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ASYMMETRIC INFORMATION AND THE ELECTORAL MOMENTUM OF PUBLIC OPINION POLLS

by

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I. Introduction

When confronted with the substantial lead given to Regan by public opinion polls during the 1984 presidential campaign Walter Mondale replied: "Polls do not vote, people vote." Candidate Mondale obviously wanted to convince the public that his true electoral backing was stronger than that suggested by public opinion polls. If nothing else, this remark reveals Mr.Mondale believes that good performance at the polls may reinforce the electoral support of the favored candidate.[1] There is a widepsread popular feeling that some individuals are more likely to vote for a candidate when they perceive the odds in his favor to be better. Such behavior is consistent with the implied view of candidate Mondale about the effect of good performance at the polls on the voting behavior of the public. However, it does not seem to be consistent with the widely accepted notion that the voting behavior of individuals is motivated by the rational calculus of self interest.

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1 The fact that Reagan and his campaign advisors did not try to deny his lead at the polls suggests that they probably hold a similar view.

This paper proposes a rational explanation for the tendency of some groups of individuals to vote more heavily for a candidate the higher the odds, in their view, that this candidate will win. It is based on the notion that the typical individual evaluates candidates not only by their positions on the issues, but also by their relative efficiency in providing the pure public good aspects of government. More generally, candidates differ in some attributes that are considered desirable by all voters. The political science literature refers to such attributes as a valence dimension. Examples of such attributes are integrity, executive ability, compassion and intelligence (Enelow and Hinich (1984), p. 174). In a world of risk averse voters one aspect of the valence dimension of a candidate is the precision of his position. The basic result of the paper applies also to the case in which a candidate's ability or efficiency is conceived solely in terms of this precision.[2/]

Some individuals have more precise information about the relative abilities of candidates than others. For briefness we refer to them as the informed and the uninformed respectively. The uninformed cannot count on the candidates pronouncements concerning their relative ability since each candidate normally claims to be the more efficient one. They rely, therefore, on credible signals that are provided by public opinion polls.[3/] Since those polls partially reflect the information of the informed they convey new knowledge about the relative efficiency of the candidates to the uninformed. High approval for a candidate at the polls signals that he is more likely to be abler. It is, therefore, rational for some of the uninformed to follow the polls and vote for him. However, this information is not perfect since it also reflects changes in the distribution of the public on the issue space. The major result of the paper is that public opinion polls tend to reinforce

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[2/] Models with risk averse voters facing candidates whose positions are not known with certainty have been developed recently by Bernhardt and Ingberman (1984) and Ingberman (1985).

[3/] This is analogous to the widespread notion in economics that market prices perform a signalling function. An example is Grossman and Stiglitz (1980). A good introduction to the effects of asymmetric information in economics appears in Grossman (1981).

the effects of shifts in the public's preferences on the election results. A shift to the right that shows up as more support for the right wing candidate at the polls is partly interpreted as a higher efficiency of this candidate. As a result, he wins with a margin that is higher than the margin he would have obtained either in the absence of polls or under perfect information. Polls reinforce the momentum of the candidate in whose favor public opinion is shifting.

The notion that public opinion polls can influence the outcome of elections is not new. As early as the beginning of the fifties Simon (1954) investigated under what circumstances a published prediction will be confirmed even if there is reaction to the prediction.[4/] In a sense the present paper can be viewed as a rational, informationally based, explanation for the "bandwagon effect" postulated by Simon. More recently McKelvey and Ordeshook (1984) have presented a general equilibrium model in which uninformed voters are uncertain about the positions of candidates and candidates are uncertain about voter's utility functions. They show that in equilibrium "poll" and "endorsement" information reveals all the relevant information to candidates and uninformed voters. In this paper by contrast the information provided by polls is not sufficiently refined to transmit all the relevant information from the informed to the uninformed. As a consequence, the uninformed confound changes in the distribution of voters' preferences with the relative efficiency of the candidates.[5/]

The basic model including the structure of information and voting behavior is presented in Section II. The general effects of a single error free poll (for any distribution of ideal points) on the election outcome are discussed in Section III. The interpretation of those results when ability is measured in terms of the candidates' precision in a world of risk averse voters appears in Section IV. The momentum effects of polls are illustrated and amplified in Section V by considering particular families of distributions of ideal points

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[4/] A similar problem was investigated in the context of public price predictions by Grunberg and Modigliani (1954).

[5/] This is similar to the framework in Grossman and Stiglitz (1980) in which some informational differences between the informed and the uninformed persist even when the market for information is in equilibrium.

and of voters' utility functions. This section also incorporates a poll sampling error and allows sequential learning from a series of successive polls. Concluding remarks and observations on the applicability of the paper's hypothesis follow.

II. The Model and Voters' Behavior

a. Basic Structure

Administrations and candidates differ from each other not only in their positions over the issue space but in their ability to provide efficient government as well. A candidate takes a position on a unidimensional issue space. Voters judge him both by his position and by his ability to deliver the pure public good aspects of government. Formally the benefit or utility that voter i gets if candidate j is elected is

$$r_{ji} = w(x_j) - L(|p_j - p_i|), \; w' > 0, \; L' > 0 \qquad (1)$$

where p_j is the position of candidate j, p_i is the ideal position of voter i and w' and L' are, respectively, the first derivatives of w and L with respect to their arguments. x_j is a measure of the candidate's ability to provide the public good. The better his ability the higher uniformly is the utility all voters get from the candidate. This formulation of voters' utility captures the notion that administrations differ not only in the positions they take on issues but in their ability to deliver the public good as well. As a consequence, the typical voter's evaluation of a candidate depends both on the candidate's position and on his ability.

There are two candidates indexed by 1 and 2. Their positions are public knowledge but their abilities and, therefore, the terms x_j are imperfectly known. In particular, x_j, j = 1, 2 are random variables whose distributions are public knowledge. There are two types of individuals. The informed who know the actual efficiencies, x_j, of the candidates and the uninformed who only know the distributions of x_j, j = 1, 2.

Let f(p) be the density of voters with ideal point p in the population. Obviously

$$f(p) \geq 0, \qquad \int_{-\infty}^{\infty} f(p)dp = 1. \qquad (2)$$

Elections are held at discrete intervals and the distribution of ideal points usually changes from one election to another. Those changes reflect shifts in the structure of the voting population due to new voters and deaths as well as changes in the ideal points of individuals who remain in the voting population from one election to another.[6/] Shifts in the distribution of ideal points probably occur by small increments continuously through time. Individual voters know how their positions change but do not know, a priori, how the ideal points of other voters have changed. However, periodic public opinion polls provide at least partial information on the shifts that occur in the distribution of ideal points. This is probably the main reason for the existence of such polls. This state of affairs is approximated here by assuming that the distribution of ideal points changes between each preelection period and the subsequent one but not within each of those periods.[7/] During each such period one or several public opinion polls are conducted.

