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OUTPUT GROWTH AND EMPLOYMENT FLUCTUATIONS

by

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ABSTRACT

This paper develops and estimates with U.S. data a real business cycle model with endogenous long-term growth. The analysis is focused on the joint determination of output and hours of employment.

The paper is an attempt to contribute to the integration of business cycle analysis with long-term growth considerations. A practical aspect of this integration pursued in the paper is the decomposition of the output series into permanent and transitory, or cyclical, components. This decomposition is performed in a bivariate (output growth, hours of employment), theory-constrained setup.

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1. Introduction

The analysis of business cycles requires from the researcher to come to grips with the nonstationary behavior of most macroeconomic time series. A convenient and common approach is to assume that the nonstationarity arises from exogenous technological progress. If technological change is represented by a deterministic function of time the appropriate detrending procedure is the estimation of the deterministic trend in the series, obtaining the cycle as the residual component. A more general approach along these lines is to view technological progress as a stochastic process which is independent of the cyclical factors. In this setup the appropriate procedure for detrending involves the estimation of the permanent component in an unobserved components model (as in Watson (1986, model 1)).

The situation is different in business cycle models where long-term growth is endogenous, as in King and Rebelo (1986) and King, Plosser, Stock and Watson (1986). These models follow Lucas (1985) and Romer (1986) in the incorporation of mechanisms that generate sustained growth without exogenous technological change. A major implication of this framework is that temporary shocks have permanent effects. Hence, the theory itself does not separate the permanent and the cyclical components.

The present paper pursues this line of research, which integrates the analysis of business cycles with long-term growth considerations. The paper develops a real business cycle model with endogenous growth, focusing on the joint determination of output and hours of employment. Studying these two variables jointly is of particular interest in the present context, since total hours per-capita seems to be a stationary and typically cyclical time series, while output per-capita is clearly nonstationary.

A closed-form solution for both variables is obtained from the model and estimated with U.S. data. The bivariate setup makes possible to identify structural parameters related to labor supply and capital accumulation, as well as the parameters of the joint process of two exogenous shocks affecting this economy. One is a standard disturbance in the production function, as usually introduced in the real business cycle models, and the other is a disturbance to capital accumulation, reflecting innovations in the quality of capital. Using the estimates of the model the long-run impulse responses of output to the two types of shocks are calculated.

The model is then used to perform the stochastic detrending of output in a theoretically-constrained bivariate setting. In the present framework with endogenous technological progress, innovations in the permanent component are correlated with the innovations in the transitory component. The appropriate detrending procedure is a multivariate version of the method suggested by Beveridge and Nelson (1981), as discussed in King, Plosser, Stock and Watson (1986). Unlike the results in Nelson and Plosser (1982) and in other studies, which are consistent with all output movements being permanent, the present bivariate detrending procedure detects large transitory fluctuations in output.

Section 2 of the paper describes the setup of the model and section 3 presents the solution. The econometric estimation is reported in section 4. Section 5 includes the calculation of the long-run impulse responses of output and the stochastic detrending. Section 6 contains concluding remarks.

2. The Setup of the Model

The framework is a stochastic growth model of the type used in Kydland and Prescott (1982), Long and Plosser (1983) and King, Plosser, Stock and Watson (1986). As in the latter two papers the present model has a log-linear structure, so that a closed-form solution can be obtained. An important difference with those models, however, is that hours of work are not constant in equilibrium.

The economy is composed of a large and constant number of identical households and identical firms interacting in a competitive environment. The representative firm produces output according to the technology

$$(1) \qquad Y_t = F(K_t, H_t L_t, z_{1t}) = A_o K_t^{\alpha}(H_t L_t)^{1-\alpha}\exp(z_{1t}), \quad 0 < \alpha < 1,$$

where z_{1t} is a a productivity shock, K_t is physical capital in productivity units, H_t is an index of knowledge and L_t is labor input in time units. Hence, the accumulation of human capital has the effect of Harrod-neutral technological progress. The shock z_{1t} follows a stationary process, to be specified below.

The capital stock variable evolves as

$$K_{t+1} = K_t G(I_t/K_t)\exp(z_{2t+1}), \qquad G' > 0, \quad G'' < 0,$$

where I_t is the amount of resources devoted both to investment that increases the quantity of capital and to research that improves its quality. Capital accumulation is subject to the stationary disturbance z_{2t}. This is the type of capital evolution equation used by Lucas and Prescott (1971). Unlike the standard linear form $(K_{t+1} = K_t(1-d) + I_t)$ it exhibits decreasing returns, which can be interpreted as reflecting adjustment costs in increasing the volume of capital or diminishing returns in research activities. The specific form adopted for the function G is $(I_t/K_t)^{1-\delta}$, $0 < \delta < 1$, so that the capital evolution equation becomes[1]

$$(2) \qquad K_{t+1} = A_1 K_t (I_t/K_t)^{1-\delta} \exp(z_{2t+1}).$$

The shocks z_{1t} and z_{2t} are assumed to follow the vector autoregressive process $\phi(B)z_t = a_t$ where $z_t \equiv (z_{1t}, z_{2t})'$, a_t is the vector white noise $(a_{1t}, a_{2t})'$ and $\phi(B)$ is a 2x2 matrix polynomial of order p in the backshift operator B.

An alternative interpretation of (2) is the following. Rewriting (2) as $K_{t+1} = A_1 K_t^{\delta} I_t^{1-\delta} \exp(z_{2t+1})$, δ can be associated with the relative quality of old capital relative to new investment goods.

Similarly as in Arrow (1962) and Romer (1985), knowledge is assumed to grow proportionally to and as a by-product of the accumulated investment and research activities in the economy:

$$(3) \qquad H_t = A_2 \bar{K}_t, \qquad A_2 > 0,$$

where $\bar{K}_t$ is the average stock across firms.[2] Thus, the production function of the representative firm--equation (1)--displays increasing returns at the social level, but the model will be consistent with competitive equilibrium since each firm takes H_t as given.

The representative household maximizes the intertemporal utility function

$$E_t \Sigma_{j=0}^{\infty} \beta^j U[C_{t+j}, H_{t+j}(L - L_{t+j})], \quad 0 < \beta < 1,$$

where C_t is the flow of consumption goods per-household. For simplicity it is assumed that the number of households and the number of firms are the same, so that $L-L_t$ is the flow of leisure per household. Following Ghez and Becker (1975) and Heckman (1976), we incorporate the notion that knowledge increases the productivity of the input of time in home activities, which in turn produces utility. Hence, H_t not only increases market labor productivity, but also the marginal disutility from labor supply. This effect is captured by the parametrization

$$(4) \qquad U = \ln[C_t - H_t L_t^{1+\omega}], \qquad \omega > 0.$$

This form, chosen for convenience, implies that the direct positive effect of H_t, which is not a choice variable, on utility is neglected.

