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INFLATIONARY CONSEQUENCES OF ANTICIPATED MACROECONOMIC POLICIES. PART II: BUDGET CUTS

by

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1. INTRODUCTION

In Drazen and Helpman (1985) we began examining the effects of the government following monetary and fiscal policies which were known to be infeasible, except over the short run. We considered an economy in which current policies implied a budget deficit such that in the absence of a change in policy government debt would grow without bound. We assumed government expenditures were constant, so that stabilization required either an increase in taxes or in seignorage revenues (via an increased rate of money growth), or both. Though the abandonment of current policies was inevitable, both the timing of a regime switch and the policy mix that would follow might be uncertain. We investigated the effects of these two types of uncertainty on the time path of economic variables before the regime switch in the context of a dynamic optimizing model. Among other things we found that regime switch uncertainty in itself may induce fluctuations or overshooting in the inflation rate. Though the model makes clear that the budget deficit is the ultimate cause of inflation, there need be no simple correlation between rising deficits (inclusive of debt service) and rising inflation.

In the present paper, we extend our earlier analysis to the case where a regime change is effected via a cut in government spending. Stabilization via budget cuts warrants separate treatment for two reasons. First, in the earlier case, where government spending and total output were held constant, private consumption and real interest rates were constant as well. Here, when government spending is cut to effect a stabilization, the cut being larger the later is the date of stabilization, consumption is not constant over the whole path. Uncertainty about the timing of stabilization will introduce uncertainty about when and by how much consumption will jump. In an economy where the real interest rate clears the output market, this will imply a non-constant real interest rate, where a real risk premium built into the real interest rate reflects the probability of jumps. The present setup is therefore not only more complex than our earlier setup, but it is qualitatively different, because it includes a crucial element of real uncertainty that was previously absent.

A second reason for separate treatment of budget cuts is the possibility of replicating with this model hyperinflationary time paths in a way which may be more convincing than earlier attempts. Hyperinflations are typically ended by sharp cuts in government expenditures (eliminating government reliance on borrowing and money creation), where the timing of the fiscal reform is uncertain. This type of model therefore is particularly relevant for studying hyperinflations. A major somewhat intriguing characteristic of hyperinflationary time paths is that after sharp falls in real balances and sharp increases in inflation, in most cases both variables "turned around" (that is, real balances rose and inflation fell) before the rate of money growth was actually reduced (more details about these facts may be found in Bental and Eckstein [1985]). The usual explanation is that when expectations of a successful reform became prevalent, expectations of lower future inflation induced a fall in current inflation. The main problem with this explanation is that it does not work within the framework of standard macroeconomic models with rational expectations. We are able to replicate the turnaround in inflation and real balances with our model without arbitrary assumptions about expectations.

Our main results are as follows. Even when the date of stabilization is known, there will be a price jump and a real interest rate blip at the time of stabilization. Real balances may rise, stay constant, or fall before a stabilization, with inflation moving in the opposite direction. The third possibility applies when the interest elasticity of demand for money is smaller than one. When the date of stabilization is uncertain, the nominal interest rate will include two risk premia, a real one reflecting the probability of a jump in the real interest rate and a nominal one reflecting the probability of a price jump. The real interest rate will be rising over time before a stabilization. Several paths for real balances and inflation are consistent with individual optimization and market clearing. These include a path which replicates the broad empirical characteristics of a hyperinflation. We conclude by presenting an example of such a path.

2. The Case of Certainty

We consider an economy where, as in Drazen and Helpman (1985), present macroeconomic policy -- which consists of a fixed level of public spending, fixed taxes, and a fixed rate of money growth -- implies that government debt will grow without bound, and, hence, the policy is steady state infeasible. In our earlier paper we considered stabilization effected by increases in money growth rates or taxes. Here, we consider stabilization effected by cuts in government expenditures. Stabilization implies freezing the level of per capita debt achieved at some time T by cutting government expenditures so that the new, lower level of government spending, plus debt service can be financed by the existing taxes (inflation and regular) with no further growth in debt. We thereby freeze the economy at a steady state equilibrium.