Let k_I and k_U be, respectively, the proportions of informed and uninformed individuals in the population. Obviously $k_I + k_U = 1$. I assume that the distribution of voters across those two groups is independent of their distribution by ideal points so that the density of informed voters with ideal point p is given by

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[6/] Dramatic shifts in the public's preferences along the liberal conservative dimension occurred in 1932 when Roosevelt was elected and in 1980 when Reagan was first elected. Romer and Rosenthal (1984) document the strong shift to the right that occurred in the U.S. senate in 1980. Presumably this shift was induced at least partly by a shift to the right in the voting population.

[7/] This is a reasonable approximation provided changes in the distribution of ideal points occur gradually through time and provided the interval between elections is sufficiently large in comparison to the length of the preelection period during which the public is intensively exposed to public opinion polls. Both conditions seem to hold for the U.S. political system. In any case the main result of the paper does not depend on this assumption.

$$k_I\ f(p) \tag{3}$$

and similarly for uninformed voters.

b. Voters' Behavior

Whether they respond to a public opinion poll or actually vote individuals cast their ballot in favor of the candidate that delivers the highest expected utility. More precisely individual i votes for candidate 1 if and only if the expected utility from this candidate, given the information available to the individual, is larger than the expected utility from candidate 2. This condition is given for both informed (I) and uninformed (U) voters in equation (4)

$$y \equiv w(x_1) - w(x_2) > L(|p_1-p_i|) - L(|p_2-p_i|) \equiv -D(p_i), \quad i \in I \quad (a)$$

$$(4)$$

$$E(y|F_U) > L(|p_1-p_i|) - L(|p_2-p_i|) \equiv -D(p_i), \quad i \in U \quad (b)$$

where $E(y|F_U)$ is the expected value of y given the information set, F_U, of the uninformed. Equation (4) states that a voter may prefer candidate 1 to candidate 2 even if the latter is nearer to his ideal point on the issue space provided he perceives the first candidate to be sufficiently more efficient than the second. y measures the portion of the utility differential from the candidates that is due soley to their different abilities. I will refer to it as the efficiency induced utility differential or as the efficiency differential for brevity. For informed individuals the perceived efficiency differential is equal to the actual differential. For uniformed individuals it is equal to their best guess of the differential given the information available to them at the time. Before any poll is taken the only useful information available to an uninformed individual is the distribution of the efficiency induced utility differential. This distribution is induced by the underlying distribution of x_j, j=1,2 and by the form of the w(.) function.

Let

$$h(y) \tag{5}$$

be the probability density of y before any poll is taken. Hence, before any poll is taken the efficiency induced expected utility differential between the candidates as perceived by the uninformed is

$$\mu_y \equiv {}_y\int yh(y)\, dy \tag{6}$$

If both candidates were always identical in their ability, the typical voter's utility differential from the two candidates would result only from the difference in the positions of the two candidates in the issue space. This differential is given by the term $-D(p_i)$ on the right-hand-side of equation (4). More precisely $D(p_i)$ is the difference between the utility that individual i gets from candidate 1 and the utility that he gets from candidate 2 when there are no differences in ability between the two candidates. We shall refer to it as candidate's 1 positional advantage with voter i. Obviously this advantage depends on the individual's ideal point. $-D(p_i)$ is the positional advantage of candidate 2 with voter i. Equations (4) state that an individual voter will vote for candidate 1 when the efficiency induced excess expected utility from this candidate is larger than the positional advantage of the other candidate.

In order to characterize the voting behavior of different individuals it is convenient to characterize the relationship between the positional advantages a candidate has with different voters. For that purpose it is assumed without loss of generality that

$$p_1 \leq p_s < p_i \leq p_2 \tag{7}$$

It is intuitively plausible that the positional advantage of the left wing candidate (1) with the more liberal voter(s) is larger than his positional advantage with the more conservative voter (i). This is formalized in the following proposition.

<u>Proposition 1</u>: For any two voters s and i whose ideal points satisfy condition (7)

$$D(p_s) > D(p_i)$$

Proof: Since $L' > 0$ and $p_s < p_i$

$L(|p_s - p_2|) > L(|p_i - p_2|)$ and $L(|p_s - p_1|) < L(|p_i - p_1|)$

The proof is completed by using those two inequalities in the definition of $D(p_i)$ in equation (4). Q.E.D.

Intuitively as we shift to the left along the distribution of ideal points in the range $[p_1, p_2]$ the "loss" of utility from candidate's 2 position increases and the loss of utility from candidate 1 position diminishes so the positional advantage of the left wing candidate increases. This is true only as long as p_s and p_i are bounded by the positions of the two candidates. When the ideal points of the voters that are compared are not in the range $[p_1, p_2]$ it is generally conceivable, although not very likely, that the positional advantage of the left wing candidate with the more liberal voter is smaller than his positional advantage with the less liberal voter. The following assumption rules such cases out.

Assumption 1:

(i) $D(p_s) > D(p_i)$ for all i and s such that $p_s < p_i < p_1$.
(ii) $D(p_s) < D(p_i)$ for all i and s such that $p_s > p_i > p_2$.

Assumption 1 is likely to be satisfied by a large class of utility functions. In particular it is satisfied when L ( ) is quadratic.[8]

Proposition 1, assumption 1 and equation 4a imply that if an informed individual weakly[9] prefers the left wing candidate all informed individuals with ideal points to the left of the ideal point of this individual prefer the left wing candidate. This is so because they all share the same information about the ability differential between the candidates and because the positional advantage of the left wing candidate increases the more liberal is the voter under consideration. A similar statement holds, for similar reasons, within the class of uninformed voters. Those results are summarized more precisely in the following proposition.

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[8] For $L(|p_i-p_j|) = (p_i-p_j)^2$, $D(p_i) = (p_i-p_2)^2 - (p_i-p_1)^2 = (p_2-p_1)[p_2-p_1 + 2(p_1-p_i)]$. It follows that $D(p_s) - D(p_i) = 2(p_2-p_1)(p_i-p_s)$ which implies that $D(p_s) > D(p_i)$ if $p_s < p_i$ and $D(p_s) < D(p_i)$ if $p_s > p_i$.

[9] Weak preference means that the voter either prefers the left wing candidate or is indifferent between the two candidates.

Proposition 2: (i) If informed (uninformed) individual i weakly prefers the left wing candidate all informed (uninformed) individuals with more liberal ideal points prefer the left wing candidate.

(ii) If informed (uninformed) individual i weakly prefers the right wing candidate all informed (uninformed) individuals with more conservative ideal points prefer the right wing candidate.

