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EXPORTING FIRM AND FORWARD MARKETS:
THE MULTIPERIOD CASE

by

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1. Introduction

This paper studies the behavior of a competitive exporting firm which faces uncertain exchange rate in a multiperiod framework. This firm chooses at the outset its optimal level of capital stock, which remains fixed throughout all the production periods. However, at the beginning of each date it chooses the optimal level of the variable input before the realization of the uncertain exchange rate.

This paper combines two strands of literature. First, the behavior of competitive firm under uncertainty with fixed and variable factors of production (see, for example Turnovsky (1973), Stewart (1978) and Wright (1984)). Second, the literature concerning the impact of forward markets upon the firm's optimal production and hedging decisions (e.g., Feder, Just and Schmitz (1980), Holthausen (1979), Ethier (1973), Benninga, Eldor and Zilcha (1985) and Kawai and Zilcha (1986)).

Our model differs from the first type of literature in several respects. The variable input in our case must be chosen before the realization of the exchange rate (in each period). Moreover, we introduce risk sharing arrangements in the form of forward currency markets. We differ from both strands of literature by treating a model with multiperiod production and hedging decisions.

As a result of the more general framework, in particular, the multiperiod analysis, we obtain results which are different from certain well-known properties when we consider the long-run. In the short-run, i.e. when

the capital stock is fixed historically, in the presence of forward currency markets, the output of the firm in all periods does not depend upon its attitude towards risk and its beliefs about the probability distributions. Moreover, when these markets are unbiased its production (exports) increases, in all periods.

In the long run, i.e., when the firm chooses the capital level in the first date, but it remains fixed for the following production periods, under constant returns to scale the firm's optimal capital/labor ratio declines (in all periods) in the presence of forward markets and these ratios are independent of the utility function and its subjective beliefs. However, the usual "separation property", regarding the absolute level of capital and labor, does not hold since the employment of the variable input in the future periods, and hence its output, remains uncertain. Under certain assumptions about the utility function and the production technology we show that the introduction of unbiased forward markets increases the capital investment and the production (exports). However, this is not always the case.

Usually one finds, in the case of unbiased forward markets, that the risk-averse firm fully hedges its output (see, for example, Holthausen (1979) and Feder-Just-Schmitz (1980)). In our model this risk averse firm sells currency forward more than the value of its output (except the last period), i.e., overhedges. This is a result of the dependence of its future profits upon the realizations of the exchange rates in the intermediate periods.

In section 2 we introduce the model. Section 3 deals with the short-run case in the presence of forward markets. The long-run analysis is brought in

section 4 with section 5 treating the optimal hedging behavior, and lastly section 6 concludes the paper.

2. The Model

Consider a competitive risk-averse firm which exports all its output. This firm is a price-taker in the international markets and faces uncertain exchange rates. We assume that the firm produces and exports in several periods and thus faces uncertain revenues in a few dates. To simplify our analysis let us assume that the number of periods is two.

The firm employs capital K and labor L using a neoclassical production function $F(K,L)$. We assume that $F(K,L)$ is homogeneous of degree one, and satisfies $F_K > 0$, $F_L > 0$, $F_{KK} < 0$, $F_{LL} < 0$, $F_{KL} > 0$.

The capital in this model represents all the fixed inputs to be determined at the beginning of the production periods, while labor is a variable input; however, the latter is determined at the beginning of each period before the resolution of uncertainty for that period. Denote by r the capital price in period 1 and by w_1, w_2 the labor wages in periods 1 and 2. The firm is a price-taker in the factors markets in the domestic country.

When a currency forward market exists we denote by e_1^f the forward exchange rate from date $t = 0$ to date $t = 1$ and by e_2^f the forward exchange rate from date $t = 1$ to $t = 2$. This firm maximizes the expected utility of the sum of its profits (in both periods) denominated in local currency. Let U represent its von-Neumann Morgenstern utility function, hence it satisfies, $U' > 0$ and $U'' < 0$. Therefore the firm chooses inputs and forward commitments such that (E is the expectation operator)

$$\begin{aligned} & \max_{K, L_1, L_2} E U(\tilde{\pi}) \\ & X_1, X_2 \end{aligned} \quad (1)$$

s.t.

