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IMPERFECT COMPETITION UNDER UNCERTAINTY

by

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Abstract

The behavior of firms in the Cournot-Nash model is examined under uncertainty in market demand and in their production. The emerging result is that if one of the firms is strongly more risk averse than the other its output will increase but total output declines (even when the other is risk neutral). In the Stackelberg model however if the follower is risk neutral he might compensate for the decline of output by the leader.

One way to classify studies that examined the behavior of firms under various sources of uncertainty is according to the market structure they assumed. As of now we find only two market structures: a competitive market (Sandmo (1971), Leland (1972), Turnovsky (1969) and (1973), Batra and Ullah (1974), Hartman (1975) and (1976), Rati and Ullah (1976), Rati (1978), Pope and Just (1978), Stewart (1978), Epstein (1979), Holthausen (1979), Perrakis (1980), Feder et. al. (1980), and Fishelson (1984)) and a monopoly (Blair (1974), Holthausen (1976), Das (1980), and Fishelson (1986)).

In the present study an intermediate structure is examined, the oligopoly, and in particular a duopoly. We first analyze the Cournot-Nash model under uncertainty and compare the results with those under certainty for risk neutral and risk averse firms (risk loving is a priori excluded). We then continue with the Stackelberg model. We conclude with comments on the case of a leading firm and many small competitive firms. For the sake of simplicity all models are as linear as possible.

I. The Cournot-Nash Duopoly Model

The behavior underlying this model is that each of the two firms is a follower in the sense that each firm regards its competitor's supply as fixed and then it determines its own output that maximizes its profits. This model for the certainty case is common knowledge and is presented in textbooks of intermediate microeconomics (e.g., Henderson and Quandt (1971), Ch. 6). The maximization of profits by each firm yields its reaction function, $q_i = f(q_j)$. The solution of the two reaction functions for the two unknowns q_i^* and q_j^* yields the optimal outputs, the total market output, $q_i^* + q_j^*$, and the market price, P, (following market demand). The simplest case that yields an explicit solution is that in which market demand is linear in total output and the marginal cost of each firm is linear in its output (total costs are quadratic). In order to set the grounds we thus assume the overall linearity to hold. Also given uncertainty the maximization is obviously that of utility, which is a function of profits. Correspondingly:

Market demand: $P = a + b(q_1 + q_2)$

Firm 1 costs: $c(1) = \alpha_1 q_1 + \frac{\beta_1}{2} q_1^2$

Firm 2 costs: $c(2) = \alpha_2 q_2 + \frac{\beta_2}{2} q_2^2$

Firms 1 utility: $U(1) = U_1(\pi(1)) = U_1(q_1 P - c(1))$

Firms 2 utility: $U(2) = U_2(\pi(2)) = U_2(q_2 P - c(2))$.

The Cournot-Nash first order condition for profit maximization by firm 1 is

1) $E\{U_1'(\cdot)[a + 2bq_1 + bq_2 - \alpha_1 - \beta_1 q_1]\} = 0$.

The same condition applies to firm 2:

2) $E\{(U_2'(\cdot)[a + 2bq_2 + bq_1 - \alpha_2 - \beta_2 q_2]\} = 0$.

i) <u>Uncertainty of Market Demand</u>

Uncertainty of market demand is introduced by first assuming its shift parameter, a, to be random, i.e. $a = a^0 + \epsilon$ where $\epsilon \sim \text{symmetric}(0, \sigma_\epsilon^2)$. Hence

$E(a) = a^0$, $Var(a) = \sigma_\epsilon^2$. Each firm has to decide on output before the market is realized. The reaction function of firm 1 becomes

$$3)\quad q_1 = \frac{1}{\beta_1 - 2b}\left(a^0 + bq_2 - \alpha_1 + \frac{Cov(U_1'(\cdot),a)}{EU_1'(\cdot)}\right) .$$

Similarly for firm 2,

$$4)\quad q_2 = \frac{1}{\beta_2 - 2b}\left(a^0 + bq_1 - \alpha_2 + \frac{Cov(U_2'(\cdot),a)}{EU_1'(\cdot)}\right) .$$

As expected (profits are linear in the random variable) a risk neutral firm is not changing its reaction function due to uncertainty since $Cov(U'(\cdot),a) = 0$.

