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DEMAND FOR INPUTS UNDER UNCERTAINTY; A METHODOLOGICAL-GRAPHICAL NOTE

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In this study we present, graphically, the effects of uncertainty on the demand for factors of production by a firm, in each of the following market structures: perfect competition, monopoly and monopsony.[1] The analysis is restricted to the two-variable inputs case. We assume that the production activity requires at least one additional input but that its quantity is predetermined (ex-ante input). We thus do not discuss the issues involved in its determination and view its quantity in the short run, to be given (fixed input).

The graphical presentation is based upon the Iso value of the marginal product curves in the perfect competition model, the Iso marginal revenue product in the monopoly model, and the geometric location of the equalities between the marginal factor costs and the values of marginal product in the monopsony case. For each structure, two relations between the inputs are analyzed, the first in which they are assisting and the second in which they are rivals. The first is defined by a positive cross derivative of the production-function w.r.t. the inputs while the second by a negative cross deritative. We first present the curves and solutions in a certain world and then the corresponding ones under uncertainty. In all cases the firms are assumed to be risk averse and the random effects linear.[2]

Competitive Firm

Under certainty the two conditions that have to be jointly fulfilled are the equalities of the value of the marginal product to the price of the input.

(1) $VMP_a = P_b$

(2) $VmP_b = P_b$

where $VMP_i = P_x MP_i$.

Let $MP_{ab} > 0$. Then, in the two-dimensional inputs space (Figure 1), the slope of the curve corresponding to condition 1 above (denoted by aa) is

$$-MP_{aa}/MP_{ab} > 0.$$

The slope of the curve corresponding to condition 2 above (denoted by bb) is

$$-MP_{ab}/MP_{bb} > 0.$$

The second-order conditions for profit maximization require that

$$\frac{-MP_{aa}}{MP_{ab}} > \frac{-MP_{ab}}{MP_{bb}} \quad \text{i.e.} \quad MP_{aa} \cdot MP_{bb} - MP_{ab}^2 > 0.$$

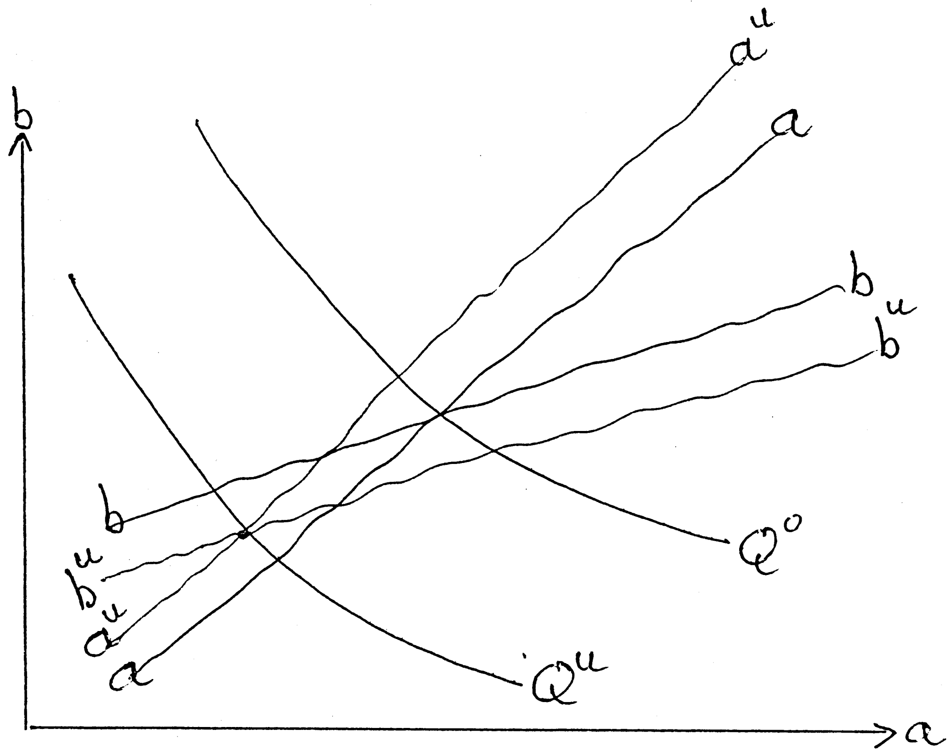

Figure 1

A Competitive Firm and the Assisting Inputs

For the case $MP_{ab} < 0$ one finds that the slope of aa is $-\frac{MP_{aa}}{MP_{ab}} < 0$, and of bb is $-\frac{MP_{ab}}{MP_{bb}} < 0$.