This utility function has the convenient property that the marginal rate of substitution between consumption and leisure is independent of the consumption level. Hence, labor supply can be solved independently of the intertemporal optimization over consumption and saving. This would also hold, though, for any utility function applied to the composite good $C_t - H_t L_t^{1+\omega}$. The logarithmic form has the additional property that it yields a closed-form solution to all endogenous variables. However, unlike in Long and Plosser (1983) and King, Plosser, Stock and Watson (1986), who use the standard logarithmic utility function, equilibrium employment is not constant.

Finally, the resource constraint in the economy is:

$$Y_t = C_t + I_t. \tag{5}$$

3. Solution of the Model

The interaction of optimizing households and firms in competitive equilibrium is analyzed by solving the following representative agent's dynamic problem[3]

$$V(K_t, \bar{K}_t, \underline{z}_t) = \max_{\{C_t, I_t, L_t\}} \{\ln(C_t - A_2 \bar{K}_t L_t^{1+\omega}) + \beta E_t V(K_{t+1}, \bar{K}_{t+1}, \underline{z}_{t+1})\},$$

where $\underline{z}_t = (z_t, z_{t-1}, \ldots, z_{t-p})$,

subject to:

$$C_t = Y_t - I_t,$$

$$Y_t = A_o K_t^{\alpha} (A_2 \bar{K}_t)^{1-\alpha} L_t^{1-\alpha} \exp(z_{1t}), \text{ and}$$

$$K_{t+1} = A_1 K_t^{\delta} I_t^{1-\delta} \exp(z_{2t+1}).$$

Additionally, the individual agent takes the evolution of the average level of capital in the economy as described by the process:

$$(2') \qquad \bar{K}_{t+1} = A_3(\bar{K}_t)^{\tau_1}\exp[\Sigma_{j=0}^{p}\ \tau_{2j}z_{1,t+1-j} + \tau_{3j}z_{2,t+1-j}], \qquad \tau_1<1/\beta.$$

Since this process is taken as given, the individual agent does not recognize the effect of his own decisions on $\bar{K}_{t+1}$. For any arbitrary set of coefficients in (2′) (satisfying the condition $\tau_1<1/\beta$ for finiteness of utility) the model can be solved by conjecturing that the value function is of the form

$$V(K_t, \bar{K}_t, \underset{\sim}{z}_t) = D_o + D_1 \ln K_t + D_2 \ln \bar{K}_t + \Sigma_{j=0}^{p} D_j' z_{t-j},$$

where D_j' is a 1x2 vector.

The solution is:

$$(6) \quad L_t = A_\ell K_t^{\frac{\alpha}{\alpha+\omega}}(\bar{K}_t)^{\frac{-\alpha}{\alpha+\omega}} \exp(z_{1t})^{\frac{1}{\alpha+\omega}}, \qquad A_\ell=\left[A_o A_2^{-\alpha}(\frac{1-\alpha}{1+\omega})\right]^{\frac{1}{\alpha+\omega}}$$

$$(7) \quad Y_t = A_y K_t^{\frac{\alpha(1+\omega)}{\alpha+\omega}} (\bar{K}_t)^{\frac{\omega(1-\alpha)}{\alpha+\omega}} \exp(z_{1t})^{\frac{1+\omega}{\alpha+\omega}}, \qquad A_y=\left[A_o^{1+\omega}\ A_2^{\omega(1-\alpha)}(\frac{1-\alpha}{1+\omega})\right]^{\frac{1}{\alpha+\omega}},$$

$$(8) \quad I_t = bY_t, \qquad b = \frac{\beta D_1(1-\delta)}{1+\beta D_1(1-\delta)}(\frac{\alpha+\omega}{1+\omega})$$

(9) $C_t = (1-b)Y_t$,

(10) $K_{t+1} = A_k K_t^{\delta + \frac{\alpha(1+\omega)}{\alpha+\omega}(1-\delta)} (\bar{K}_t)^{\frac{\omega(1-\alpha)(1-\delta)}{\alpha+\omega}} \exp(z_{1t})^{\frac{(1+\omega)(1-\delta)}{\alpha+\omega}} \exp(z_{2t+1})$,

$$A_k = A_1(bA_y)^{1-\delta}.$$

The coefficients D_1 and D_2 are

$$D_1 = \frac{\alpha(1+\omega)/(\alpha+\omega)}{1-\beta[\frac{\alpha+\omega(\delta+\alpha(1-\delta))}{\alpha+\omega}]} > 0.$$

$$D_2 = \frac{\omega(1-\alpha)[1+\beta D_1(1-\delta)]}{1-\beta r_1} > 0.$$

Since D_1 is positive (and $0 < \alpha < 1$), this implies that $0 < b < 1$. The other coefficients of the value function, which also can be calculated, are not relevant for our purposes.

As in the similar contexts analyzed by Romer (1985), King and Rebelo (1986) and King, Plosser, Stock and Watson (1986), the model generates an endogenous "engine of growth". This role is taken in the present model by the capital stock process in equation (10). Consider first the case where the externality does not exist and hence $\bar{K}_t$ drops from equation (10). Since the exponent of K_t is less than one, this would imply that the stochastic difference equation in (10) is a stationary process. Therefore, no long-term growth would occur in this economy.

The presence of the externality implies that K_{t+1} depends not only on K_t but also on $\bar{K}_t$. Now, since all firms are identical the equality $\bar{K}_t = K_t$ holds. Then, equation (10) becomes

$$(10') \qquad K_{t+1} = A_k K_t \exp(z_{1t})^{\frac{(1+\omega)(1-\delta)}{\alpha+\omega}} \exp(z_{2t+1}).$$

Hence, the disturbances z_{1t} and z_{2t+1} have now permanent effects on the capital stock, which accumulates at a stochastic but stationary rate.

Since $\bar{K}_t = K_t$, the process for the average capital stock in (2′) coincides with the process for K_t in (10′). Hence, the arbitrary parameters of (2′) should be identical to the corresponding coefficients in (10′). In particular $\tau_1 = 1$, satisfying the condition $\tau_1 < 1/\beta$.[4]

We turn now to consider the solutions for the two variables of main interest: output and hours of employment. Since output depends on the capital stock, the nonstationarity of the latter is absorbed by output as well. With $\bar{K}_t = K_t$, equation (7) becomes

$$(7') \qquad Y_t = A_y K_t \exp(z_{1t})^{\frac{1+\omega}{\alpha+\omega}},$$

where the "trend" in output is generated by the accumulation of K_t.

When $K_t = \bar{K}_t$ the process for hours of employment in equation (6) becomes:

$$(6') \qquad L_t = A_\ell \exp(z_{1t})^{\frac{1}{\alpha+\omega}} \;.$$

The capital stock drops from the labor equation, because of the two opposing forces that it exerts on labor supply. Since knowledge affects proportionally both the market and the non-market marginal productivities of labor, productivity at home rises as much as in the market place, so that no net effect on L_t remains.

