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TECHNOLOGICAL PROGRESS AND INCOME INEQUALITY

by

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Working Paper No.23-88

June, 1 9 8 8

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Financial assistance from the Foerder Institute of Economic Research is gratefully acknowledged

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1. Introduction

Technological changes alter the distribution of incomes over time directly, through their effect on productivity, and indirectly, by affecting the rate of accumulation of factors of production. In this paper we study the relationship between changes in the technology of production and the resulting variations in income inequality. As far as we know, this is the first attempt to analyze this problem in a general equilibrium framework. For lack of established frame of reference for this investigation, special care must be taken in stating the scope of our effort. First, we do not attempt to explain the historical sources of income inequality. Rather, taking the existence of income inequality as given, we try to evaluate the changes in income inequality that will follow the introduction of new technologies of production. Second, the main source of income inequality considered here is the unequal distribution of intergenerational transfers. This should not be taken to imply that we consider other sources of inequality, such as differences in talent and education or pure luck (see, for example, Loury (1981)) less important. We disregard these factors for analytical convenience only. We believe that a more comprehensive treatment can be built upon the results presented here. We defer further discussion of this point to the last section.

We present our results within the framework of a competitive equilibrium in an overlapping generations economy with **endogenous labor supply** and a **bequest motive**. Each individual in his economy lives for two periods. During the first period he works, consumes, and saves some of his income. Saving is intended in part for bequest and in part it is used to pay for consumption during the second period of the individual's lifetime. At the end of the first period of his life each individual gives birth to a single offspring and at the same time makes the bequest transfer. During the second period he engages solely in consumption. Thus, all the relevant decisions, i.e., the consumption-saving decision, the labor-leisure decision, and the decision concerning the allocation of saving between the second period consumption and bequest, are made in the first period. The bequests are motivated by the "joy of giving" and, in our analysis, constitutes the sole source of heterogeneity among individuals in each generation. We take the historical distribution of bequests to be exogenous. The distribution of incomes in each generation, however, is determined in part by the amount of work supplied by different individuals. The technology of production is characterized by constant returns to scale.

Our analysis involves comparataive dynamic experiments in which a permanent shift of the production function occurs at a given point in time. We trace the resulting changes in the distribution of income during the period in which the new technology is introduced and in every period thereafter. We examine the consequences for income inequality of three types of shifts in the production function known in the growth literature as Hicks-neutral, Harrod-neutral, and Solow-neutral technological changes. We show that: (a) Hicks-neutral technological changes will not affect the distribution of incomes during the period when the changes occur. However, the growth of the capital stock that follows, results in greater equality in the distribution of income in each and every period following the introduction of the technological innovation; (b) Harrod-neutral technological improvements will result in a greater or smaller income inequality depending on the magnitude of the elasticity of substitution in production. More specifically, if the elasticity of substitution is greater or equal to one, a Harrod-neutral technical improvement leads to greater equality in the distribution of incomes during the period when the change occurs and in every period thereafter. If the elasticity of substitution is smaller than one, then the immediate effect of the improvement in the production technology is to increase income inequality, but the long-term effects are ambiguous. We specify sufficient conditions under which the technological improvement leads to a greater income inequality in every period following the introduction of the new technology. In the case of unitary elasticity of substitution the qualitative results are the same as in the case of Hicks-neutral changes, the magnitude of the effect, however, is smaller. (c) Generally speaking, the effects of Solow-neutral technological changes are the opposite of those of Harrod-neutral described above. In particular, if the elasticity of substitution is smaller than one, then a Solow-neutral technological improvement leads to greater equality in the distribution of incomes during the period when the change occurs and in every period thereafter. If the elasticity of substitution is smaller than one, then the direct effect of the improvement in the production technology is to increase income inequality, but the indirect effects are ambiguous. In the case of unitary elasticity of substitution the qualitative results are the same as in the case of Hicks-neutral changes. The quantitative effects, however, are smaller.

In the next section we specify the model. In section 3 we analyze the effects of technological changes on the distribution of incomes. Concluding remarks appear in section 4.