A risk averse firm, for any level of output of the other firm, its reaction function is lower since $Cov(U_i',a) < 0$. Graphically the risk neutral and risk averse cases look as in Figure 1.

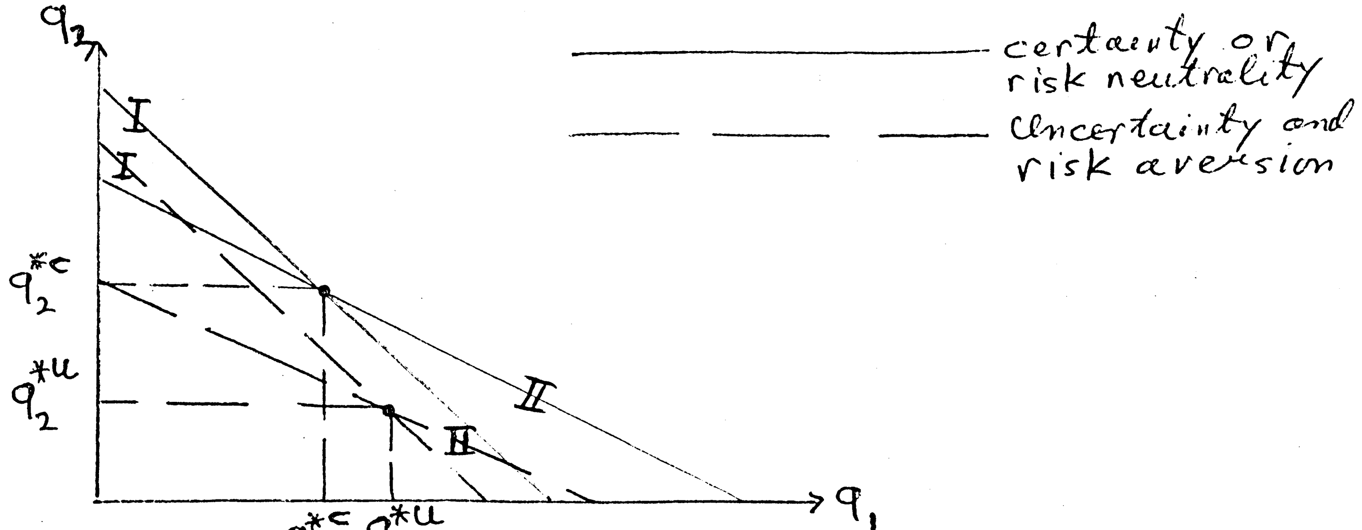

Figure 1

Note that while each reaction function depends only upon the self risk aversion the optimal output for each firm depends upon both its risk aversion and the other firm's risk aversion. Both outputs might be smaller or one smaller and the other larger than under certainty. For the latter all that is needed is a large difference between the degrees of risk aversion of the firms. In any case it can be shown (A-aversion, N-neutrality) that $q_1^A + q_2^A < q_1^N + q_2^N$. Hence, while for a monopoly and a competitive firm, risk aversion always (given linearity of profits in the random variable) lowers output, for each of the duopoly-Cournot-Nash firms this is not a necessary result, but it is for the market (for details see Appendix I). One of the interesting results that holds also under uncertainty is $|dq_i/dq_j| < 1$. We show it for linear models however it holds for all models with nondownward sloping MCs and the elasticity of demand is continuously declining along the demand.