It should be emphasized that the current model generates fluctuations in labor supply through a different channel than the intertemporal substitution mechanism stressed originally by Lucas and Rapping (1969). The latter has to do with the response of labor supply to the deviations of the real wage from the normal future level. Then, for example, if the real wage follows a random walk this channel does not operate. By contrast, in the present model also a random walk real wage generates stationary labor fluctuations. The reason is that what matters for labor supply here is the contemporaneous productivity differential in market and home activities, represented by the shock z_{1t}. As market productivity and hence the real wage change with a given realization of z_{1t}, labor supply reacts positively. Over time the transitory shock generates the accumulation of capital and knowledge, which cancel each other as to the labor supply decision. The increment in the real wage typically becomes permanent, given the accumulation of capital, but as the shock z_{1t} dissipates over time so does the labor supply fluctuation.

To establish the link between output and employment fluctuations in an empirically useful form, the solution for output is expressed in terms of its rate of change. From equations (6′), (7′) and (10′) and using lower-case letters for natural logs, it follows that[5]

$$(11)\quad \Delta y_t = \mu_y + (\frac{1+\omega}{\alpha+\omega})(1-\delta B)z_{1t} + z_{2t}, \qquad \mu_y=(\frac{1-\delta}{\alpha+\omega})\ln\left[A_o^{1+\omega}A_2^{\omega(1-\alpha)}(\frac{1-\alpha}{1+\omega})\right]$$

$$(12)\quad \ell_t = \ell n(A_\ell) + z_{1t}.$$

These two equations form the structural empirical model, to be estimated in the next section.

Equation (11) shows that the disturbance to capital accumulation, z_{2t}, has a permanent effect on output, while the effect of the production function disturbance, z_{1t}, is partially reversed next period according to the parameter δ ($0 < \delta < 1$). Note that the theory rules out the possibility that the nonstationarity in output can be represented by a deterministic time-trend. For this to be the case the variance of z_{2t} would have to be zero and, more importantly, the moving-average coefficient δ would have to be one--implying that investment does not contribute at all to future productive capacity.

Finally, equation (12) implies that the movements of hours of employment reflect the productivity shock z_{1t}. Hence, by looking at the ℓ_t series one can identify, in the Box-Jenkins sence, the process describing the production function shock in a univariate setting.

4. **Estimation**

In this section the model is estimated and tested. The econometric strategy is described first and then the data used and the results obtained are reported.

The structural form in equations (11) and (12) can be written, excluding the constants, as

$$(13) \qquad x_t = \Omega(B) z_t,$$

where $x_t = [\Delta y_t, \ell_t]'$,

$$\Omega(B) = \begin{bmatrix} 1+\omega & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \delta(1+\omega) & 0 \\ 0 & 0 \end{bmatrix} B,$$

and the factor $1/(\alpha+\omega)$ has been absorbed into z_{1t} and z_{2t} (since α turns out not to be identifiable).

As mentioned in section 2, z_t is assumed to follow a vector autoregressive process, which is given by

$$(14) \qquad \phi(B) z_t = a_t,$$

$a_t \sim \text{iin}[0,\Sigma]$, and $\Sigma = [\sigma_{ij}]$, $i,j = 1,2$, where $\phi(B)$ is a 2x2 matrix polynomial of order p with $\phi(0)=I$.

To estimate the model as a vector autoregression, the polynomial $\Omega(B)$ in (13) is normalized so as to set the first matrix coefficient to the identity matrix:

$$(15) \qquad x_t = \tilde{\Omega}(B)\tilde{z}_t, \quad \text{with} \quad \tilde{\Omega}(B) = \Omega(B)\Omega(0)^{-1} \quad \text{and} \quad \tilde{z}_t = \Omega(0)z_t.$$

From (14) it then follows that

$$(16) \qquad \tilde{\phi}(B)\tilde{z}_t = \tilde{a}_t, \qquad \tilde{a}_t = \Omega(0)a_t,$$

where

$$(17) \qquad \begin{aligned} &\tilde{\phi}(B) = \Omega(0)\phi(B)\Omega(0)^{-1}, \\ &\tilde{a}_t \sim \text{iin}[0,\tilde{\Sigma}], \qquad \tilde{\Sigma} = \Omega(0)\Sigma\Omega(0), \end{aligned}$$

and $\tilde{\phi}(B)$ is also of order p.

Using (15) and (16), the system can be expressed in the autoregressive form

$$(18) \qquad \tilde{\phi}(B)\tilde{\Omega}(B)^{-1}x_t = \tilde{a}_t,$$

where $\tilde{\Omega}(B)^{-1} = \begin{bmatrix} 1 & \lambda B \\ 0 & 1 \end{bmatrix}$, and $\lambda = \delta(1+\omega)$.

Considering (18) as the reduced form, the corresponding parameters to be estimated are the 4xp coefficients in $\tilde{\phi}(B)$, λ and the three entries in $\tilde{\Sigma}$.

Structural parameters are the 4xp in $\phi(B)$, δ, ω and the three elements in Σ. Hence, without one further restriction it is not possible to identify all the structural parameters. A natural, though arbitrary, identifying assumption adopted is that $\sigma_{12} = 0$, so that the innovations driving z_{1t} and z_{2t} are contemporaneously uncorrelated. Correspondingly, the feedback between z_{1t} and z_{2t} is assumed to work only through $\phi(B)$. Then, the relationship $\Sigma = \Omega(0)^{-1}\tilde{\Sigma}\,\Omega(0)^{-1}$, following from (17), and $\sigma_{12} = 0$ imply that

$$1+\omega = \tilde{\sigma}_{12}/\tilde{\sigma}_{22}.$$

With ω identified, δ is determined as $\delta = \lambda/(1+\omega)$. Then, $\Omega(0)$ is also known and (17) can be used to determine $\phi(B)$ and Σ.

The testing of the model requires the specification of an alternative model. Since (18) is a constrained vector autoregression of order p+1, the natural alternative is an unconstrained version of the same order. It should be stressed that this test does not require the identifying restriction $\sigma_{12}=0$.

The model was estimated with the following U.S. data. The variable Y_t is measured by real GNP and L_t is total employment times average weekly hours. Since the model is based on a representative agent and constant population, both output and total hours were divided by the working age population (between 16 and 64 years of age). The labor data are from the Current Population Survey, which is a survey of households.

The data are annual and the sample period is 1954-1985[6]. It would have been preferable to use quarterly data, modeling the seasonality as a characteristic of the exogenous shocks. However, the quarterly GNP data are available only in deseasonalized form and hence this route is not feasible. Therefore, the use of quarterly data would have required an arbitrary univariate seasonal adjustment of the hours series. We opted for using annual data, which avoids the seasonality issue.