2. The Model

2.1. Preferences and Technology

Consider an overlapping generations economy with no population growth. Each individual in this economy lives for two periods, a working period followed by a retirement period. At the end of the first period every individual gives birth to one offspring. We denote by G_t the set of individuals born at the outset of period t and refer to these individuals as generation t. The economy starts at date 0 where G_{-1} lives only at their retirement period, i.e., $t = 0$, while their only source of income is their savings.

Preferences: The preferences of individuals of generation t are represented by

$$(1) \qquad U = c_{1t}^{\alpha_1}(1 - \ell_t)^{\alpha_2} b_t^{\alpha_3} c_{2t}^{\alpha_4}$$

where c_{it} $i = 1,2,$ denotes the consumption spending of individuals in generation t during the first and second periods of their lives; ℓ_t denotes the labor supply off individuals in generation t (for simplicity of exposition we assume that $0 \leq \ell_t \leq 1$, so that $(1 - \ell_t)$ represents the amount of leisure during the working period of the individual's lifetime); b_t denotes the bequest transfer of an individual of generation t to his offspring, which, in our model, is motivated by the "joy of giving"; $\alpha_i > 0$ for $i = 1,2,3,4$, are parameters. We assume that the preferences are the same for all individuals in all generations. Thus, the only source of heterogeneity in our framework is the difference in the bequest transfers.

Technology: Production in this economy is carried out by competitive firms that produce a single commodity with the use of labor and capital. The commodity serves for consumption and investment. Following Diamond (1965), we assume that the stock of capital in each period, K_t, is determined by the level of saving in the preceding period. The aggregate production function $F(K_t, L_t)$ is assumed to exhibit constant returns to scale, where K_t is the aggregate level of capital and L_t

the aggregate labor input. We also assume that $F_{KK} < 0$, $F_{LL} < 0$ and $F_{KL} > 0$.

2.2. Equilibrium

In each period the economy features three markets; two factor markets, namely, labor and capital, and a commodity market. To define competitive equilibrium we begin by considering the state of the economy at the outset of period t. Denote by Ω the set of families in each generation; it is time-independent since there is no population growth. Although our analysis can be carried out for any finite Ω, to simplify our notations we assume there is a continuum of individuals (or families) in each period, hence we may assume that $\Omega = [0,1]$ with some density function μ on the Borel sets in [0,1]. Since we assume no population growth μ is time independent. In each period t there are two members of each family $\omega \in \Omega$, the "old" member belongs to G_{t-1} and the "young" member belongs to G_t. Suppose that the distribution of the bequests received by individuals of generation G_t is given by the function $b_{t-1}: \Omega \rightarrow [0, m]$, where $m < \infty$. Given his inheritance, $b_{t-1}(\omega)$, the wage rate, w_t, and the rates of interest, r_t and r_{t+1}, each individual ω in G_t chooses the level of saving, $s_t(\omega)$, the level of bequest, $b_t(\omega)$ and the level of labor supply, $\ell_t(\omega)$, so as to maximize his utility given in equation (1) subject to

$$(2) \qquad c_{1t} = b_{t-1}(1 + r_t) + w_t\ell_t - s_t - b_t,$$

and

(3) $$c_{2t} = s_t(1 + r_{t+1}).$$

Note that in period $t = 0$ individual ω in G_{-1} consumes $c_{-1}(\omega) = (1+r_0)s_{-1}(\omega)$.

Definition 1: Given K_0, s_{-1} and b_{-1}, a **competitive equilibrium** is a sequence of functions, $\{c_{1t}, c_{2t}, \ell_t, s_t, b_t\}_{t=0}^{\infty}$, and a sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$, such that for all t, $t = 0,1,2,\ldots$

(a) $(c_{1t}, c_{2t}, \ell_t, s_t, b_t)$ is the solution to the maximization problem (1)-(3) for almost all ω.