ii) <u>Uncertainty in Costs of Production</u>

For simplicity we again assume that the uncertainty affects the shift parameter α, i.e. $\alpha_i = \alpha^0 + \epsilon_i$, $\alpha_i \sim \text{symmetric}(\alpha^0, \sigma^2\epsilon_i)$. The firms have to make production decisions before the costs of production are realized (order nonstorable inputs). The change in the analysis compared to the one above is that now the Cov(·) is between $U_i'(\cdot)$ and α_i. Risk neutrality obviously again implies (recall linearity of profits in the random variable) that $\text{Cov}(U_i'(\cdot), \alpha_i) = 0$. Risk aversion implies that $\text{Cov}(U_i'(\cdot), \alpha_i) > 0$ (we assume $\alpha_i > 0$ even at extreme negative values of ϵ_i). The analysis presented above for product market uncertainty is repeated as do the results, except that now since the uncertainty is firm specific (before common to both) the possibility for the output of one firm decreasing while the others increasing becomes more prevalent. Also note that the likelihood of Var $\epsilon_1 \neq$ Var ϵ_2 which also calls for different effects of the uncertainty. The effect are a compounded result of the variance of ϵ_i and the degree of risk aversion (the issue of increasing risk, i.e. the variance of the random variable is dealt with later).

iii) Extension

The uncertainty in the two cases dealt with above was attached to the "constants" in the first order conditions. It might as well be attached to the slope parameters b, β_1, β_2. Again the uncertainty of b is common to both firms while that of β_i is firm specific. The difference between the previous cases and this one is that in the present also the slopes of the raction functions change. This a priori might result in unstable equilibrium solutions (slope reversals) which is unique to duopoly. If b is uncertain $(b = b^0 + \epsilon)$ the reaction functions are

$$5) \quad q_1 = [a - \alpha_1 + q_2 b^0 + q_2 \frac{\mathrm{Cov}(U_1', b)}{EU_1'}]/(\beta_1 - 2b^0 - \frac{2\ \mathrm{Cov}(\ \ )}{EU_i'})$$

$$6) \quad q_2 = [a - \alpha_2 + q_1 b^0 + q_1 \frac{\mathrm{Cov}(U_2', b)}{EU_2'}]/(\beta_2 - 2b^0 - \frac{2\ \mathrm{Cov}(\ \ )}{EU_2'})$$

Given risk aversion, $\mathrm{Cov}(U',b) < 0$. The $\mathrm{Cov}(\cdot)$ term appears both in the numerator, with a positive sign, and in the denominator with a negative sign. Hence the numerator and denominator are affected in opposite directions. Since for a risk averse firm the $\mathrm{Cov}(\cdot)$ is negative there is a double negative effect on the reaction curve. The relation dq_1/dq_2 which is negative, gets smaller in absolute terms and dq_2/dq_1 also gets smaller. The nature of the previous outcome reemerges. Either both q_1^* and q_2^* decline or one increases while the other declines but $q_1^* + q_2^*$ obviously declines (Figure 2).