The individual is assumed to derive utility from consumption and real money balances, where his instantaneous utility function is assumed separable across commodities and across time. Utility at instant t may then be written as

$$(1) \qquad u(c(t)) + v\left(\frac{M(t)}{P(t)}\right)$$

where c, M and P are real consumption, nominal balances, and the price level and $u(\cdot)$ and $v(\cdot)$ are concave. The individual can hold either real money balances m or real bonds b as assets, the real interest rate on the latter being r. He has a discount rate β and receives after-tax labor income y per period. We assume that total output y_0 is exogeneously given, so that a fixed level of taxes τ imply that after tax income is fixed as well.

The individual's objective is to maximize expected discounted utility over an infinite horizon. In the case of full certainty about timing this is easily represented. Let $V^S(\cdot)$ be the present discounted value of maximized utility from T onwards. V^S will be a function of the real value of an individual's assets at T, namely $b(T) + M(T)/P_s(T)$ (where $P_s(T)$ is the price level the instant after the policy switch), and perhaps of T as well. The present discounted value of welfare from 0 to infinity if a switch occurs at T is then

$$(2) \qquad \int_0^T e^{-\beta t}\left[u(c(t)) + v\left(\frac{M(t)}{P(t)}\right)\right]dt + e^{-\beta T}V^s\left(b(T) + \frac{M(T)}{P_s(T)};\ T\right)$$

where the marginal utility of wealth $V_w^s(\cdot)$ is equal to the marginal utility of consumption in the post stabilization steady state.

If we define by $z(x)$ the addition to nominal cash balances at time x, the individual's choice problem may be thought of as choosing functions $c(t)$, $M(t)$, and $z(t)$ to maximize (3) subject to two constraints -- one on income, the other on the relation of M and z -- and to boundary conditions. The income constraint is that the present discounted value of income between 0 and t plus the value of wealth at 0 must equal the discounted value of expenditures on c and z/P plus the value of bonds at t discounted to time zero, plus initial real money balances. This must hold for all t. For $t = T$ we obtain

$$\text{(3a)} \qquad e^{-R(T)}b(T) + \int_0^T e^{-R(t)}\left(c(t) + \frac{z(t)}{P(t)} - y\right) dt + \frac{M(0)}{P(0)} = w(0)$$

where wealth $w(0)$ equals initial bonds plus the initial value of nominal balances evaluated at the initial price level and $R(t)$ is the integral of the interest rate from 0 to t. From T onwards we have the constraint that

$$\text{(3b)} \qquad \int_T^\infty e^{-(R(t)-R(T))}\left(c_s(t) + \frac{z_s(t)}{P_s(t)} - y\right) dt = b(T) + \frac{M(T)}{P_s(T)}.$$

The second constraint is that the change in money balances between any two points in time is the integral of the $z(x)$, namely:

$$\text{(4)} \qquad M(t) = M(0) + \int_0^t z(x)dx.$$

The mathematical derivation of the solution to the individual's first-order conditions is presented in Appendix 1. The first-order condition for consumption may be written (see (A1.2)):

$$\text{(5)} \quad e^{-\beta t}u'(y_0-g) = e^{-\beta T}u'(y_0-g_s)e^{R(T) - R(t)} \quad , \quad 0 \leq t < T$$

where g is real government expenditures before the regime switch and g_s is the level of expenditures after the switch.

Equation (5) implies that the instantaneous interest rate is equal to β before the stabilization, but that there is a jump in the interest factor at the stabilization date (it can also be shown that after stabilization the interest is also equal to β). More precisely, from (5) and the initial condition $R(0) = 0$ we obtain (see A1.5):

$$(6) \quad R(t) = \begin{cases} \beta t & \text{for } 0 \leq t < T \\ \beta t + \ln \dfrac{u'(y_0-g)}{u'(y_0-g_s)} & \text{for } t = T \end{cases}$$

This may be represented as in Figure 1. (We have used the fact that market clearing requires that consumption equals output minus government spending.)