(b) $$\int \ell_t(b_{t-1}(\omega)(1+r_t), w_t, r_{t+1})d\omega = F_L^{-1}(K_t, w_t),$$

(c) $$K_t = F_K^{-1}(L_t, r_t)$$

(d) $$K_{t+1} = \int [b_t(b_{t-1}(\omega)(1+r_t), w_t, r_{t+1}) + s_t(b_{t-1}(\omega)(1+r_t), w_t, r_{t+1})] d\mu$$

Condition (a) asserts that the various demand functions in the economy are derived from optimal consumer behavior assuming that all consumers are price takers. Conditions (b) and (c), are the equilibrium conditions in the labor and capital markets, respectively. The specification of the demand functions is based on the assumption that firms are price takers in the factor markets. Condition (d) describes the dynamic adjustment of the aggregate capital stock in the economy assuming full depreciation of the capital stock in each period. These conditions, in conjunction with the constraints (2) and (3) imply the material balance condition:

$$(4) \qquad \int c_{1t}(\omega)d\mu + \int c_{2(t-1)}(\omega)d\mu + K_{t+1} = F(K_t, L_t), \text{ for } t = 0,1,\ldots$$

The existence of a competitive equilibrium in this economy can be shown using standard methods. We do not prove it here.

2.3. Demand Functions and Income

Solving the maximization problem (1)-(3) we get, for each t,

$$\text{(a)} \quad c^*_{1t} = \alpha_1 \frac{u^*}{\lambda}$$

$$\text{(b)} \quad c^*_{2t} = \alpha_4(1 + r_{t+1})\frac{u^*}{\lambda}$$

$$\text{(5)} \quad \text{(c)} \quad b^*_t = \alpha_3 \frac{u^*}{\lambda}$$

$$\text{(d)} \quad 1-\ell^*_t = \frac{\alpha_2}{w_t} \frac{u^*}{\lambda}$$

where u^* denotes the optimal value of U in equation (1) and the asterisks denote the optimal values of the variables. Note that, since $c^*_{2t} = (1 + r_{t+1})s^*_t$, it follows that $s^*_t = \alpha_4 \frac{u^*}{\lambda}$.

Using the optimality conditions (5) and the constraints (2) and (3) we obtain the reduced form solution of c^*_{1t}, s^*_t, b^*_t and $1-\ell^*_t$; namely,

$$(6)\qquad c_{1t}^{*} = \alpha_1(1 + r_t)[\frac{w_t}{1+r_t} + b_{t-1}]$$

$$(7)\qquad s_t^{*} = \alpha_4(1 + r_t)[\frac{w_t}{1+r_t} + b_{t-1}]$$

$$(8)\qquad b_t^{*} = \alpha_3(1 + r_t)[\frac{w_t}{1+r_t} + b_{t-1}]$$

$$(9)\qquad 1 - \ell_t^{*} = \alpha_2[1 + \frac{1+r_t}{w_t} b_{t-1}].$$

The income, $y_t(\omega)$ of individual ω in G_t in period t is defined by:

$$(10)\qquad y_t(\omega) = w_t\ell_t(\omega) + (1+r_t)b_{t-1}(\omega),$$

or, in reduced form,

$$(11)\qquad y_t(\omega) = (1 - \alpha_2)(1 + r_t)[\frac{w_t}{1+r_t} + b_{t-1}(\omega)].$$

The aggregate level of income in period t is given by

$$(12)\qquad Y_t = \int y_t(\omega)d\mu = (1 - \alpha_2)(1 + r_t)[\frac{w_t}{1+r_t} + B_{t-1}]$$

where $B_{t-1} = \int b_{t-1}(\omega)d\mu$.

Finally, from (9), the aggregate supply of labor in period t, $L_t = \int \ell_t(\omega) d\mu$, is given by:

$$L_t = 1 - \alpha_2 - \alpha_2 \frac{1+r_t}{w_t} B_{t-1}. \tag{13}$$

3. The Distributional Effects of Technological Innovations

3.1. *The Measurement of Income Inequality*

A formal analysis of the distributional effects of technological changes requires a formal measure of income inequality. To define such a measure we need the following notation. Let X and Z be two random variables with values in a bounded interval in $\mathbb{R}$, and let m_x and m_z denote their respective means. Define $\hat{X} = X/m_x$ and $\hat{Z} = Z/m_z$ and denote by F_x and F_z the cumulative distribution functions of $\hat{X}$ and $\hat{Z}$, respectively. Let $[a,b]$ be the smallest interval containing the supports of $\hat{X}$ and $\hat{Z}$.