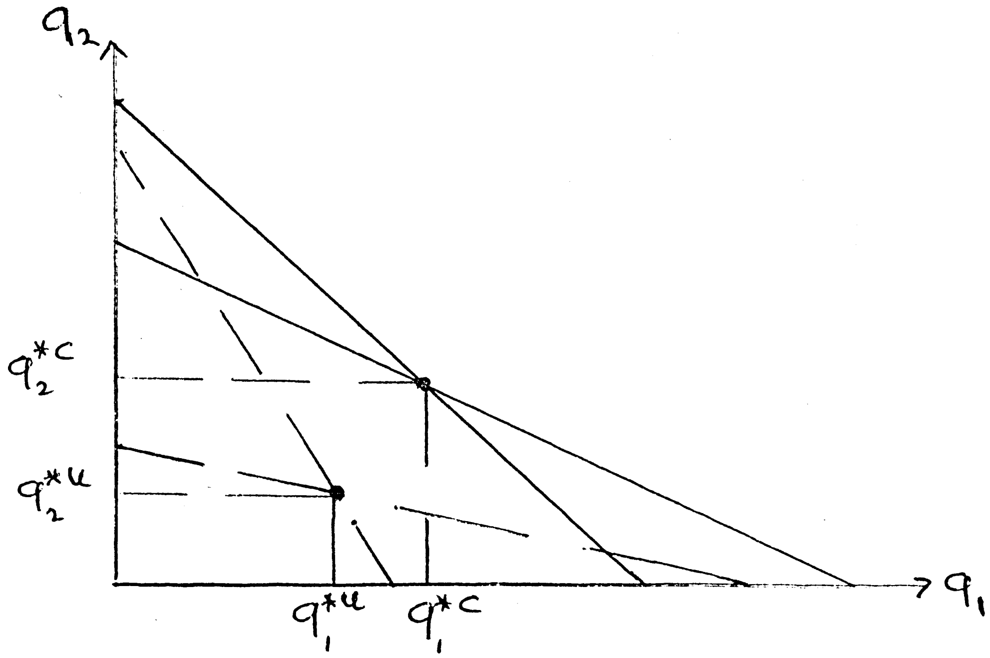

Figure 2

The same result is obtained when β_1 or β_2 or both are random except that again it is sufficient for one firm to be risk neutral or for its β to be nonrandom to pick up some of the decline in output of the other firm but it would never make up for all of the decline. We also note that the driving force of the uncertainty is the nonzero sign of the term Cov(U',v) where v is the random variable itself or a function of it. Thus even if the market equations are nonlinear or the effect of the random variable is nonlinear (given that its behavior is correctly specified, i.e. when it equals its expected value the market solution equals that of a certain world) risk aversion would lower output compared to risk neutrality. There are however market equations that would lead to a change in behavior also of a risk neutral firm due to uncertainty, i.e. when profits are nonlinear in the random variable ($E\pi \neq \pi/\epsilon = E\epsilon$). But also in these cases the distinct behavior of a risk averse firm will stay as described above.

II. Duopoly-Collusion

The known outcome for the certainty world analysis is that a collusion between the firms yields a solution that is identical to that of a multiplant monopoly, i.e. profits are maximized at the level of output where $\sum MC = MR$ and for this output costs are minimized when $MC_1 = MC_2$. Thus at equilibrium $MR = MC_1 = MC_2$. This result obviously does not change under uncertainty when the newly formed monopoly is risk neutral and profits are linear in the random variable. The outcome is however a priori undetermined when the cartel is not risk neutral. The first question to be asked is with regard to the attitude of the newly formed organization towards risk. How is the new attitude formed given that previous to the collusion the firms were risk averse. It is conceivable that the risk aversion would be some weighted sum of the degrees of risk aversion where the weights are the shares in profits or output or any other criteria, or that the extreme risk aversion dominates, or that due to collusion risk aversion is eliminated.

The theory of the formation of utility functions due to mergers or collusion is practically nonexistent in economics. Thus, we assume it to be with some degree of risk aversion (e.g., risk pooling). If the uncertainty is in market demand then the distribution of production among the partners would be as under certainty but total output will decline,

$$7)\quad C'(q_1) = C'(q_2) = EMR + \frac{Cov(U',\epsilon)}{EU'}$$

i.e. efficiency in production is maintained. If however the uncertainty is in the costs of production and is partner specific then

8) $$C'(q_1) - \frac{\text{Cov}(U',\epsilon_1)}{EU'} = C'(q_2) - \frac{\text{Cov}(U',\epsilon_2)}{EU'} = MR$$

Thus the "full" marginal costs of production equal across partners. The marginal costs in production will equal only if $\text{Cov}(U',\epsilon_1) = \text{Cov}(U',\epsilon_2)$ implying that the random effect on plant 1 is identical to the random effect on plant 2. Again total market output and the output of each partner will be less than under certainty.

III. The Stackelberg Model

The model is a duopoly in which one firm (the leader) takes the reaction function of the other firm (the follower) as given. Thus, the leader incorporates this information in his profit maximization objective function. The leader solves for the optimal output to produce. Hence the Stackelberg model is a quantity setting firm model (e.g. Holthausen (1976)). In an uncertain world the sources of uncertainty might be the market, the reaction function of the follower, and the costs of production of the leader. Uncertainty in market demand, with deterministic behavior of the follower calls for a change in optimal quantity compared to certainty. Similarly when the follower behavior or the leaders production costs are uncertain. In all the cases where the leader is risk averse the determined optimal output would be smaller than under certainty. The follower's output would then be larger since his output is inversely related to that of the leader. Hence whether total output declines depends entirely upon the reaction function of the follower (which is already the result of his subjective profit maximization given his utility function, cost function and his view of market demand).