TABLE 1: ESTIMATED AUTOCORRELATIONS AND PARTIAL AUTOCORRELATIONS FOR Δy_t AND ℓ_t
(Sample 1954-1985)

(i) Δy_t lag/	1	2	3	4	5
Autocorrelations	0.11	-0.18	-0.14	-0.05	0.09
(Standard Errors)	(0.18)	(0.18)	(0.19)	(0.19)	(0.19)
Partial Auto-correlations	0.11	-0.20	-0.10	-0.06	0.06
(Standard Errors)	(0.18)	(0.18)	(0.18)	(0.18)	(0.18)
(ii) ℓ_t					
Autocorrelations	0.49	-0.16	-0.37	-0.21	-0.02
(Standard Errors)	(0.18)	(0.21)	(0.22)	(0.24)	(0.24)
Partial Auto-correlations	0.49	-0.52	-0.01	-0.02	-0.09
(Standard Errors)	(0.18)	(0.18)	(0.18)	(0.18)	(0.18)

Observation of the autocorrelations and partial autocorrelations of Δy_t and ℓ_t in Table 1 indicates that both variables look stationary. Neither the correlations nor the partial autocorrelations of Δy_t are significantly different from zero (using a two sigma rule for significance). For ℓ_t only the first autocorrelation and the first two partial autocorrelations are significant. Since ℓ_t is proportional to z_{1t}, the first two partial correlations for ℓ_t suggest that the productivity shock, when considered in a univariate setting, can be well described as an AR(2) process.

Finally, the order of the VAR polynomial $\phi(B)$, which is identical to the order of $\tilde{\phi}(B)$, was set to 2. Given the apparent second order of z_{1t} and the size of the sample we did not experiment with any higher order processes.

The maximum likelihood estimates of the reduced form parameters in equation (18) are given in table 2, and the estimates of the structural form in table 3.

The estimate of ω indicates a high elasticity of labor supply with respect to the real wage. This elasticity corresponds to $1/\omega \simeq 3.3$. In micro-data studies (see for example MaCurdy (1981) and Ashenfelter (1984)) the estimates of a similar elasticity are much smaller. However, as Heckman (1984) points out, most of the variation in total hours comes from movements in the number of workers rather than in hours per worker. Since the macro-data used here captures the total variation in hours, one would expect a higher estimate for that elasticity. The estimate of δ does not have, to our knowledge, comparable values in the literature. Recall that the capital accumulation equation is $K_{t+1} = A_1 K_t^{\delta} I_t^{1-\delta}$, and hence $1-\delta$ represents the elasticity of the next period capital stock with respect to current investment. A rough comparison, though, can be made with the corresponding elasticity in the standard linear capital evolution equation. In the latter

TABLE 2

REDUCED FORM PARAMETER ESTIMATES

$$\hat{\lambda} = \underset{(0.35)}{0.82}$$

$$\hat{\tilde{\phi}}(B) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \underset{(0.28)}{0.30} & \underset{(0.53)}{0.04} \\ \underset{(0.17)}{0.19} & \underset{(0.23)}{0.72} \end{bmatrix} B - \begin{bmatrix} \underset{(0.69)}{0.18} & \underset{(0.33)}{-0.33} \\ \underset{(0.17)}{-0.20} & \underset{(0.18)}{-0.40} \end{bmatrix} B^2$$

$$\hat{\tilde{\Sigma}} = \begin{bmatrix} 0.00040 & \\ 0.00021 & 0.00016 \end{bmatrix}$$

D.W. equation Δy: 2.05

D.W. equation ℓ: 2.15

TABLE 3

STRUCTURAL PARAMETER ESTIMATES

$$\hat{\omega} = \underset{(0.16)}{0.30}$$

$$\hat{\delta} = \underset{(0.28)}{0.63}$$

$$\hat{\phi}(B) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.78 & 0.04 \\ -0.86 & 0.04 \end{bmatrix} B - \begin{bmatrix} -0.84 & -0.33 \\ 0.64 & 0.45 \end{bmatrix} B^2$$

$$\hat{\Sigma} = \begin{bmatrix} 0.00016 & \\ 0.00000 & 0.00013 \end{bmatrix}$$

case, the elasticity equals the ratio of gross investment to the usual measure of the capital stock. This ratio is in the order of magnitude of 0.1, much smaller than the $1-\delta$ value of 0.37 obtained here. Hence, the present model indicates a higher relative importance of current investment for future productive capacity than in the usual capital accounting.[7]

Finally in this section, the restrictions implied by the model on the reduced form were tested using the alternative of an unrestricted VAR of third order. The likelihood ratio test yields the statistic 1.6 much lower than the 5% critical χ^2 value of 7.8, corresponding to 3 degrees of freedom. Hence, the the theoretically constrained model cannot be rejected in favor of the unrestricted alternative.

5. **Long-Term Impulse Responses and Stochastic Detrending**

The estimated model can be used to calculate the long-run effects on output of current innovations of a given size. In previous work (see Watson (1986) and Campbell and Mankiw (1986)) this type of calculation was performed in a univariate, model-free setting. In the present bivariate context there are two disturbances which have specific structural interpretations. Hence, we can isolate the effects of a_{1t}, the innovation to the production function disturbance z_{1t}, and the effects of a_{2t}, the innovation to the effective capital formation disturbance z_{2t}. One can expect that for any given exogenous persistence of z_{1t} and z_{2t}, the latter will have stronger long-term output effects because it directly affects the efficiency units volume of capital.[8]

The long-term impulse response of a unit size innovation corresponds to the sum of coefficients in the possibly infinite moving average representation of the growth rates. To obtain such a representation one can rewrite equation (18) as

$$(19) \qquad x_t = \psi(B)\tilde{a}_t,$$

where $\psi(B) = \tilde{\Omega}(B)\tilde{\phi}(B)^{-1}$ and $\psi(0) = I$. Let $\psi_{ij}(B) = \Sigma_{k=0}^{\infty}\psi_{ij}^{k}B^{k}$ be the $(i,j)^{th}$ component of $\psi(B)$, i,j=1,2. Then, for output we have

$$(20) \qquad \Delta y_t = \psi_{11}(B)\tilde{a}_{1t} + \psi_{12}(B)\tilde{a}_{2t}$$

with $\tilde{a}_{1t} = (1+\omega)a_{1t} + a_{2t}$ and $\tilde{a}_{2t} = a_{1t}$ (from equation (16)). Hence, (20) can be expressed as

$$\Delta y_t = \left[\psi_{11}(B) + \frac{\psi_{12}(B)}{1+\omega}\right](1+\omega)a_{1t} + \psi_{11}(B)a_{2t}.$$

Since $\psi_{11}(0) = 1$ and $\psi_{12}(0) = 0$ (from $\psi(0) = I$), an innovation in output of, say, one percent corresponds either to $(1+\omega)a_{1t}$ or a_{2t} of the same magnitude (or a combination of both). The calculation of the long-term effect of each one of these shocks requires the values of $\psi_{1j}(1) = \Sigma_{k=0}^{\infty}\psi_{1j}^{k}$, j = 1,2, which take into account the total impact of a current innovation on growth rates over the entire infinite future. These values are calculated using the estimated parameters as $\psi(1) = \tilde{\Omega}(1)\tilde{\phi}(1)^{-1}$.