Definition 2: F_x is more equal than F_z if:

$$\int_a^t [F_x(s) - F_z(s)] ds \leq 0 \qquad \text{for all} \quad t \in [a,b].$$

This definition, due to Atkinson (1970), is equivalent to the requirement that the Lorentz curve corresponding to X is everywhere above that of Z. Thus, if F_x is more equal than F_z according to definition 2, then it has a lower Gini index. We say that X is more equal than Z if the c.d.f. of $\hat{X}$ and $\hat{Z}$ satisfy: F_x is more equal than F_z.

Applying definition 2 to income inequality in the model of section 2 we observe that, by equations (11) and (12), and using the above notation,

$$(14) \qquad \hat{y}_t(\omega) = \frac{w_t/(1+r_t) + b_{t-1}(\omega)}{w_t/(1+r_t) + B_{t-1}}.$$

Consequently, given the distribution of bequests $b_{t-1}(\cdot)$ an increase in $w_t/(1+r_t)$ leads to a greater equality in the distribution of income in period t. Thus, the immediate distributional effects of technological changes depend on the effects of these changes on the relative factor prices. In the long-run the relative factor prices depend also on the changes in the capital-labor ratio induced by the new technologies and on the effects of the changing technology on the intergenerational transfers.

3.2. *Technological Changes - Definitions*

To examine the effects of improved technology on income inequality we conduct the following comparative dynamics analysis. We take the distribution of incomes at the time when the technological innovation is introduced as given. We also assume that the new technology is unanticipated. We shall consider three kinds of exogenous changes in period t=0 representing permanent shifts in the production technology.

These shifts are expressed formally as follows. Let $F(\gamma_{1t} K_t, \gamma_{2t} L_t)$ be the production function in period t. Then, a **Hicks-neutral** technological improvement is characterized by: $\gamma_{1t} = \gamma_{2t} = \gamma_t$, $\gamma_t = 1$ for $t < 0$, $\gamma_t = \gamma > 1$ for $t \geq 0$. A **Harrod-neutral** technological improvement is characterized by $\gamma_{1t} = 1$ for all t, $\gamma_{2t} = 1$ for $t < 0$ and $\gamma_{2t} = \gamma > 1$ for $t \geq 0$. A **Solow-neutral** technological improvement is characterized by $\gamma_{2t} = 1$ for all t, $\gamma_{1t} = 1$ for $t < 0$ and $\gamma_{1t} = \gamma > 1$ for $t \geq 0$. We shall consider each of these in turn.

3.3. *The Effects of Hicks-Neutral Technological Changes*

Let $Q_t = F(\gamma K_t, \gamma L_t)$. Then, competitive equilibrium implies

$$\text{(15)} \qquad X_t \equiv \frac{1+r_t}{w_t} = \frac{\partial Q_t/\partial K_t}{\partial Q_t/\partial L_t}.$$

Thus, in the case of Hicks-neutral technological change,

$$\text{(16)} \qquad X_t = \frac{\gamma F_K(K_t, L_t)}{\gamma F_L(K_t, L_t)} = \frac{F_K(K_t, 1 - \alpha_2 - \alpha_2 B_{t-1} X_t)}{F_L(K_t, 1-\alpha_2 - \alpha_2 B_{t-1} X_t)},$$

where the second equality follows from equations (13) and (15). Consequently, $\partial X_t/\partial\gamma = 0$. Hence, by equation (14) a Hicks-neutral technological change does not affect the distribution of income during the period in which the change occurs.