The first-order conditions also imply (combine (A1.3) with (A1.4) and (6)):

$$(7) \qquad \frac{1}{P(t)} = \int_t^T e^{-\beta(x-t)} \frac{v'(m(x))}{\theta} \frac{1}{P(x)} dx + e^{-(R(T)-R(t))} \frac{1}{P_s(T)} \quad , \; 0 \leq t < T$$

where $\theta = u'(y_0 - g)$ and $m(t)$ is real balances at time t. This is a standard asset pricing equation showing that the value of one unit of money at time t (the left-hand-side) equals the sum of the discounted flow of earnings (here, marginal utilities) and the discounted resale value. An immediate implication of this pricing equation is that at T there is a price jump whose magnitude is determined by the jump in the interest factor, because (7) implies:

$$\frac{P(T)}{P_s(T)} = e^{R(T)-R(T^-)}$$

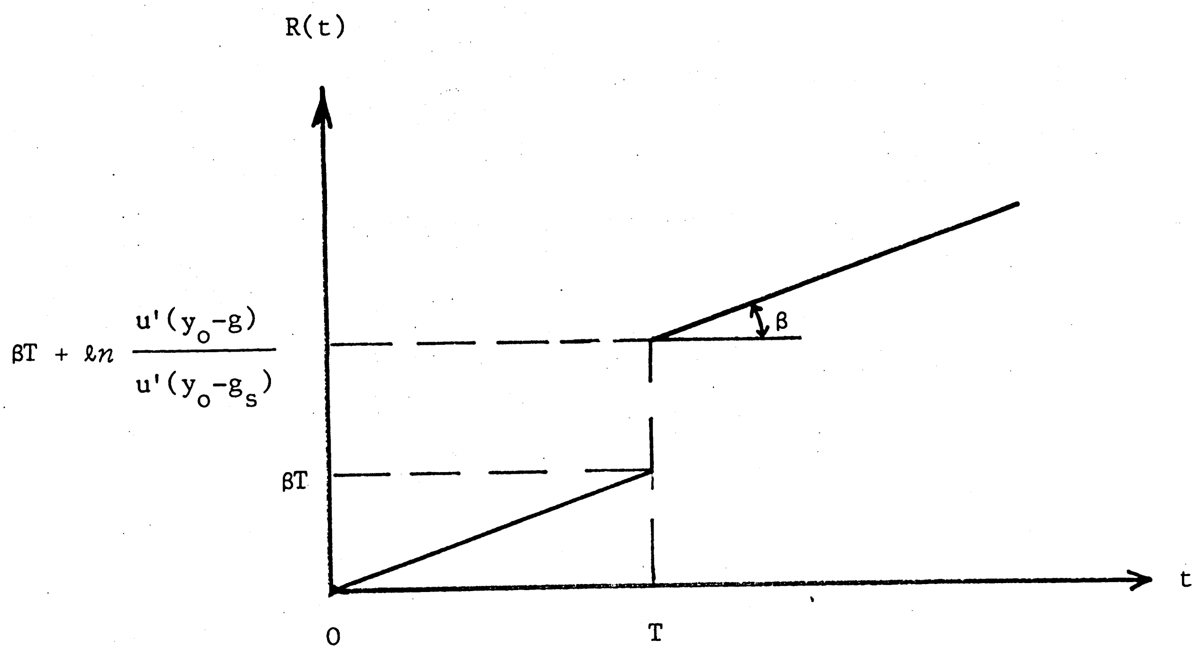


FIGURE 1

Using (6) this implies:

$$\frac{P(T)}{P_s(T)} = \frac{\theta}{\theta_s} \tag{8}$$

where $\theta_s = u'(y_0 - g_s)$. Since $g_s < g$; that is, there is an upward jump in the interest factor, this implies a downward jump in the price level. In addition, differentiation of the asset pricing equation with respect to t yields:

$$\frac{v'(m)}{\theta} = \beta + \frac{\dot{P}}{P} \quad , \text{ for } 0 \leq t < T \tag{9}$$

