of $h(y-r, y-r, d, \pi) = \pi$, with $\pi^1 < \pi^2$).

Proof: The shape of $h(\cdot)$ ($h_4 > 0$ and $h_{44} > 0$), implies that given $\pi_T < \pi^2$ the sequences $(\pi_{T-i})_{i=0}^{T-1}$ are strictly monotone and satisfy $|\pi_{T-j-2} - \pi_{T-j-1}| < |\pi_{T-j-1} - \pi_{T-j}|$ for $j = 0, \dots, T-3$. Therefore, given any $\epsilon > 0$, there exists a T sufficiently large such that $|\pi_1 - \pi^1| < \epsilon$. By continuity of $h(\cdot)$ $\lim_{T \rightarrow \infty} \pi_1 = \pi^1$ (a graphical exposition is given in figure 2). □

The expected future stabilization not only rules out multiple equilibria (proposition 1) but also reverses the classification of the "stable" and the "unstable" inflation paths. That is, the only equilibrium path converges to the low inflation if the stabilization date is delayed indefinitely. Therefore, the explanation of runaway inflations based on the stability of the path converging to the high steady-state inflation (π^2) is inconsistent with expected stabilization.

Notice that if the stabilization policy implies $\pi_T > \pi^2$, then the stabilization date cannot be postponed indefinitely because inflation rates become arbitrarily large as time progresses. In this case, a

stabilization must occur within a finite time and inflation must be decreasing towards stabilization. Another case in which inflation is decreasing towards stabilization is characterized next:

Corollary 1: If $d_t = d$ and $r_t = r$ for $t = 0, 1, \dots, T-1$ with $0 < d < \bar{d}$ and $\pi_T = 1$, then $\pi_{T-i} > \pi_{T-i-1}$ for $i = 1, \dots, T-3$.

Proof: The low steady-stage root π^1 for $0 < d < \bar{d}$ is greater than

1. Therefore, $\pi_{T-1} = h(y-r, y-r, d, 1) > 1$ and

$\pi_{T-i} = h(y-r, y-r, d, \pi_{T-i+1}) > \pi_{T-i+1}$ for $i = 2, \dots, T-3$. $\square$

The corollary argues that if $\pi_T = 1$, a hyperinflation is ruled out. A sufficient condition to obtain $\pi_T = 1$ is to maintain $r_t = r$ for all t with $d_t = 0$ for $t \geq T$. In this case $\pi_T = h(y-r, y-r, 0, 1)$, and by the assumption made on $h(\cdot)$, $\pi_T = 1$. Consequently, increasing inflation rates towards stabilization are not consistent with a policy that keeps post-stabilization taxes fixed at their pre-stabilization levels and reduces the deficit solely by decreasing government consumption. This conclusion can be reformulated as follows.

Proposition 4: Let $r_t = r_1$ and $d_t = d$ with $0 < d < \bar{d}$ for $t \leq T-1$. Further, let $d_t = 0$ and $r_t = r_2$ for $t \geq T$. Then $r_2 > r_1$ is a necessary condition for inflation rates to be rising towards stabilization.

Proof: To get $\pi_t < \pi_{t+1}$ for $t \leq T-1$ it is necessary and sufficient to have $\pi^1 < \pi_T < \pi^2$. But $\pi_T = h(y-\tau_1, y-\tau_2, 0, 1)$. As $\pi^1 > 1$, we must have $\pi_T > 1$ is necessary to get $\pi_T > 1$. Since $h(y-\tau_1, y-\tau_1, 0, 1) = 1$ and $h_2 < 0$, $\tau_2 > \tau_1$ is necessary to get $\pi_T > 1$. Figure 3 shows an appropriate configuration of τ_1 and τ_2 . $\square$

Proposition 4 states that a positive inflation at the stabilization date is necessary if inflation is to increase prior to stabilization. This inflation must be due to reduced demand for real balances because the government deficit has been eliminated. The required reduction in real balances is, in this model, a result of increases in post-stabilization taxes. As we argued above, the historical stabilization programs have indeed involved tax increases and subsidy increases (rather than cuts in government consumption). Furthermore, post-stabilization real balances have not returned to their original levels. These observations are consistent with our model.