III. The Effect of a Single Poll on Votes

In order to decide how to vote uninformed individuals must evaluate the ability induced expected utility differential between the candidates. In this section I consider how this evaluation and, therefore, the outcome of the elections is affected when one poll is taken prior to the election. Before the poll is taken

$$E[y \mid F_U^b] = \mu_y = \int_y yh(y)\, dy \tag{8}$$

where F_U^b is the information set of the uninformed before the poll is taken. Equation (4b) implies that all uninformed individuals with ideal points such that $\mu_y > -D(p_i)$ vote for candidate 1. Let p_μ^* be defined by

$$-D(p_\mu^*) = \mu_y \tag{9}$$

p_μ^* is the critical ideal point at which the positional advantage of candidate 2 and the ability differential between the candidates, as perceived by the uninformed, precisely offset each other. An individual with ideal point at p_μ^* is indifferent between the two candidates. Proposition 2 implies that

$$\begin{aligned} p_i < p_\mu^* &\rightarrow v_i = 1 \\ p_i > p_\mu^* &\rightarrow v_i = 2 \end{aligned} \tag{10}$$

where v_i is the voting action of uninformed individual i. Similarly let p_y^* be defined by

$$- D (p_y^*) = y \qquad (11)$$

p_y^* is the ideal point of an informed individual who is indifferent between the two candidates. Proposition 2 implies that

$$p_i < p_y^* \rightarrow v_i = 1$$
$$p_i > p_y^* \rightarrow v_i = 2 \qquad (12)$$

where v_i stands now for the voting action of informed individual i. Thus an uninformed individual votes for candidate 1 or 2 depending on whether his ideal point is to the left or to the right of p_μ^*. Similarly an informed individual votes for candidate 1 or 2 depending on whether his ideal point is to the left or to the right of p_y^*.

Prior to the election one perfect poll is taken and its results are disseminated to the entire voting population. Since the poll is perfect and in view of equations (10) and (12) the proportion of voters who express a preference for candidate 1 is given by

$$q = k_I F(p_y^*) + k_U F(p_\mu^*) \qquad (13)$$

where

$$F(p) \equiv \int_{-\infty}^{p} f(P)\, dP \qquad (14)$$

is the cumulative distribution function of voters' ideal points.

Equation (11) defines p_y^* as an implicit function of y. p_y^* is a monotonically increasing function of y. This can be seen by noting that when y goes up the critical ideal point has to change so as to cause an increase in the positional advantage of the conservative candidate. Proposition 1 and assumption 1 imply that this is achieved by an increase in p_y^*. It follows from equation (13) and the positive dependence of p_y^* on y that q is positively related to the difference in ability between the left wing candidate and the right wing candidate. Thus knowledge of the poll result as embodied in q gives the uninformed voters information about the relative efficiency of the two candidates. The higher the relative efficiency of candidate 1 the larger the fraction of informed individuals who prefer candidate 1. As a result q is positively correlated with the relative efficiency of candidate 1 so that the uninformed may deduce from a high q that the relative efficiency of this candidate is high.

But in the presence of shifts in the distribution of ideal points the poll result is an imperfect indicator for the relative ability of the two candidates. A high q may reflect either a high relative efficiency of candidate 1 or a shift to the left in the distribution of ideal points or both. It is therefore rational, on the part of the uninformed, to attribute high q-s to both factors. As a result whenever q assumes an extreme value, only because of a change in the distribution of ideal points, the uninformed partly interpret this as a greater than average relative efficiency of one of the candidates. For example a low q that is caused by a shift to the right is partly interpreted by the uninformed as a greater relative efficiency of the right wing candidate. After seeing the poll results the fraction of the uninformed that vote for the right wing candidate is therefore larger than before the publication of those results. Thus, in periods that are dominated by strong changes in the distribution of ideal points, public opinion polls tend to increase the plurality of the favored candidate beyond the prepoll plurality of this candidate. On the other hand in periods of relative stability in the distribution of ideal points the uninformed tend to underinterpret the actual efficiency differential between the candidates.

Both phenomena arise because, even with statistically perfect polls, the uninformed cannot perfectly interpret the information provided by the poll. They confound shifts to the left or to the right with the relative efficiency of the two candidates. This confusion is not restricted to extreme cases. As a matter of fact the uninformed normally confound those two effects. This can be illustrated by considering a distribution preserving change in the mean of ideal points. Such a change can be parametrized by shifting the entire distribution f(p) by a constant but stochastic factor d. In particular let

$$g(p,d) \equiv f(p + d) \text{ for all } p \text{ and } d \tag{15}$$

and let the expected value of d be zero. When d is at its expected value g(.) is equal to the original density function f(p). A positive (negative) d means that there has been a shift to the left (right). Let G(p,d) be the cumulative distribution function of g(p,d). It follows from (15) that

$$G(p,d) = F(p + d) \text{ for all p and d} \tag{16}$$

using (16) in (13)

$$q(y,d) = k_I F[p_y^*(y) + d] + k_U F[p_\mu^* + d] \tag{17}$$

where the notation $p_y^*(y)$ is designed to stress the (positive) dependence of p_y^* on y. Let

$$s(d) \tag{18}$$

be the probability density function of d. By Bayes theorem

$$\Pr[y|q] = \frac{h(y) \Pr[q|y]}{\int_Y h(Y) \Pr[q|Y]\, dY} \tag{19}$$

where Pr [y|q] is the conditional probability of the ability differential given the poll result and Pr [q|y] is the conditional probability of the poll result given an observation on the ability differential. Equation (17) defines d as an implicit function of q and y. Let

$$d = d(y,q),\ d_y < 0,\ d_q > 0 \tag{20}$$

be this function. Here d_y and d_q are, respectively, the partial derivatives of d(.) with respect to y and q. The signs of those partial derivatives are implied by the fact that, from equation (17), q is an increasing function of both y and d. Given y the poll result will be exactly q if the value of d is equal to that given by equation (20). It follows that

$$\Pr[q|y] = s[d(y,q)] \tag{21}$$

In words: the conditional probability that, given y, the poll result is q, is equal to the probability that the shift in the distribution of ideal points is exactly equal to the shift necessary to produce a poll result of q when the realization of the ability differential is y. Substituting (21) into (19)

$$\Pr[y|q] = \frac{h(y)\,\varepsilon[d(y,q)]}{\int_Y h(Y)\,\varepsilon[d(Y,q)\,dY} \tag{22}$$

which implies that the after poll expected value of the ability induced utility differential for the uninformed is[10/]

$$\mu_y^a \equiv E[y|F_U^a] = \int_y y \Pr[y|q]\,dy \tag{23}$$

Since $d_y < 0$ and $d_q > 0$ an increase in q shifts the entire posterior probability density function of y in (22) to higher values of y. As a consequence an increase in q results through (23) in an increase in μ_y^a. Thus the higher is q the higher is the posterior expected value of the ability induced utility differential in favor of the left wing candidate. As a consequence the fraction of uninformed individuals who vote for the left wing candidate is larger. Note that this is true whatever the origin of the increase in q. Thus if q increases only because there is a shift to the left in the distribution of ideal points ($d > 0$) the uninformed partly interpret this increase as a higher relative efficiency of the left wing candidate. As a consequence this candidate gets more votes than what he would have obtained in the absence of polls or with perfect information. The same statement is true when most of the increase in q is due to an increase in d. This is the momentum effect of public opinion polls. The electorate advantage of the candidate whose position is becoming more popular is enhanced by the existence of public opinion polls.