The dynamic equations for debt and real balances are derived from the government's behavioral equations. The government's budget constraint implies that the regular deficit $d = g-\tau$ plus debt service must be financed either by issuing bonds or printing money. We therefore have:

$$\begin{aligned} \dot{b} &= \beta b + d - \dot{M}/P \\ &= \beta b + g-\tau - \mu m \end{aligned} \tag{10}$$

for $0 \leq t < T$, where μ is the constant rate of money growth, while at T there is a jump in the interest rate. Though $\dot{b}$ is not continuous, b will be. This may be seen by writing the budget constraint in integral form

$$b(t) = b_0 + \int_0^t e^{R(x)}(g(t) - \tau - \mu m(x))dx \qquad \text{for all } t$$

where

$$g(t) = \begin{cases} g & \text{for } 0 \leq t < T \\ g_s & \text{for } t \geq T \end{cases}$$

We add to this the definitional relation for the change in per capita real balances, namely

(11) $\dot{m} = (\mu - \dot{P}/P)m$

where we assume that population growth is equal to zero. Substituting for $\dot{P}/P$ from (9) into (11), we obtain

(12) $\frac{\dot{m}}{m} = (\beta + \mu) - \frac{v'(m)}{\theta}$

as the dynamic equation until T.

The motion of the system until T can be described via phase diagrams. For given μ and τ, the $\dot{b} = 0$ locus is defined by (see (10))

(13) $b = \frac{1}{\beta}(\mu m + \tau - g)$

This locus has slope β/μ in m-b space (where b is on the horizontal axis), with b rising to the right of the locus, falling to the left. The $\dot{m} = 0$ locus is defined by (see (12))

(14) $\frac{v'(m)}{\theta} = \mu + \beta$

where m is falling below this horizontal locus, rising above it. The motion of the system is represented by the phase diagram in Figure 2. The steady state is a point of unstable equilibrium. For any values of g, μ and τ there is only one value of b which is consistent with steady state. If for given b and m the values of g, μ, and τ are such that we are to the right of the $\dot{b} = 0$ locus, b will grow without bound in the absence of a regime switch.

We may now derive the transversality condition at T. When stabilization is effected via cuts in government expenditures, consumption and hence the