5. Concluding Remarks

The theoretical structure analyzed in this paper succeeds in providing a unified framework within which all major phenomena common to high inflation episodes are present. Specifically, we show that a period of increasing inflation rates that ends abruptly, with falling real balances and (more-or-less) constant inflation tax revenues can be a result of the expected stabilization program. We identified two

conditions that must be present in order to generate the inflationary period. First, the inflation tax revenues should be smaller than the maximum sustainable in steady state, and second, the expected stabilization program must reduce the demand for real balances by increasing taxes.⁸ We argue that both conditions were fulfilled in the actual historical episodes. In this way we show that Sargent's (1982) view about the end of big inflations is consistent with the inflationary process itself when the end is anticipated by the population.

The post stabilization period in all the historical episodes is characterized by an increase in the supply of high powered money that did not affect the price level (see the appendix). This phenomenon is explained by Sargent (1982) as a result of the change in the asset composition of the Central banks that accompanied the increased money supply. Essentially, the central banks exchanged money for private and public debt. Several authors have shown that such open market operations are neutral (Wallace (1981), Chamley and Polemarchakis (1984), Peled (1985)). All of these models are compatible with our second example, in which the demand for money is derived from an overlapping generations structure, and in which money is essentially undominated in return. It is clear that the assumption on the lack of rate of return dominance is reasonable for the post stabilization period, whereas during the inflation money was clearly dominated by other assets (such as foreign currency). Accordingly, Sargent's (1982) explanation of this particular phenomenon is consistent with our

description of the pre-stabilization path of the economy. In fact, the post stabilization open market operations are neutral not only with respect to current and future prices, but also with respect to past prices even when they are fully anticipated.

The expected stabilization induces a unique equilibrium in our model. If the stabilization is expected to occur in the distant future, the inflation rate in the economy will be close to the lower of the two inflation rates that generate the required seignorage. In models that do not include the anticipated stabilization feature, this low inflation is dynamically unstable and therefore sustainable only under very special specification of the policy parameters (see Sargent and Wallace (1987)). Our result that the low inflation rate is the stable one conforms with similar conclusions derived by Marcet and Sargent (1987). The fact that the latter models are quite different from ours makes the conclusion more robust and calls into question previous explanations of high inflation that depended on the multiple-steady-state theory.

APPENDIX

Common characteristics of high inflation episodes

We review in this appendix the literature on high inflation episodes and provide evidence that shows the similarity of the behavior of prices, real balances and inflation-tax revenues that we summarized in the introduction.

First, the fact that inflation is increasing and then stops abruptly is displayed by graphs in Sargent (1982) for the cases of Austria, Hungary, Germany and Poland. The same phenomenon can be observed in Hungary and Greece after the second World War and is discussed by Bomberger and Makinen (1983) and Makinen (1984). In Israel the half-year average monthly inflation rates were increasing from about 4-5 percent in 1980 to about 14 percent in the second half of 1984 and the first half of 1985. Since the stabilization program of July 1985 the monthly inflation rate is about 1.5 to 2 percent per month. The data on the Israeli case are available in the Bank of Israel Annual Reports, in Bruno (1986) and Bental and Eckstein (1987).

Second, real balances of high powered money are presented in Sargent (1982) and in figures 4-7. They clearly show the fact that monthly real value of money was decreasing continuously up to the date of stabilization. Afterwards, real balances increase, but not to the level that prevailed before the inflation. Similar trends existed in Hungary and Greece after the second World War. In Israel, the

substantial decline in the real value of high-powered money occurred during the period of 1978-1981. In addition, there was some decline in real balances as monthly inflation rates reached the level of 14 percent in 1984. After stabilization real balances increased substantially, exceeding their 1979-1980 level, but not that of the mid 70s.