$$\begin{aligned} \tilde{\pi} = & \tilde{e}_1 P_1 F(K, L_1) + (e_1^f - \tilde{e}_1) X_1 - rK - w_1 L_1 + \\ & + \delta \left[\tilde{e}_2 P_2 F(K, L_2) + (e_2^f - \tilde{e}_2) X_2 - w_2 L_2 \right] \end{aligned} \quad (2)$$

where $\tilde{\pi}$ is the total profit of the firm (in these two periods), $\tilde{e}_t$ is a random variable representing the spot exchange rate at the end of date t , P_t the product price in date t denominated in foreign currency, X_t is the forward sale of foreign currency by the firm in date $t-1$ to be delivered in date t , $t = 1, 2$, and δ is the discount factor for date 2 profits. We assume that P_1 , P_2 , w_1 , w_2 and r are such that the firm produces a positive output in both periods.

An important feature in this model is the dependence of the conditional distribution of the random variable $\tilde{e}_2$ upon the realization of $\tilde{e}_1$. We assume that if $\hat{e}_1 > e_1'$ then $\tilde{e}_2(\hat{e}_1) \geq \tilde{e}_2(e_1')$ for all states of nature. As a result, the currency forward price from date 1 to date 2, e_2^f , will also depend upon the realization of $\tilde{e}_1$, i.e., $e_2^f = e_2^f(e_1)$. Therefore, in maximizing (1), the firm's decision about labor (the only variable input) to be employed in period 2 will depend upon the realization of the exchange rate $\tilde{e}_1$ for period 1, i.e., $L_2 = L_2(e_1)$. Similarly, since e_2^f and L_2 depend

on the realization of $\tilde{e}_1$, the forward sale of foreign currency in period 1, X_2 , will be dependent upon the value of e_1 in period 1. We shall assume throughout the paper that $\bar{e}_2(e_1) = E[\tilde{e}_2|e_1]$ is a nondecreasing function of e_1 , i.e., nonnegative correlation between e_1 and $\bar{e}_2(e_1)$. Denote by $\bar{e}_1 = E\tilde{e}_1$. We say that the forward markets are unbiased if $e_1^f = \bar{e}_1$ and $e_2^f(e_1) = \bar{e}_2(e_1)$ for all e_1 . Finally, we denote by E_1 the expectation with respect to $\tilde{e}_1$. E_2 stands for the conditional expectation on $\tilde{e}_2$ for a given value of e_1 , i.e., the expectation with respect to $\tilde{e}_2(e_1)$.

In the next section we study the impact of uncertain exchange rates on the production levels in the case where there exist some fixed inputs.

3. Forward Markets: The Short-Run Case

We consider now the case where the capital stock of the firm is already determined. This includes, for example, the cases where the exchange rate regime has been changed from fixed to flexible or the regime has been flexible but the firm's technology is such that the capital cannot be varied.

Let $K = \bar{K}$ be given hence the firm maximizes (1) under $K = \bar{K}$ fixed. The first-order conditions are:

$$E[\tilde{e}_1 P_1 F_L(\bar{K}, L_1) - w_1] U'(\bar{\pi}) = 0 \quad (3)$$

$$E_2[\tilde{e}_2 P_2 F_L(\bar{K}, L_2(e_1)) - w_2] U'(\bar{\pi}) = 0 \quad (4)$$

$$E(e_1^f - \bar{e}_1) U'(\bar{\pi}) = 0 \quad (5)$$

$$E_2(e_2^f(e_1) - \tilde{e}_2)U'(\bar{\pi}(e_1)) = 0, \text{ for all } e_1. \quad (6)$$

Therefore we derive the following separation result:

Theorem 1: When the capital $\bar{K}$ is fixed, and currency forward markets exist, the labor inputs are determined by the following equations:

$$e_1^f P_1 F_L(\bar{K}, L_1) = w_1 \quad (7)$$

$$e_2^f(e_1) P_2 F_L(\bar{K}, L_2(e_1)) = w_2, \text{ for all } e_1. \quad (8)$$

In particular, L_1 and $L_2(e_1)$ are independent of the attitude towards risk of the firm and its subjective belief about the probability distributions of $\tilde{e}_1$ and $\tilde{e}_2$. Thus the "separation property" (see, for example Feder-Just-Schmitz (1980)) holds in our model in the short run. Condition (7) is attained from (3) and (5) while (8) is derived from (4) and (6). We now turn to the impact of unbiased forward markets.