The long-term effect of the innovation $(1+\omega)a_{1t}$ turns out to be 0.39. For comparison, for a process following a deterministic trend plus stationary fluctuations, the long-term response is zero. In the random walk case the corresponding value is one, since with the long-run level of the variable shifts with the actual level. The value of 0.39 indicates an intermediate case where about 60% of the productivity innovation is "undone" over the future. Hence, production function shocks are mostly of transitory, or cyclical nature.

The situation is very different with respect to the capital innovation a_{2t}. For this shock the long-term response is 1.96. Hence, the effect of a given innovation in output due to this source builds up over time to approximately twice the size of its initial impact.

Whether most of the innovations in output are transitory or not depends on the relative variances of the two types of structural innovations. From the estimates of ω and Σ in table 3 it follows that the variance of $(1+\omega)a_{1t}$ is 0.00027, while the variance of a_{2t} is only 0.00013. Hence, direct innovations to productivity, which have a mostly transitory effect, seem to be the more important source of output innovations.

The model was also used to perform the stochastic detrending of output, which is closely related to the calculation of the long-term impulse responses. The procedure is based on a bivariate counterpart of that suggested by Beveridge and Nelson (1981) as discussed in King, Plosser, Stock and Watson (1986).

The stochastic detrending is achieved by the decomposition of the level of y_t into a permanent and a cyclical component:

$$y_t = y_t^p + y_t^c,$$

where y_t^p is a random walk (plus drift) representing the stochastic trend and y_t^c is a stationary cycle. The decomposition is obtained by rewriting equation (20) as

$$(21) \qquad y_t = \left[\psi_{11}(1)\,\frac{\tilde{a}_{1t}}{1-B} + \psi_{12}(1)\,\frac{\tilde{a}_{2t}}{1-B}\right] + \left[\frac{(\psi_{11}(B) - \psi_{11}(1))}{1-B}\,\tilde{a}_{1t} + \frac{(\psi_{12}(B) - \psi_{12}(1))}{1-B}\,\tilde{a}_{2t}\right].$$

The expression in the first brackets is a random walk and is thus defined as the permanent component y_t^p. It is related to the long-run impulse responses because it represents the long-run level of output (except for the deterministic growth).

The expression in the second brackets of (21) is defined as y_t^c. Using the fact that the residuals $\tilde{a}_{1t}$ and $\tilde{a}_{2t}$ can be written in terms of the observable variables, it is shown in the appendix that y_t^c can be expressed as

$$(22) \qquad y_t^c = \Gamma_1(B)\Delta y_t + \Gamma_2(B)\ell_t,$$

where $\Gamma_1(B)$ and $\Gamma_2(B)$ are finite order polynomials whose coefficients can be calculated from the estimated parameters. Since Δy_t and ℓ_t are stationary, equation (22) implies that y_t^c is also stationary. Once y_t^c is calculated from (22), the stochastic trend can be obtained as $y_t^p = y_t - y_t^c$.

This method of calculating the permanent and the transitory components differs from the procedure suggested by Beveridge and Nelson. That procedure is to calculate y_t^p by recursively producing forecasts of the future levels of the time series, which approach the trend value as the forecast period increases. Here we can obtain and calculate closed-form expressions for the trend and cycle.

Figure 1 depicts the stochastic trend along with the actual values of output. The resulting values for y_t^c and the cyclical movements in total hours, which is ℓ_t itself minus a constant, are plotted in figures 2 and 3. The cycles in hours exhibit troughs in 1958, 1961, 1964, 1971, 1975 and 1982, most of them matching the conventional chronology of business cycles. These fluctuations tend also to be reflected, although roughly, in the cycles of output depicted in figure 2. The importance of the present bivariate procedure for the computation of transitory output movements can be stressed by the fact that in a univariate setting output seems to follow a random walk--see table 1, where Δy_t looks like white noise. Hence, no transitory movements would exist in this case.

To illustrate the model's interpretation of the detrending and impulse response calculations it is of particular interest to consider the markedly different behavior of detrended output and hours in the 1974-1975 episode. Figure 3 shows that hours turn downwards in 1974, and then they decline sharply in 1975. In contrast, the output cycle surprisingly turns upwards in 1974 and only in 1975 it declines (figure 2).

The source of the difference lies in the sharp negative innovation to effective capital accumulation in 1974. Note in figure 5 that the value of

a_{2t} is about -2.6%, the lowest value in the sample[9]. Given the long-run impulse response of 1.96 to capital accumulation innovations (see above) the long-run value of output -- or trend -- declines by about 5% in 1974 (this is the calculated permanent loss of annual output due to that particular negative shock). Now, for the measurement of the temporary component, what matters is that the trend declines by more than actual output (see figure 1), producing the recorded positive deviation from trend in 1974. Then, the observation of 1975 is dominated by the productivity innovation of -4%, also the largest in the sample (see figure 4) which generates a sharp decline in output relative to trend. Recall that for this type of disturbance the long-run effect is 0.39, so that the trend responds by much less than actual output.

6. Concluding Remarks

This paper studied the joint determination of output and hours of work in a real business cycle model with endogenous growth. As in Arrow (1962) and Romer (1986), a main characteristic of the model is a positive externality associated with investment activity. Investment is seen as producing the accumulation of knowledge which in turn increases productivity at the social level. This form of technological progress is thought of as affecting also the productivity in home activities, which produce utility, and hence modelled here as increasing the disutility from labor supply. The latter mechanism is responsible for hours being a stationary variable in the model.

The question of the importance of transitory or cyclical movements in output was addressed by performing a decomposition of output into a permanent and a transitory component in a bivariate, theory constrained, setting. The model interpretes output fluctuations as being caused by two different types of shocks. One is a disturbance to the production function, which turns out to have mostly transitory effects. The other is a shock to the effective capital stock, having large permanent effects on output. The results from the decomposition indicate the presence of large temporary fluctuations, which stand in contrast with results from a model-free univariate setting where the log of output appears to be well described as a random walk.

APPENDIX

In order to prove equation (22), we express equation (18) from the text as

$$\text{(A.1)} \qquad \phi^*(B)x_t = \tilde{a}_t, \qquad \phi^*(B) = \tilde{\phi}(B)\tilde{\Omega}(B)^{-1},$$

and rewrite for convenience equation (21):

$$\text{(A.2)} \qquad y_t = [\psi_{11}(1)\,\frac{\tilde{a}_{1t}}{1-B} + \psi_{12}(1)\,\frac{\tilde{a}_{2t}}{1-B}]$$

$$+ [\frac{\psi_{11}(B)-\psi_{11}(1)}{1-B}\,\tilde{a}_{1t} + \frac{\psi_{12}(B)-\psi_{12}(1)}{1-B}\,\tilde{a}_{2t}],$$

$$= y_t^p + y_t^c,$$

where $\psi_{ij}(B)$ is the $(i,j)^{th}$ component of $\psi(B) = \phi^*(B)^{-1}$. As shown below, the fact that the constant terms are ignored will not affect the generality of the argument.

