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POPULATION POLICY AND INDIVIDUAL CHOICE

by

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ABSTRACT

In this essay we explore systematically the general equilibrium implications of endogenous fertility for some of the social issues of population policy, including the optimal level or rate of growth of population, real and false externalities, and issues of inter- and intragenerational income distribution. Endogenous fertility simply means that parents care about the numbers and welfare of their children and respond to economic constraints and opportunities in their choices affecting their children and/or that parents bear children as a provision for old age security. It is remarkable that this simple and obvious concept has such far reaching and significant normative implications. What is even more remarkable, however, is that the idea that parents care about their children does not seem to have found any place in the current ethical and philosophical debates about optimal population.

1. Introduction

Since Becker's (1960) analysis, the implications of endogenous fertility in the sense of parental altruism towards their own children, for consumption, labor supply and household employment decisions have been explored extensively in the literature. The purpose of this essay is to examine the general equilibrium implications of endogenous fertility for a number of social issues of population policy. We are thus concerned with the normative rather than the positive implications of endogenous fertility. In our analysis, we adopt the simplest possible formulation: In addition to their own consumption, the number of children and the utility of each child is assumed to enter the utility function of the parents. Thus, subject to whatever economic opportunities and constraints which they face, parents are assumed to maximize their own utility functions (one per couple) in making choices with respect to numbers of children and investments in them. Non-coercive tax and subsidy policies may be devised to affect these decisions; in the absence of such policies, a *laissez-faire* solution will generally exist. We first review some elementary concepts of welfare economics, concerning efficient and optimal allocations of society's resources. We then ask whether the *laissez-faire* solution will be efficient from the standpoint of the present generation, that is, whether individual choice in the absence of social intervention will lead to a Pareto-optimal solution. At this point we digress to consider other, or additional, motives for having children, such as to provide for parents' consumption in old age. Finally, we introduce the notion of an intergenerational social welfare function and ask whether *laissez faire* leads to a social optimum under various criteria and, if not, what non-coercive social policies may be introduced to achieve one.

2. Review of Welfare Economics

Most economic or social policies involve conflicts of interests among different members of society. Most frequently, some gain while others lose by any particular social policy. Thus, welfare economics -- the branch of economics dealing with normative issues — cannot escape the difficult task of weighing the gain of some members of society against the loss to others. For example, in the current debate about tax reform, we often hear the claims that the middle income groups will be better off due to the reduction in tax rates, while high income groups will be the losers as a result of the elimination of various tax loopholes and the imposition of the minimum tax liability. The design of a social policy typically involves interpersonal comparison of utilities (gains and losses).

One criterion which does not depend on such comparisons for different individuals is as follows: a measure which improves the wellbeing of some (or all) individuals without making anyone else worse off is socially desirable. Such an improvement is called a Pareto improvement. Related to such improvements is the notion of Pareto efficiency. An allocation of economic resources (or, more generally, a social state) is said to be Pareto efficient if no Pareto improvement is possible. That is, an allocation is Pareto efficient if no reshuffling of resources can make some people better off without making at least one person worse off. Once the society attains a Pareto efficient allocation, no proposed change can attract a unanimous vote. In general, the term efficiency, as used by economists, means Pareto efficiency.

The question naturally arises as to whether our economic system or other economic systems can attain Pareto efficiency. We can describe our system as a market economy in the following sense: In the market place, self-motivated, utility-maximizing consumers express their demands for various goods and services. Similarly, self-motivated, profit-seeking firms express their willingness to supply these commodities. Competitive markets clear when prices are such that supplies meet demands. It was Adam Smith (1776) who first noted that the outcome of this market clearing process is efficient. He wrote picturesquely about the "invisible hand," which guides the many consumers and firms in a laissez-faire environment, each looking for its own benefit and without coordinating its actions with anyone else, to an outcome which is good for society. In modern language, we say that market allocations, not always but under some conditions to be discussed later, are Pareto efficient.

An elementary explanation of this result is as follows. Suppose there is only one individual. Consider Figure 1 where the demand of this individual for a particular commodity is represented by the curve D = MB. The demand curve (as well as the demand curves for all the other commodities valued by the individual) is the outcome of the maximization of utility subject to the consumer's budget constraint. This constraint is that expenditures on all commodities should not exceed income from all sources. The price along the demand curve indicates the amount of money the consumer is willing to pay for the marginal unit of the good. Therefore, the demand price is just the gross benefit/utility, measured in monetary equivalents, the consumer extracts from the marginal unit. Hence, we can view the demand curve as the marginal benefit curve, and denote it by MB. MB falls as the quantity increases. The sum of the marginal benefits derived from all the units up to a certain quantity, say, Q_0 (more precisely, the area under the demand curve) measures the total gross benefit.[1] For example, the area $DAHQ_1$ is the total benefit the consumer enjoys when consuming Q_1 units of the good.

Figure 1: Consumer's Surplus

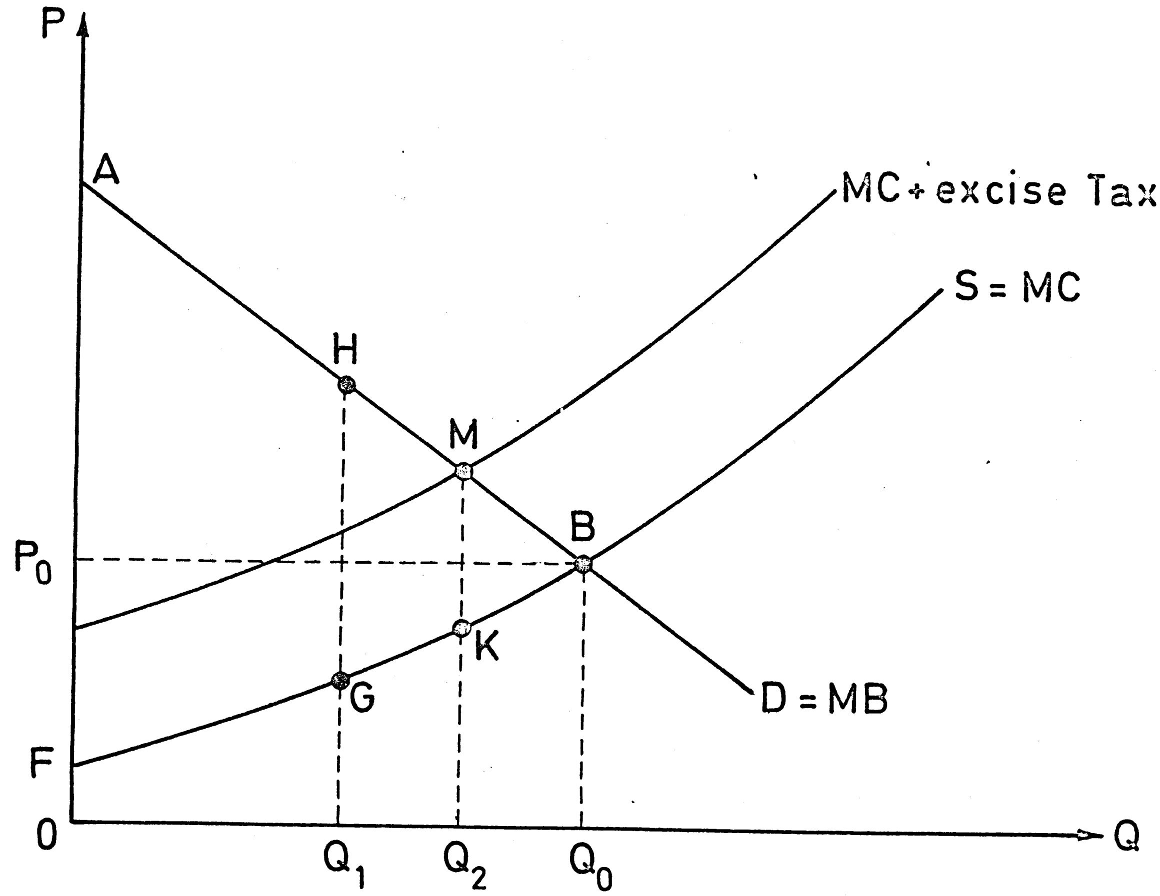


Figure 1 depicts also the supply curve for this good. Elementary microeconomic theory tells us that the supply schedule is just the marginal cost curve which is denoted S = MC. As with marginal and total benefit, the area under the S = MC curve measures total cost. For example, the total cost of producing Q_1 units is given by the area $OFGQ_1$.

The net benefit to society of producing and consuming the commodity in question is the excess of total benefit over total cost and is represented by the area between the demand and supply schedules. This is called <u>net economic surplus</u>. For example, the net economic surplus at Q_1 is FAHG. As can be easily seen from Figure 1, Adam Smith's theorem says that net economic surplus is maximized at the competitive market-clearing price P_0, where MB = MC.

Now, suppose there are many individuals and assume that the curve D = MB represents the aggregate demand (the horizontal sum of individual demands) for the good. Then, the area under the demand curve now represents the total benefits enjoyed by all members of society, and hence the area between the demand and supply curves is the total net benefit accruing to all members of society. In this case we still conclude that total net benefit is maximized at the competitive market-clearing price P_0. This result, which essentially states that competitive market equilibria are efficient, is the first optimality theorem of welfare economics.

Notice also that if the government intervenes in the marketplace by, say, imposing an excise tax, the competitive equilibrium will no longer be efficient. To see this, observe that an excise tax raises the marginal cost curve of the firm by the magnitude of the tax rate, and we obtain a new equilibrium at Q_2. Notice, however, that the cost of production to society does not rise as a result of the tax and hence the marginal cost is still represented, as before the tax, by the old MC curve. Thus, the net economic surplus remains the area between the MC and the MB curves. Therefore, the net economic surplus at Q_2 is only FAMK, which is lower than the competitive surplus, FAB. Society loses KMP, called Harberger's triangle (Harberger, 1964). The excise tax is thus distortionary. The cause of the distortion lies in the wedge that the tax drives between the true (social) marginal cost as given by the MC curve and the marginal cost from the firm's standpoint which includes also the excise tax. On the other hand, a tax which does not drive a wedge between private and social marginal costs or benefit is not distortionary. A lump-sum tax is an example of such a tax which preserves efficiency.

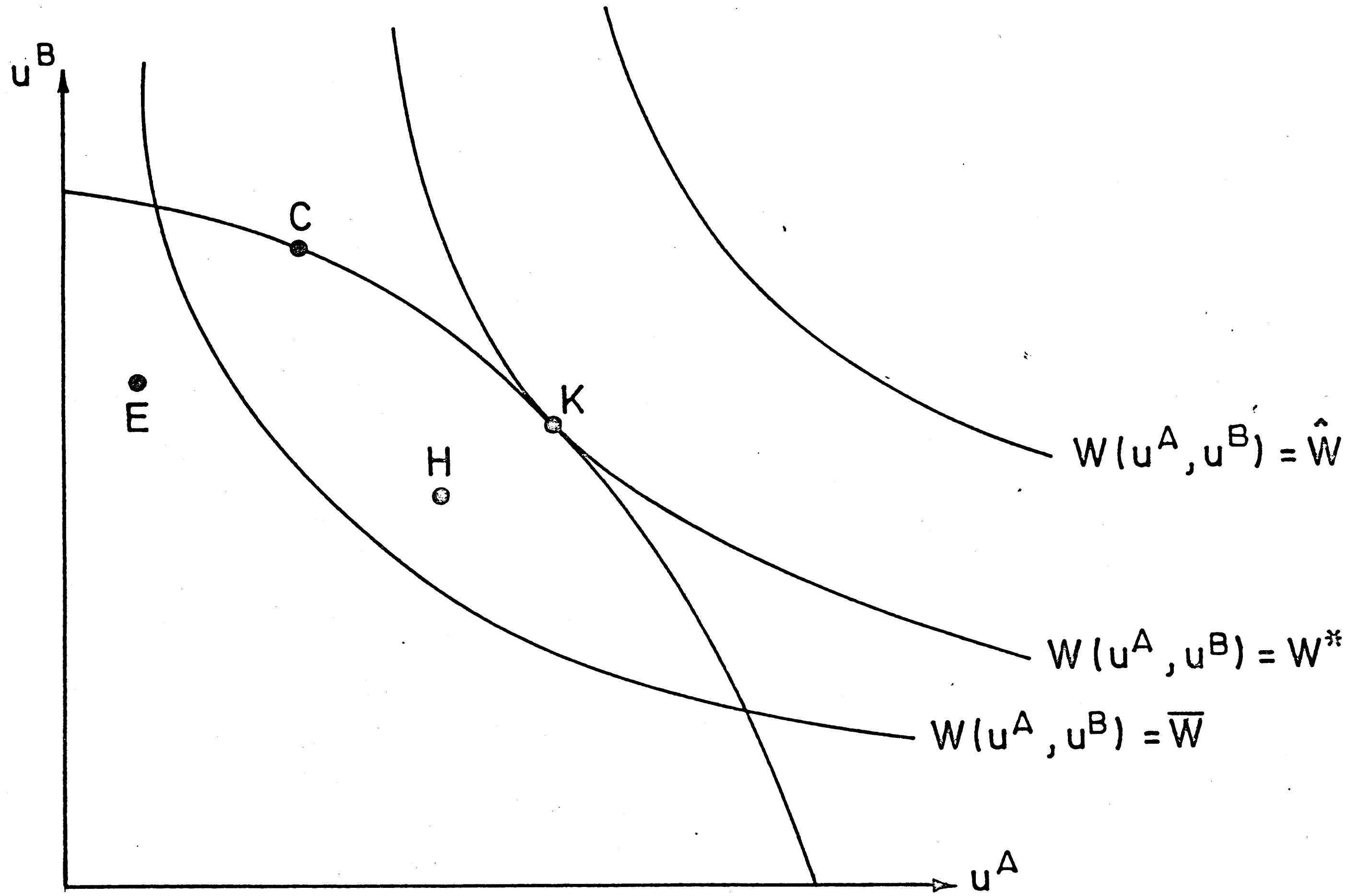
The distribution of the total net benefits among the members of society depends on the underlying distribution of ownership of the society's economic resources (e.g., labor endowments, capital, land, shares in firms' profits, etc.). This distribution of endowments determines the distribution of individual demands. The areas under these demand curves represent individual gross benefits as well as aggregate demand. The demand curves determine the market-clearing price together with the supply curves. A redistribution of endowments will therefore change not only the market-clearing price P_0, but also the distribution of net benefits. The "invisible hand" ensures that the new competitive equilibrium allocation is also Pareto efficient. Thus, by continuously redistributing initial endowments, we obtain a continuum of market allocations which are all Pareto efficient. In fact, under certain conditions every possible Pareto-efficient allocation can be attained by a proper redistribution of initial endowments and by letting the competitive markets clear. This result is the second optimality theorem of welfare economics.