In the Cournot-Nash model (both firms behave as followers) we noticed that a necessary condition for an internal stable solution is that $\left|\frac{dq_i}{dq_j}\right| < 1$. In the Stackelberg model this condition for the follower is not a necessary condition (see however below for the market structure of a leading firm and many small firms). Thus one might find that if the leader is risk averse and the follower is not, uncertainty might yield a larger total output. To show this we employ Figure 3 and save the formal analysis. We however argue that the case of $\frac{dq(\text{follower})}{dq(\text{leader})} > 1$ is economically nonconceivable since if the leader knows it he will behave accordingly (see Appendix 2).

With regard to efficiency in production when the uncertainty originates in market demand or in the reaction function of the follower the deterministic rule for cost minimization continues to hold. Hence, the leader is efficient in production.

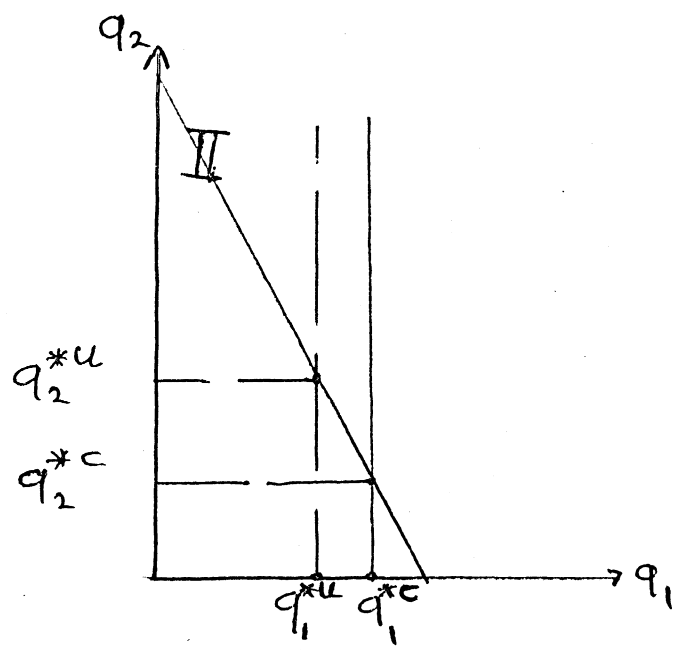

Figure 3

IV. The Market Leading Firm Case (One large firm and many small ones)

The small firms follow market price, i.e. $\sum s = q(P)$. They however act before the market is realized but with full information on the behavior of the leader. The leader knowing their behavior subtracts their supply from market demand. Hence, the "net" demand he faces is $D(P) - \sum s$ where both D and $\sum s$ are functions of P, $NQ = D(P) - q(P)$. The leader maximizes profits given the net demand, i.e. he equates the MR of NQ with his MC to find the quantity to produce and then announces the price according to NQ. The small firms respond to this price by following their $\sum s$. In a certain world this price is consistent with market demand and the total supply. Hence this leading firm model fits the price setting firm model (see again Holthausen (1976)). The uncertainty with regard to the large firm is due to either of the following: market behavior, the behavior of the small firms, and to his production and cost functions. The nature of the effects of the first two uncertainties is the same, they result in uncertainty of the net demand NQ. It can be shown that a risk averse firm that is price setting will set a price above the one a risk neutral firm would set (see Appendix 3).

Uncertainty in the production and thus cost functions would also have the conventional effect, i.e. a higher price than under certainty. Thus the behavior of this market structure under uncertainty is the commonly expected one. Yet for completion one should note that uncertainty in the market might lower also the supply of the small firms if they are risk averse, i.e. the net demand the large firm faces might increase. However, if the large firm is also risk averse it also reacts to market uncertainty and its "net and full" demand and the corresponding marginal revenue might be below or above the demand and the corresponding marginal revenue that emerge when the market is certain. It can however be shown that if the small firms are risk averse and the large one risk neutral the leader will never make up for the decline in supply of the small firms. Hence, the effect of uncertainty on market equilibrium quantity is a priori determined (see Figure 4). Furthermore, if the large firm is risk averse and the small firms are risk neutral they can not compensate the market for the risk aversion behavior of the large firm. Hence, total supply would again be smaller than under certainty.