An analogous result holds for the post poll expected value of d. A relatively large q that is caused mostly by a relatively large y is partially interpreted by the uninformed as a leftward shift in the preferences of the electorate. As a result the relatively efficient left wing candidate draws less votes than he would have in the absence of polls or under perfect

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10/ F_U^a designates the information set of the uninformed after they have seen the poll results.

information. Part 1 of the appendix provides a rigorous demonstration of this intuitively appealing result.

More generally fluctuations in q are attributed to fluctuations in y and in d in direct proportion to the average variabilities of those two variables.[11] Since the actual realizations of y and d are usually not exactly in proportion to their respective average variabilities there is normally a certain degree of confounding between relative abilities and changes in the preferences of the electorate. More precisely when y and d assume their apiori respective expected values equations (9), (11) and (17) imply

$$q(\mu_y, 0) = F[p_y^*(\mu_y)]$$

Deviations of q(.) from $q(\mu_y, 0)$ are caused by deviations of y from μ_y as well as by deviations of d from zero. The uninformed therefore rationally attribute deviations of q from $q(\mu_y, 0)$ to the two sources in proportion to the relative variabilities in this source. Since the actual realizations of y and d usually differ from their respective variabilities the poll result usually induces some confounding between deviations of y from its mean and deviations of d from its mean among the uninformed. The nature of this confusion as well as the dependence of perceptions on relative variabilities are made more explicit in section V by introducing simple parametrization of the utility function and of f(p).

IV. Interpretation of Ability in Terms of a Candidate's Perceived Riskiness.[12]

Voters are usually uncertain about the position a candidate will take if actually elected. This uncertainty has two origins. First the candidate may not be thoroughly precise in public statements of his position. Second the public is uncertain about the extent to which the candidate will be able to implement his program even when this program is precisely stated. In either case the position of candidate j, j = 1,2, is a random variable.

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[11] Obviously this statement applies only to the uninformed. Given q the informed can deduce the precise value of d from (17).

[12] I am indebted to Dan Ingberman for suggesting this interpretation.

Let

$$u(p_j, p_i) \equiv -(p_i - p_j)^2 \qquad j = 1,2$$

be the utility derived by individual i from candidate j when the realization of this candidate's position is p_j. The expected utility from the candidate is

$$Eu(p_j, p_i) = -[V_j + (p_i - \bar{p}_j)^2].$$

where $\bar{p}_j$ and V_j are respectively the mean and the variance of candidate's j position.[13] Provided he knows $\bar{p}_j$ and V_j individual i votes for candidate 1 if

$$-[V_1 + (p_i - \bar{p}_1)^2] > -[V_2 + (p_i - \bar{p}_2)^2].$$

Letting $y \equiv V_2 - V_1$ and rearranging we obtain that individual i votes for candidate 1 whenever

$$p_i < \frac{1}{2}\left[\frac{y}{\bar{p}_2 - \bar{p}_1} + \bar{p}_2 + \bar{p}_1\right] \equiv p_y^{**}$$

provided he knows what is the variance differential, y, between the two candidates.

The mean positions of the candidates ($\bar{p}_j$, $j = 1,2$) are public knowledge. The actual difference in riskiness between the two candidates as measured by y is known with certainty by the informed. But the uninformed only know the distribution of the riskiness differential. Since they are expected utility maximizers the uninformed vote for candidate 1 at the poll if [14]

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[13] Formally the general case of section III specializes to the one here for $x_j = V_j$, $w(x_j) = -V_j$ and $L(|p_i - \bar{p}_j|) = -(p_i - \bar{p}_j)^2$ for $j = 1,2$ and with p_j from equation (1) reinterpreted as the mean--$\bar{p}_j$.

[14] $E\,V_j$, $j = 1,2$ is the expected value of V_j.

$$-[E\,V_1 + (p_i - \bar{p}_1)^2] > -[E\,V_2 + (p_i - \bar{p}_2)^2].$$

Rearranging this is equivalent to

$$p_i < \frac{1}{2}\left[\frac{\mu_y}{\bar{p}_2 - \bar{p}_1} + \bar{p}_2 + \bar{p}_1\right] \equiv p_\mu^{**}$$

Thus this problem maps into that of section III with p_y^* and p_μ^* replaced by p_y^{**} and p_μ^{**} respectively. Obviously the basic confusion, by the uninformed, between shifts in the distribution of ideal points and the relative efficiency of the candidates remains as in Section III. The main difference is that now the relative efficiency of the candidates is conceived in terms of their relative riskiness. Since individuals are risk averse they prefer, ceteris paribus, the less risky candidate. Not knowing the precise value of the riskiness differential, the uninformed use poll results as an indicator for this differential. Since poll results also reflect shifts in the distribution of ideal points in the population the uninformed partly interpret a shift to the right as a relatively lower riskiness of the right wing candidate. This creates a momentum effect in favor of the right wing candidate.

V. Sequential Polling and the Momentum Effect of Polls

This section focuses on the particular case in which the distribution of ideal points is uniform and the utility function linear. Besides providing a more concrete illustration of the elements discussed in the previous section this specialization makes it possible to analyze the cumulative effects of many polls prior to elections on the election results.

It was assumed for simplicity in the previous sections that poll results are not subject to error. This assumption is relaxed here by allowing a sampling error. Thus this section is less general than section III in some dimensions but more general in other dimensions.

a. A Uniform Distribution of Ideal Points and a Linear Utility

Let the density function of ideal points be

$$f(p) = \begin{cases} \frac{1}{2a} & c-a \le p \le c+a,\ a > 0 \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

The mean of the distribution of ideal points is equal to c. In order to capture expected shifts in the distribution of ideal points c is specified as a random variable whose distribution is given by

$$c \sim N(\mu_c, \sigma_c^2) \tag{25}$$

The utility function in equation 1 is specialized to

$$r_{ji} = x_j - |p_j - p_i|, \quad j = 1,2 \tag{26}$$

and it is assumed that y has a normal distribution with prior mean μ_y and variance σ_y^2. With the utility function in (26) the conditions for a vote in favor of the left wing candidate in equations (4) specialize to

$$\begin{aligned} y \equiv x_1-x_2 &> -D(p_i) = |p_1-p_i| - |p_2-p_i|, \quad i \in I \qquad (a) \\ E(y|F_U) &> -D(p_i) = |p_1-p_i| - |p_2-p_i|, \quad i \in U \qquad (b) \end{aligned} \tag{27}$$