To obtain additional insight into these distributional issues, suppose there are only two persons in society (A and B). Consider Figure 2. A distribution of net benefits (utilities) is a pair (u^A, u^B), represented by some point in this figure. A point representing an efficient allocation is, by definition, a point from which no movements towards the Northeastern boundary are feasible. Thus, the locus of all possible Pareto-efficient points must be downward sloping, indicating that increasing u^A inevitably involves decreasing u^B, and vice versa. The locus of Pareto-efficient points is called the Pareto frontier. It should be clear that any point below this frontier is not efficient and any point above it is not feasible.

Figure 2 also serves to demonstrate the weakness, or the limited usefulness, of the Pareto-efficiency concept in comparing different allocations. For instance, we can definitely say that C is an improvement over E in the sense of increasing a Pareto efficiency, and should therefore definitely be chosen over E. But we cannot resort to the Pareto criterion in order to make a choice between C and H because going from C to H improves the welfare of A, but decreases the welfare of B. The choice between C and H must therefore involve an interpersonal comparison of utilities (weighing the gain to A against the loss to B). In practice, most policy options involve exactly this kind of a choice as between C and H.

In order to make choices involving interpersonal comparisons of utilities, we introduce the notion of a social welfare function, $W = W(u^A, u^B)$, which measures the welfare of society from each pair of utility levels (u^A, u^B). A choice among allocations (pairs of utility levels) may be made according to the level of social welfare (measured by W) that society derives from each pair. In Figure 2, we describe society's ranking over allocations (pairs of utility levels). This is done by drawing a map of social indifference curves. Welfare level is constant along any such curve. Point K represents the highest level of welfare attainable by society. It is therefore called the socially optimal allocation. Note that, by the second optimality theorem, this allocation is also the outcome of *laissez-faire* competition in the market place, provided a certain redistribution of initial endowments is made.

Figure 2



3. Externalities and Public Goods

In the preceding section, the validity of the two theorems concerning the relationship between competitive equilibria and Pareto efficiency is qualified by certain conditions. Principally these are the absence of externalities and nonexistence of public goods.

a. Externalities

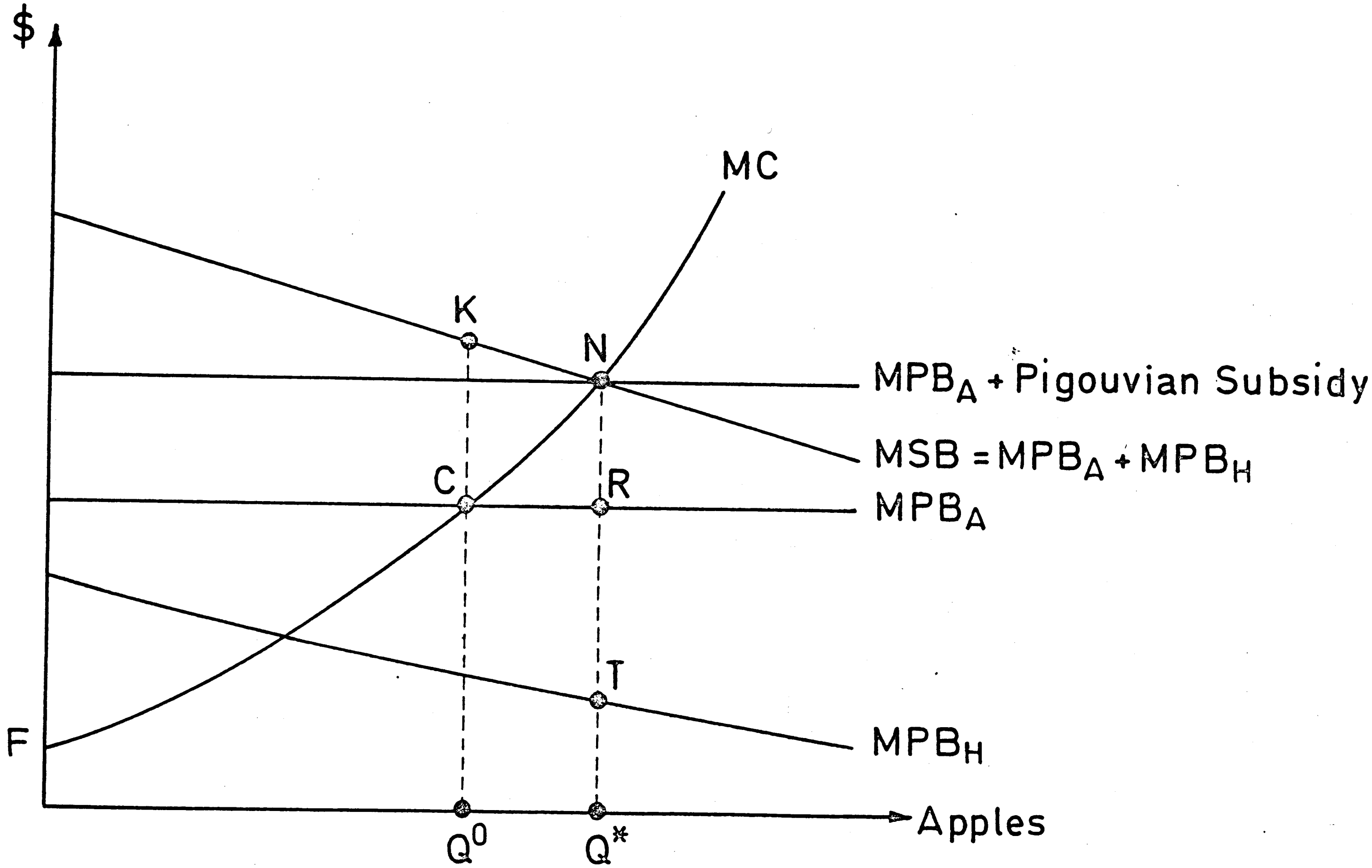
So far, we have implicitly assumed that an action taken by any one of the agents (consumers or firms) in the market directly affects only that agent's utility or profit and that of no one else. In other words, all effects which result from the action of an agent are such that all costs and benefits resulting from the actions of that agent are fully perceived, and accrue to that agent. We say these costs and benefits are fully internalized. In this case, self-interested individual decisions led to an efficient outcome.

However, there are many important instances where actions taken by one agent have effects which are external to that agent. The action of one agent may directly affect the utility or profit of some other agent (or agents). In such cases, we say that externalities exist. Perhaps the most famous example of externalities is that of the fable of the bees (James Meade, 1952). The owner of an apple orchard produces, of course, apples. But as a by-product, apple trees yield also apple blossoms. The neighbor farmer raises bees in order to produce honey. However, his bees consume nectar from apple blossoms. In this case, the apple blossoms are an unpriced input into the production process of honey. The action of the orchard owner generates (via the production of apple blossoms) an external effect on another agent, the honey producer. The apple grower, in this case, does not capture the full benefit arising from his activity because he sells only the apples but not the apple blossoms. His actions, motivated by maximizing his own profit, overlook the benefit to the honey producer. (And, of course, the bees also pollinate the apple trees, which is an unpriced benefit to the apple grower.) In any case, the marginal private beneift (MPB) from apple production (accruing to the orchard owner) falls short of the marginal social benefit (MSB) which is the sum of marginal private benefit of the orchard owner and the marginal benefit accruing to the honey producer. Thus, the action of the apple producer, while "correct" from his perspective is "wrong" from society's standpoint. A market failure occurs: a competitive laissez-faire market fails to achieve an efficient level of output of apples. In the apple blossom/honey bee example, the external effect is beneficial and is, therefore, called a positive externality or an external economy. Of course, in other cases the external effect may be harmful, in which case it is called a negative externality or an external diseconomy.[2]

Figure 3 describes graphically why a market failure arises in the apple blossom/honey bee example. The curve, MPB_A represents the marginal private benefit (revenue) accruing to the apple grower. His marginal cost curve is the curve MC. The competitive decision with respect to how many apples produce is done solely by the apple grower. His private profit is maximized at Q^0 where his marginal private benefit is equal to his marginal cost (his profit is given by the area FEC). Q^0 is thus the competitive output of apples. However, apples also benefit honey production. The marginal value product of apples (via apple blossoms) in honey production is depicted by the curve, MPB_H. Thus, the marginal social benefit of apples is given by the curve MSB which is the vertical sum of MPB_A and MPB_H. The efficient output of apples is, therefore, Q^* where MSB = MC and total net benefit to society is FMN. (Compare this to FMKC, which is the net social benefit at the competitive equilibrium). The cause of the market failure can be easily pinpointed: the apple producer does not take into account (and

Figure 3: The Apple-Blossom/Honey-Bee Example

$NR = TQ^* =$ Pigouvian Subsidy



justifiably so, from his standpoint) the benefit, MPB_H, accruing to the honey producer.

This discussion immediately suggests two kinds of remedies for the market failure. The apple grower can be induced to produce more if his MPB curve is raised by a subsidy. If the per-unit subsidy is equal to $NR = TQ^*$ (which is exactly the marginal value product of apples in honey production at the efficient level of output, Q^*), then the apple grower will produce Q^* because his marginal private benefit curve will be now the curve labelled "MP_B + Pigouvian Subsidy," and it intercepts his MC curve at Q^*. Such a subsidy is called a Pigouvian subsidy.[3] Another remedy is for the two producers to merge. The marginal private benefit of the new firm will be just MSB and it will produce Q^*. In this case the external effect is fully internalized and no market failure arises.

b. Public Goods

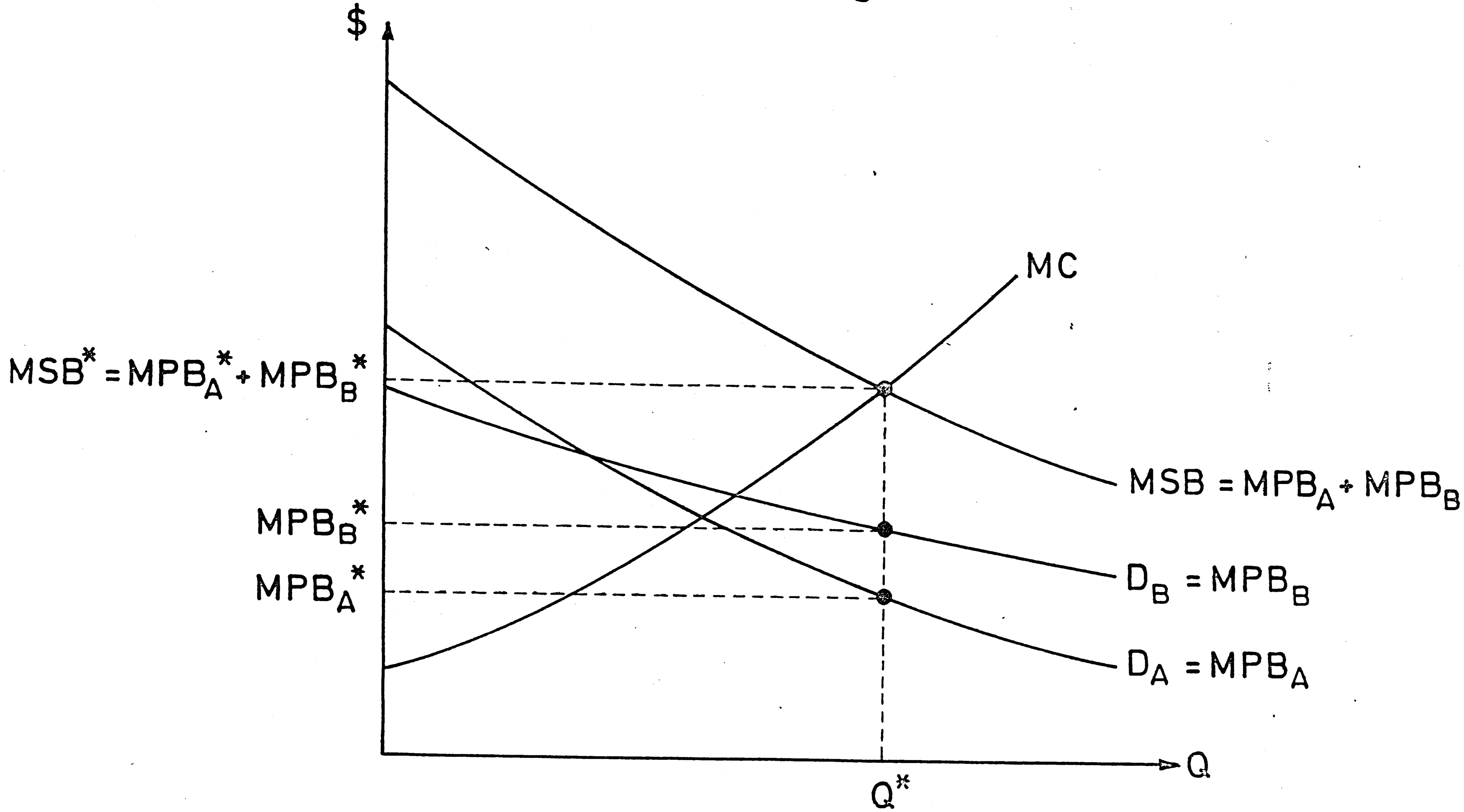
Most goods with which we deal are ordinary private goods in the sense that they can be parceled out among different individuals or, at least, among different families. But there are many examples, such as national defense, television or radio broadcasts, which ". . . all enjoy in common in the sense that each individual's consumption of such a good leads to no subtraction from any other individual's consumption of the good" (Samuelson, 1954, p. 387). Such goods are called public goods.