With regard to efficiency in production one should note that a price setting firm does not follow the deterministic rule of cost minimization. It was shown (Holthausen, eq. (18)) that if capital is the ex ante determined input and labor the ex post determined input and the source of uncertainty is in the market's demand (we extend it to also include the supply of the small firms) then the expected marginal rate of technical substitution by a risk averse firm diverts from that of cost minimization ($E\{MP_K/MP_L\} \neq i/w$) under certainty.

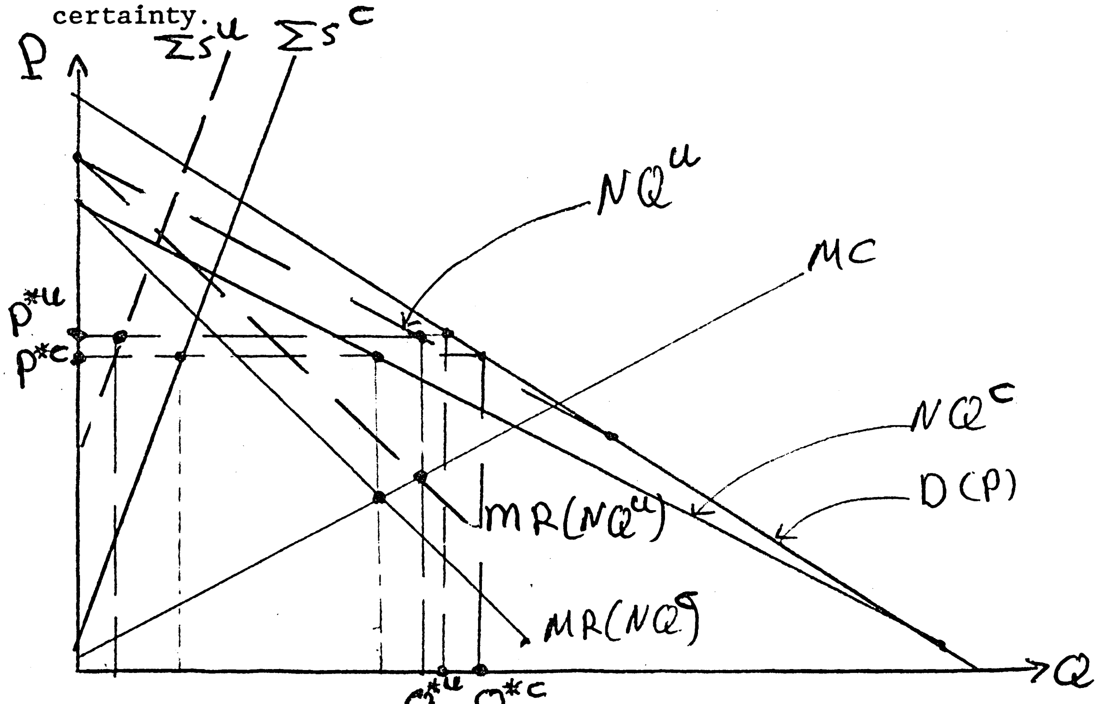

Figure 4

V. Increasing Risk

We did not discuss the efffect of the increase of risk on the behavior of the risk averse firm. From section I and II it is obvious that the effect depends upon the reaction of the term Cov(U',v)/EU' to changes in risk where v is the random variable. A convenient and traditional view of risk is the variance of the random variable. In addition to the assumptions that profits are linear in the random variable and the utility concave w.r.t. profits one has also to assume with regard to the third derivative of utility w.r.t. profits (the second is obviously negative). A positive third derivative leads EU' to increase with a mean preserving spread while a negative third derivative to a decline of EU' in response to a mean preserving spread.

This is still insufficient for a final answer since Cov(U',v) might and is very likely to change with a mean preserving spread. Investigating the behavior of Cov(U',v) yields

9) $\mathrm{Cov}(U',v) = E\{(U' - EU')(v - EV)\}$.