Theorem 2: Assume that the capital level is fixed. Introducing unbiased forward markets will increase the labor inputs in all periods. Thus, for a fixed level of capital, export increases.

Proof: Without currency forward markets, hence $X_1 = 0$ and $X_2 = 0$, only equations (3) and (4) are necessary and sufficient conditions for optimality. Denote the optimal solution in this case by $\hat{L}_1$ and $\hat{L}_2(e_1)$, therefore $\bar{\pi}(\tilde{e}_1) = \tilde{e}_1 P_1 F(\bar{K}, \hat{L}_1) + \tilde{e}_2(e_1) P_2 F(\bar{K}, \hat{L}_2(e_1)) - w_1 \hat{L}_1 - w_2 \hat{L}_2(e_1)$.

By our assumptions $\bar{\pi}(e_1)$ is a nondecreasing function of e_1 , since higher realizations of $\bar{e}_1$ cannot reduce the total profits (note that all realizations of $\bar{e}_2(e_1)$ are larger, or the same, if e_1 assumes higher value). Thus, by a well-known argument (see for example Benninga-Eldor-Zilcha(1985)), equation (3) implies that

$$\bar{e}_1 P_1 F_L(\bar{K}, \hat{L}_1) - w_1 > 0. \quad (9)$$

Comparing (9) to (7) we easily derive that $\frac{\hat{L}_1}{\bar{K}} < \frac{L_1}{\bar{K}}$ since F_L is decreasing in L . By a similar argument, noting that $\bar{\pi}$ is a nondecreasing function in $\bar{e}_2(e_1)$, we obtain from (4), when $e_2^f(e_1) = \bar{e}_2(e_1)$ for all e_1 ,

$$\bar{e}_2(e_1) P_2 F_L(\bar{K}, \hat{L}_2(e_1)) - w_2 > 0 \text{ for all } e_1 \quad (10)$$

which also yields, when compared to equation (8), that $\frac{\hat{L}_2(e_1)}{\bar{K}} < \frac{L_2(e_1)}{\bar{K}}$ for all e_1 . This completes the proof of the theorem.

4. Forward Markets: The Long-Run Case

In this section we consider the impact of forward markets upon the optimal decisions of the firm regarding all inputs. This, in fact, can be studied from the first order conditions of problem (1) when K is variable and X_1, X_2 are determined optimally. Denote by $*$ the optimum when we solve such a problem. Therefore,

$$E \left[\tilde{e}_1 P_1 F_K(K^*, L_1^*) + \delta \tilde{e}_2 P_2 F_K(K^*, L_2^*(e_1)) - r \right] U'(\pi^*) = 0 \quad (11)$$

$$E \left[\tilde{e}_1 P_1 F_L(K^*, L_1^*) - w_1 \right] U'(\pi^*) = 0 \quad (12)$$

$$E_2 \left[\tilde{e}_2(e_1) P_2 F_L(K^*, L_2^*(e_1)) - w_2 \right] U'(\pi^*) = 0, \quad \text{for all } e_1. \quad (13)$$

$$E(e_1^f - \tilde{e}_1) U'(\pi^*) = 0 \quad (14)$$

$$E_2(e_2^f(e_1) - \tilde{e}_2(e_1)) U'(\pi^*) = 0, \quad \text{for all } e_1. \quad (15)$$

Combining equations (12) and (14) we derive

$$e_1^f P_1 F_L(K^*, L_1^*) = w_1 \quad (16)$$

Similarly from equations (13) and (15) we obtain

$$e_2^f(e_1) P_2 F_L(K^*, L_2^*(e_1)) = w_2, \quad \text{for all } e_1. \quad (17)$$

Conditions (16) and (17), together with the homogeneity (of degree 0) of F_L , imply that the firm determines the optimal labor/capital ratio regardless of its attitude towards risk and regardless of its beliefs about the distributions of $\tilde{e}_1$ and $\tilde{e}_2$. However, in determining the absolute levels of

capital and labor, the firm takes into account its attitude towards risk and the distribution functions of $\tilde{e}_1$ and $\tilde{e}_2$. This can be seen from condition (11) where $U'(\pi^*)$ is an integral part of this condition.