From the point of view of the individual consumer, a consumption of a public good entails utility in exactly the same way as a consumption of an ordinary private good. Therefore, one can derive, as before, for each individual a demand curve for a public good. This demand curve is also the marginal benefit curve. Consider two individuals, A and B, whose demand curves for a public good are, respectively, the curves $D_A = MPB_A$ and $D_B = MPB_B$

in Figure 4. In this case, any output of the public good is consumed simultaneously by both A and B. Hence, the marginal social benefit (MSB) at each level of output is the (vertical) sum of the marginal private benefit accruing to A (MPB_A) and the marginal private benefit accruing to B (MPB_B). Plotting the marginal cost curve as the curve MC in Figure 4, the efficient level of output of the public good can be seen to be Q^* where the $MSB \equiv MPB_A + MPB_B = MC$. This condition of optimality is known as the Lindahl-Samuelson condition.

Can this outcome be reached in a competitive market? Generally, the answer is no. To induce a firm to produce Q^*, it must be given the price $P^* \equiv MSB^*$. Individuals A and B will both purchase the quantity Q^* if they pay the prices $P_A^* \equiv MPB_A^*$ and $P_B^* \equiv MPB_B^*$, respectively. That is, each individual must pay a price which is equal to that person's marginal private benefit (at the efficient level of output) and the firm must pocket the sum of all the individual prices. Such an equilibrium is called a Lindahl equilibrium. However, the problem of "free riding" makes the Lindahl equilibrium virtually impossible. Usually once the public good is produced, exclusion of an individual from consuming it is prohibitively costly. Realizing this, each individual attempts to free-ride and the firm will not be able to collect very much revenue for its product. Therefore, Lindahl equilibria are not viable. Of course, the government can provide public goods paid for by imposition of taxes. Whether it can provide efficient levels of public goods depends on its ability to measure the true individual valuations (the MPB curves) of the public goods.

Figure 4



4. Endogenous Fertility and Potential Market Failure: False Issues

So far, it has been assumed that utility is derived only from the consumption of ordinary commodities. To come more to the point of this essay, we now allow utility to be generated from a consumption of another "good": children. We assume that parents are altruistic towards their own children so that they extract utility from the number and from the welfare of their offspring. Furthermore, the decision-making of the consumer is now extended to include also the number and welfare of offspring (endogenous fertility). In this section, we examine two potential sources of market failure that may arise when fertility is endogenous.

First, if there are _pure_ public goods, such as national defense, basic research, weather forecasts, etc., the per capita costs of providing these goods fall as the population size is increased. Since all enjoy these goods at no additional cost, it is possible that there exists a market failure in relation to population size resulting in the inefficiency of _laissez faire_. Second, a fixed resource, such as land which must be combined with labor to produce goods for consumption, could lead to Malthusian diminishing returns to a larger population size. This suggests a potential source of external diseconomies and market failure in relation to population size. It is a remarkable fact that neither of these two potential sources lead to market failure when fertility is endogenous; competition leads to Pareto efficiency from the standpoint of the present generation.

Consider for the sake of simplicity, a two-period model with one parent in the first period. We assume a fixed resource, land, and a fixed supply of labor per capita (i.e., no labor-leisure decisions). Land is used in each period, together with labor, to produce a single good which can be used as private consumption (c^i) and public consumption (p^i) in period $i = 1,2$. c^1 is the consumption of the parent in the first period while c^2 is the consumption of each child in the second period. Due to the Malthusian fixed factor (land), there is a diminishing marginal product of labor. Assuming that the labor endowment is one unit, output is f(1) in the first period, where f is a production function, which we assume exhibits diminishing marginal product $f' > 0$ and $f'' < 0$. The parent in the first period bears n children. Therefore, output is f(n) in the second period.

The consumption possibilities of this economy can be described by the following two resource constraints:

$$(1) \qquad c^1 + p^1 + b = f(1).$$

$$(2) \qquad nc^2 + p^2 = b + f(n),$$

where b is the quantity of consumption transferred from the parent in the first period to children in the second period. Constraint (1) states that total output in period 1, f(1), is used for private good consumption, c^1, public consumption, p^1, and bequest, b. Constraint (2) has a similar interpretation. Implicitly we assume that the private good can be stored from the first to the second period without cost. These two constraints are combined to yield a single constraint:

$$(3) \qquad c^1 + nc^2 + p^1 + p^2 = f(1) + f(n).$$

which specifies the overall resource constraint faced by society.

The government provides the public goods in each period and finances them by a lump-sum tax (T) which is imposed on the parent and all of the parent's progeny; that is, the dynasty as a whole. Notice that in our model, a head tax is not a lump-sum tax since the number of children is endogenous; hence, a head tax should be regarded as an excise tax on children. This is the reason for imposing a fixed tax, T, on the whole dynasty rather than a head tax on each of its members. The government budget constraint is written as:

$$(4) \qquad p^1 + p^2 = T.$$

The government is thus restricted to a balanced budget over the whole horizon rather than at each period.[4]

We consider here any <u>arbitrary</u> pair (p^1, p^2) of public good provisions, including also the efficient pair. It can be shown that there is no market failure despite a seemingly non-internalized benefit that a greater population size has a lower cost per capita of providing the public good. Since we are considering <u>any</u> pair of public good provisions, our result holds, therefore, whether or not the government provides the efficient level of public goods.

The parent in period 1 maximizes utility which depends on private consumption (c^1 and c^2), public consumption (p^1 and p^2), and the number of children (n):

$$(5) \qquad u = u(c^1, c^2, p^1, p^2, n).$$

Obviously, p^1 and p^2 are not choice variables by the parent but rather are provided by the government. Thus, the parent's utility maximization is carried out with respect to c^1, c^2, and n, subject to the budget constraint:

$$(6) \qquad c^1 + nc^2 = w^1 + nw^2 + \pi^1 + \pi^2 - T.$$

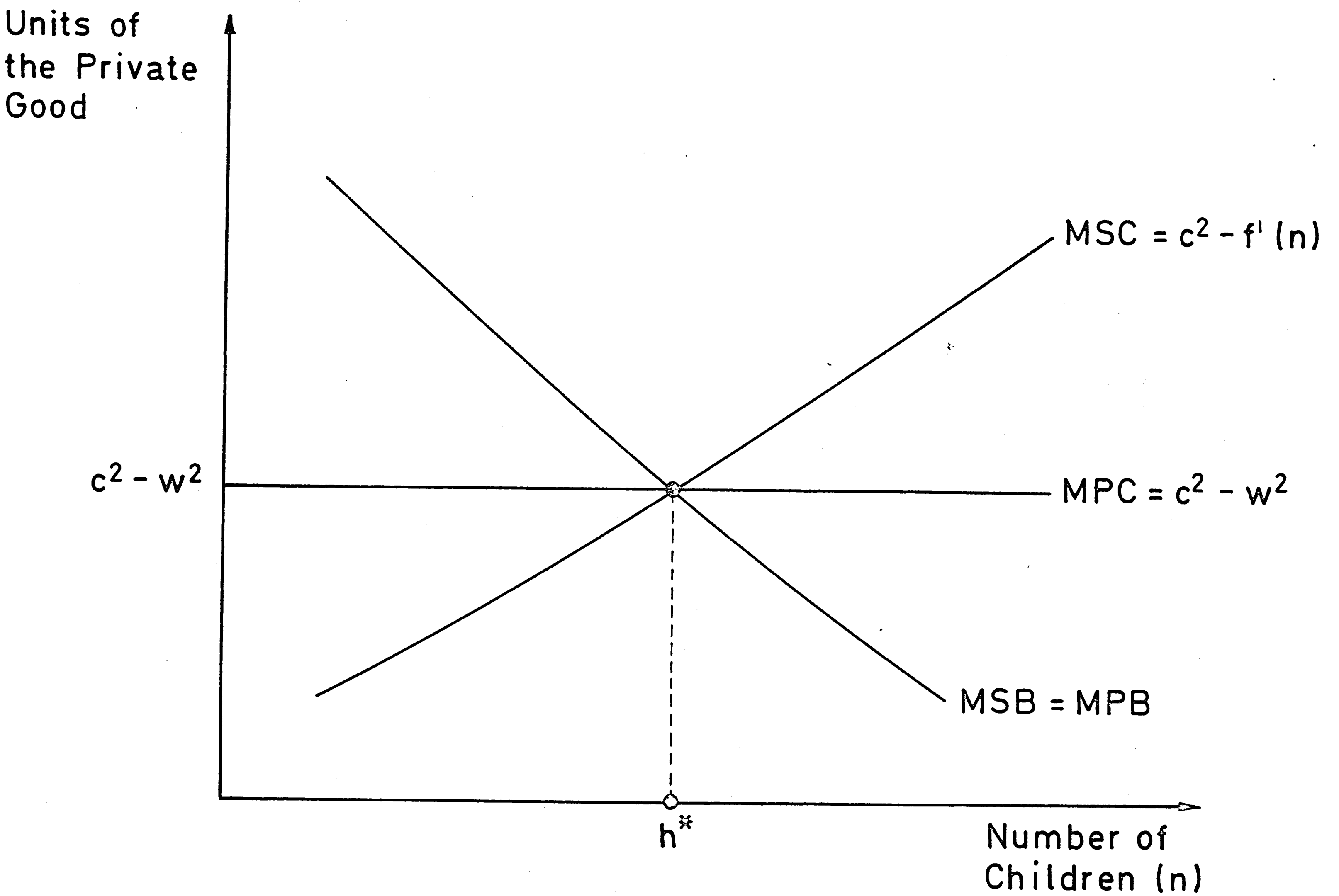
w^i is the equilibrium wage rate in period i, and π^i is the equilibrium land rent (i = 1,2) which accrue to the owners of the land, i.e., the parent in period 1, and the children in period 2. The parent who cares about her children makes plans for their consumption, taking into account their earnings (nw^2) and the land rent (π^2) accruing to them in the second period. The parent also takes into account the entire tax bill (T) of the dynasty. The fact that the children as a group receive both labor income and land rent is really the key to our conclusion.

Now, let us examine whether the competitive equilibrium is efficient. To do this, consider Figure 5. In period 1, society consists of the single parent living then. To focus our attention on the issue at hand, we assume that the objective of society coincides with the objective of this parent which is to maximize the agent's utility (5).[5] Hence, the marginal social benefit (MSB) from the number of children coincides with the private one (MPB). Next, consider the marginal social cost of children. As can be seen from the overall resource constraint of society (3), the marginal cost of n is $c^2 - f'(n)$, where f' is the marginal product of labor. The marginal child consumes c^2 but also produces a marginal product of $f'(n)$. Hence, the parent's net marginal cost is $c^2 - f'(n)$. Since the marginal produce is diminishing, the curve $MSC = c^2 - f'(n)$ is upward sloping. Now, consider the marginal private cost of n. It can be seen from the parent's budget constraint (6) that the marginal child consumes c^2, but also earns a wage of w^2. Hence, the marginal private cost of children to the parent is $MPC = c^2 - w^2$. Since the parent and the children are assumed to behave as perfect competitors, i.e., as price (and wage)-takers, they consider w^2 as a constant parameter. Hence, the MPC curve is horizontal.

Thus, the equilibrium number of children is n^*. The key point to observe here is that profit-maximization requires that labor (children) be employed up to the point where marginal produce is equal to the wage. Since w^2 is the equilibrium wage rate, we must, therefore, have $f'(n^*) = w^2$. Hence, MSC = MPC at the equilibrium number of children, n^*. Therefore, MSC is equal to MSB at n^*, so that n^* is also the efficient number of children.