Substituting the left-hand-side of (A.1) for $\tilde{a}_{1t}$ and $\tilde{a}_{2t}$ in (A.2), one obtains the y_t^p as a function of $(1-B)y_t$ and ℓ_t:

$$(A.3) \qquad y_t^p = \psi_{11}(1)\left[\frac{\phi_{11}^*(B)(1-B)y_t + \phi_{12}^*(B)\ell_t}{1-B}\right]$$

$$+ \psi_{12}(1)\left[\frac{\phi_{22}^*(B)\ell_t + \phi_{21}^*(B)(1-B)y_t}{1-B}\right]$$

$$= \left[\psi_{11}(1)\phi_{11}^*(B) + \psi_{12}(1)\ \phi_{21}^*(B)\right]y_t$$

$$+ \left[\frac{\psi_{12}(1)\phi_{22}^*(B) + \psi_{11}(1)\phi_{12}^*(B)}{1-B}\right]\ell_t$$

$$= \Gamma_1^*(B)y_t + \frac{\Gamma_2^*(B\ )}{(1-B)}\ell_t,$$

where $\phi_{ij}^*(B)$ is the $(i,j)^{th}$ component of $\phi^*(B)$.

Now, from the fact that $\psi(1)\cdot\phi^*(1) = I$ it follows that

$$(A.4) \qquad \Gamma_1^*(1) = 1 \qquad \Gamma_2^*(1) = 0,$$

so that $\Gamma_2^*(B) = \tilde{\Gamma}_2(B)(1-B)$, where $\tilde{\Gamma}_2(B)$ is a finite order polynomial. We thus have:

$$(A.5) \qquad y_t^p = \Gamma_1^*(B)y_t + \tilde{\Gamma}_2(B)\ell_t.$$

Now, using the fact that $y_t^c = y_t - y_t^p$ it follows that

$$\text{(A.6)} \qquad y_t^c = (1 - \Gamma_1^*(B))y_t - \tilde{\Gamma}_2(B)\ell_t.$$

Since $\Gamma_1^*(1) = 1$ it follows that $(1-\Gamma_1^*(B)) = \Gamma_1(B)(1-B)$, where $\Gamma_1(B)$ is a finite order polynomial. We thus have:

$$\text{(A.7)} \qquad y_t^c = \Gamma_1(B)\cdot(1-B)y_t + \Gamma_2(B)\ell_t, \qquad \Gamma_2(B) = -\tilde{\Gamma}_2(B),$$

which expresses the cyclical component as a finite linear combination of present and past values of $(1-B)y_t$ and ℓ_t. If $E[(1-B)y_t] = \mu_y \neq 0$, then one can obtain a zero mean cycle by substituting $(\Delta y_t - \mu_y) = \Delta y_t'$ into (A.7). This proves (22).

FOOTNOTES

1 A drawback of the specification in (2) is that if I_t is zero, K_{t+1} is zero as well. Hence, this functional form can be taken as valid only for $I_t > 0$. In any event, the present model will always generate positive investment.

2 The assumption of proportionality in equation (3) is crucial for generating balanced growth in this model.

3 Since, in general, z_t is not a Markov process, the value function does not depend on z_t only.

4 Comparing (2′) and (10′), it follows that $\tau_{2,1}=(1+\omega)(1-\delta)/(\alpha+\omega)$, $\tau_{3,o}=1$, $A_3=A_k$ and the rest of the coefficients in (2′) are equal to zero.

5 Another interesting feature of the model, which is an aside in the context of this paper, is that the constant in the production function, A_o, appears as a determinant of the <u>growth rate</u> of output (see equation (11)). If A_o is interpreted as the general degree of efficiency in the economy -- i.e., for given values (and units) of K_t and L_t, it determines the level of output -- this setup implies that a more efficient economy will also grow faster. This implication follows from the present version of the endogeneous growth mechanism. Higher A_o implies higher output and hence higher investment, resulting in more future output and so on. Since the capital stock depends positively on A_o, this framework implies that the level of output per worker, or in

general income per capita, should be positively correlated with the growth rate of the economy. This effect does not appear in Lucas(1985) because in his model human capital is augmented by the input of time only, and not of produced resources. In the present model, the opposite case was adopted.

6 See below about the results when the sample includes also the 1948-1953 observations.

7 If the model is estimated including the observations 1948-53, the fit of the model worsens dramatically. In particular the variance of the labor equation almost doubles (from 0.00016 to 0.00029). The main difference in the parameter estimates is that $\hat{\omega}$ declines to about zero. The estimate $\hat{\omega}$ is particularly sensitive because it is identified from the covariance matrix. Also, $\hat{\delta}$ declines from 0.63 to 0.51. The standard error of $\hat{\omega}$ decreases to 0.13 and the standard error of $\hat{\delta}$ increases to 0.52.

8 As mentioned above in section 3, this can be seen in the output equation (11), where z_{2t} has a permanent effect on output but the effect of z_{1t} is partially offset next period according to the parameter δ.

9 In more general terms, one may think of this measured negative innovation in the capital stock as reflecting the drastic increase in oil prices, which rendered part of the capital stock economically obsolete.