The positional advantage of the left wing candidate, $D(p_i)$, is given in equation (28) and is plotted as a function of voters' ideal points in figure 1. It is a nonincreasing function of p_i.

$$D(p_i) = \begin{cases} (p_2-p_1) & p_i \le p_1 < p_2 \\ p_2+p_1-2p_i & p_1 \le p_i \le p_2 \\ -(p_2-p_1) & p_1 < p_2 \le p_i \end{cases} \tag{28}$$

In the absence of ability differentials all individuals with ideal points to the left of A (see figure) vote for candidate 1 and the rest vote for candidate 2. With an ability differential of size A'B in favor of candidate 1 all individuals to the left of A' vote for candidate 1 and the rest vote for candidate 2. In this case the ability differential in favor of candidate 1 more than compensates for his positional disadvantage with voters with ideal points between A and A'. As a consequence all individuals with ideal points between A and A' vote for candidate 1 in spite of the fact that candidate 2 is

Left Wing Candidate's Positional Advantage as a

Function of Voters' Ideal Points

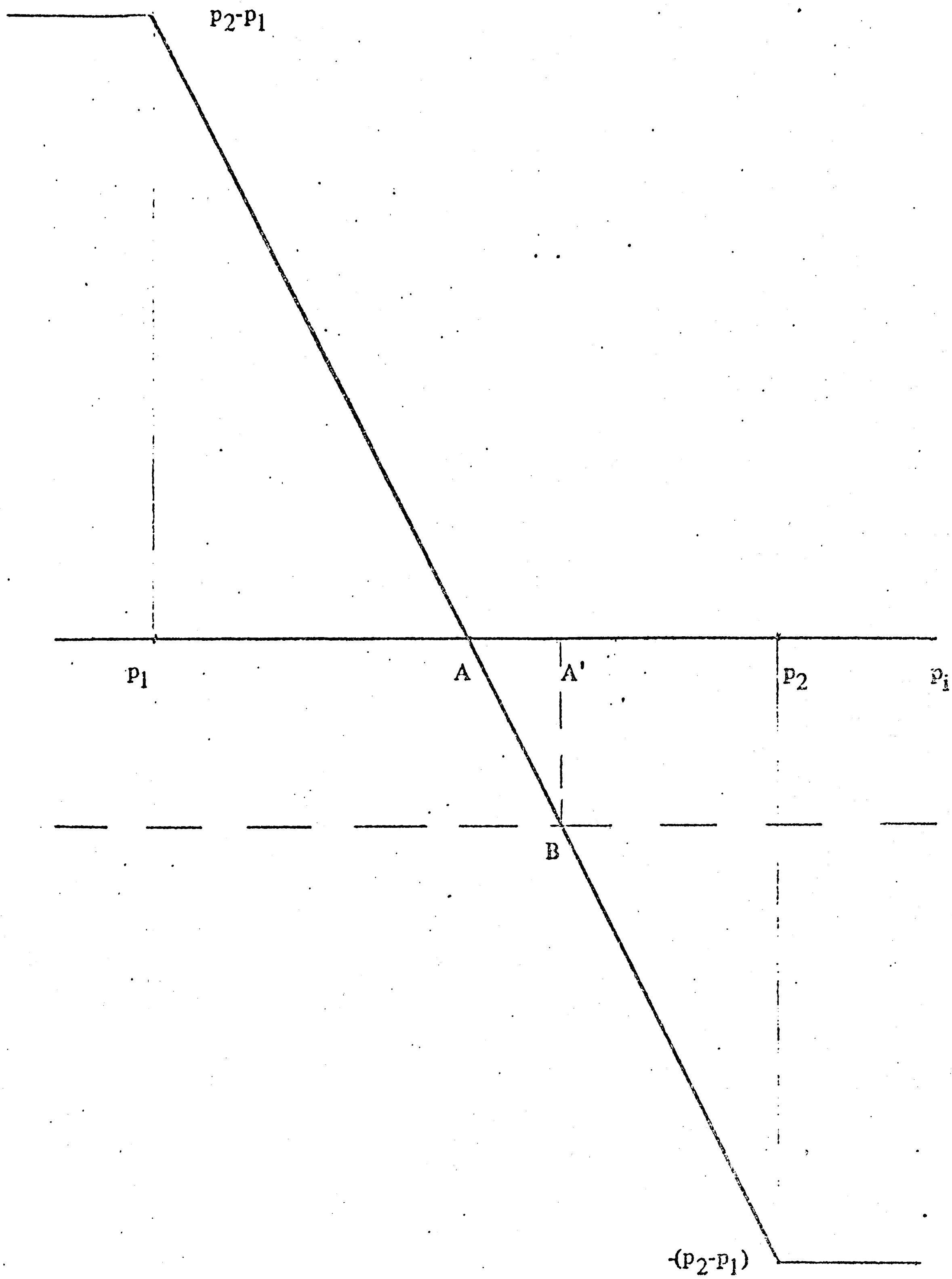

Figure 1

nearer to them in the issue space. The larger the ability advantage of candidate 1 the larger the number of individuals that are nearer to candidate 2 who vote for candidate 1. When $\mu_y > p_2 - p_1$ the mean efficiency advantage of candidate 1 is so large that all uninformed individuals vote for him. Conversely when $\mu_y < -(p_2 - p_1)$ the mean efficiency advantage of the other candidate is so large that all uninformed individuals vote for him. We shall assume away such extreme cases by requiring that

$$-(p_2-p_1) < \mu_y < p_2-p_1 \tag{29}$$

so that a priori each candidate gets some of the votes of the uninformed.

Let q_i be the proportion of respondents to the i-th public opinion poll that prefer candidate 1. Let $E[y|q_1,\ldots,q_{n-1}]$ be the expected value of the efficiency differential between candidates conditional on the results of the first n-1 polls. This expected value is the perception of the ability differential by the uninformed after they have been exposed to the results of the first n-1 polls. Given this perception the straw vote behavior of the uninformed at the n-th poll is given by

$$\begin{aligned} p_i &< p^*_{n-1} \rightarrow v_{ni} = 1 \\ p_i &> p^*_{n-1} \rightarrow v_{ni} = 2 \end{aligned} \tag{30}$$

where

$$p^*_{n-1} \equiv \frac{1}{2}\left(E[y|q_1,\ldots,q_{n-1}] + p_2 + p_1\right) \tag{31}$$

and v_{ni} is the straw vote of individual i at the n-th poll. Equation (31) is obtained by substituting (28) into (9), reinterpreting μ_y as the prior mean of y after n-1 polls, replacing p^*_μ by p^*_{n-1} and solving for p^*_{n-1}. Provided the realization of y is in the range $(-(p_2-p_1),\ p_2-p_1)$ equations (11) and (28) imply similarly that[15/]

[15/] If $y < -(p_2-p_1)$ all informed individuals vote for candidate 2 and if $y > p_2-p_1$ they all vote for candidate 1. Although the main point of this section holds for those extreme cased too their explicit discussion is omitted for simplicity. When $y = -(p_2-p_1)$ all informed individuals with ideal points to the right of p_1 vote for candidate 2 and those to the left of p_1 are indifferent between the two candidates. A symmetric statement holds when $y = p_2-p_1$.

$$p_y^* = \frac{1}{2}(y + p_2 + p_1) \tag{32}$$

The voting behavior of the informed at all polls and at the elections is given by equations (12) with p_y^* given by (32).