Again

$$-\frac{MP_{aa}}{MP_{ab}} < -\frac{MP_{ab}}{MP_{bb}}.$$

The respective curves for $MP_{ab} < 0$ are given in figure 2.

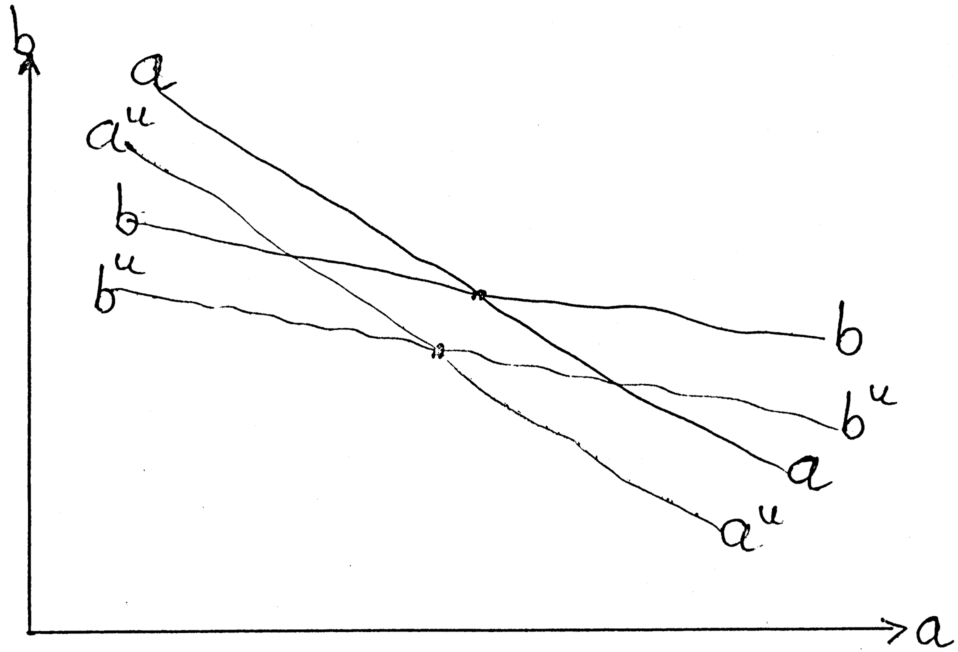

Figure 2

Competitive Firms where Inputs are Rivals

If the source of uncertainty is the product market, and the production decisions are made before the product market is realized, certainty would affect both the aa and bb curves. They will be shifted to the left and down respectively.

The shifts are due to the effect of uncertainty on equations (1) and (2). The two equations, under uncertainty, become (given profit maximization)

(1a) $$VMP_a^o + F_a \frac{Cov(U', \epsilon)}{EU'} = P_a$$

(2a) $$VMP_b^o + F_b \frac{Cov(U', \epsilon)}{EU'} = P_b$$

where $P_x = P_x + \epsilon$, ϵ is a random variable.

Assuming risk aversion, Cov(·) < 0. Hence, for a constant level of $P_a(P_b)$ the quantity demanded of a(b) under uncertainty is smaller than under certainty $(P_x = P_x^0)$ or given risk neutrality (Cov(·) = 0). Hence, the employment of both inputs declines regardless of whether they are assisting or rivals, and thus output obviously declines. This conclusion is partly modified when the source of uncertainty is the inputs markets.

If both inputs markets are uncertain and we assume the random effects to be uncorrelated and that employment decisions are made before they are realized the previous result is repeated.