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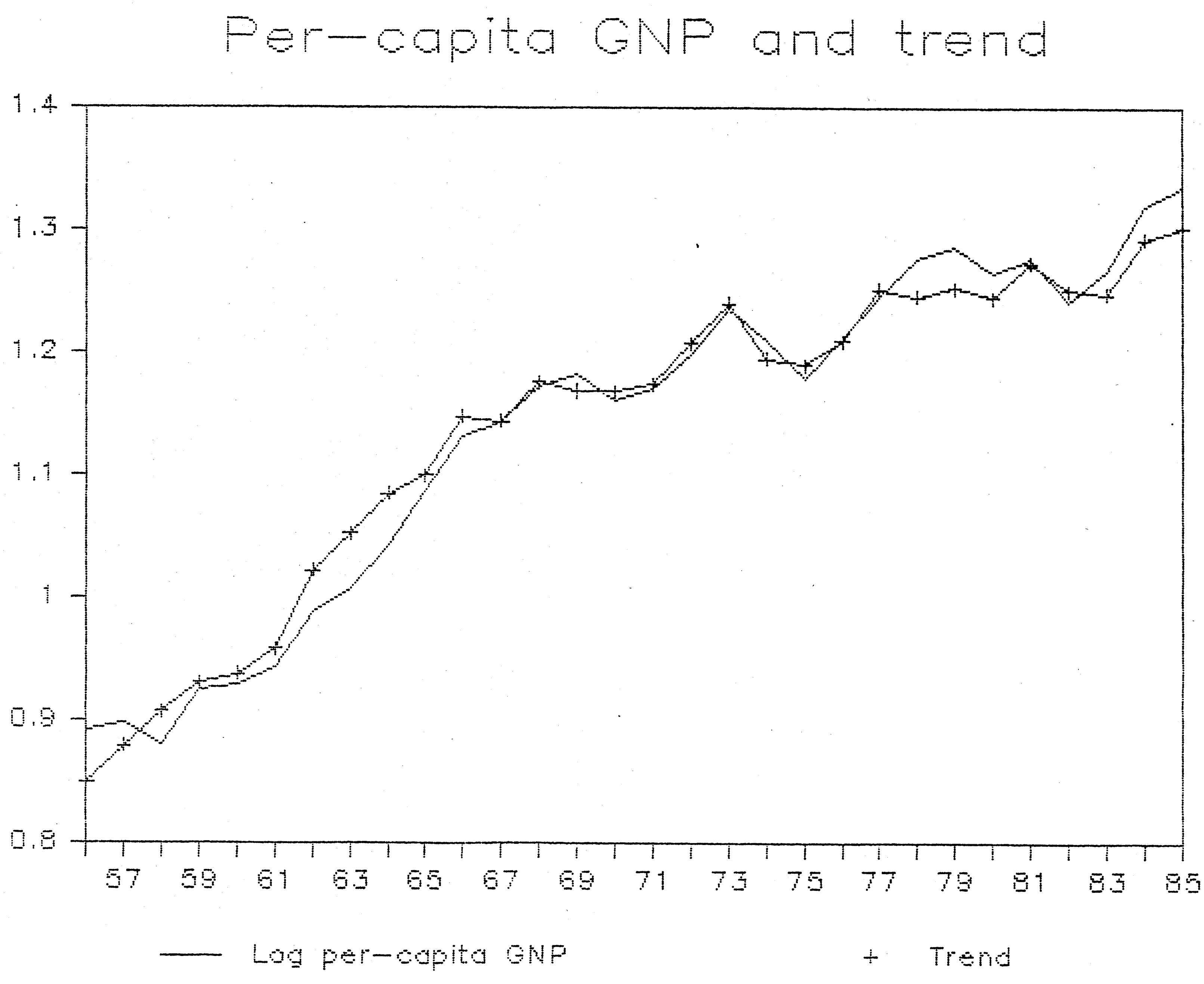
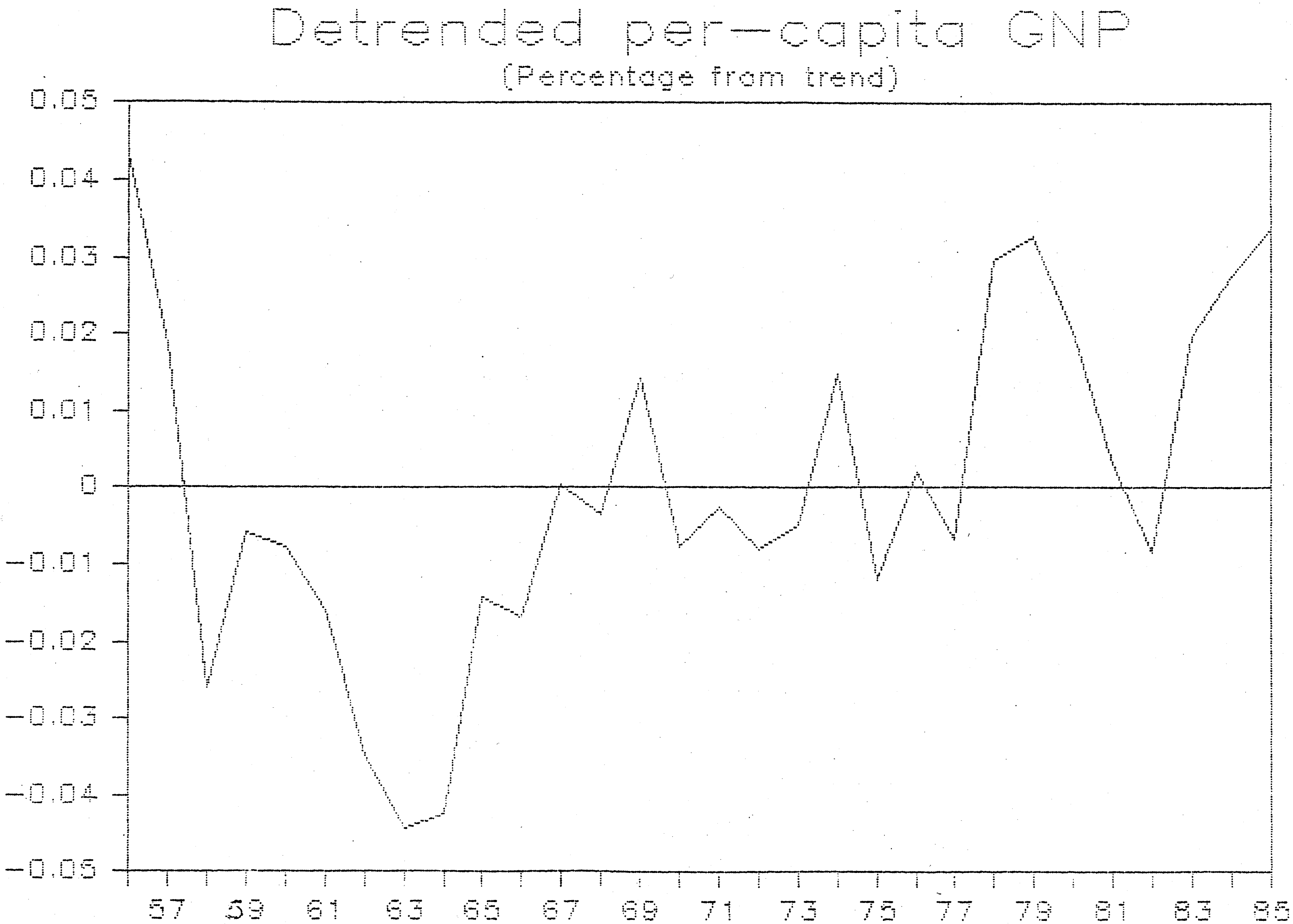


Per-capita GNP and trend
1.4
1.3
1.2
1.1
1
0.9
0.8
57
59
61
63
65
67
69
71
73
75
77
79
81
83
85
Log per-capita GNP
+ Trend

Figure 1



Detrended per-capita GNP
(Percentage from trend)
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
57
59
61
63
65
67
69
71
73
75
77
79
81
83
85