In view of (12), (24) and (30) the proportion of individuals that indicate a preference for candidate 1 at the n-th poll is

$$q_n = k_I \int_{c-a}^{p_y^*} \frac{dp}{2a} + k_U \int_{c-a}^{p_{n-1}^*} \frac{dp}{2a} + \eta_n \tag{33}$$

where η_n is a random sampling error whose distribution is given by

$$\eta_i \sim N(0, \sigma_\eta^2) \text{ for all } i \tag{34}$$

I assume for simplicity that the quality of the different polls is identical so the variance, σ_η^2, of the sampling error is independent of the poll index.[16] The random variables η_i, $i = 1,2,3\ldots$, y and c are all mutually uncorrelated. Substituting (31) and (32) into (33) and rearranging

$$q_n - A_0\, E[y \mid q, \ldots, q_{n-1}] = A + A_y\, y - A_c\, c + \eta_n \tag{35}$$

where

$$A_0 \equiv \frac{k_U}{4a} ;\ A \equiv \frac{1}{2}\left(1 + \frac{p_2 + p_1}{2a}\right);\ A_y \equiv \frac{k_I}{4a};\ A_c \equiv \frac{1}{2a} \tag{36}$$

After the first n-1 polls the uninformed form the conditional expectation $E[y \mid q_1, \ldots, q_{n-1}]$. Since they know the parameters A_0 and A, the publication of q_n amounts through equation (35) to an observation on the sum

$$s_n \equiv A_y y - A_c c + \eta_n. \tag{37}$$

Thus after the n-th poll all the new information provided by the polls to the uninformed can be summarized in the vector $(s_1, \ldots, s_n)$. It follows that

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[16] The main result does not depend on this assumption.

$$E[y|q_1,\ldots q_n) = E(y|s_1,\ldots,s_n) \quad (38)$$

It is shown in part 2 of the appendix that the mean of the s_i is a sufficient statistic for both y and c. This implies in conjunction with equation (38) that

$$E[y|q_1,\ldots,q_n] = E[y|\bar{s}_n] \quad (39)$$

where

$$\bar{s}_n = \frac{1}{n}\sum_{i=1}^{n} s_i = A_y y - A_c c + \frac{1}{n}\sum_{i=1}^{n} \eta_i \quad (40)$$

Since it is a linear combination of normal variates $\bar{s}_n$ is also distributed normally. Hence[17]

$$E[y|\bar{s}_n] = Ey + \rho_{y\bar{s}_n} \frac{\bar{s}_n - E\bar{s}_n}{\sigma_{\bar{s}_n}} \sigma_y \quad (41)$$

where $\sigma^2_{\bar{s}_n}$ is the variance of $\bar{s}_n$ and $\rho_{y\bar{s}_n}$ is the coefficient of correlation between y and $\bar{s}_n$. Using equation (40) to calculate those parameters and the term $\bar{s}_n - E\bar{s}_n$, substituting the results into (41) and rearranging

$$E[y|\bar{s}_n] = \theta_n \frac{\sum_{i=1}^{n} r_i}{n} + (1-\theta_n)\mu_y \quad (42)$$

where

$$\theta_n \equiv \frac{A_y^2\ \sigma_y^2}{A_y^2\ \sigma_y^2 + A_c^2\ \sigma_c^2 + \sigma_\eta^2/n} = \frac{k_I^2\ \sigma_y^2}{k_I^2\ \sigma_y^2 + 4\sigma_c^2 + 16a^2\sigma_\eta^2/n} \quad (43)$$

and

$$r_i \equiv \frac{s_i}{A_y} + \frac{A_c}{A_y}\mu_c = y - \frac{A_c}{A_y}(c-\mu_c) + \frac{\eta_i}{A_y} \qquad \text{for all } i \quad (44)$$

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[17] See for example Brunk (1965) pp. 212-18.

Equation (42) summarizes the belief of the uninformed about the relative efficiency of the two candidates after the uninformed have been exposed to the results of n polls. This belief or perception is a weighted average of the observations on r_i $(i = 1,\ldots,n)$ provided by the polls[18] and of the prepoll mean perceived ability differential μ_y. It can be seen from equation (43) that the uninformed take the poll results more seriously (θ_n is larger) the larger the variance of the ability differential between the candidates and the larger the proportion of informed individuals in the population. Both results are intuitively clear. Other things the same the larger the variance σ_y^2, the more likely it is that a deviation of the poll results from their unconditional mean is caused by a deviation of the ability differential from its mean rather than by other reasons. It is therefore rational to pay more attention to the poll results. The larger the fraction of informed individuals in the population the stronger the correlation between the information those individuals have about the ability differential and the poll results. It is therefore rational for the uninformed to give more weight to poll information in such cases.

The larger the variability, σ_c^2, in the distribution of ideal points and the larger the polling error, σ_η^2, the smaller the weight given by the uninformed to poll results and the larger their reliance on the prepoll mean μ_y. The intuition is that with higher σ_η^2 and or σ_c^2 it is less likely that the poll results are good indicators for an ability differential between the candidates so the uninformed pay less attention to the polls.

Finally as can be seen from equation (43) the larger the number of polls the higher the weight given by the uninformed to the mean information from the polls. This is a direct consequence of the fact that the sampling error of the mean poll result, σ_η^2/n, is a decreasing function of the number of polls.

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[18] Since individuals know the parameters in the polling equation (35), as well as μ_c, an observation on s_i is equivalent to an observation on r_i for all i.