To trace the effect of the technological change in subsequent periods we observe that equations (7) and (8), the definition of K_1 and the fact that $(1 + r_t) = \gamma F_K(K_t, L_t)$ imply that $K_1' = \gamma K_1$, where the prime superscript indicates the value of the variable following the introduction of the technological innovation. Furthermore, for all $t \geq 1$ the effect of an increase in K_t on X_t may be inferred from equation (16), i.e., since $F_{KL} > 0$

$$(17) \qquad A \frac{dX_t}{dK_t} = \frac{F_L F_{KK} - F_K F_{KL}}{F_L^2} < 0 \qquad \text{for all } t,$$

where $A = 1 + \frac{F_{KL}F_L - F_{LL}F_K}{F_L^2} \alpha_2 B_{t-1} > 0$. In addition, by the above argument, $B_0' = \gamma B_0$, and, for all t,

$$(18) \qquad \frac{\partial X_t}{\partial B_{t-1}} = \frac{F_K F_{LL} - F_L F_{KL}}{F_L^2} \alpha_2 X_t < 0.$$

Hence, $K_1' > K_1$ in conjunction with equation (17) implies that as a consequence of the improved technology $X_1' < X_1$. By the argument following equation (14), this, in itself, implies a greater equality in the distribution of incomes in period $t=1$. Finally, the bequests received by members of generation G_1, $b_0^*(\omega)$ is such that $\frac{b_0^*(\omega)}{B_0^*}$, when viewed as a random variable, is the same as $\hat{y}_0(\omega)$, and, as in the case of $\hat{y}_0(\omega)$ its distribution is unaffected by this technological change. Therefore, due to the accumulation of capital, a **Hicks-neutral** technological progress leads to greater equality in the distribution of income in the first period following the introduction of the new technology.

To show that this conclusion applies to every subsequent period we proceed by induction. Assume that as a result of the new technology introduced in period $t = 0$ there is a greater equality in the distribution of incomes in period $t = \tau - 1 \geq 1$, and that $K_\tau' > K_\tau$. Then, by equation (17), $X_\tau' < X_\tau$ which contributes to greater equality in the distribution of incomes in period τ. Moreover, by equation (8), $\hat{b}_t(\omega) = \frac{b_t^*(\omega)}{B_t^*}$ is the same random variable as $\hat{y}_t(\omega)$ for all $t \geq 1$. Thus, by the induction assumption, the bequest transfers to generation G_τ, $b_{\tau-1}'(\omega)$, are more equally distributed following the introduction of the technological innovation in period $t=0$. Consequently, by equation (14), the income distribution in period τ, $\hat{y}_\tau'(\omega)$, is more equal than $\hat{y}_\tau(\omega)$. Finally, we show that $K_{\tau+1}' > K_{\tau+1}$. To see this we note that $K_\tau' > K_\tau$ implies (see equations (7)-(8)) that $b_{\tau-1}'(\omega) > b_{\tau-1}(\omega)$ for almost all ω in G_τ. Since $X_\tau' < X_\tau$ it follows from equations (7) and (8) that if $K_{\tau+1}' \leq K_{\tau+1}$ it must be because $(1+r_\tau') \leq (1 + r_\tau)$. We now show that this implies $L_\tau \le L_\tau'$. Note that $(1+r_\tau') \le (1+r_\tau)$ implies $\frac{K_\tau'}{L_\tau'} > \frac{K_\tau}{L_\tau}$ and, hence, $w_\tau' > w_\tau$. We now write equations (7) and (8) as follows:

$$(7') \qquad s_\tau^* = \alpha_4 w_\tau \left[1 + X_t b_{\tau-1}^*(\omega)\right]$$

and

$$(8') \qquad b_\tau^* = \alpha_3 w_\tau \left[1 + X_\tau b_{\tau-1}^*(\omega)\right].$$

Integrating equations (7') and (8') with respect to μ, and using the definition of $K_{\tau+1}$ we get:

$$(\alpha_4+\lambda_3)w_\tau\left[1 + X_\tau B_{\tau-1}^*\right] \ge (\alpha_4 + \alpha_3)w_\tau'\left[1 + X_\tau' B_{\tau-1}'\right].$$

But $w_\tau' > w_\tau$, hence $X_\tau' B_{\tau-1}' \le X_\tau B_{\tau-1}$. By equation (13) this implies $L_\tau \le L_\tau'$.