Thus, public goods and Malthusian fixed land do not cause market failure from the standpoint of the present generation when fertility is endogenous.

Figure 5: Marginal Private and Marginal Social Costs of Children



5. Endogenous Fertility and Potential Market Failures: Real Issues

Although the obvious cases of externalities when fertility is endogenous do not appear to occur, we identify two real sources of market failure arising from bequests in a model in which parents care about their children. In the absence of such care, parents will never transfer (bequeath) anything to their children in a world of perfect foresight and lack of any uncertainty about the time of death.

a. Marriage and Bequests

Consideration of bequests and of marriage suggests a potential source of market failure as follows: If bequests benefit both partners in a marriage (as a public good within marriage), parents may fail to include benefits to other children's parents in deciding on the amount of bequests to make to each of their own children. This suggests that there will be external economies generated by bequests within marriage.

For the sake of simplicity, assume there are only two families in the current generation and only two generations (periods). Let us use the following notation:

c_i = the consumption of the ith family in the first period,

n_i = the number of children of the ith family,

b_i = the per child bequest of the ith family,

k_i = the resources available to the ith family for consumption and bequest, $i = 1,2$.

The total bequest of two children who marry one another will be the sum of the bequests to each child, i.e., $b_1 + b_2$. We assume that this sum is also the consumption of the second generation. For the sake of simplicity, assume that the two families bring the same number of children into the world so that the number of children available to marry each other will be identical.

Consider first the parents of family 1 in the first period. These parents derive utility from their own consumption (c_1), number of children (n_1), and the consumption $b_1 + b_2$ of the newly formed family of each child in the second period:

(7) $$u^1 = u^1(c_1, n_1, b_1 + b_2).$$

Observe that b_2 is bequeathed by the parents of family 2, and is beyond the control of the parents of family 1. Therefore, the latter treats b_2 as a constant parameter. They choose only c_1, n_1, and b_1 so as to maximize (7), subject to the budget constraint.

(8) $$c_1 + b_1 n_1 = k_1.$$

Similarly, the parents of family 2 choose c_2, n_2, and b_2 (treating b_1 as an exogenous parameter), so as to maximize:

(9) $$u^2 = u^2(c_2, n_2, b_1 + b_2)$$

subject to the budget constraint

(10) $$c_2 + b_2 n_2 = k_2.$$

Denote the competitive equilibrium obtained by the above maximizations of the two families by:

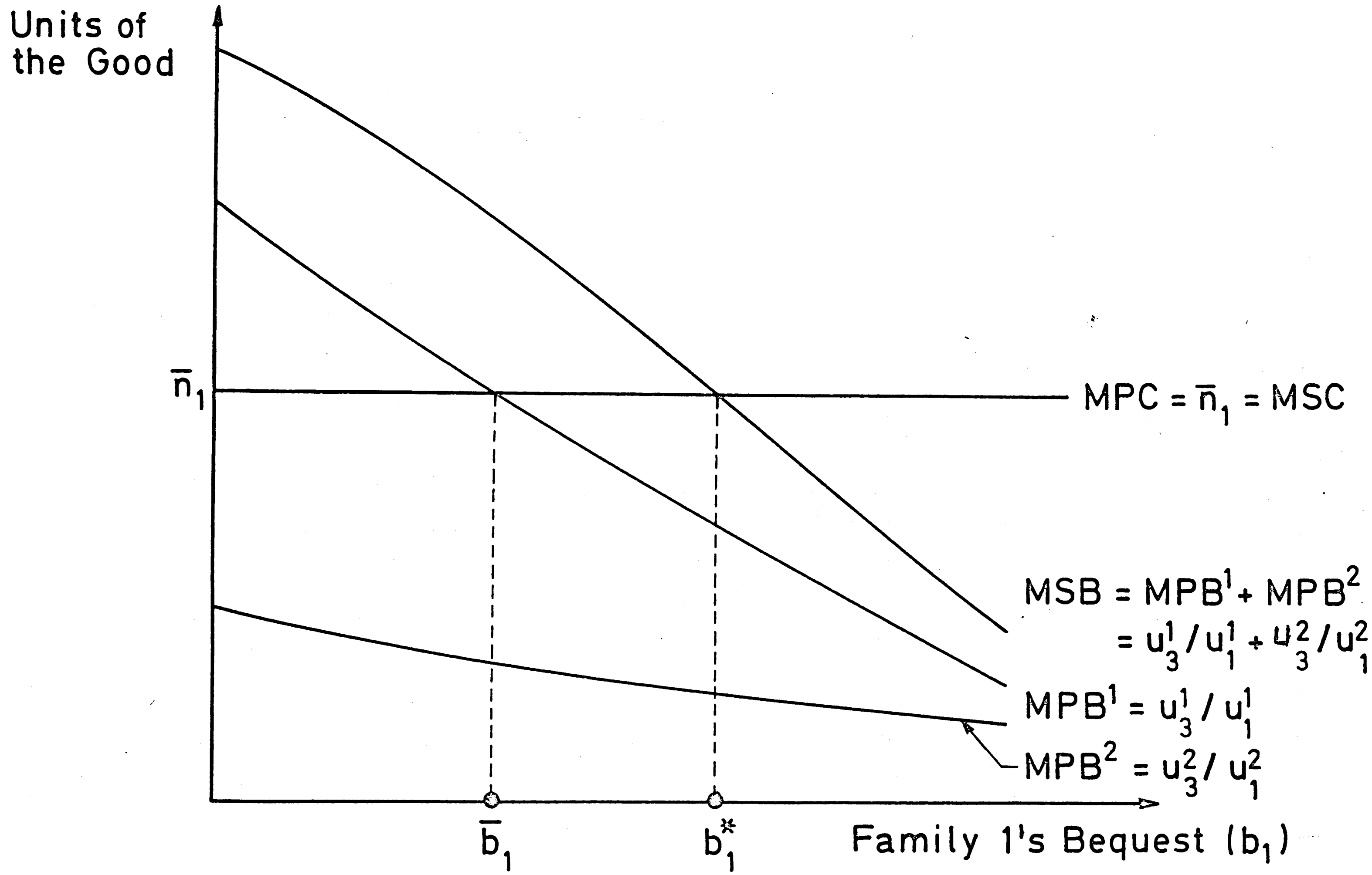
(11) $$\bar{c}_1, \bar{c}_2, \qquad \bar{u}_1 = \bar{u}_2 \equiv \bar{u}, \qquad \bar{b}_1, \bar{b}_2.$$

Now, we examine whether this allocation is Pareto efficient. We show that there are external economies associated with the amounts of bequests and hence the unconstrained equilibrium allocation cannot be efficient. Consider Figure 6 where the per child bequests made by the parents of family 1 (b_1) is plotted on the horizontal axis and the units of the all-purpose good are plotted on the vertical axis. From the budget constraint (8) of parents 1, we can see that the marginal cost of b_1 is $MPC = \bar{n}_1$, because increasing the bequest made to each child by one unit increases total cost by the equilibrium number of children $\bar{n}_1$. Increasing b_1 by one unit increases utility by its marginal utility of children's consumption u_3^1. To express this marginal utility in units of the good, we divide it by the marginal utility of the parents' consumption u_1^1. Hence, marginal private benefit is $MPB^1 = u_3^1/u_1^1$ of the parents of family 1. Therefore, the unconstrained equilibrium amount of b_1 is $\bar{b}_1$, where $MPC = MPB^1$.

Now, let us draw the marginal social cost and benefit curves in the same diagram. The cost of a bequest to society is the same as to the parent, i.e., $MSC = MPB = n$. However, b_1 benefits also the parents of family 2 because it increases consumption of their children who marry the children of the parents of family 1. Hence, the marginal private benefit of b_1 is $MPB^2 = u_3^2/u_1^2$ of the parents of family 2. The marginal social benefit of b_1 is the sum of MPB^1 and MPB^2: $MSB = MPB^1 + MPB^2 = u_3^1/u_1^1 + u_3^2/u_1^2$. Hence, the efficient level of b_1 will be b_1^* and not $\bar{b}_1$. Thus, a market failure exists.

The above discussion also suggests the Pigouvian remedy: MPB^1 should be raised by a proper subsidy to bequests so that it will intersect MPC at the efficient level of bequest b_1^*. The rate of the subsidy (s^*) should be exactly equal to the rate of the external effect $u_3^2/u_1^2/[u_3^1/u_1^1 + u_3^2/u_1^2]$ at the efficient level of bequest b_1^*. A similar subsidy applies to b_2 so as to induce the parents of family 2 to bequeath an efficient amount of b_2.[6]

Figure 6: Marginal Private and Social Costs and Marginal Utility of Bequest

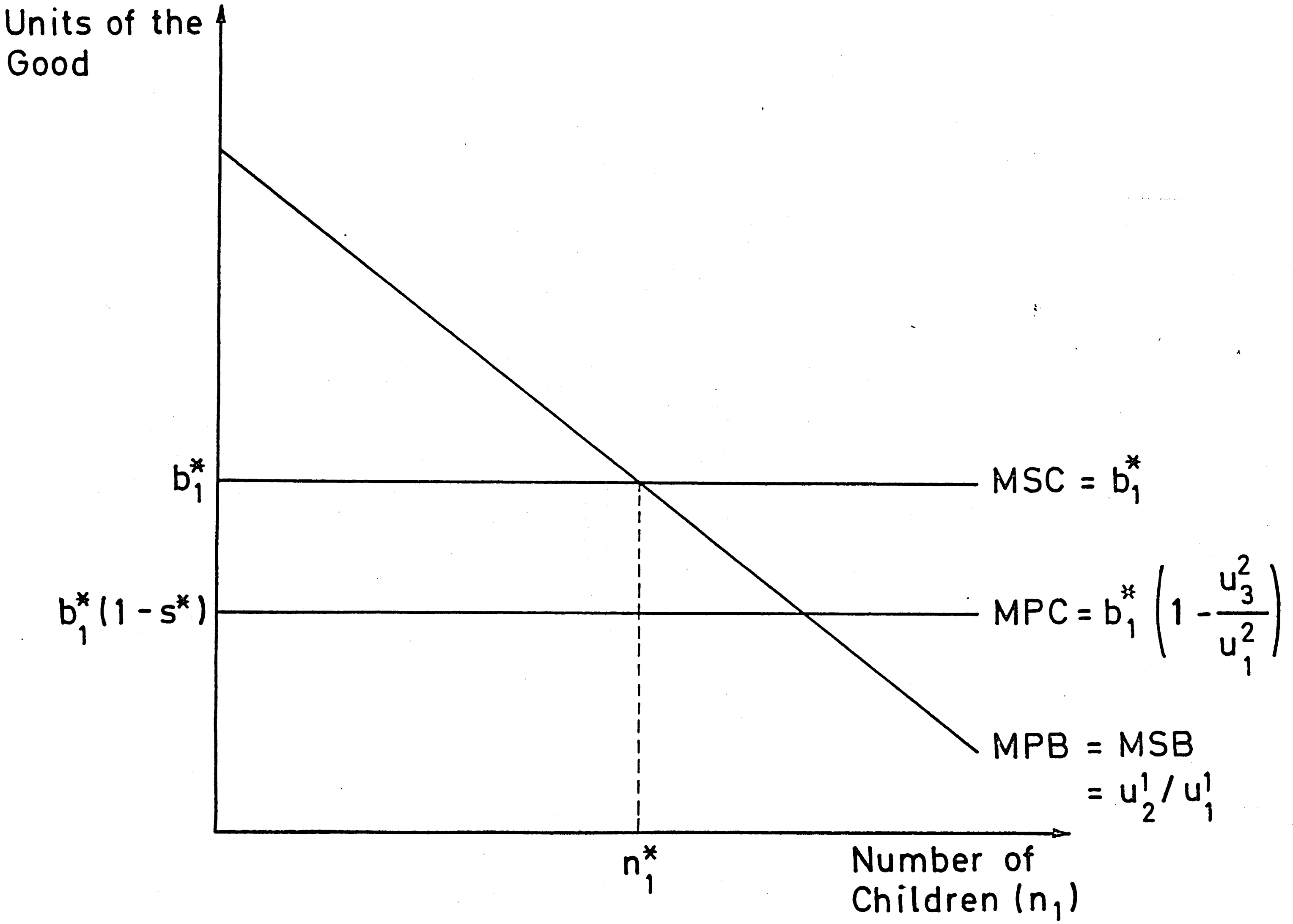


In the standard case of externalities discussed in Section 3, the Pigouvian subsidy was all that was needed. Here, things are a bit different. We can see from the budget constraint (8) or (10) that b is the price of n because each additional child costs his or her parent the amount b bequeathed to him or her.[7] Hence, subsidizing b distorts the parents' decision with respect to the number of children because it lowers their private (but not social) cost. To correct this distortion, we have to tax children directly so that marginal private cost of a child will equal the marginal social cost. To find the appropriate amount of the tax, consider Figure 7 where the optimization with respect to the number of children of the parents of family 1 is considered. The marginal private benefit of n_1 is the marginal utility of n_1, i.e., u_2^1. Expressed in terms of units of the all-purpose good, it is MPB = u_2^1/u_1^1. This is also the marginal social benefit of n_1, MSB, since number of children of the parents of family 1 do not affect the welfare of the other family. The marginal cost of n_1 to the parents of family 1 is the cost of the bequest to them which is MPC = $b_1^*(1 - s^*)$. However, the marginal cost of n_1 to society is just b_1^*, i.e., MSC = b_1^*. Therefore, to induce the parents of family 1 to bear the efficient number of children, n_1^*, we must put a tax on children of $b_1^* s^*$ per child.

b. Transfers among Siblings

When children have different abilities, investments in the human capital of each one are not equally productive. If parents cannot enforce transfers among their siblings, an egalitarian attitude toward children may lead to inefficient investment in human and non-human capital. For example, the parents may be led to invest too much in the human capital of low-ability children so that they will be equal (in utility sense) to their more able siblings. Thus, a second source of potential market failure which we have identified arises from the fact that parents cannot control the actions of

Figure 7: Marginal Private Benefits versus Marginal Private and Social Costs of a Child



their offspring after a certain point. In particular, they cannot enforce transfers among siblings. Parents who care about their children may wish to transfer resources to them. These transfers can take various forms: direct transfers of consumption (bequests), or indirect transfers by investing in the human capital of the children, investments which increase the future consumption-possibility sets of the children, etc. The most efficient method of transfer may depend on the specific characteristics of the child. Thus, parents may wish to use different methods of transfer for different children. Furthermore, it may happen that it will be more efficient to make transfers (via investment in human capital) only to some of the children and force them to transfer later on in life to the siblings who did not receive transfers from the parents. But this possibly most efficient mode of transfers to children depends on the parents' ability to enforce the required transfers among them. This poses a difficulty which cannot be eliminated, for instance, by appeal to Becker's "rotten-kid" theorem or by appeal to vaguely defined social norms (see Becker, 1974, 1976; and Hirschleifer, 1977). Becker and Tomes (1976) note the difficulty but suggest in passing that ". . . social and family 'pressures' can induce . . . children to conform to the terms of implicit contracts with their parents." Such norms might be effective in some circumstances in some societies but they have certainly not generally been effective even in ancient societies (as the Biblical episode of Cain and Abel attests), let alone in modern societies.