In this respect equations (1) and (2) become

$$(1)' \qquad VMP_a = P_a^0 + \frac{Cov(U', \epsilon_a)}{EU'}$$

$$(2)' \qquad VMP_b = P_b^o + \frac{Cov(U', \epsilon_b)}{EU'}$$

where

$$P_a = P_a^0 + \epsilon_a, \quad \epsilon_a \text{ is random}$$

$$P_b = P_b^o + \epsilon_b, \quad \epsilon_b \text{ is random}$$

Since for a risk averse form Cov(·) > 0, the similarity to uncertainty in the product market is perfect. One has just to move the Cov(·) terms to the left-hand-side of equations (1)' and (2)'.

If, however, the market of only one input is uncertain only the Iso curve of that input would shift. In the case of the inputs being assisting, the quantities of both inputs still decline. However, in the rivals case the quantity of the input whose market is uncertain declines while the employment of the other input increases. This outcome might also emerge when both markets are uncertain but to a different extent (see the $a^u a^u$ and $b^u b^u$ curves in figures 1 and 2). In this case there is a difference in the magnitudes of the terms $\frac{Cov(U', \epsilon_i)}{E(U')}$, $i = a,b$ (ϵ_i is the random effect specific to market i).

Monopoly

The only change needed when moving from the competitive firm to a monopoly is that the marginal revenue product, MRP_i, replaces the value of marginal product, VMP_i. However, as is shown below this change is crucial. We first assume $MP_{ab} > 0$. One notes immediately that the under certainty the slope of the corresponding aa is

$$- \frac{MP_{aa} + MP_a^2}{MP_{ab} + MP_a MP_b}.$$

Thus its sign is indetermined. The indeterminancy is due to the numerator ($MP_{aa} < 0$). However, given that MP_{aa} is a second-order change, one would expect $MP_a^2 > |MP_{aa}|$, thus the aa curve is likely to slope downward.

With regard to bb the story is the same. Its slope is

$$-\frac{MP_{ab} + MP_a MP_b}{MP_{bb} + MP_b^2}.$$

The indeterminancy is due to the denominator and for $MP_b^2 > |MP_{bb}|$ it is negative.

Comparing the slopes of aa and bb for the monopoly yields that aa is steeper than bb (figure 3).

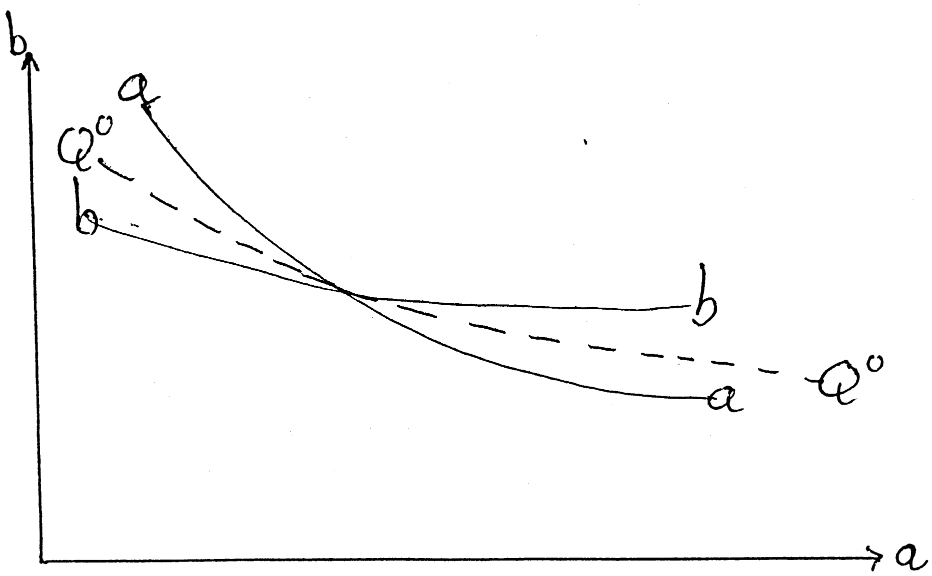

Figure 3

Monopoly, Inputs are Assisting

Hence, although the inputs are assisting, the behavior is as for rival inputs when the firm is competitive.

For $MP_{ab} < 0$ also the signs of the denominator of the slope of aa and the sign of the numerator of the slope of bb are indetermined. All one can do is to resort again to the idea that since MP_{ab} is a second-order derivative. Then $|MP_{ab}| < MP_a \cdot MP_b$. Hence, the denominator of the slope of aa is positive and aa slopes downward while the numerator of the slope of bb is negative and bb also slopes downward, i.e. figure 3 is repeated also for rival inputs.