From equation (6) and the definition of $c_{2(\tau-1)}$ it follows that the left-hand-side of equation (4) when evaluated using the variables $c_{1\tau}'$, $c_{2(\tau-1)}'$ and $K_{\tau+1}'$ is reduced relative to the left-hand-side of equation (4) with $c_{1\tau}$, $c_{2(\tau-1)}$ and $K_{\tau+1}$. But $K_\tau' > K_\tau$ and $L_\tau' \ge L_\tau$ imply that $\gamma F(K_\tau', L_\tau') > F(K_\tau, L_\tau)$. Consequently, the right-hand-side of equation (4) is larger when evaluated after the technological change.

Thus, the material balance condition does hot hold. A contradiction and, therefore, $K'_{\tau+1} > K_{\tau+1}$. We established the following result:

Proposition 1: *Given the structure of the economy specified in section 2, an unanticipated* ***Hicks-neutral*** *technological improvement in period t=0 will have the following effects:*

(i) In period t=0 the inequality in the distribution of incomes remains unchanged and in every period thereafter there will be greater equality in the distribution of incomes.

(ii) The aggregate supply of capital and labor increases in every period following the introduction of the new technology.

3.4. ***The Effects of Harrod-Neutral Technological Changes***

Let $Q_t = F(K_t, \gamma L_t)$, $\gamma > 1$. By definition,

$$X_t = \frac{F_K(K_t, (1-\alpha_2)\gamma - \alpha_2\gamma B_{t-1} X_t)}{\gamma F_L(K_t, (1-\alpha_2)\gamma - \alpha_2 \gamma B_{t-1} X_t)}. \tag{19}$$

Holding K_t and B_{t-1} constant and differentiating X_t with respect to γ we get

$$\frac{dX_t}{d\gamma} = \frac{F_K F_L}{\gamma^2} \cdot \frac{\gamma L_t [\frac{F_{KL}}{F_K} - \frac{F_{LL}}{F_L}] - 1}{F_L^2 + (F_{KL}F_L - F_K F_{LL})\alpha_2 B_{t-1}}. \tag{20}$$

It is easy to verify that, for all t,

$$(21) \qquad \gamma L_t \left[\frac{F_{KL}(K_t, \gamma L_t)}{F_K(K_t, \gamma L_t)} - \frac{F_{LL}(K_t, \gamma L_t)}{F_L(K_t, \gamma L_t)} \right] = \frac{1}{\sigma_t},$$

where σ_t is the elasticity of substitution in period t. Thus, from equations (20) and (21), (noting that in period t K_t and B_{t-1} are given),

$$(22) \qquad \frac{dX_t}{d\gamma} \gtreqless 0 \iff \sigma_t \lesseqgtr 1.$$

Since K_0 is predetermined in period $t=0$ it follows from equations (14) and (22) that the introduction of a Harrod-neutral technological innovation in period $t=0$ results in a greater (smaller) equality in the distribution of incomes in the same period if $\sigma_0 > 1$ ($\sigma_0 < 1$).

Next observe that if $\sigma_0 > 1$ then, by equations (22) and (14), $\hat{y}_0'(\omega)$ is more equal than $\hat{y}_0(\omega)$. By equations (22) and (9) $L_0' > L_0$. Thus, $F(K_0, \gamma L_0') > F(K_0, L_0)$. By the equilibrium condition (4) and equations (7) and (8) we derive that $K_1' > K_1$. Moreover, $F_{KL} > 0$ implies $\frac{\partial X_t}{\partial K_t} < 0$ for all t. Hence, if $\sigma_1 > 1$ the increase in the capital stock following the introduction of the improved technology reinforces the decline in X_1 which, by equation (14), has the effect of increasing the equality of income. Finally, because $\hat{b}_t(\omega)$ is the same random variable as $\hat{y}_t(\omega)$, the bequest transfers of generation G_0, $\hat{b}_0(\omega)$, are more equally distributed. This, together with the increase in $1/X_1$ will guarantee a greater equality in the distribution of incomes in period $t=1$. Hence, $\sigma_1 > 1$ implies greater income equality in period 1 and also that $K_2' > K_2$.