Figure 2

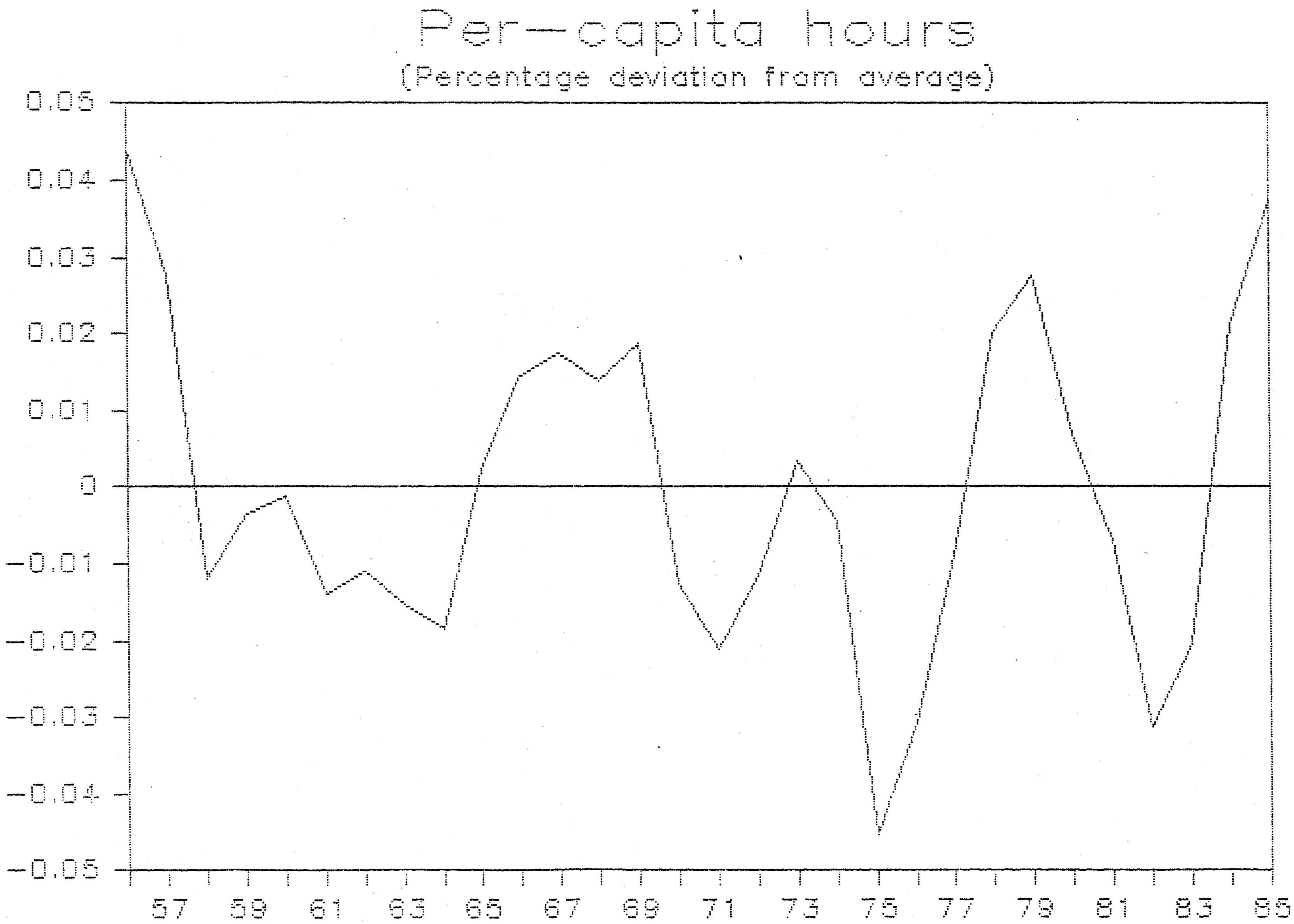

Per-capita hours
(Percentage deviation from average)
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
57
59
61
63
65
67
69
71
73
75
77
79
81
83
85

Figure 3

Figure 4

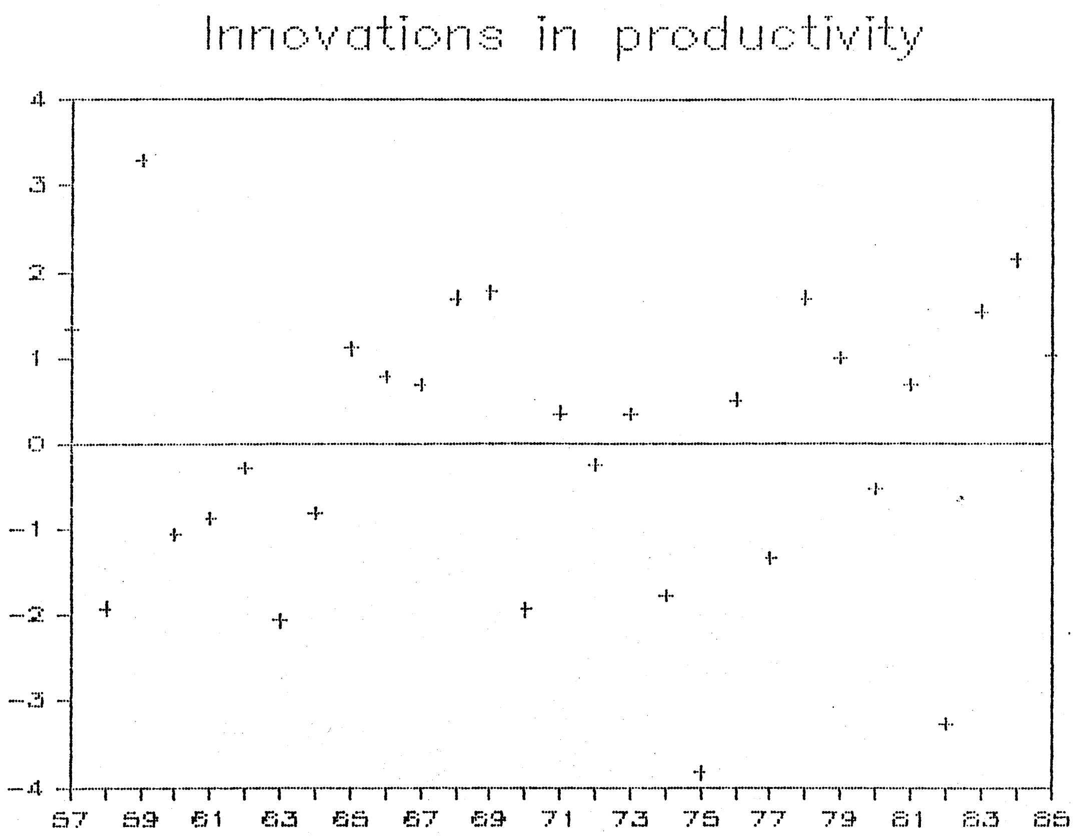


Figure 5