The most important case in which equal transfers to siblings are not efficient even for an equity-among-children conscious parent is when children differ in their abilities. In this case it might be most efficient to invest only in the human capital of the able children if parents could guarantee that these children would later on transfer part of the return to this investment to their less able siblings. However, if transfers among siblings cannot be enforced by the parents, then they may not be able to take advantage of high rates of return to investment in the human capital of their more able children. In this case, transfers in the form of investment in human capital from parents to children will be too low relative to bequests in the form of physical capital. Moreover, the investment in human capital will be inefficiently allocated among the children in the sense that the rates of return are not the same for all children.

When ability can be identified by the social planner, he can devise a system of taxes and transfers based on ability in order to achieve an efficient allocation of resources. However, when identification of more able and less able children is impossible or prohibitively costly except for the parents themselves, a Pareto-efficient solution to the problem of optimal investment in human capital and bequests cannot be achieved.

It can be shown that a linear tax on earned income and a subsidy to inheritance are Pareto improvements even though they do not lead to Pareto efficiency. They are, therefore, called second-best corrective policies. Such policies make the parents better off because they redistribute income from able to less able siblings and allow parents to allocate more efficiently investments in human and physical capital which they make on their children's behalf. Other policies, such as public investment in human capital or a tax/subsidy for education, can be shown to be Pareto inferior relative to the laissez-faire solution. Public investment in human capital (e.g., free education) is redundant as long as parents are investing positive amounts in children of all ability because parents can always undo the effects of each policy by reducing their investments in the human capital of their children dollar for dollar. Instead of direct government investment in human capital, we might consider a subsidy to education. Such a subsidy in the first period must be financed by a lump-sum tax in the same period if the government cannot transfer resources from the future to the present. Moreover, it creates a distortion by artificially lowering the cost of education to the parents. Indeed, since it can be shown that parents could have achieved the post-subsidy allocation under laissez faire, it is apparent that the subsidy must be Pareto inferior relative to the laissez-faire solution.

6. Children as a Capital Good

In this section we consider the "old age security hypothesis" which essentially views children as a capital good. In the words of Schultz (1974), "children are ...the poor man's capital" in developing countries. Becker (1960) writes that "it is possible that in the mid-nineteenth century children were a net producer's good, providing rather than using income." Neher (1971) and Willis (1980) develop the idea that parents in less developed countries are motivated, in part, to bear and rear children because they expect children to care for them in old age.

The "old age security hypothesis" states that better access to capital markets unambiguously reduces the demand for children because children are then less essential as a means of transferring income from the present to the future. For instance, Neter (1971) writes that "...the good asset (bonds) drives out the bad asset (children)." We examine this hypothesis and show that it does not generally hold.

a. A Simple Model of Old Age Security with No Capital Markets

Suppose that parents live for two periods. There is a single all-purpose good which is produced by labor alone. Each parent is endowed with an amount of labor capable of producing K_1 units of the all-purpose good. A parent brings children in the first-period, each endowed with an amount of labor capable of producing K_2 units of the goods in the second-period when the child grows up. Each child is allowed a consumption of x_1 units in the first period and x_2 units in the second-period. For the moment, we assume that the parent does not care about the welfare of the children. She merely views them as a capital good intended to provide her with old age consumption in the second period of her life when she can no longer work. Consequently, her utility (u) is assumed to depend only on $\bar{c}$ and $\tilde{c}$, her first-period and second-period consumption, respectively:

(12) $u = u(\bar{c}, \tilde{c})$.

Similarly, x_1 and x_2 are assumed to be exogenously given (say, at conventional or subsistence levels).

The parent can use the output she produces in the first-period for consumption, $(\tilde{c})$, for investment in n children, (nx_1), and for investment in physical or financial capital, (s). Thus she faces the following budget constraint in the first-period:

(13) $K_1 = \bar{c} + nx_1 + S$.

The investment in each child yields a return of $K_2 - x_2$ units of the all-purpose good in the second-period. This return is simply the output of the child less its consumption. We assume that $K_2 - x_2 > 0$, for otherwise it obviously does not pay to invest in children. Thus, the parent's consumption in the second-period, $(\tilde{c})$, is given by

(14) $\tilde{c} = n(K_2 - x_2) + (1 + r) S$,

where r is the real rate of interest.

In this subsection, we assume no capital markets exist so that children become the sole form of capital, and are the only means of transferring consumption from the present to the future. Consequently, we set S = 0.

Solving for n from (13),

(15) $n = \frac{k_1 - \bar{c} - S}{x_1}$,

and from (14)

$$(16) \qquad n = \frac{\tilde{c} - (1+r)\,S}{K_2 - x_2},$$

and equating (15) to (16) (recalling that S = 0) yields the parent's consumption possibility frontier (between $\bar{c}$ and $\tilde{c}$):

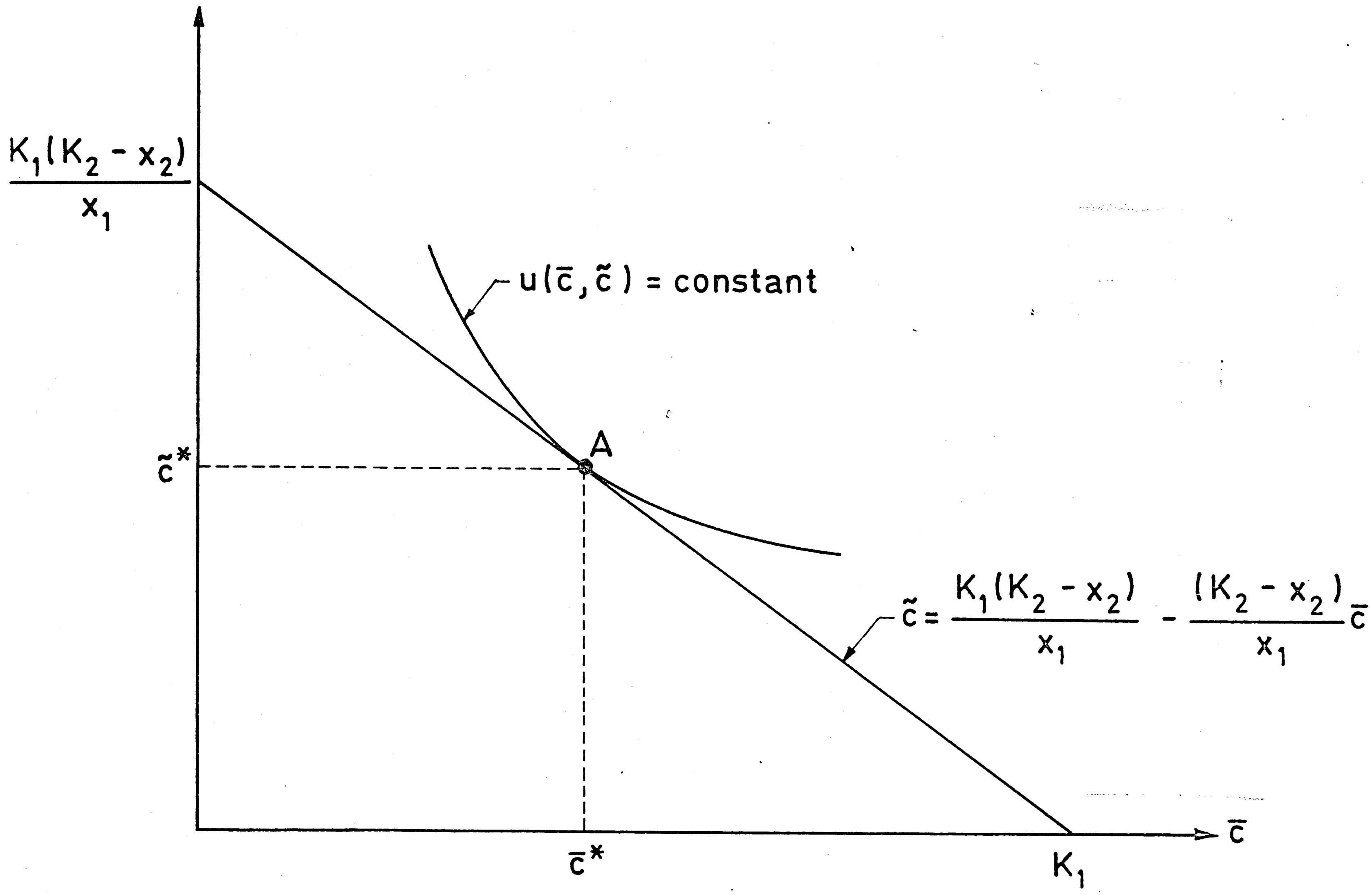
$$(17) \qquad \tilde{c} = \frac{K_2 - x_2}{x_1}(K_1 - \bar{c}).$$

The interpretation of (17) is straightforward: the maximum level of second-period consumption for the parent is achieved by consuming nothing in the first-period, $(\bar{c} = 0)$, and investing all of the endowment, K_1, in K_1/x_1 children which, in turn, yield $(K_2 - x_2)K_1/x_1$ in the second-period. Notice also that $(K_2 - x_2)K_1/x_1$ is the future value of the parent's life-time income measured in units of future consumption. A unit of present consumption, $(\tilde{c})$, is a unit of foregone investment in children and has, therefore, an opportunity cost in terms of future consumption of $(K_2 - x_2)/x_1$, which is the return to a unit of investment in the all-purpose good in children: investing one unit of the all-purpose good in children means bearing $1/x_1$ children who each yield $K_2 - x_2$. The parent's consumption-possibility frontier (17) is depicted in Figure 8.

The parent chooses that point on the consumption-possibility frontier (17) which maximizes her utility function. This is the point, $(\bar{c}^*, \tilde{c}^*)$, in Figure 8 where an indifference curve, $u(\bar{c}, \tilde{c})$ = constant, is tangent to the consumption-possibility frontier. Once the parent chooses the optimal consumption bundle, $(\bar{c}^*, \tilde{c}^*)$, the optimal number of children, (n^*), is determined from (15) or (16):

$$(18) \qquad n^* = \frac{K_1 - \bar{c}^*}{x_1} = \frac{\tilde{c}^*}{K_2 - x_2}.$$

Figure 8: Equilibrium Demand for Consumption in Periods 1 and 2



The following examples illustrating the result may be given:

Example A: Suppose the utility function (12) is of the Cobb-Douglas form

$$u(\bar{c}, \tilde{c}) = \bar{c}^{\alpha}(1 - \tilde{c})^{1-\alpha}, \tag{19}$$

where a fraction, α, of life-time income, $(K_1(K_2 - x_2)/x_1)$, is spent on present consumption, $(\bar{c})$, and the remaining fraction, $(1 - \alpha)$, is spent on future consumption, $(\tilde{c})$. Consequently, the parent chooses:

$$\bar{c}^* = \alpha \frac{K_1(K_2 - x_2)}{x_1} \Big/ \frac{K_2 - x_2}{x_1} = \alpha K_1 \tag{20a}$$

$$\tilde{c}^* = (1 - \alpha) \frac{K_1(K_2 - x_2)}{x_1} \tag{20b}$$

$$n^* = \frac{K_1 - \bar{c}^*}{x_1} = \frac{(1 - \alpha)K_1}{x_1} . \tag{20c}$$

b. A Capital Market with an Exogenously Given Interest Rate and No Borrowing

Let us now drop the constraint that $S = 0$. Suppose there is a capital market in which parents can invest their savings. There is now a means alternative to children for transferring present to future consumption. But no borrowing is yet allowed, i.e., S could now be positive but not negative: $S \geq 0$. The real return to savings is r.[8]

Recall that for the moment the parent does not derive utility from the number or welfare of her children. Therefore, she will not invest in children if the return which they yield, $((K_2 - x_2)/x_1)$, is lower than the alternative return $1 + r$ in the capital market. We assume that K_2, x_2 and x_1 vary across families. Parents for whom x_2 or x_1 are sufficiently high or K_2 is sufficiently low, so that

(21a) $$\frac{K_2 - x_2}{x_1} < 1 + r,$$

will choose to have no children. They will transfer consumption from the present to the future via the capital market (i.e., they will choose a positive S), instead. On the other hand, parents for whom the inequality (21a) is reversed, so that

(21b) $$\frac{K_2 - x_2}{x_1} > 1 + r,$$

would actually like to borrow in the capital market at the low rate of interest, r, in order to invest in the high yield, $((K_2 - x_2)/x_1) - 1)$, asset (namely, children); but they cannot. Hence, they will choose S = 0. These parents are not affected by the introduction of this one-sided (lending only) capital market. Consequently, they will choose the same number of children as before.