b. The Momentum Effect of Polls

Obviously uninformed individuals have more information with polls than without them. Also the larger the number of poll results they are exposed to - the better their information since the cumulative sampling error σ_η^2/n decreases with the number of polls. However even with a very large number of polls (and afortiori with a small number) uninformed voters cannot avoid confounding shifts in the distribution of ideal points with the relative efficiency of the two candidates. I turn now to the demonstration of this claim. Substituting (44) into (42)

$$E[y|\bar{s}_n] = \theta_n(y - \frac{A_c}{A_y}(c-\mu_c) + \frac{\bar{\eta}_n}{A_y}) + (1-\theta_n)\mu_y \tag{45}$$

where $\bar{\eta}_n = \sum_{i=1}^{n} \eta_i/n$. It is apparent from equation (45) that a shift to the right $(c-\mu_c>0)$ is partly interpreted by the uninformed as a higher relative efficiency of the right wing candidate. To illustrate this effect in isolation consider the particular case in which both y and $\bar{\eta}_n$ are equal to their respective means μ_y and 0. For this case equation (45) reduces to

$$E[y|\bar{s}_n] = \mu_y - \theta_n \frac{A_c}{A_y}(c-\mu_c) \tag{46}$$

Equation (46) suggests that the uninformed interpret some of the shift to the right as a higher relative efficiency of the right wing candidate. As a consequence a larger fraction of the uninformed votes for the right wing candidate after being exposed to the polls. In spite of the fact that the ability differential between the candidates does not deviate from its mean, μ_y, the polling information convinces the uninformed that y is below μ_y. The coexistence of uninformed voters and public opinion polls thus reinforces the effect of shifts in the distribution of ideal points on the election results creating a momentum effect of polls. Without this effect the electoral gains of the right wing candidate from a rightward shift in the distribution of ideal points would be smaller. Obviously this effect is not limited to the case $y = \mu_y$. Furthermore it does not vanish as the number of polls increases. When the number of polls tends to infinity equation (45) reduces to[19]

$$\lim_{n\to\infty} E[y|\bar{s}_n] = \mu_y + \theta[y-\mu_y - \frac{A_c}{A_y}(c-\mu_c)] \quad , \tag{47}$$

where

$$\theta \equiv \lim_{n\to\infty} \theta_n = \frac{k_I^2\,\sigma_y^2}{k_I^2\,\sigma_y^2 + 4\,\sigma_c^2}$$

As can be seen from the last term on the right-hand-side of (47) shifts in the distribution of ideal points are still confounded with the relative efficiency of the candidates. Even in the absence of a sampling error a shift in ideal points towards one side along the issue space increases the plurality of the candidate on this side of the space by more than the increase he would have experienced under perfect information. For example given $y - \mu_y$ a shift to the right ($c-\mu_c>0$) is partly interpreted by the uninformed as a relatively more efficient right wing candidate. As a consequence he wins with a larger plurality. Note that the momentum effect of polls is stronger the more precise, ceteris paribus, are the polls since in such a case σ_η^2 is smaller and θ_n therefore larger.

c. Sequential Learning from Public Opinion Polls

Let $\bar{r}_n = \sum_{i=1}^{n} r_i/n$. Simple algebra implies that $\bar{r}_n = [(n-1)\,\bar{r}_{n-1} + r_n]/n$.

Using this relation in equation (42) and noting equation (44)

$$E[y|\bar{s}_n] = E[y|\bar{s}_{n-1}] + \frac{\theta_n}{A_y n}(s_n - \bar{s}_{n-1}) \tag{48}$$

19/ $\bar{\eta}_n$ tends in probability to zero since it has an expected value of zero and a variance, σ_η^2/n, which collapses to zero as $n \to \infty$.

Equation (48) is a sequential updating equation for y. It states that if $s_n = \bar{s}_{n-1}$ the uninformed do not change their views about the ability differential after observing the results of the n-th poll. Otherwise they update their forecast up or down depending on whether s_n is larger or smaller than $\bar{s}_{n-1}$. Essentially the uninformed give all polls equal weight since they know that the values of y and c do not change from one poll to the next. Had we allowed continuous but possibly persisting changes in the distribution of ideal points between polls the weights given to the information from different polls would not necessarily be equal. But the inability to separate changes in the distribution of ideal points from the candidates' relative ability would carry over to this case as well. This suggests that the momentum effect of polls exists also in a world that is characterized by a high volatility of voters' positions prior to elections.

d. Qualifications and General Considerations

In the absence of polling errors each poll defines a line in the ability differential-electorate preferences plane that is consistent with the observed value of the poll result. Each poll generates such a line. In general the slope of each line depends on the expected value of y conditioned on the observations on the previous poll results. Since this expected value changes from one poll to the next one the first two polls generate, in general, two intersecting lines. The coordinates of the intersection point yield exact solutions to the ability differential and the current position of the electorate preferences. Thus, in general, the results of two separate polls can, in the absence of polling errors, fully reveal the information of the informed to the uninformed.

This is not true in the uniform distribution of ideal points example discussed above because with such a distribution the slopes of the lines that correspond to different polls do not depend on the expected value of y and are, therefore, all identical. As a consequence, in the absence of polling errors, the first poll yields all the information that can be obtained from any number of polls and this information is imperfect. Additional polls add new equations but those equations are all linearly dependent and do not add new information. In the presence of polling errors additional polls help sharpen the information of the uninformed even in the case of the uniform distribution since with more polls the variance of the polling error is smaller. But as demonstrated by the discussion of equation (47) even when this variance goes to zero some confounding between y and c remains.

This confounding would not necessarily persist with a large number of polls in general. In this sense the uniform distribution of ideal points example is a special case. But in a wider sense it is, I believe, illustrative of the basic difficulty some voters have in separating the relative quality of the candidates from shifts in public preferences by using public opinion polls. I elaborate on this points in what follows. Suppose the distribution of ideal points is such that the lines defined by each poll are linearly independent but also that all polls are subject to error. Prior to elections a finite number of polls (n) is taken. Those n polls produce n equations with n+2 unknowns—n errors, the ability differential and the current electorate's distribution of preferences. The uninformed are still unable to perfectly infer the ability differential from the polls in spite of the fact that the slopes of the poll lines differ. The existence of poll errors maintains the confusion between the ability differential and the current distribution of voters preferences. In particular good performance at the polls that is caused uniquely by a shift to the right is partly interpreted as a relatively higher ability of the right wing candidate. Thus as long as the number of polls is finite it is reasonable to expect that polls will generate a momentum effect for any distribution of ideal points.[20/] The uniform distribution example illustrates this general principle in a particular case.[21/]

[20/] The technical details needed for a precise demonstration of this claim are probably more complex.

[21/] It is instructive to compare and contrast the results here with those of McKelvey and Page (1984). They show that, under certain conditions, the existence of publicly announced statistics lead to consensus beliefs about posterior probabilities. This would have been the case here too with at least two perfect polls for distributions of ideal points that lead to independent poll equations and, therefore, to full revelation of candidates' activities. But since full revelation of the private information possessed by the informed is normally not achieved, some differences between the posterior probabilities of informed and uninformed individuals persist even after the publication of poll results.

VI. Concluding Remarks

The main result of the paper can be summarized as follows: For any ability or risk differential between the candidates the existence of public opinion polls reinforces the effect of changes in the public's preferences on election results. In the presence of one perfect poll this result holds for any distribution of ideal points and for a fairly wide class of voters' utility functions. The effects of many noisy polls was illustrated by using a uniform distribution of ideal points and linear utility functions.