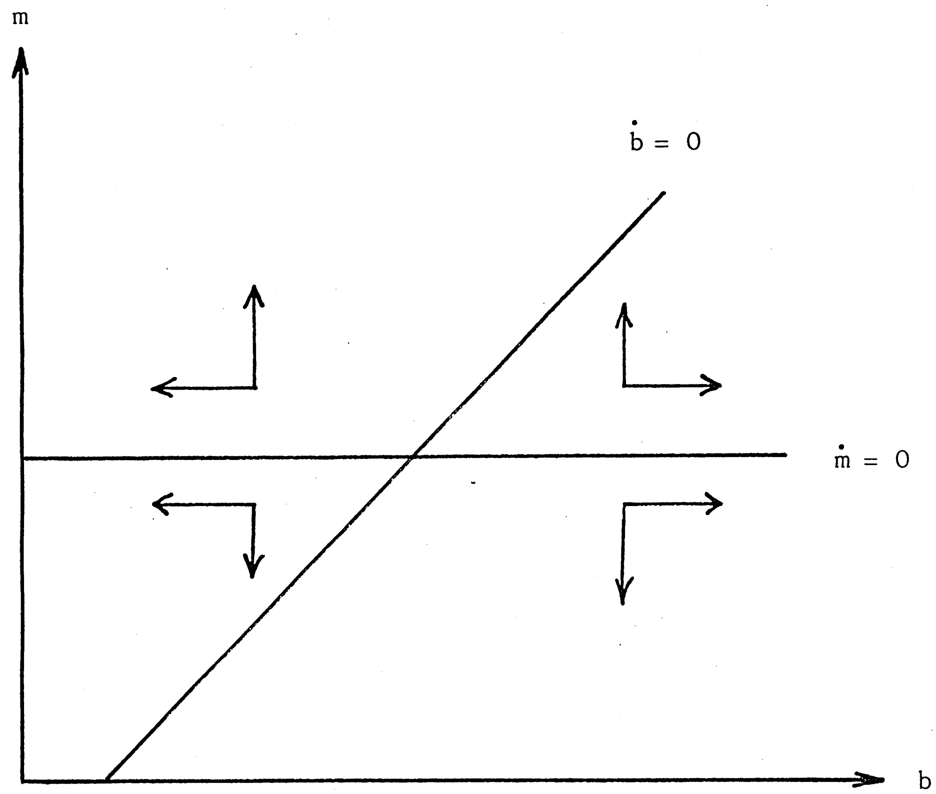


FIGURE 2

marginal utility of consumption, are not constant before and after the stabilization. For constant taxes and money growth rate the relation between expenditures and debt in steady state is given by (13), implying that consumption after T is

$$\begin{aligned} c_s &= y_0 - g_s \\ &= y_0 + \beta b - \tau - \mu m_s \end{aligned}$$

Using this relationship and $\theta_s = u'(c_s)$ we obtain from (14) the steady state implicit relationship between b and m_s

$$\beta + \mu = \frac{v'(m_s)}{u'(y_0 + \beta b - \tau - \mu m_s)} \tag{15}$$

Along this curve, which we denote by $m_s^g(b)$, real balances unambiguously rise with increases in debt, as in Figure 3. This can be seen geometrically as follows. The larger is the government's debt, the larger is the required budget cut. But the lower is public spending, the lower is the $\dot{b} = 0$ locus and the higher is the $\dot{m} = 0$ locus. The curve $m_s^g(b)$ consists of the collection of the intersection points of these loci for different levels of government spending.

As in the case of stabilization based on changes in money growth or taxes (see Drazen and Helpman [1985]), there is a maximum feasible level of steady state debt which can be supported by budget cuts, because g can drop at most to zero. This upper limit, which we denote by $\bar{b}$, depends on the rate of money growth and taxes.

The key difference between the case where stabilization is effected via budget cuts and the cases of stabilization via tax or money growth increases