By an induction argument similar to that used in the proof of Proposition 1 this conclusion can be extended to all future periods provided $\sigma_t > 1$ for all $t \geq 0$. Thus we have:

Proposition 2: *Given the structure of the economy described in section 2, if $\sigma_t > 1$ for all $t \geq 0$, then an unanticipated* ***Harrod-neutral*** *technological improvement in period $t=0$ results in a greater equality in the distribution of incomes in the period during which the new technology becomes available and in every period thereafter. Furthermore, the technological improvement results in larger aggregate capital stock and labor supply in every period following the introduction of the new technology.*

Remark 1: The case where $\sigma_t < 1$ for all $t \geq 0$ is not symmetric. As we already observed a **Harrod-neutral** technological innovation implies in this case that $X_0' > X_0$ and, consequently, greater inequality in the distribution of incomes in period $t = 0$. As we trace the evolution of the economy through period $t=1$ we observe that, by equation (9), $X_0' > X_0$ implies $L_0' < L_0$. However, if $\gamma L_0' > L_0$ then $(1+r_0') > (1+r_0)$ and by equations (7) and (8) $K_1^{'} > K_1$. The larger capital stock in period 1 may offset the effects of the improved technology and the resulting income distribution may become more equal. In general, therefore, if $\sigma_t < 1$ the effect of a **Harrod-neutral** technological improvement on income inequality in subsequent periods is ambiguous.

Remark 2: If $\sigma_t = 1$ for all t, (i.e., F is a Cobb-Douglas production function), then the analysis of **Hicks-neutral** technological changes applies, and, except for the magnitude, which is smaller in the case of **Harrod-neutral** technological changes, the characterization of the effects of **Harrod-neutral** technological progress is as specified in Proposition 1.

3.5. *The Effects of Solow-Neutral Technological Changes*

By definition in this case,

$$(23) \qquad X_t = \frac{\gamma F_K(\gamma K_t, L_t)}{F_L(\gamma K_t, L_t)}, \qquad \text{for all} \quad t \geq 0.$$

Fixing K_t and B_{t-1} and differentiating X_t with respect to γ, we get:

$$(24) \qquad \frac{dX_t}{d\gamma}M = F_K F_L \left[1 + \gamma K_t \left(\frac{F_{KK}(\gamma K_t, L_t)}{F_K(\gamma K_t, L_t)} - \frac{F_{LK}(\gamma K_t, L_t)}{F_L(\gamma K_t, L_t)} \right) \right],$$

where $M = F_L^2 + \gamma\alpha_2 B_{t-1}(F_L F_{KL} - F_K F_{LL}) > 0.$

But,

(25) $$\frac{1}{\sigma_t} = -\frac{\gamma K_t}{L_t}\left[\frac{F_{KK}(\frac{\gamma K_t}{L_t}, 1)}{F_K(\frac{\gamma K_t}{L_t}, 1)} - \frac{F_{LK}(\frac{\gamma K_t}{L_t}, 1)}{F_L(\gamma K_t/L_t, 1)}\right] =$$

$$= -\gamma K_t\left[\frac{F_{KK}(\gamma K_t, L_t)}{F_K(\gamma K_t, L_t)} - \frac{F_{LK}(\gamma K_t, L_t)}{F_L(\gamma K_t, L_t)}\right]$$

Hence, $\frac{dX_t}{d\gamma}M = F_K F_L[1 - \frac{1}{\sigma_t}]$ and

(26) $$\frac{dX_t}{d\gamma} \gtreqless 0 \iff \frac{1}{\sigma_t} \lesseqgtr 1.$$

Suppose that a **Solow-neutral** technological improvement is introduced in period $t=0$ and that $\sigma_0 < 1$. By condition (26) this implies that $X_0' < X_0$. Since K_0 and the distribution of bequests received by generation G_0 are predetermined by the saving and bequest decisions of the preceding period, the decline in X_0 implies greater equality in the distribution of incomes in period $t=0$.