In this model the widely recognized separation result does not hold even though the production function is deterministic. The reason is that the second period output depends upon the employment of labor in the second period, which in turn depends upon the realization of $\tilde{e}_1$. Therefore, when the firm determines its optimal capital stock, the output in the second period is uncertain and hence its attitude toward risk and the distribution of the random variables must be incorporated into its decision process.

Without loss of generality, we shall take $\delta = 1$ from now on.

Let us denote the optimal levels of inputs in the long-run case when forward markets do not exist (i.e., $X_1 = 0$, $X_2 = 0$ in problem (1)) by $\hat{K}$, $\hat{L}_1$, $\hat{L}_2(e_1)$. Now let us show that production becomes more labor intensive as a result of introducing forward markets.

Theorem 3: The introduction of unbiased currency forward markets increases the capital/labor ratio in all periods for industries where capital is determined once and for all and labor is a variable factor; Formally,

$$(a) \quad \frac{K^*}{L_1^*} < \frac{\hat{K}}{\hat{L}_1}$$

$$(b) \quad \frac{K^*}{L_2^*(e_1)} < \frac{\hat{K}}{\hat{L}_2(e_1)}, \quad \text{for all values of } e_1.$$

Proof: The necessary and sufficient conditions, when no forward markets are available, in the long-run case are

$$E[\tilde{e}_1 P_1 F_K(\hat{K}, \hat{L}_1) + \tilde{e}_2 P_2 F_K(\hat{K}, \hat{L}_2(e_1)) - r] U'(\hat{\pi}) = 0 \quad (18)$$

$$E[\tilde{e}_1 P_1 F_L(\hat{K}, \hat{L}_1) - w_1] U'(\hat{\pi}) = 0 \quad (19)$$

$$E_2(\tilde{e}_2(e_1) P_2 F_L(\hat{K}, \hat{L}_2(e_1)) - w_2) U'(\hat{\pi}) = 0, \quad \text{for all } e_1. \quad (20)$$

We now use similar arguments as in the proof for Theorem 2. Clearly, $\hat{\pi}(e_1)$ is an increasing function of e_1 , hence $\text{Cov}(\tilde{e}_1, E_2 U'(\hat{\pi})) < 0$ since U' is a decreasing function. Thus from equation (19) and $e_1^f = \bar{e}_1$ we obtain

$$\bar{e}_1 P_1 F_L(\hat{K}, \hat{L}_1) - w_1 > 0 \quad (21)$$

By the exact same argument, since $e_2^f(e_1) = \bar{e}_2(e_1)$, we have

$$\bar{e}_2(e_1) P_2 F_L(\hat{K}, \hat{L}_2(e_1)) - w_2 > 0, \quad \text{for all } e_1. \quad (22)$$

Comparing equations (21) and (22) to equations (16) and (17) and using the homogeneity of degree zero of F_L we easily derive (a) and (b).

The proof of Theorem 3 shows that when forward markets are not available, as a result of the uncertain exchange rates the expected value of the marginal product of labor exceeds the wage rate. However, when the firm uses forward contracts it equates the value of the marginal product of labor (using the forward prices) to the wage rate.

Unfortunately, in general, one cannot strengthen Theorem 3 in the sense of indicating what precisely happens to the level of capital invested as a result of introducing unbiased forward markets. We can derive such a result only under the following assumption:

Assumption (A.1). The utility function $U(x)$ satisfies: $U'(x)$ is convex and $xU'(x)$ is concave.

Assumption (A.1) holds for example when $U(x) = Bx^\alpha$, $0 < \alpha \leq 1$, or when $U(x)$ is a logarithmic or quadratic function. Now we prove, assuming (A.1), that introducing unbiased forward markets results in higher capital investment and higher labor, hence the output increases in both periods. We shall also assume that $F(K,L)$ is a Cobb-Douglas production function for the sake of simplifying the proof of Theorem 4. This result can be proved, assuming (A.1), for most CES production functions. Although our assumptions are sufficient conditions, we believe that this theorem does not hold without some additional restrictions on the utility and the production functions.

Theorem 4: Assume that (A.1) holds and that $F(K,L)$ is a Cobb-Douglas production function. Introducing unbiased forward markets results in

higher capital investment and higher production; i.e. higher exports in both periods. In particular, $K^* \geq \hat{K}$, $L_1^* \geq \hat{L}_1$ and $L_2^*(e_1) \geq \hat{L}_2(e_1)$ for all values of e_1 .