Since some families (those for whom (21a) holds) will choose to have no children in the presence of a capital market, while some families (those for whom (21b) holds) will choose to have the same number of children as in the absence of a capital market, it follows that total population must be lower with a capital market than without one. This is the essence of the "old age security" hypothesis.

c. The "Old Age Security Hypothesis" Reconsidered: An Endogenously Determined Interest Rate.

The analysis of the preceding subsection demonstrates that the "old age security" hypothesis is a partial-equilibrium result: the interest rate, being exogenously given, did not clear the capital market; some families had positive savings while some others who wanted to dissave were constrained not to do so. In this section, we introduce a perfect capital market in which people can both lend and borrow and in which the interest rate is determined at equilibrium to clear the market. Total savings equal total dissavings: the total supply of funds of those who save is equal to the total demand for funds of those who dissave.

In this case some families may indeed, as in the preceding subsection, choose to have no children if the capital market offers them a higher yield investment opportunity. But, in contrast to the analysis of the preceding section, some other families may now use the capital market for borrowing in order to further invest in children. Thus, the introduction of a perfect (two-sided) capital market may well increase rather than decrease the number of children, contrary to the "old age security" hypothesis. In the absence of a capital market, all families transfer resources from the present to the future via children. With a capital market, only families with high rates of return on children (relative to the interest rate) continue to use children as a means of transferring resources from the present to the future. The other families--with low rates of return on their children--can also enjoy the high rates of return of the former families by lending to them and letting them invest in children. Thus, families with high rate of return on children invest in children not only for themselves, but also for families with low rate of returns. Thus, the introduction of a capital market allows the economy to use children as a capital good more efficiently. Consequently, the economy may invest more in children. This is exactly what happens in the next example.

Example B: Suppose there are only two types of families (A and B) who both have the Cobb-Douglas utility function (19) of example A above. The two families have the same endowments, (K_1 and K_2), the same second-period child consumption, (x_2), but different first-period child consumption, (x_1^A and x_1^B). We assume that $x_1^A > x_1^B$, so that the return on investment in children is higher for family B than for family A:

$$\frac{K_2 - x_2}{x_1^A} < \frac{K_2 - x_2}{x_2^B}.$$

In the absence of a capital market, we know from Example A that the number of children in each family (see 20c) is given by:

$$(22) \qquad n^{*i} = \frac{(1 - \alpha)K_i}{x_1^i}, \qquad i = A, B.$$

(Note that the family with the higher return on children chooses to have more children.) The aggregate number of children in this case, (N^*), can be found from (22):

$$(23) \qquad N^* = n^{*A} + n^{*B} = (1 - \alpha)\, K_1 \left(\frac{1}{x_1^A} + \frac{1}{x_1^B}\right).$$

Now, let us introduce a capital market with the possibility of both lending $(S > 0)$ and borrowing $(S < 0)$ at the market determined real interest rate of r. We find the equilibrium r at which the savings of one family are equal to the dissavings of the other family. If $1 + r$ is lower than the return on investment in children, $((K_2 - x_2)/x_1^i)$, for some family, then it pays that family to borrow (and invest in children) indefinitely, thereby increasing indefinitely future consumption. But this cannot be an equilibrium. Thus, the equilibrium interest rate, (r^{**}), cannot fall short of the rate of return on investment in children for any family:

$$(24) \qquad 1 + r^{**} \geqq \frac{K_2 - x_2}{x_1^B} > \frac{K_2 - x_2}{x_1^A}.$$

Now, if the first inequality in (24) is strict, then both families have zero demand for children (because the rate of return on children is lower than the rate of interest). In this case, both families will want to save as this becomes the only means of securing a positive level of second-period consumption which is essential (given the Cobb-Douglas specification of the utility function). But we cannot have a capital-market equilibrium when both families save. Thus, at equilibrium, we must have:

$$(25) \qquad 1 + r^{**} = \frac{K_2 - x_2}{x_1^B} > \frac{K_2 - x_2}{x_1^A}.$$

In this case, family A will choose to have no children, $n^{**A} = 0$, because it is better to invest in the capital market. The first-period and second-period budget constraints, (13) and (14), respectively, become now:

$$(13a) \qquad K_1 = \bar{c} + S$$

and

$$(14a) \qquad \tilde{c} = (1 + r)S.$$

These two constraints can be consolidated into one present value, life-time budget constraint:

$$(26) \qquad K_1 = \bar{c} + \tilde{c}/(1 + r).$$

A maximization of the utility function (19), subject to the budget constraint (26), yields the optimal levels of $\bar{c}$, $\tilde{c}$, and S for family A:

$$(27a)\qquad \bar{c}^{**A} = \alpha K_1,$$

$$(27b)\qquad \tilde{c}^{**A} = (1 - \alpha)K_1(1 + r),$$

and

$$(27c)\qquad s^{**A} = K_1 - \bar{c}^{**A} = (1 - \alpha)K_1.$$

Family B is indifferent between investing in the capital market and investing in children (because $(K_2 - x_2)/x_1^B = 1 + r^{**}$). Consumption levels of this family are given by

$$(28a)\qquad \bar{c}^{**B} = \alpha K_1,$$

and

$$(28b)\qquad \tilde{c}^{**B} = (1 - \alpha)K_1\ (1 + r^{**}).$$

For equilibrium in the capital market, family B must dissave, i.e., family B must borrow in the capital market in order to invest in children (because family A has a positive s). Accordingly,

$$(29)\qquad s^{**B} = -s^{**A} = -(1-\alpha)K_1.$$

In order to find the number of children of family B, we substitute (28a) and (29) into the first-period budget constraint, (13), to obtain:

$$(30)\qquad n^{**B} = \frac{K_1 - \tilde{c}^{**B} - s^{**B}}{x_1^B} = \frac{2(1-\alpha)K_1}{x_1^B}.$$

The aggregate number of children (N^{**}) in this case is found from the condition that $n^{**A} = 0$ and (30),

$$N^{**} = n^{**A} + n^{**B} = \frac{2(1-\alpha)K_1}{x_1^B}. \tag{31}$$

Comparing N^* to N^{**}, from (23) and (31), we see that

$$N^{**} = \frac{2(1-\alpha)K_1}{x_1^B} = (1-\alpha)K_1\left(\frac{1}{x_1^B} + \frac{1}{x_1^B}\right) > (1-\alpha)K_1\left(\frac{1}{x_1^A} + \frac{1}{x_1^B}\right) = N^*. \tag{32}$$

Thus, the introduction of a capital market <u>increases</u>, rather than decreases, the number of children, contrary to the "old age security hypothesis."

d. <u>Income and Substitution Effects with Endogenous Fertility</u>

So far it was assumed that neither numbers of children nor children's welfare entered the parents' utility function. If, however, parents do care about their children, which is an essential ingredient of our approach, we can again show that there is no presumption that the existence of a capital market will lead to a lesser demand for children than in its absence.

Suppose, then, that the utility function is

$$u = u(\bar{c}, \tilde{c}, x_1, x_2, n), \tag{33}$$

so that parents care about the number of their children, n, and their children's welfare, which, in turn, depends on the children's consumption, x_1 and x_2. The parents now choose x_1 and x_2 as well as n, $\bar{c}$, and $\tilde{c}$.

In this case, the introduction of a capital market for transferring present to future consumption may plausibly increase the demand for children even in a partial equilibrium setting of the kind employed in subsection b above, where the interest rate is exogenously given and there is no dissaving. This is because a better access to capital markets increases welfare and thus may create a positive income effect on the desired number of children. This effect may dominate the negative substitution effect (as shown in subsection b above) that a better access to capital markets may have on the number of children.

7. Socially Optimal Population Size: Beyond the Pareto Principle

Criteria for a social optimum usually concern choices in which the number and identity of the individuals are given; in this case, although many difficulties of comparability are involved, the criteria are otherwise unambiguous. The classical utilitarian criterion is to maximize the sum of individual utilities:

$$W^B(u^1, \ldots, u^n) = \sum_{h=1}^{n} u^h .$$

We call W^B a Benthamite social welfare function. Since scaling all utilities up or down by a constant multiplicative factor doesn't affect any essential property of W, if n is known, this criterion does not differ from the maximization of average or per capita utility:

$$W^M(u^1, \ldots, u^n) = \frac{1}{n} \sum_{h=1}^{n} u^h .$$

We call W^B a Millian social welfare function. But in a situation in which different choices produce a different population level then the two criteria can lead to different conclusions. For example, suppose that the question is whether to add an additional person to the existing population. If the utility of the additional person called into existence is positive but less than the average of the population in the status quo ante, then adding the person will produce a greater total utility, but a smaller average.

Our purpose here is not to decide the issue of which criteria, or if some other should be used, but rather to compare the two with each other and with the laissez faire solution when fertility is endogenous. We show that the Benthamite social welfare function always leads to a larger population than the Millian criterion, but that the laissez faire solution may yield a population larger than the Benthamite or less than the Millian. We carry out the analysis for a two-generation case, but the result can be extended to any finite number of generations and to an infinite number of generations, provided only that in the infinite-generation case we restrict ourselves to stable population growth paths (so that the relationship between two consecutive generations is always the same).

Consider an economy with two generations, each consisting of just one type of consumer. In the first period, there is only one adult person. She consumes (together with her children) a single private good (c^1). She also raises identical children who will grow up in the second period. She dies at the end of the first period and bequeaths b to each one of her children. The number of children (n) that are born in the first period is a decision variable of the parent living then. The number of persons living in the second period is n. Each one consumes a single private good (c^2). We assume that the parent cares about both the number and welfare of her children. Therefore, we include the children's utilities in the parent's utility function. In a reduced form, we can write the parent's utility as

$$(34) \qquad u^1(c^1, n, u^2(c^2)).$$

u^1 is concave in c^1 and u^2; u^2 is increasing and concave in c^2; both u^1 and u^2 are non-negative (people enjoy positive happiness). u^1 is also increasing in c^1 and u^2, but it is not necessarily increasing in the number of children, n. Assume that the parent lives only one period and that her budget constraint is

$$(35) \qquad c^1 + nb = K; \; c^1, \; n > 0,$$

where K is her initial endowment which is nonrenewable and does not depreciate over time. This is like having an exhaustible resource capable of producing K units of consumption.

The exact specification of the supply side is not very important for this problem although for some issues it would be important to introduce production and capital accumulation.

Assume that children are born with no endowments. Thus, the exhaustible resource has to suffice for the consumption of the current and all future generations. The children's per capita consumption is therefore equal to their per capita inheritance:

$$(36) \qquad c^2 = b.$$

Although we do not restrict the bequest, b, to be nonnegative, it is immediately seen from (36) that it will never be negative. Thus, institutional arrangements which do not allow b to be negative - parents cannot obligate their children to pay their debts - are superfluous here.

Constraints (35)-(36) can be consolidated into one budget constraint for the parent:

$$(37) \qquad c^1 + nc^2 = K.$$

A competitive or <u>laissez-faire</u> allocation (LFA) is obtained when the parent's utility function (34) is maximized with respect to c^1, c^2, and n, subject to the budget constraint (37). Denote this allocation by (c^{1L}, c^{2L}, n^L).

In our model the Benthamite social welfare function is defined by:

$$(38) \qquad B(c^1, c^2, n) = u^1(c^1, n, u^2(c^2)) + nu^2(c^2).$$

As mentioned, it is assumed that there is diminishing marginal utility of c^1 and c^2, i.e., u^1_{11}, $u^2_{11} < 0$, where subscripts stand for partial derivatives. A Bentham optimal allocation (BOA) is obtained by maximizing (38) with respect to c^1, c^2, and n, subject to the economy-wide budget constraint (37). Denote this allocation by (c^{1B}, c^{2B}, n^B).

The Millian social welfare function, namely the per capita utility, is

$$(39) \qquad M(c^1,c^2,n) = \frac{u^1(c^1,n,u^2(c^2)) + nu^2(c^2)}{1 + n} = B(c^1,c^2,n)/(1+n).$$

The Millian Optimal Allocation (MOA) is obtained by maximizing (39) with respect to c^1, c^2, and n, subject to the resource constraint (37). Denote this allocation by (c^{1M},c^{2M},n^M).