The post-stabilization increase of the real balances is documented by figures 1-4, as well as by Sargent (1982), Bomberger and Makinen (1983) and Makinen (1984). In Bental and Eckstein (1987), as well as the Bank of Israel Report of 1985, it is shown that during the second half of 1985 (following the stabilization), there was a significant increase in real base-money. Sargent argues that in the European countries the increase in base-money was matched by increases in real assets of the central bank or decreases in the real debt. The Israeli data clearly show that the increase in real base-money was almost exactly matched by a decrease in the value of the outstanding government debt, so that the deficit was approximately zero. Therefore, one can argue that the post stabilization increases in real balances are a result of neutral open-market operations that leave the price level unchanged.

Thirdly, we present graphically the monthly levels of seignorage

$\left(\frac{M_t - M_{t-1}}{P_t} = \frac{\Delta M}{P} \right)$ for the four European cases in figures 1-4. There is no significant trend in these values. Sargent and Wallace (1973) calculate the deficit for these, as well as additional inflationary episodes such

as Russia in the twenties (1922-1923) and Hungary and Greece after the second World War. Their figure 1 is based on the calculation of $\Delta M [1/2(P_t + P_{t-1})]$ as a measure for the deficit. They argued that... "these data are generally without noticeable trends."

The same argument is made by Bruno (1986) and Bruno and Fischer (1986) about the Israeli economy. In the Israeli case, both the revenues from printing base-money and total deficit do not show a noticeable trend (see also the appendix to Bental and Eckstein (1987)).

The fourth common characteristic of the high inflation episode is the particular set of policies that achieved a balanced budget during and after the stabilization period. It seems that in most cases the main measure undertaken was to decrease sharply direct subsidies to consumers and increase tax collection. The increase in taxes is mainly due to the fact that taxes were not indexed and therefore the real value of tax collection was deflated during the period of increasing inflation. When prices stabilized, this deflation stopped. Sargent (1982) emphasized that in all the four cases reviewed by him there were substantive increases in direct taxes and almost all subsidies and free government services were eliminated.

For example, in 1922, the only tax that was collected at the source and on an ad valorem basis in Germany was a flat 10% tax on wages. All other direct and indirect taxes involved long delays in assessment and collection, and were not indexed. In October 1923 the taxes were indexed to a daily gold value. The effect on revenues was dramatic:

from 14.5 million gold-marks in October 1923 to 63.2 and 312.3 in the last two months of 1923. The monthly tax revenues in 1924 averaged 550.3 million gold marks (see Sargent (1982)). The reduction in government expenditures in Germany was accompanied mainly by dismissing a large number of government employees (about 1/3 of the personnel), and increasing the prices of government services.

Because of the large share of subsidies in the government expenditures, it is hard to assess whether the share of government consumption in GNP has changed much after the stabilization in any of the European episodes. In Israel it is clear, however, that government consumption in the second half of 1985 and 1986 has not changed, and that the government budget was balanced solely by sharp reductions in subsidies and increases in tax collection (see Bruno (1986), and Bank of Israel Annual Reports (1985, 1986)).

FOOTNOTES

- 1 See appendix for a more detailed description of these features.
- 2 Another class of models explains increasing inflation rates as a result of changes of the amount of seignorage raised by the government. While allowing agents to anticipate a change in the budget policies (for example, Flood and Garber (1986), Sargent and Wallace (1981), La Haye (1985) and Drazen and Helpman (1985)) some of these models are inconsistent with the observed behavior of the inflation tax revenues during and after the inflation in the historical episodes.
- 3 A more general model would incorporate uncertainty concerning both the date and composition of the stabilization program, as suggested by Drazen and Helpman (1985). However, we believe that our main conclusions are robust with respect to the introduction of uncertainty.
- 4 This specification is not common to all studies of hyperinflation. In Sargent and Wallace (1987), Dornbusch and Fischer (1986), as well as Bruno (1986), inflation may occur even when there are no government deficits and the nominal money supply is held constant. We believe that the evidence from the behavior of economies with zero monetary growth favors strongly the view that the price level remains fixed in such circumstances.