However, there is a difference in the magnitude of the slopes. The slope of aa for $MP_{ab} < 0$ is larger than that when $MP_{ab} > 0$. The slope of bb for $MP_{ab} < 0$ is smaller than for $MP_{ab} > 0$ (figure 4).

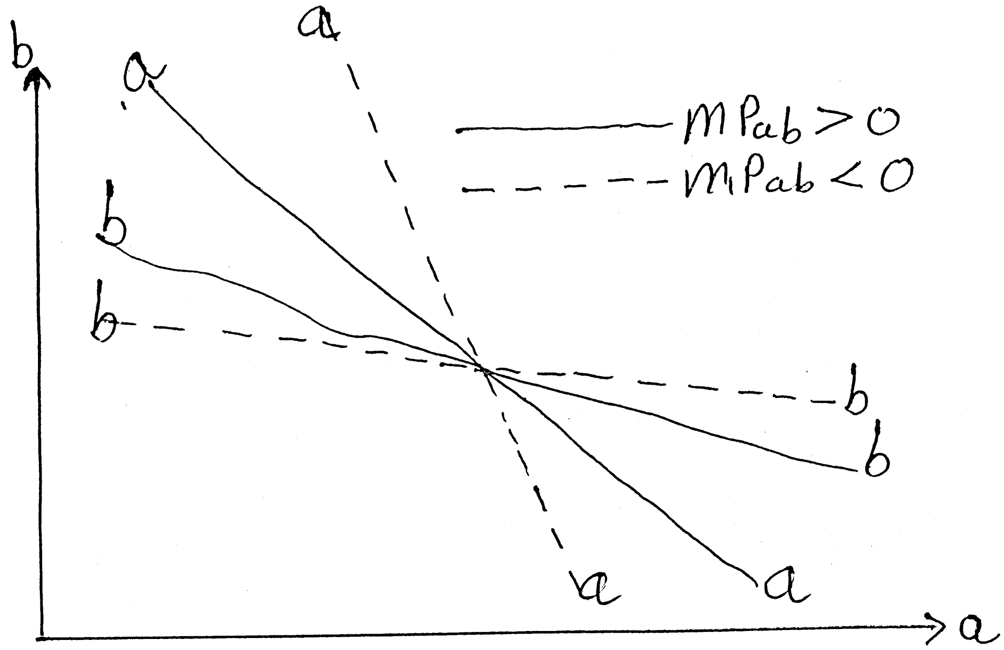

Figure 4

Monopoly Assisting and Rival Inputs

Once Figure 4 is accepted, the effect of uncertainty in the product market is well determined. The aa curves shift leftward and the bb curves shift downward. Thus, output declines regardless of whether the inputs are assisting or rivals. The reason for the leftward and downward shifts are due, as in the competitive firm case, to the presence of additional terms in equations (1) and (2) when the source of uncertainty is the product market. (Equations (1)' and (2') now become

$$(1)^{mu} \qquad MRP_a + \frac{F_a \, Cov(U', \epsilon_a)}{EU'} = P_a$$

$$(2)^{m}u \qquad MRP_b + \frac{F_b \, Cov(U', \epsilon_b)}{EU'} = P_b$$

where ϵ_a and ϵ_b are the random components of the marginal revenue of the monopoly in the product market. For risk aversion the two Cov(·) are negative.

When the randomness originates in the inputs markets positive Cov(·) terms appear on the r.h.s. of $(1)^{mu}$ and $(2)^{mu}$ and the arguments given for the shifts of aa and bb of a competitive firm are repeated.

A small variation of the monopoly case presented above is again the one in which uncertainty is present only in the market of one of the inputs, i.e. compared to certainty either only aa shifts to the left or only bb downwards. If the first takes place the employment of a obviously declines while that of b increases. If the second takes place then a and b switch roles. The effect on output cannot be determined only by the two curves. Since the isoquants also slope downward the isoquant has to be at a specific shape compared to aa and bb (see figure 3 - the heavy line $q^o q^o$). Now it is clear that in both cases discussed above output declines.