Since the proof of Theorem 4 is long and technical we bring it in the Appendix, in order to ease the reading of the paper. Note that we do not assume (A.1) in the sequel.

5. Optimal Hedging

The purpose of this section is to study the optimal engagements in forward contracts of a competitive firm with some fixed and some variable inputs. We derive here that the optimal hedging rules are different significantly from the case where all inputs are variable. One main reason for this result is that the returns attributed to the fixed input are correlated with the first period exchange rate. To emphasize this difference let us consider first the unbiased forward markets case.

Theorem 5: Assume that the currency forward markets are unbiased, i.e.,

$e_1^f = \bar{e}_1$ and $e_2^f(e_1) = \bar{e}_2(e_1)$ for any e_1 , and that $\bar{e}_2(e_1)$ is increasing in e_1 , then

$$X_1^* > P_1 F(K^*, L_1^*) \quad (23)$$

$$X_2^*(e_1) = P_2 F(K^*, L_2^*(e_1)) \quad \text{for all } e_1. \quad (24)$$

We observe that the first period forward sale (in the unbiased markets case) is not a full hedge if $\tilde{e}_1$ and $\tilde{e}_2$ are correlated. However, in the second period (in general, on the last date) the firm fully hedges its output.

Proof: Let us show first that equation (24) holds. From equation (13) we obtain, given $e_2^f(e_1) = E(\tilde{e}_2|e_1) = \bar{e}_2(e_1)$, that the firm chooses $X_2^*(e_1)$ such that

$$\text{Cov}(\tilde{e}_2(e_1), U'(\pi^*(e_1)) | e_1) = 0 \quad \text{for all } e_1 \quad (25)$$

where

$$\begin{aligned} \pi^*(e_1) = & \tilde{e}_1[F(K^*, L_1^*) - X_1^*] + \tilde{e}_2(e_1)[F(K^*, L_2^*(e_1)) - X_2^*(e_1)] + \\ & \bar{e}_1 X_1^* + \bar{e}_2(e_1) X_2^*(e_1) - w_1 L_1^* - w_2 L_2^*(e_1) - rK^* \end{aligned} \quad (26)$$

Thus $\pi^*(e_1)$ is either monotone increasing in $\tilde{e}_2$ (when $F(K^*, L_2^*(e_1)) > X_2^*(e_1)$) or non-increasing (if $F(K^*, L_2^*(e_1)) \leq X_2^*(e_1)$). But from equation (25) it is clear that $\pi^*(e_1)$ cannot be either increasing in $\tilde{e}_2(e_1)$ or decreasing. Therefore $F(K^*, L_2^*(e_1)) = X_2^*(e_1)$ for all values of e_1 .

To show equation (23) we obtained from equations (12) and (16) that the firm chooses X_1^* such that equation (28) (in the Appendix) holds. But, using equation (24) of this theorem, $\pi^*(e_1)$ reduces to

$$\begin{aligned} \pi^*(e_1) = & \tilde{e}_1 P_1 [F(K^*, L_1^*) - X_1^*] + \bar{e}_2(e_1) P_2 F(K^*, L_2^*(e_1)) - w_2 L_2^*(e_1) \\ & + e_1 P_1 X_1^* - w_1 L_1^* - rK^*. \end{aligned}$$

The derivative of $\Delta(e_1) = P_2 \bar{e}_2(e_1) F(K^*, L_2^*(e_1)) - w_2 L_2^*(e_1)$ with respect to e_1 is,

$$P_2 \frac{d\bar{e}_2(e_1)}{de_1} + \left[P_2 \bar{e}_2(e_1) F_L(K^*, L_2^*(e_1)) - w_2 \right] \frac{dL_2^*(e_1)}{de_1}.$$

However, from the F.O.C. we find that $\Delta'(e_1) = P_2 \frac{d\bar{e}_2(e_1)}{de_1}$. Since $\bar{e}_2(e_1)$ is increasing in e_1 if $F(K^*, L_1^*) - X_1^* \geq 0$ it implies that $\pi^*(e_1)$ is a monotone increasing function of e_1 . Hence the covariance in equation (28) should be negative which is a contradiction. Therefore $F(K^*, L_1^*) < X_1^*$.