Next we claim that $K_1' > K_1$. Suppose that this is not the case. First we observe that $X_0' < X_0$ in conjunction with equation (9) implies

$L_0' > L_0$. Consequently, $F(\gamma K_0, L_0') > F(K_0, L_0)$. If $K_1' < K_1$, this requires that $\int s_0' d\mu = S_0' < S_0$ and $B_0' < B_0$. Since $\frac{1}{X_0'} > \frac{1}{X_0}$, equation (7) implies that $(1 + r_0') < (1 + r_0)$. But, by definition, this implies $\int c_{2,-1}'(\omega) d\mu < \int c_{2,-1}(\omega) d\mu$. Furthermore, from equations (6) and (7), $S_0' < S_0$ implies $\int c_{10}'(\omega) d\mu < \int c_{10}(\omega) d\mu$. Substituting in the material balance condition (4), we see that as a result of the change of technology the expressions on the left-hand-side decrease while the output on the right-hand-side increases. A contradiction and, therefore, $K_1' > K_1$.

Observe that $K_1' > K_1$ implies $X_1' < X_1$. This, in itself, contributes towards greater equality in the distribution of income in period $t=1$. As in the previous cases, the introduction of the new technology in period $t=0$ implies a greater equality in the distribution of bequest transfers, $b_0(\omega)$, to members of generation G_1. Therefore, on both counts income is more equally distributed in period $t=1$.

By induction this argument extends to all future periods. We thus proved the following:

Proposition 3: *Given the structure of the economy described in section 2, if $\sigma_t \leqq 1$ for all $t \geq 0$ then an unanticipated* **Solow-neutral** *technological improvement in period $t=0$ results in greater equality in the distribution of incomes in the period during which the technological change is introduced and in every* *period thereafter. Furthermore, the technological improvement results in a larger aggregate capital stock and labor supply in every period following the introduction of the new technology.*

Remark 3: The statement of propositions 2 and 3 intended to highlight the symmetry of the effects of **Harrod-neutral** and **Solow-neutral** technological improvements. Clearly, if $\sigma_t = 1$ for all t the **Solow-neutral** technological change has the same effects as Hicks-neutral technological change, Also if $\sigma_t > 1$ for all $t \geq 0$ then, by the same argument as in Remark 1, beginning with period $t=1$ the distributional effects of **Solow-neutral** technological progress are ambiguous.

4. Concluding Remarks

In this paper we raise the issue of the effects of technological innovations on the distribution of incomes in a dynamic general equilibrium framework. We analyzed these effects within the context of a competitive overlapping generations economy with endogenous labor supply and a bequest motive. We set ourselves the ambitious goal of tracing the effects of technological changes on the distribution of income in each and every period following the introduction of the new technologies. In doing so we were able to identify the direct effect of a changing technology on the distribution of incomes emanating from the changes in productivity of the factors of production and the indirect effects emanating from the accumulation of these factors.

The analysis in this paper may be extended in several directions: First, it is possible to extend the production technology to include several types of labor and to analyze the effects of technological progress that results in differential shifts in the productivity of the different types of workers. It will also permit a meaningful discussion of income inequality resulting from different skills. Second, it is possible to use our framework to analyze an economy with individuals who differ in their tastes for leisure, thereby taking into consideration another source of income inequality. This extension may be carried out using the functional form of the utility function in equation (1). This functional form is responsible for the fact that we obtained an explicit reduced form solution of the model. Without this assumption the analysis becomes less tractable.

Another extension involves the analysis of the effects of production uncertainty on the distribution of incomes. For instance, a multiplicative uncertainty as in Diamond (1967) is similar to uncertain Hicks-neutral technological innovation in our model. The analysis of uncertain technology, however, requires a specification of the insurance and financial markets available for the allocation of risk bearing.

Finally, the model can be used to analyze the effects of diffferent policies on the distribution of incomes. An example of such an analysis is provided in Karni and Zilcha (1988) where we analyze the distributional effects of social security. On the basis of the analysis in this paper it is clear that the effects of any economic policy on the distribution of incomes must take into account the influence of the policy on the accumulation of factors of production.

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