It is important to emphasize that we assume that the parent's utility function represents her <u>interest</u> (e.g., happiness from being a parent, guilt relief in providing for the children, etc.) rather than her <u>moral</u> (social) preferences (e.g., believing that it would be wrong to have children and let them starve). This is why we add $nu^2(c^2)$ to $u^1(c^1,n,u^2(c^2))$ when we define Benthamite and Millian social welfare criteria. Otherwise, were we to adopt the second interpretation that parents get no happiness at all from caring for their children, adding $nu^2(c^2)$ to $u^1(c^1,n,u^2(c^2))$, is superfluous. However, in this case we would not have a theory of <u>endogenous</u> fertility.

One would expect that when maximizing total happiness of society rather than average happiness, optimal population size will be larger. This is indeed the case: $n^B > n^M$. To prove this, observe that both the BOA and the MOA satisfy the same resource constraint (37). Since the Millian allocation maximizes M and since M = B/(1+n), it follows that

$$(40) \qquad \frac{B(c^{1M},c^{2M},n^M)}{1 + n^M} > \frac{B(c^{1B},c^{2B},n^B)}{1 + n^B}.$$

Since (c^{1B}, c^{2B}, n^B) maximizes B, it follows that

$$(41) \qquad B\,(c^{1B}, c^{2B}, n^B) > B\,(c^{1M}, c^{2M}, n^M)\ .$$

Therefore

$$\frac{1 + n^M}{1 + n^B} < \frac{B\,(c^{1M}, c^{2M}, n^M)}{B\,(c^{1B}, c^{2B}, n^B)} < 1,$$

from which it follows that $n^B > n^M$.[9]

Since the Millian criterion calls for a maximization of the average utility, intuition suggests that laissez faire results in overpopulation. However, although this may be true under some circumstances, it does not hold in general.

Since the LFA satisfied the same resource constraint (37) as does the MOA, it follows from the definition of the MOA that

$$(42) \qquad M\,(c^{1M}, c^{2M}, n^M) > M\,(c^{1L}, c^{2L}, n^L).$$

Since $M = B/(1+n)$, it is implied by (39) that

$$(43) \qquad B\,(c^{1M}, c^{2M}, n^M) > \left(\frac{1 + n^M}{1 + n^L}\right) B\,(c^{1L}, c^{2L}, n^L).$$

Since $u^2 > 0$, it also follows that

$$\begin{aligned} B\,(c^{1L}, c^{2L}, n^L) &= u^1(c^{1L}, n^L, u^2(c^{2L})) + n^L u^2(c^{2L}) \\ (44) \qquad &> u^1(c^{1L}, N^L, u^2(c^{2L})) \\ &> u^1(c^{1M}, n^M, u^2(c^{2M})), \end{aligned}$$

because (c^{1L}, c^{2L}, n^L) maximizes u^1 subject to the overall resource constraint (37). Thus, we conclude from (43) and (44) that

$$B(c^{1M}, c^{2M}, n^M) > \left(\frac{1 + n^M}{1 + n^L}\right) u^1(c^{1M}, n^M, u^2(c^{2M})),$$

so that

$$\frac{1 + n^M}{1 + n^L} < \frac{B(c^{1M}, c^{2M}, n^M)}{u^1(c^{1M}, n^M, u^2(c^{2M}))}$$

(45)

$$= \frac{u^1(c^{1M}, n^M, u^2(c^{2M})) + m^M u^2(c^{2M})}{u^1(c^{1M}, n^M, u^2(c^{2M}))} = 1 + \frac{n^M u^2(c^{2M})}{u^1(c^{1M}, n^M, u^2(c^{2M}))}.$$

Since the extreme right-hand side of (45) is strictly greater than 1, it is impossible to say anything about the ratio on the extreme left-hand side; in particular, we cannot conclude that $n^L > n^M$.

Since the Benthamite criterion calls for a maximization of total utility of parents and children while the competitive allocation maximizes the parent's utility only, intuition suggests that laissez faire leads to a smaller than Bentham-optimal population. However, this is not necessarily true: when $nu^2(c^2)$ is added to the parent's utility, as suggested by the Benthamite criterion, increasing the product $nu^2(c^2)$ is indeed desirable; but it does not follow that we have to increase both n and c^2.

To see this, observe that it follows from the definition of the LFA and the BOA that

$$u^1(c^{1L}, n^L, u^2(c^{2L})) > u^1(c^{1B}, n^B, u^2(c^{2B})),$$

and

$$u^1(c^{1B}, n^B, u^2(c^{2B}) + n^B u^2(c^{2B}) > u^1(c^{1L}, n^L, u^2(c^{2L})) + n^L u^2(c^{2L}).$$

Hence,

$$n^B u^2(c^{2B}) > n^L u^2(c^{2L}).$$

Thus, indeed the total utility from children (nu^2) must be larger at the BOA than at the LFA. But it does not follow that $n^B > u^L$.

The assumption that fertility is endogenous enables us to consider noncoercive policies aimed at moving the economy from the LFA to either the BOA or the MOA by changing the incentives (prices) which parents face. Notice that the need here for a government action, unlike the externality case, is not warranted because of a market failure. The LFA is indeed efficient. It is located on the Pareto frontier of Figure 2. The government action is needed only in order to move to the socially optimal allocation which is another point on this frontier.

We consider all possible direct and indirect taxes and subsidies as candidates for the optimal policy. Notice that, in our case, children themselves are a "commodity" and may be subject to a tax or a subsidy. As explained in Section 4, a tax which is a head tax is <u>not</u> a lump-sum nondistortionary tax as in the traditional economic literature with exogenous population. Here such a head tax affects fertility decisions on the margin.

Among the set of possible direct and indirect taxes and subsidies to achieve a social optimum, we find that it is necessary to subsidize future consumptions and grant child allowances (positive or negative to encourage or discourage having children). It can be shown that a subsidy to future consumption (c^2) is warranted under both the Benthamite and the Millian criteria; a positive child allowance is necessary under the Benthamite criterion, but the child allowance needed under the Millian criterion may be positive, zero, or negative.

Although the remedies that are needed here are not necessitated by the existence of externalities, we can nevertheless employ a similar apparatus to derive them. The fact that the social welfare function is different from the parent's utility implies that marginal social benefits (as derived from the social welfare function) will, in general, be different than marginal private benefits (as derived from the parent's utility). This is why we can employ techniques here which are similar to those employed in the externality case.

Consider first the BOA. It is obtained by maximizing

$$u^1(c^1,n,u^2(c^2)) + nu^2(c^2),$$

subject to the resource constraint:

$$k - c^1 - nc^2 = 0$$

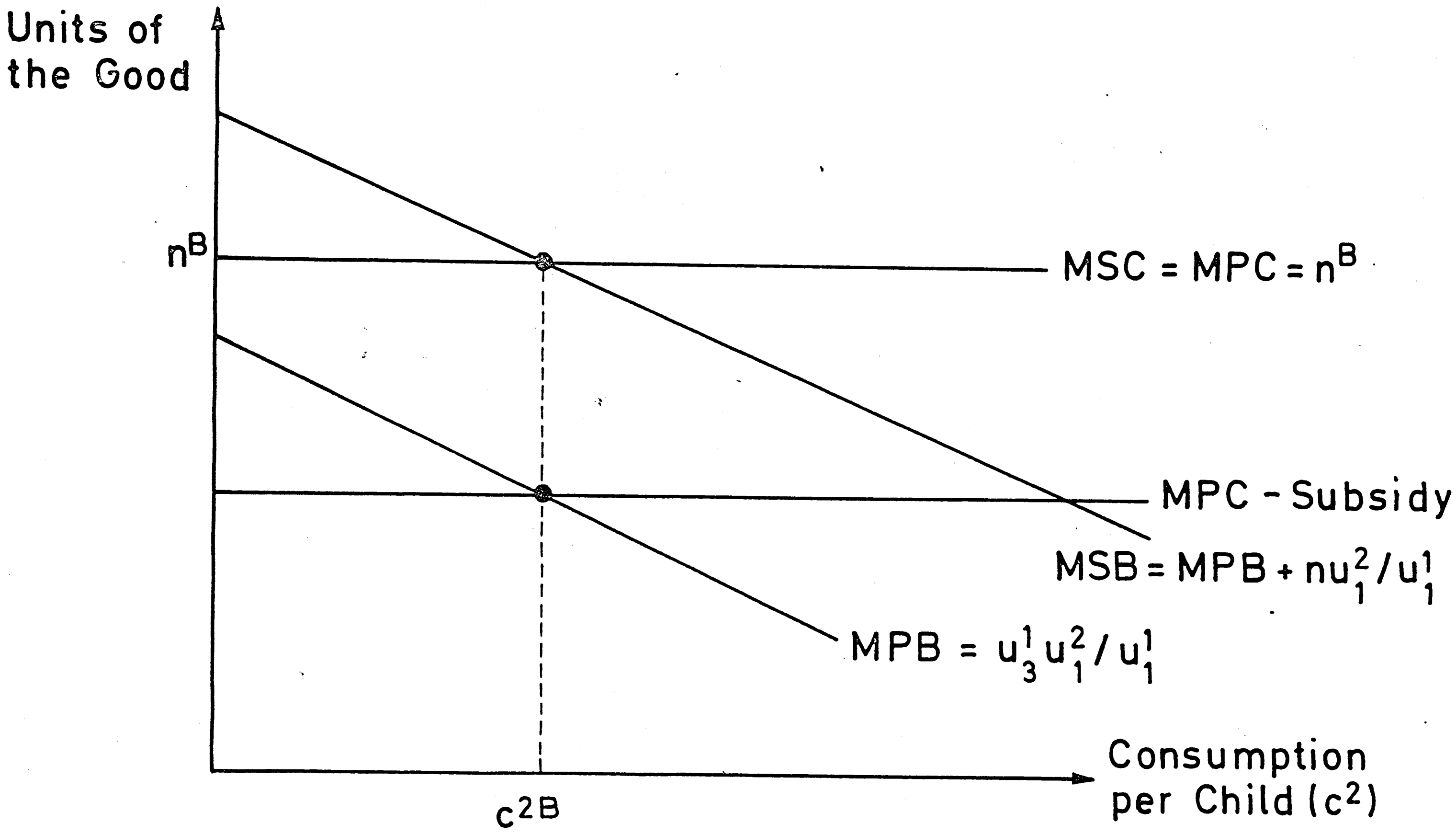
(see (37) and (38) above).

The marginal cost of c^2 is the same for the parent and society. It is derived from the budget constraint (37) or the number of children. The MSC = MPC curve is drawn in Figure 9. However, the benefits differ. For the parent, the marginal benefit is the marginal utility that the parent derives from c^2 which is $u^1_3\ u^2_1$. Expressed in units of the all-purpose good, it becomes MPB = $u^1_3\ u^2_1/u^1_1$. However, society extracts utility from children not only via the parent's ability, but also directly via the term nu^2. Hence, MSB = MPB + nu^2_1/u^1_1. Thus, the Benthamite optimal level of per-child consumption is at c^{2B}, where MSB = MSC. To support this consumption level, the parent's cost of c^2 should be lowered by an appropriate subsidy, so that the MPC curve will intersect the MPB curve at c^{2B}. Similarly, the marginal social benefit of children exceeds the marginal private benefit of children because of the term nu^2 which is added to the parent's utility, u^1. Hence, a child allowance is needed in order to support the BOA. In a similar fashion, one can derive the appropriate measures needed to support the MOA. The exact formulae for the policies needed to support the BOA and MOA are given in Table 1. The derivation of these formulae is in the Appendix.

TABLE 1: OPTIMAL POLICIES

	BOA	MOA
Subsidy to c^2	$u_1^2/u_1^1 > 0$	$u_1^2/u_1^1 > 0$
Child Allowance	$u^2/u_1^1 - c^{2B} u_1^2/u_1^1 > 0$	$u^2/u_1^1 - c^{2M} u_1^2/u_1^1 + M/u^1$

Figure 9: Determination of the Optimal Numbers of Children and Their Consumption with a Benthamite Social Welfare Function



8. The Implications of Endogenous Fertility for Population Policy

There is currently much debate over policies designed to achieve socially optimal population size or rates of growth and, more particularly, what such socially desirable goals should. But little attention has been paid to the question of why we members of the present generation care about future generations. Presumably we do care because each of us, individually, cares about our progeny. If this is the case, it raises the possibility that the result of individual choice, in contrast to a collectively imposed solution, may indeed achieve socially optimal results, at least according to some criteria of social optimality.

A minimal criterion of social optimality is that of so-called Pareto efficiency. We show that individual choice in a free-market context does, in general, lead to an equilibrium which is Pareto-efficient. But there are circumstances which lead to a failure of *laissez faire* to attain an efficient solution from the standpoint of members of the present generation. We illustrate such circumstances in two cases: First, when bequests of parents to their children are a public good within the second-generation family, *laissez faire* leads to market failure because parents may fail to take into account the additional utility which their bequest brings to the parents of their child's spouse. A subsidy to bequests, coupled with a tax on children, can correct this failure. Second, if children have differing ability and parents cannot enforce transfers among siblings, market failure may occur because parents are led to underinvest in their more able children in order to achieve greater equality of consumption among their offspring. Plato's solution (*The Republic*) was to abolish the family. We, however, suggest a less drastic second-best policy, namely a progressive income tax, coupled with a subsidy to the bequest of financial assets. Public investment in education is shown to be redundant and a subsidy to education to be welfare reducing.