□

To understand the overhedging in period 1, let us analyze the firm's position after one period. In the case where the spot exchange rate is lower than $e_1^f = \bar{e}_1$ the firm profits from its position in the forward market and its level of output by $(X_1^* - P_1 F(K^*, L_1^*))(\bar{e}_1 - e_1) > 0$. However, low realization of e_1 implies lower expected profits in the following period. Thus, the overhedge, $X_1^* - P_1 F(K^*, L_1^*)$, can be considered as a hedge against uncertain returns in the second period. Note that if all factors of production are variable, in particular K can be chosen for the second period after the realization of $\bar{e}_1$, it can be shown that X_1^* must equal the value of output $P_1 F(K_1^*, L_1^*)$, using the unbiased forward exchange rate market. Hence the first period overhedging is attributed to the uncertain return to

the fixed input (i.e. capital) in the second period which is correlated with the realization of $\tilde{e}_1$. This rationale holds also in commodity forward markets. For example, if an agricultural product's prices $\tilde{P}_1$ and $\tilde{P}_2$ are correlated, a farmer may use this period forward commodity market not only to hedge its profits in the current period, but also against the impact of the realization of $\tilde{P}_1$ upon the profits in the following periods. The present value of all future profits equals to the market value of his farm (= his assets). Thus, in using the current forward market, he is hedging against change in the value of his assets.

In the case of biased forward markets, if $e_1^f > \bar{e}_1$ then part (a) of our theorem clearly holds since there is an additional incentive to sell forward more than the output. In the normal backwardation (i.e. $e_1^f < \bar{e}_1$) the difference between X_1^* and $P_1 F(K^*, L_1^*)$ depends on the tradeoff between the premium that the firm pays in the forward market and the value of hedging future profits.

6. Discussion

The floating of major world currencies has caused, in recent years, an increase in the activity of forward (and futures) currency markets. A number of countries (including Finland, France, Germany, Israel, Italy, Spain and Sweden) have instituted exchange-rate guarantee programs.¹ These arrangements are designed to aid firms which produce for export markets to face exchange-rate risks. This paper is another contribution to analyzing the impact of such forward markets upon the competitive firms production and optimal hedging decision in a multiperiod model.

It is traditional in this literature to compare the firm's behavior under uncertainty with the "certainty equivalent" case (see for example Sandmo (1971), Leland (1972)). The classical result in the one-period models is that the behavior of risk neutral firm is identical to the certainty-equivalent case. Here we observe, in the multiperiod framework (with fixed and variable inputs), that this result does not hold. In the certainty-equivalent model the first-order conditions are: (Let $\bar{e}_2 = E\tilde{e}_2$),

$$\bar{e}_1 P_1 F_K(K^c, L_1^c) + \bar{e}_2 P_2 F_K(K^c, L_2^c) - r = 0 \quad (A)$$

$$\bar{e}_1 P_1 F_L(K^c, L_1^c) = w_1 \quad (B)$$

$$\bar{e}_2 P_2 F_L(K^c, L_2^c) = w_2. \quad (C)$$

However, the optimality conditions for risk-neutral firms are,

$$E(\bar{e}_1 P_1 F_K(K, L_1) + \bar{e}_2(e_1) P_2 F_K(K, L_2(e_1)) - r) = 0 \quad (A')$$

$$\bar{e}_1 P_1 F_L(K, L_1) = w_1 \quad (B')$$

$$\bar{e}_2(e_1) P_2 F_L(K, L_2(e_1)) = w_2 \quad \text{for all } e_1. \quad (C')$$

It is clear that these two sets of conditions yield different solutions since, for example, in (A') $\text{Cov}(\bar{e}_2(e_1), F_K(K, L_2(e_1))) > 0$ in general. Hence $\bar{e}_2 P_2 E F_L(K, L_2(e_1)) - w_2 < 0$.

Most of the results in this paper hold without the assumption of constant returns to scale technology. However, Theorems 3 and 4 do not hold without this assumption.

Let us show that the firm's overhedging behavior in the first period holds even if one introduces unbiased long-term forward markets. Suppose that in our model we add an unbiased forward market from date 0 to date 2. This will not affect the firm's optimal production decisions and its first period overhedging behavior because of the following reasons. First, at time 0 the second period output is uncertain and secondly the firm can hedge against the impact of low realization of the exchange rate in the first period on profits in all periods only through one-period forward markets. Formally, it can be shown that the additional first-order condition due to this extra unbiased forward market can be derived from the other conditions.²

Let us, briefly, consider some policy implications of our results. A number of countries have instituted exchange-rate guarantee programs for longer ranges of time relative to the existing forward markets. Our result shows that, except for the subsidy that might be involved in such a program, there is no effect upon the production decisions of the firms described here (assuming that unbiased forward markets are available).