Monopsony

Assuming monopsony in both variable input, the first-order conditions under certainty require that jointly

$(1)^{mn}$ $\quad VMP_a = MFC_a$ (the aa curve),

and

$(2)^{mn}$ $\quad VMP_b = MFC_b$ (the bb curve),

where MFC denotes the marginal factor costs, i.e. $MFC_i = P_i(1 + 1/\phi_i)$ where ϕ_i is the elasticity of supply of i. Traditionally, $\phi_i > 0$ and $MFC_a > P_a$ for all prices of a. The slope of the aa curve is

$$\frac{MFC_{aa} - P_x MP_{aa}}{P_x MP_{ab} - MFC_b} .$$

For $MP_{ab} > 0$ the slope is indetermined.

For $MP_{ab} < 0$ the slope is definitely negative.

Along bb the slope is

$$\frac{MFC_{aa} - P_x MP_{ba}}{P_x MP_{bb} - MFC_{bb}}.$$

Here again, for $MP_{ba} > 0$ the slope is indetermined while for $MP_{ba} < 0$ it is definitely negative. The indefinitness of slopes for $MP_{ab} > 0$ also generates a problem in determining their relative positions. The relative position is also indetermined for $MP_{ab} < 0$ when both aa and bb slope downward. One way to get out of the indefinitness is to assume monopsony only in one market, e.g. the market for input a. Hence, $MFC_{bb} = 0$. Then for $MP_{ab} > 0$ the aa is sloping upward and

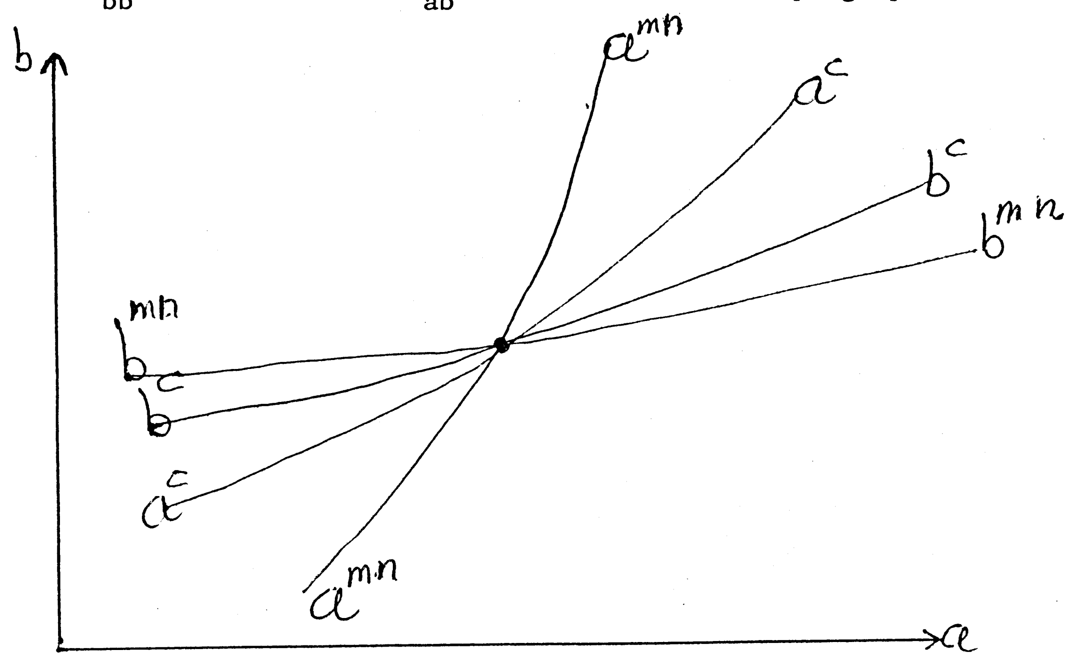

Figure 5a

is steeper than that of aa of a competitive firm, while bb can slope both ways, but if it is positive, the slope is smaller than that of a competitive firm (figures (5a) and (5b)).