To go beyond the issue of efficient allocations from the standpoint of only the present generation, it is necessary to introduce a social welfare function which aggregates, and therefore compares, utilities of the present and future generations. In this essay we consider two: the sum of individual utilities and the average of individual utilities and show that the former always leads to a larger optimal population than the latter. When fertility is endogenous, a laissez faire solution is well-defined, but it cannot be shown to lie between the results produced by the two social criteria or, indeed, to bear any particular relation to them. When fertility is endogenous, a positive subsidy to second-generation consumption, and a child allowance (which may be negative) can be used to achieve a social optimum in a non-coercive, price-based manner.

Footnotes

1. For this statement to be exact, one must assume that the marginal utility of money income is constant so that we can add together marginal benefits which are measured in monetary units.
2. As an example, think about a chemical firm which dumps its waste into a river which, in turn, reduces the catch of a downstream fisherman.
3. In the case of an external diseconomy, the remedy will be a Pigouvian tax.
4. In fact, it does not matter here whether the government is restricted to a balanced budget at each period or only over the whole horizon because the parent cares for his children. As long as the bequest is strictly positive, the parent can always use the bequest to undo any intergenerational distribution of taxes by the government.
5. We discuss alternative social objectives in a subsequent section.
6. Another remedy is to internalize the externality. This can be accomplished if marriages are pre-arranged by parents who also negotiate with each other the collective amount of bequest of both families.
7. Similarly, n is also the price of b.
8. One may envision this as an opening of an access to an international capital market with a given world rate of interest in which the residents of the small underdeveloped country can invest their savings but cannot borrow.
9. This proof is due to T. N. Srinivasan.

Appendix

The BOA is obtained by maximizing

$$u^1(c^1, n, u^2(c^2)) + nu^2(c^2)$$

subject to the budget constraint:

$$K - c^1 - nc^2 = 0$$

Letting $\lambda > 0$ be the Lagrange multiplier, the following first-order conditions for an interior solution may be derived:

$$\text{(A1)} \qquad u^1_1 = \lambda,$$

$$\text{(A2)} \qquad u^1_2 + u^2 = \lambda c^2,$$

$$\text{(A3)} \qquad u^1_3 u^2_1 + nu^2_1 = \lambda n.$$

Dividing (A2) and (A3) by (A1), we obtain

$$\text{(A4)} \qquad \frac{u^1_2 + u^2}{u^1_1} = c^2,$$

$$\text{(A5)} \qquad \frac{u^1_3 u^2_1 + nu^2_1}{u^1_1} = n$$

Equation (A4) asserts that the <u>social</u> marginal rate of substitution of c^1 for n (the willingness of society to give up parent's consumption for an additional child), which is $(u^1_2 + u^2)/u^1_1$, must be equated to the <u>social</u> "cost" of an additional child, which is equal to its consumption c^2. Similarly, equation (A5) asserts that the social marginal rate of substitution of c^1 for c^2 must be equated to the social "cost" of a unit of the child's consumption, which is n, since every one of the n children consumes this unit.

In order to achieve the BOA allocation (via the market mechanism), it may be possible for the government to subsidize c^2 at the rate of α, to give child allowances (possibly negative) of β per child, and to balance its budget by a lump-sum tax (possibly negative) in the amount T. In this case, the parent's budget constraint becomes

$$\text{(A6)} \qquad c^1 + nc^2(1-\alpha) = K + \beta n - T.$$

Given this budget constraint, the parent maximizes

$$u^1(c^1, n, u^2(c^2))$$

by choosing c^1, n, and c^2. Letting $\theta > 0$ be the Lagrange multiplier for this problem, we obtain the following first-order conditions for an interior solution:

$$\text{(A7)} \qquad u_1^1 = \theta,$$

$$\text{(A8)} \qquad u_2^1 = -\theta\beta + \theta c^2(1-\alpha),$$

$$\text{(A9)} \qquad u_3^1 u_1^2 = \theta n\,(1-\alpha).$$

Dividing (A8) and (A9) by (A7), we obtain:

$$\text{(A10)} \qquad \frac{u_2^1}{u_1^1} = c^2(1-\alpha) - \beta,$$

(A11) $$\frac{u_3^1 u_1^2}{u_1^1} = n(1-\alpha).$$

Equation (A10) states that the marginal rate of substitution of c^1 for n (i.e., parent's willingness to give up her own consumption for an additional child) must be equated to the "price" of a child as perceived by the parent from the budget constraint (A6). The "price" consists of two components: (i) the cost of providing the child with c^2 units of consumption which is only $c^2(1-\alpha)$ due to the subsidy α, and (ii) the tax on children which is $-\beta$. Equation (A11) states that the marginal rate of substitution of c^1 for c^2 must be equated to the "price" of c^2 which is the number of children, times $1-\alpha$.

If it is possible to achieve a BOA in this way, we can find the optimal level of α and β by comparing the first-order conditions for the BOA (namely, (A4)-(A5)) with those of the individual parent's optimization problem (A10)-(A11). First, compare (A5) with (A11) to conclude that

$$n(1-\alpha) = n\left(1-\frac{u_1^2}{u_1^1}\right).$$

so that the optimal subsidy to children's consumption under the Benthamite criterion is

(A12) $$\alpha^B = \frac{u_1^2(c^{1B}, n^B, u^2(c^{2B}))}{u_1^1(c^{1B}, n^B, u^2(c^{2B}))}.$$

Next, compare (A4) with (A10) to conclude that

$$c^2 \frac{u^2}{u_1^1} = c^2(1-\alpha) - \beta,$$

so that the optimal child allowance under the Benthamite criterion is

$$\text{(A13)} \qquad \beta^B = \frac{u^2(c^{2B})}{u_1^1(c^{1B},n^B,u^2(c^{2B}))} - \alpha^B c^{2B}.$$

The interpretation of the formulae for α and β is straightforward. Since the term $nu^2(c^2)$ of the Benthamite criterion (38) is ignored by the parent objective (11), we have a case where c^2 generates a difference between private and social evaluations; hence, it ought to be subsidized in order to achieve the BOA. The optimal magnitude of this subsidy has to be determined according to what the parent ignores (at the margin). When the parent considers increasing c^2, she ignores the social benefit nu_1^2 at the margin. This benefit is measured in utility units. Its equivalent in terms of the numeraire consumption good is nu_1^2/u_1^1. From the parent's budget constraint (A6), we can see that if we subsidize c^2 at the rate α, then each unit of c^2 receives a subsidy of $n\alpha$. Thus, the subsidy ought to be set at a level such that $n\alpha = nu_1^2/u_1^1$ which explains the magnitude of the optimal α in (A12).

For the same reason, n ought to be subsidized by u^2/u_1^1, so that the price of n for the parent will be $c^2 - (u^2/u_1^1)$. Since, by the parent's budget constraint (A6), the price of n is $c^2(1-\alpha) - \beta$, we have to equate $c^2 - (u^2/u_1^1)$ to $c^2(1-\alpha) - \beta$. Thus, it follows that $\beta^B = (u^2/u_1^1) - \alpha^B c^2$, as in (A13).

Note that $\alpha^B > 0$. To find the sign of β^B, observe that

$$\beta^B = \frac{u^2}{u_1^1} - \alpha^B c^2 = \frac{u^2 - c^2 u_1^2}{u_1^1}.$$

by substituting (A12) into (A13). Since u^2 is concave, it follows that

$$u^2(c^2) - u^2(0) > u_1^2(c^2)(c^2-0).$$

Since u^2 is assumed nonnegative, it follows that

$$u^2(c^2) > c^2 u_1^2(c^2),$$

so that $\beta^B > 0$: The optimal child allowance under the Benthamite criterion must be positive.

Fixed α and β may not in fact lead to the BOA because the parent's optimization problem is not convex; therefore, the second-order conditions may not hold. In case the second-order conditions do not hold with fixed α and β, it is possible to achieve a BOA with nonlinear taxes, i.e., with instruments α and β which are functions of c^1, c^2, and n. In other words, we can always satisfy the second-order conditions by functions $\alpha(\cdot)$ and $\beta(\cdot)$. The values of $\alpha(\cdot)$ and $\beta(\cdot)$ at the optimum will be exactly α^B and β^B as given in (A12) and (A13), i.e.,

$$\alpha^B = \alpha(c^{1B}, n^B, c^{2B}),$$

and

$$\beta^B = \beta(c^{1B}, n^B, c^{2B}).$$

Exactly the same kind of analysis may be carried through for the Millian social welfare function. In this case, one finds that the subsidy to children consumption, namely α^M, is positive as in the Benthamite case. But the sign of the optimal child allowance (β^M) is ambiguous in this case. The reason for this ambiguity can be seen by comparing the Millian objective, which is $(u^1+nu^2)/(1+n)$, with the parent's objective, which is just u^1. On the one hand, the Millian objective adds nu^2 to the parent's objective and in this way n increases MSB above MPB; but, on the other hand, we also divide $u^1 + nu^2$ by $(1+n)$, and in this way n lowers MSB. Thus, one cannot determine *a priori* whether n should be taxed or subsidized.

References

Arrow, K.J. (1978), "Extended Sympathy and the Possibility of Social Choice," Philosophia, 7: 223-37.

Becker, G.S. (1960), "An Economic Analysis of Fertility," in Easterlin, R.A. (ed.) Demographic and Economic Change. Princeton: Princeton University Press.

Becker, G.S. (1960), "An Economic Analysis of Fertility," in R. Easterlin (ed.), Demographic and Economic Change in Developing Countries. Princeton: Princeton University Press.

__________ (1965), "A Theory of the Allocation of Time," Economic Journal, 75: 493-517.

__________ (1976), "Altruism, Egoism and Genetic Fitness: Economics and Sociobiology," Journal of Economic Literature, 14: 817-826.

__________, and H.G. Lewis (1973), "On the Interaction between the Quantity and Quality of Children." Journal of Political Economy, 81: 279-288.

__________, and N. Tomes (1976), "Child Endowments and the Quantity and Quality of Children," Journal of Political Economy, 84: S142-63.

Coase, R.H. (1960), "The Problem of Social Cost," Journal of Law and Economics, 3: 1-44.

Diamond, P.A. (1973), "Consumption Externalities and Imperfect Competitive Pricing," Bell Journal of Economics, 4: 526-538.

__________, and D.L. McFadden (1974), "Some Uses of the Expenditure Function in Public Finance," Journal of Public Economics, 3: 3-21.

Goldman, S. and H. Uzawa (1964), "A Note on Separability in Demand Analysis." Econometrica, 32: 387-98.

Harasanyi, J. (1955), "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility," Journal of Political Economy, 63: 309-21.

Harberger, A. (1964), "Measurement of Waste," American Economic Review, 54: 58-76.

Hirschleifer, J. (1977), "Shakespeare vs. Becker on Altruism: The Importance of Having the Last Word," Journal of Economic Literature, 15: 500-502.

Lane, John S. (1977), On Optimal Population Paths, Lecture Notes in Economics and Mathematical Systems No. 142. Berlin, Heidelberg, New York: Springer-Verlag.

Malthus, T.R. (1970), An Essay on the Principle of Population and a Summary View of the Principle of Population. Baltimore: Penguin. (Originally published in 1798 and 1830.)

Meade, J. (1952), "External Economies and Diseconomies in a Competitive Situation," Economic Journal, 62: 54-67

Mitchell, W.C. (1912), "The Backward Art of Spending Money," American Economic Review, 2: 269-81.

Neher, P.A. (1971), "Peasants, Procreation and Pensions." American Economic Review, 61: 380-389.

Nerlove, M. (1974), "Household and Economy: Toward a New Theory of Population and Economic Growth," Journal of Political Economy, 82: S200-18.

__________, A. Razin, and E. Sadka (1987), Household and Economy: Welfare Economics of Endogenous Fertility. New York: Academic Press.

Pigou, A.C. (1947), A Study in Public Finance (3rd Edition). London: Macmillan.

Pitchford, J.D. (1974), Population in Economic Growth. Amsterdam: North-Holland.

Rawls, J. (1971), The Theory of Justice. Cambridge, MA: Harvard University Press.

Reid, M.G. (1934), Economics of Household Production. New York: Wiley.

Samuelson, P.A. (1954), "The Pure Theory of Public Expenditure," The Review of Economics and Statistics, 36: 387-389.

__________ (1958), "An Exact Consumption Loan Model of Interest with or without the Social Contrivance of Money," Journal of Political Economy, 66: 467-82.

Schultz, T.W. (1974), ed., Economics of the Family: Marriage, Children and Human Capital. Chicago and London: NBER.

Sato, R. and Davis, E.G. (1971), "Optimal Savings Policy When Labour Grows Endogeneously," Econometrica, 39: 877-97.

Sen, A. (1977), "Social Choice Theory: A Re-examination," Econometrica, 45, 53-89.

Solow, R.M. (1956), "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics, 70: 65-94.

Swan, T.W. (1956), "Economic Growth and Capital Accumulation," Economic Record.

Willis, R.J. (1980), "The Old Age Security Hypothesis and Population Growth," in T. Burch, ed., Demographic Behavior: Interdisciplinary Perspectives on Decision Making. Boulder: Westview Press.