Our result regarding the overhedging should be taken into account by the tax authorities in various countries. For example, in the U.S. for tax purposes one is considered as a speculator if he sells short in the forward market and he is not long in the commodity (currency) itself. Our result shows that selling forward more than one's output (or its value) may stem from hedging purposes in a many-period model.

APPENDIX

Proof of Theorem 4: Assume that $e_1^f = \bar{e}_1 = E\tilde{e}_1$ and $e_2^f(e_1) = \bar{e}_2(e_1) = E(\tilde{e}_2 | \tilde{e}_1 = e_1)$. Note that $X_2^*(e_1) = F(K^*, L_2^*(e_1))$ for all e_1 (see, for example, the proof of Theorem 5), hence

$$\begin{aligned} \hat{\pi}^* &= \hat{\pi}(K^*, L_1^*, L_2^*) = \tilde{e}_1 P_1 F(K^*, L_1^*) + \tilde{e}_2 P_2 F(K^*, L_2^*(e_1)) \\ &\quad - rK^* - w_1 L_1^* - w_2 L_2^* = \pi^* + X_1^* (\tilde{e}_1 - \bar{e}_1) + \\ &\quad + P_2 F(K^*, L_2^*(e_1)) \cdot (\tilde{e}_2 - \bar{e}_2(e_1)). \end{aligned} \quad (27)$$

In particular, under the assumption of unbiasedness, $\hat{\pi}^*$ is riskier than π^* , since $E\hat{\pi}^* = E\pi^*$ (see Rothschild and Stiglitz (1970)). From equation (15) we obtain that $L_2^*(e_1)$ is a nondecreasing function of e_1 . Also, note that equations (12) and (16) imply, when $e_1^f = \bar{e}_1$, that

$$\text{Cov}(\tilde{e}_1, U'(\pi^*)) = 0. \quad (28)$$

By our assumption about the production function we know that for some

$$\text{constant } \bar{\theta}, \quad \frac{F_K(K^*, L_2^*(e_1))}{F_L(K^*, L_2^*(e_1))} = \bar{\theta} \frac{L_2^*(e_1)}{K^*} \quad \text{and also}$$

$$F(K^*, L_2^*(e_1)) = F_K(1, \frac{L_2^*(e_1)}{K^*}) + L_2^*(e_1) F_L(1, \frac{L_2^*(e_1)}{K^*}).$$

Therefore, π^* can be expressed as follows:

$$\begin{aligned}
 \pi^*(e_1) &= \tilde{e}_1(P_1 F(K^*, L_1^*) - X_1^*) + \bar{e}_2(e_1)P_2 F(K^*, L_2^*(e_1)) - \\
 &\quad - w_2 L_2^*(e_1) - w_1 L_1^* - r = \\
 &= \tilde{e}_1(P_1 F(K^*, L_1^*) - X_1^*) + \bar{e}_2(e_1)L_2^*(e_1)F_L(K^*, L_2^*(e_1)) + \\
 &\quad + \bar{e}_2(e_1)P_2 F_K(K^*, L_2^*(e_1)) - w_2 L_2^*(e_1) - w_1 L_1^* - rK^* = \\
 &= \tilde{e}_1(P_1 F(K^*, L_1^*) - X_1^*) + \lambda^* L_2^*(e_1) + \text{const.}
 \end{aligned} \tag{29}$$

For some positive constant λ^* . We have used the fact that $\bar{e}_2(e_1)F_L(K^*, L_2^*(e_1)) = w_2 = \text{const.}$ for all e_1 (see (17)). Using equations (28) and (29) we now derive:

$$\begin{aligned}
 \text{Cov}(L_2^*(e_1), U'(\pi^*)) &= \frac{1}{\lambda^*} \text{Cov}[\pi^*(e_1) + (X_1^* - F(K^*, L_2^*))\tilde{e}_1, U'(\pi^*)] \\
 &= \frac{1}{\lambda^*} \text{Cov}(\pi^*, U'(\pi^*)) \leq 0
 \end{aligned} \tag{30}$$