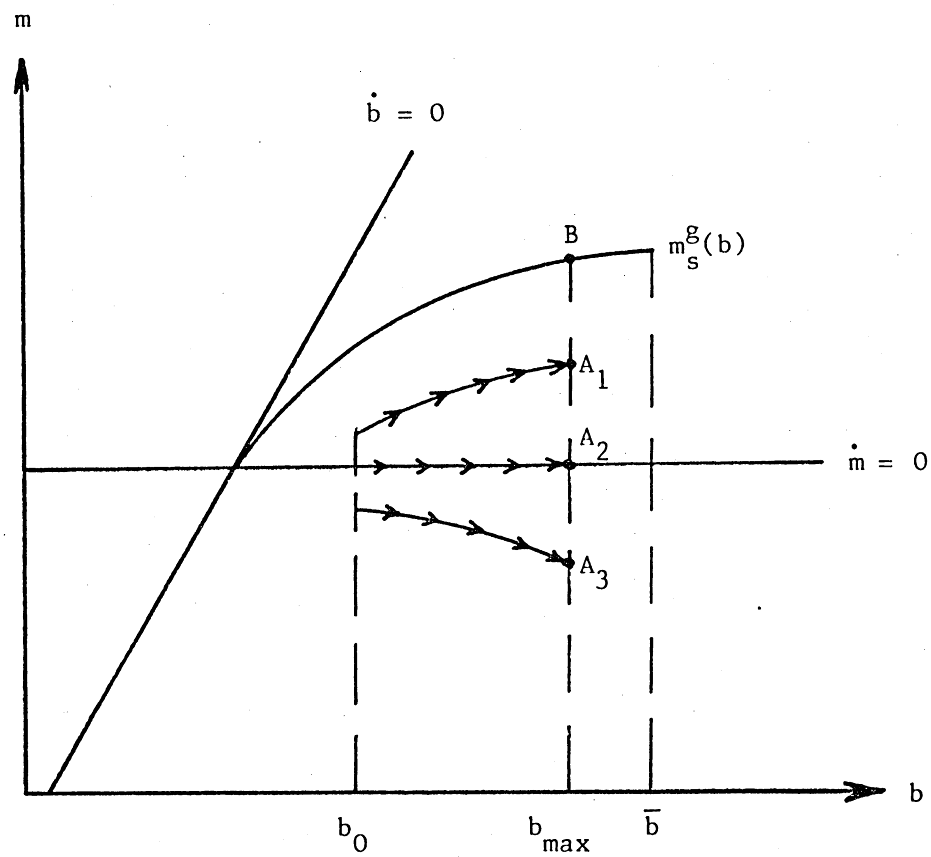


FIGURE 3

considered in our earlier paper is the behavior of the price level and the real interest rate at T. In the case of tax or money growth increases, there is no price level jump at T, so that the dynamic path must intersect the relevant terminal surface. In the case of stabilization via budget custs there will be a jump in the real interest rate and in the price level, and hence, in real balances at T. This has important implications for the dynamic path of real balances and inflation, and also for the real interest when the timing of the policy switch is uncertain. (We will see later on that in this case the real interest is not constant.)

To derive the dynamic path we begin by writing the endpoint condition (8) as

$$\frac{m_s(T)}{m(T)} = \frac{\theta}{\theta_s} \tag{16}$$

If we denote by $\bar{m}$ the value of real balances which solves (14), we have that

$$\frac{v'(\bar{m})}{\theta} = \beta + \mu = \frac{v'(m_s)}{\theta_s}$$

Therefore, using this relationship together with (16) we may write

$$\frac{v'(m_s)}{v'(\bar{m})} = \frac{m(T)}{m_s} \tag{17}$$

or, that

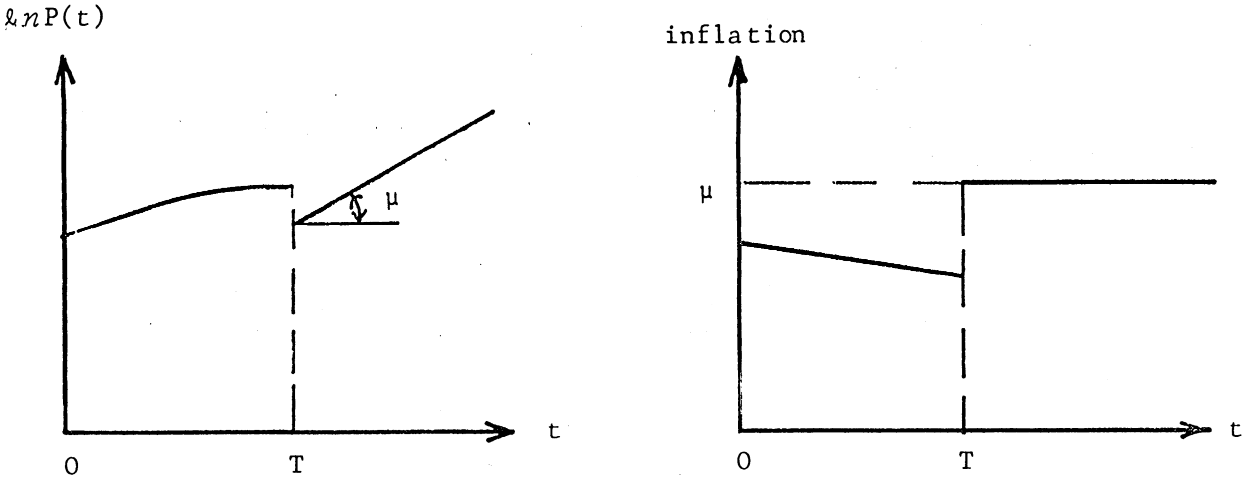
$$\frac{m(T)}{\bar{m}} = \frac{v'(m_s)m_s}{v'(\bar{m})\bar{m}} \tag{18}$$