Let Ev = 0. Then,

10) $\mathrm{Cov}(U',v) = E\{U' \cdot v\}$

Since $U = U(\pi)$, $\pi = f(v)$ and we assume

11) $U' > 0,\ f' > 0,\ U'' < 0,\ f'' \lesseqgtr 0,\ U''' \lesseqgtr 0.$

Then

12) $$\frac{d(U' \cdot v)}{dv} = v \cdot U'' \cdot f' + U' , \quad \text{(sign indetermined)}$$

and

13) $$\frac{d^2(U' \cdot v)}{dv^2} = U''f' + vU'''f' + vU''f'' + U''$$

In 13) the sign is determined only for $U''' < 0$ and $f'' > 0$. On the other hand if $f' < 0$ the sign of 12) is positive but the sign of 13) is indetermined regardless of the assumptions on the signs of f'' and U'''. Thus, in spite of previous conclusions regarding consumers and firms behavior as risk changes we argue that a priori it is indetermined.

Summary

The results of this study indicate that the market structure of imperfect competition is not homogeneous with regard to the effects of introducing uncertainty. It turns out that depending on the behavior and cost parameters the follower might compensate the market in terms of quantity produced, for the decline in output by the leading firm. On the other hand all other results due to uncertainty are the conventional ones, i.e. regardless of the source of uncertainty risk aversion always results in a lower output by the risk averse firm, and by the market.

Appendix 1 – Cournot–Nash

Let:

$$\mathrm{Cov}(U_1'(\cdot),a)/EU_1' = A_1$$

$$\mathrm{Cov}(U_2'(\cdot),a)/EU_2' = A_2$$

Then,

$$q_1 = \frac{1}{\beta_1 - 2b}(a^0 + bq_2 - \alpha_1 + A_1) = D_1 + B_1 q_2$$

$$q_2 = \frac{1}{\beta_2 - 2b}(a^0 + bq_1 - \alpha_2 + A_2) = D_2 + B_2 q_1$$

and

$$q_1^* = (D_1 + B_1 D_2)/(1 - B_1 B_2)$$

$$q_2^* = (D_2 + B_2 D_1)/(1 - B_1 B_2)$$

$$q_1^* + q_2^* = (D_1 + D_2 + B_2 D_1 + B_1 D_2)/(1 - B_1 B_2)$$

Since $B_1 < 0$ and $B_2 < 0$ then $B_1 B_2 > 0$. However note that $|B_1| < 1$, $|B_2| < 1$ thus $B_1 B_2 < 1$. Furthermore the stability of the internal solution requires that $|\frac{1}{B_1}| < |B_2|$ and vice versa. Hence, $q_1^{*A} + q_2^{*A} < q_1^{*N} + q_2^{*N}$ where A denotes risk aversion and N risk neutral.

Appendix 2 - Stackelberg

The followers reaction function is

$$q_2 = \gamma + \delta q_1$$

The leader's objective function is thus

$$\text{Max } q_1[a + b(q_1 + \gamma + \delta q_1)] - \alpha q_1 - \frac{\beta}{2} q_1^2 .$$

The solution of the leader's first order condition is

$$q_1^* = \frac{a + b\gamma - \alpha}{\beta - 2b - 2b\gamma}$$

Since $b < 0$, $\delta < 0$, if $|\delta| > 1$ the denominator might be negative.

Appendix 3 - One Large Firm and Many Small Ones

In a perfectly linear world where net demand faced by the large firm is

$$NQ = \gamma + \delta P + v \qquad \gamma > 0 , \quad \delta > 0 , \quad v \sim (0,\sigma_v^2)$$

and its costs of production are

$$C(q) = \alpha q + \beta/2q^2$$

its optimal solution for market price is

$$P = \frac{\delta\alpha + \gamma(\delta\beta - 1)}{2\delta - \beta\delta^2} + \frac{Cov(U',v)}{E\{U'\}}$$

where Cov(U',v) < 0 for a risk averse firm.

In a world where profits are nonlinear in the random variable also a risk neutral firm would set a price that differs from that given certainty. Yet the price set by a risk averse firm would always be above that of a risk neutral firm.

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