The last equality has been obtained due to equation (28). Now we use the fact that $X_1^* \geq F(K^*, L_1^*)$ (see the proof of Theorem 5) to derive that

$$\left| \text{Cov}(L_2^*(e_1), \frac{U'(\pi^*)}{EU'(\pi^*)}) \right| \leq \left| \text{Cov}(L_2^*(e_1), \frac{E_2 U'(\pi^*)}{EU'(\pi^*)}) \right| \tag{31}$$

First note that equation (31) holds when U' is linear since $E\pi^* = E\pi^*$. When U' is convex and $xU'(x)$ concave, since π^* is riskier than π^* , we find that

$$EU'(\pi^*) \leq EU'(\hat{\pi}^*) \quad (32)$$

$$E_1 \pi^* U'(\pi^*) \geq E \hat{\pi}^* U'(\hat{\pi}^*) \quad (33)$$

which imply that equation (31) holds because

$$\left| \text{Cov}(\pi^*, \frac{U'(\pi^*)}{EU'(\pi^*)}) \right| \leq \left| \text{Cov}(\hat{\pi}^*, \frac{U'(\hat{\pi}^*)}{EU'(\hat{\pi}^*)}) \right|$$

and the fact (which follows from the monotonicity of $\hat{\pi}^*$ in e_1)

$$0 = \text{Cov}(\tilde{e}_1, U'(\pi^*)) \geq \text{Cov}(\tilde{e}_1, U'(\hat{\pi}^*)).$$

Now let us prove the assertion of the theorem. Let us rewrite condition (11). Since π^* does not depend upon the realization of $\tilde{e}_2$, for each given e_1 we can rewrite equation (11) as follows: For some positive constant q^* ,

$$E_1 \left[\tilde{e}_1 P_1 F_K(K^*, L_1^*) + q^* L_2^*(e_1) - r \right] U'(\pi^*) = 0 \quad (34)$$

Now, using equation (31) we obtain from equation (34) that

$$E_1 \left[\tilde{e}_1 P_1 F_K(K^*, L_1^*) + q^* L_2^*(e_1) - r \right] E_2 U'(\hat{\pi}^*) \leq 0 \quad (35)$$

Similarly, we obtain from equations (12) and (13),

$$E_1 (\tilde{e}_1 P_1 F_L(K^*, L_1^*) - w_1) E_2 U'(\hat{\pi}^*) < 0. \quad (36)$$

$$E_2 (\tilde{e}_2(e_1) P_2 F_L(, L_2^*(e_1)) - w_2) E_2 U'(\hat{\pi}^*) < 0. \quad (37)$$

Now consider the first-order conditions for $\hat{K}$, $\hat{L}_1$, $\hat{L}_2$ (which are necessary and sufficient conditions for optimality when $X_1 = 0$ and $X_2 = 0$). Since the maximand $EU(\tilde{\pi})$ is concave in K , L_1 and L_2 , comparing conditions (35), (36) and (37) with equations (18), (19) and (20) we derive, using the results in Theorem 3 and the assumption that $F_{KL} > 0$, that $K^* \geq \hat{K}$, $L_1^* \geq \hat{L}_1$ and $L_2^*(e_1) \geq \hat{L}_2(e_1)$ for all e_1 .

FOOTNOTES

1. An exchange-rate guarantee program is designed to afford exporters the benefits of a forward market where none exist. Countries which have established such programs and which also have forward markets for their currencies - among them France, Germany and Japan - limit participation in the programs to commitments which cannot be covered by the forward markets (generally in excess of two years). In addition, some of these programs incorporate a distinct subsidy element.
2. Let $e_2^0 = E\tilde{e}_2$ be the forward price for contracts between $t = 0$ and $t = 2$. X_2^0 is the corresponding forward sale in this market. Then the additional condition is:

$$E(e_2^0 - \tilde{e}_2)U'(\pi^{**}) = 0$$

where $\pi^{**} = \pi^* + X_2^0(e_2^0 - \tilde{e}_2)$. This condition is implied from the unbiasedness assumption and conditions (14) and (15) since

$$\text{Cov}(\tilde{e}_2, U'(\pi^{**})) = \text{Cov}(\tilde{e}_2(e_1), U'(\pi^{**})|e_1) = 0.$$

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