Uncertainty affects either the left-hand-side of equations $(1)^{mn}$ and $(2)^{mn}$, the uncertainty originates in the product market, or their right-hand-side, uncertainty originates in the inputs markets. Uncertainty that affects the left-hand-side implies a lower perceived VMP for each input mix than under certainty. Uncertainty that affects the right-hand-side implies a higher perceived marginal factor cost at each input mix than under certainty. This interpretation for the effect of uncertainty implies that regardless of the source of uncertainty the aa curve moves to the left compared to that under certainty while the bb curve moves downward. The implications for the levels of employment of the inputs are as follows:

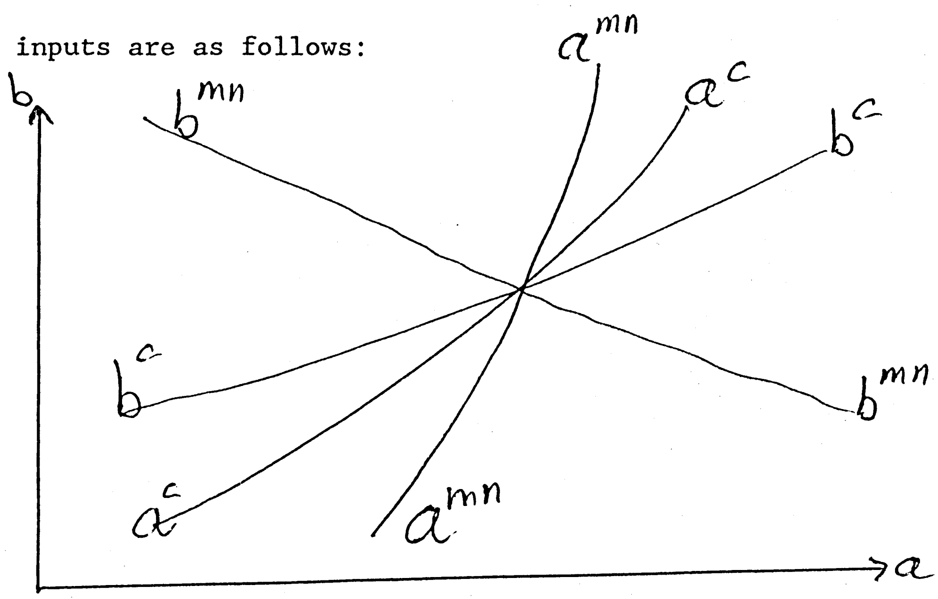

Figure 5b

When bb slopes upward the effect of uncertainty is as for a competitive firm, i.e. uncertainty lowers the employment of both inputs. When bb slopes downward the effect of uncertainty on the quantities of a and b is only partly determined. The quantity of a always declines while that of b might decrease or might increase or stay the same.

Conclusions

The graphical presentation of the first-order conditions for the employment of inputs provides a very convenient way to demonstrate the effects of uncertainty either in the product market or the inputs market on the quantity demanded of the inputs and the level of output. We provided the analysis for the three extreme market structures, i.e. perfect competition, monopoly and monopsony. A first glance on the role of uncertainty in a duopoly market is presented in Fishelson (1988). The next step to take is to merge the graphical methodology of this study with Fishelson (1988).

FOOTNOTES

1. A rather detailed list of studies on firms' behaviour under uncertainty is provided in Appendix 1.

2 The linearity restriction is made for the sake of convenience of analysis. This is illustrated in the example below which also saves similar analyses in the text below.

Maximization of expected utility from profits by a competitive firm under uncertainty in the product market implies

$$\text{Max } E\{U(P_x f(a,b) - P_a \cdot a - P_b \cdot b)\}.$$

The first-order conditions where $P_x = P_x^o + \epsilon$ $(E\epsilon = 0)$ are

$$VMP_a^o + \frac{f_a Cov(U', \epsilon)}{EU'} = P_a$$

$$VMP_b^o + \frac{f_b Cov(U', \epsilon)}{EU'} = P_b.$$

The first-order conditions where $P_x = P_x^o \cdot \epsilon(E\epsilon = 1)$, are:

$$VMP_A^0(1 + \frac{Cov(u', \epsilon)}{Eu'}) = P_a$$

$$VMP_b^0(1 + \frac{Cov(u', \epsilon)}{Eu'}) = P_b$$

Hence, the nonlinearity of the random variable also affects the VMP multiplicatively although in principle it does not make a qualitative change in the analysis.

APPENDIX I

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