The information transmitted by polls can explain the existence of bandwagon effects similar to those discussed by Simon (1954). But it is important to point out that the momentum effect of polls discussed here does not generate the precise form of bandwagon effects postulated by Simon. He defines a bandwagon effect as a situation in which some people are more likely to vote for a candidate when they expect him to win than when they expect him to lose. The momentum effects described here are not restricted to such situations. In the presence of polls individuals vote more heavily for the candidate whose position is becoming more popular independently of whether this candidate is or is not more likely to win. The momentum is in reference to a world in which there are no polls or in which poll results are not published.

Empirical detection of the momentum effect of polls is not easy because that is not the only effect which operates prior to elections. It seems, however, that preelection periods which are characterized by substantial and unidirectional changes in the relative support for the candidates at the polls are more likely to have been subject to sizable momentum effects. Two presidential elections are of particular interest in this respect. The 1948 race between Truman and Dewey and the 1980 contest between Reagan and Carter. Immediately after the last convention the Gallup poll from the beginning of August 1948 gave Truman and Dewey 37% and 48% of the vote respectively. The corresponding percentages were 40 and 46 in mid October and 44.5 and 49.5 on November 1, 1948 just prior to the election.[22] Twelve percent of the

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[22] Source: The Gallup Poll Public Opinion 1935-1971, Random House New York. The actual election results were 49.6 percent for Truman and 45.1 percent for Dewey.

voters made up their minds only in October 48.[23/] This evidence is consistent with the existence of a sizable momentum effect during the last month of the campaign. During this period public attention is usually more focused on the campaign than at any other time. Polls published at that time are, therefore, more likely to have a stronger impact on voting behavior.

During the 1980 campaign a similar pattern emerges. After the democratic convention in August 1980 the polls gave Reagan 38% and Carter 39% of the vote. Both candidates were tied at 39% each in the mid-September polls. By mid-October Reagan led with 44% of the vote versus Carter's 41%. The actual election results were 50.7% for Reagan and 41% for Carter.

A sharper test of the momentum hypothesis could be based on independent evidence on changes in the distribution of the public along the liberal-conservative dimension. A positive correlation between shifts to (say) the right and subsequent increases in support for the right wing candidate would be evidence in favor of the hypothesis. At least for the 1980 election there is some indirect evidence that there has been between 1976 and 1980 an increase in the fraction of voters who take traditionally conservative positions on some of the issues. The percentage of voters who consider inflation the most important single issue rose from around 40 in 1976 to around 60 in 1980. Simultaneously the percent of those who consider unemployment the most important issue declined by about 10%.[24/] Hibbs provides evidence that by the second quarter of 1980 "the relative impact of inflation on political support had increased enormously" (Hibbs (1982), p. 224). He mentions that one of the factors contributing to Reagan's landslide was Carter's belated realization of this change. Concern about the U.S. standing in the world was also higher in 1980 than in 1976. This evidence in conjunction with the pattern of poll returns during the last few months of the 1980 campaign as well as the actual election returns are consistent with the existence of a momentum effect. A more exact test of the momentum hypothesis is beyond the scope of this paper.

[23/] The Gallup Poll Op. Cit. p. 771.

[24/] Source: the Gallup Poll Op. Cit.

Finally, it was implicitly assumed that everybody votes. In a wider framework which incorporates the costs of voting[25/] poll results, by changing the perceived differential benefit from the candidates, will affect the degree of participation in the election.

[25/] As in Ledyard (1984) or Palfrey and Rosenthal (1984) for example.

Appendix

1. The Effect of a Poll on the Expected Value of d

Prior to the poll the expected value of d is

$$\mu_d = {}_d\int d\,s(d) \qquad \text{(A1)}$$

After the poll results have been published the probability density of d is

$$\Pr[d|q] = \frac{s(d)\ \Pr[q|d]}{{}_D\int s(D)\ \Pr[q|D]\ dD} \qquad \text{(A2)}$$

Solving y as a function of d and q from equation (17)

$$y = y(d,q),\ y_d < 0,\ y_q > 0 \qquad \text{(A3)}$$

where the signs of the partial derivatives y_d and y_q are implied by equation (17). By an argument that is analogous to that which led to equation (21)

$$\Pr\ [q|d] = h\ [y(d,q)] \qquad \text{(A4)}$$

Substituting (A4) in (A2)

$$\Pr[d|q] = \frac{s(d)\ h\ [y(d,q)]}{{}_D\int s(D)\ h\ [y(D,q)]\ dD} \qquad \text{(A5)}$$

(A3) implies that an increase in q shifts the posterior distribution of d in (A5) towards lower values of d. Since

$$E[d|F_U^a] = {}_d\int d\ \Pr[d|q]$$

it follows that an increase in q decreases the posterior expected value of d. If the increase in q is caused mostly by an increase in y the uninformed erroneously attribute part of this change to a leftward shift in the preferences of the electorate.

2. Proof that $\bar{s}_n$ is a Sufficient Statistic for y and c

Let $\psi(s_1,\ldots,s_n/y,c)$ be the joint density function of $s_1,\ldots,s_n$ given y and c. For given values of y and c equation (37) implies that

$$\psi(s_1,\ldots,s_n/y,c) = \psi(\eta_1,\ldots,\eta_n/y,c) \tag{A6}$$

Since for given y and c, η_i are identical, independently distributed normal variates it follows from (A6) that

$$\psi(s_1,\ldots,s_n/y,c) = k(n)\exp\Big[-\frac{\sum_{i=1}^{n}\eta_i^2}{2\sigma_\eta^2}\Big]$$

$$= k(n)\exp\Big[-\frac{\sum_{i=1}^{n}[s_i-A_y y+A_c c]^2}{2\sigma_\eta^2}\Big] \tag{A7}$$

where

$$k(n) \equiv (2\pi)^{-\frac{n}{2}}\sigma_\eta^{-n} \tag{A8}$$

But

$$\sum_{i=1}^{n}[s_i-A_y y + A_c c]^2 = \sum_{i=1}^{n} s_i^2 + n[A_y y-A_c c][A_y y-A_c c-2\bar{s}_n] \tag{A9}$$

Substituting (A9) into (A7) and rearranging

$$\psi(s_1,\ldots,s_n/y,c) = \phi(s_1,\ldots,s_n)\,V(\bar{s}_n, y, c) \tag{A10}$$

where

$$\phi(s_1,\ldots,s_n) \equiv k(n)\exp\Big[-\frac{\sum_{i=1}^{n}s_i^2}{2\sigma_\eta^2}\Big] \tag{A11}$$

$$V(\bar{s}_n, y, c) = \exp[-\frac{n}{2\sigma_\eta^2}(A_y y - A_c c)(A_y y - A_c c - 2\bar{s}_n)] \qquad (A12)$$

So the joint density of $s_1,\ldots,s_n$ given y and c decomposes into a product of two functions. The first one depends on $s_1,\ldots s_n$ but not on y and c. The second depends on y and c and its dependence on $s_1,\ldots,s_n$ is only through $\bar{s}_n$. It follows that $\bar{s}_n$ is a sufficient statistic for y and c.

QED.

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