- 5 This setup is consistent with an environment in which the per capita capital stock is fixed. With this assumption the real interest rate is constant, and per capita income is fixed.
- 6 The derivatives of the money demand equation are obtained from the first-order condition for money holding. Accordingly, the properties of this bordered Hessian matrix for the utility function $u(\cdot)$ are crucial for the analysis. The normalcy and gross-substitution assumptions provide the necessary information about these properties.
- 7 See, for example, Wallace (1981) for a schem on tax payments and the end of section 2 for the implications.
- 8 The importance of the tax scheme at a regime-change is demonstrated in Aiyagari (1985). There it is shown that Sargent and Wallace's (1981) monetarist arithmetics depend crucially on the tax policy at the end of the bond-financing phase.

REFERENCES

- Aiyagari, S. Rao, "Deficits, Interest Rates and the Tax Distribution,"
Federal Reserve Bank of Minneapolis Quarterly Review, Winter 1985,
5-14.
- Barro, Robert, J., "Inflationary Finance and the Welfare Cost of
Inflation," Journal of Political Economy, 80 (September/October
1972), 978-1001.
- Bental, B. and Z. Eckstein, "Inflation, Deficit and Seignorage with
Expected Stabilization," (1987).
- Bomberger, William A. and Gail E. Makinen, "The Hungarian Hyperinflation
and Stabilization of 1945-46," Journal of Political Economy, 91
(October 1983), 801-24.
- Bruno, Michael, "Israel's Stabilization, The End of the 'Lost Decade'?",
unpublished, The Hebrew University (June 1986).
- _____ and Stanley Fischer, "The Inflationary Process: Shocks
and Accommodation," in Yoram Ben-Porath, editor, The Israeli
Economy: Maturing through Crisis, Harvard University Press
(1986), 347-371.
- Cagan, Phillip, "The Monetary Dynamics of Inflation," Studies in the
Quantity Theory of Money, edited by Milton Friedman, Chicago
University Press (1956).
- Chamley, C. and H. Polemarchakis, "Assets, General Equilibrium and
the Neutrality of Money," Review of Economic Studies 51 (1984),
129-130.

- Drazen, Alan and Elhanan Helpman, "Inflationary Consequences of Uncertain Macroeconomic Policies," Foerder Institute for Economic Research, Working Paper No.21 (July, 1985).
- Dornbusch, Rudiger and Stanley Fischer, "Stopping Hyperinflations Past and Present," NBER Working Paper No.1810, (1986).
- Flood, Robert P. and Peter M. Garber, "An Economic Theory of Monetary Reform," *Journal of Political Economy*, 88 (February 1980), 24-58.
- La Haye, Laura, "Inflation and Currency Reform," *Journal of Political Economy*, 93 (June 1985), 537-560.
- Lucas, R.E. and N.L. Stokey, "Money and Interest in a Cash in Advance Economy," *Econometrica* 55(3), (May 1987), 491-514.
- Makinen, Gail, E., "The Greek Stabilization of 1944-46," *IER* 74 (December 1984), 1067-1074.
- Marcet, A. and T.J. Sargent, "Least Squares Learning and the Dynamics of Hyperinflation," manuscript (1987).
- Peled, D., "Stochastic Inflation and Government Provision of Indexed Bonds," *Journal of Monetary Economics* (1985), 291-307.
- Quah, D., "Stable and Unstable Inflationary Dynamics and Exogenous Seignorage Financing," MIT (1987), unpublished.
- Sargent, Thomas, J., "The Demand for Money during Hyperinflation under Rational Expectations: I." *International Economic Review*, 18 (February 1977), 59-82.
-
- _____, "The Ends of Four Big Inflation," in *Inflation: Causes and Effects*, edited by Robert E. Hall. Chicago University Press (1982).

and Neil Wallace, "Some Unpleasant Monetarist
Arithmetics," Federal Reserve Bank of Minneapolis Quarterly Review
(Fall 1981), 1-15.

, "Inflation and the Government
Constraint," Economics in Theory and Practice, edited by A.Razin
and E.Sadka, MacMillan Press (1987).

Wallace, Neil, "A Modigliani-Miller Theorm for Open Market Operations,"
American Economic Review, 71 (1981), 267-74.