It is clear from (18) that the value of its right-hand-side determines whether real balances just before the price jump at T are above, on, or below the horizontal $\dot{m} = 0$ locus. If this value is larger than one, real balances have to be above the $\dot{m} = 0$ locus; if it equals one, real balances have to be on this locus; and if it is smaller than one, real balances have to be below it. Obviously, since m_s depends on the stabilization date T, so does this value. It is, however, clear from the directions of movement indicated in Figure 2 that if the economy is above $\dot{m}=0$ at some time it has to be above $\dot{m}=0$ at all times, and similarly for being on or below $\dot{m}=0$. Therefore, there exist three possibilities of economic dynamics according to whether the right-hand-side of (17) is larger, equal, or smaller than one:

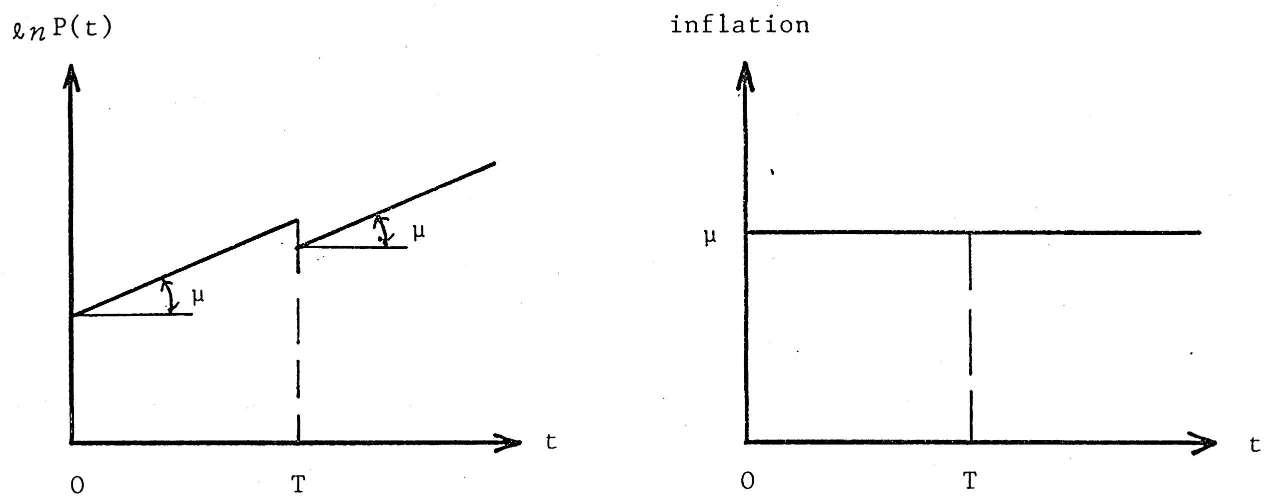
(i) Real balances are rising over time, as indicated by the upward sloping arrow path in Figure 3, until the policy switch takes place when $b = b_{max}$. Then real balances jump up from A_1 to B as a result of the downward price jump. Given (9), it is clear that in this case the rate of inflation is falling over time before the policy switch and it has to be lower than the rate of money growth (because real balances are rising). After the policy switch the rate of inflation equals the rate of money growth. Panel (i) in Figure 4 displays the resulting time path of the price level and the rate of inflation.

(ii) Real balances are constant over time -- as shown by the horizontal arrow path in Figure 3 -- until point A_2 is reached at which the policy switch takes place and real balances jump up to B. In this case the rate of inflation is constant over time except for the price jump at T, as described in panel (ii) of Figure 4.

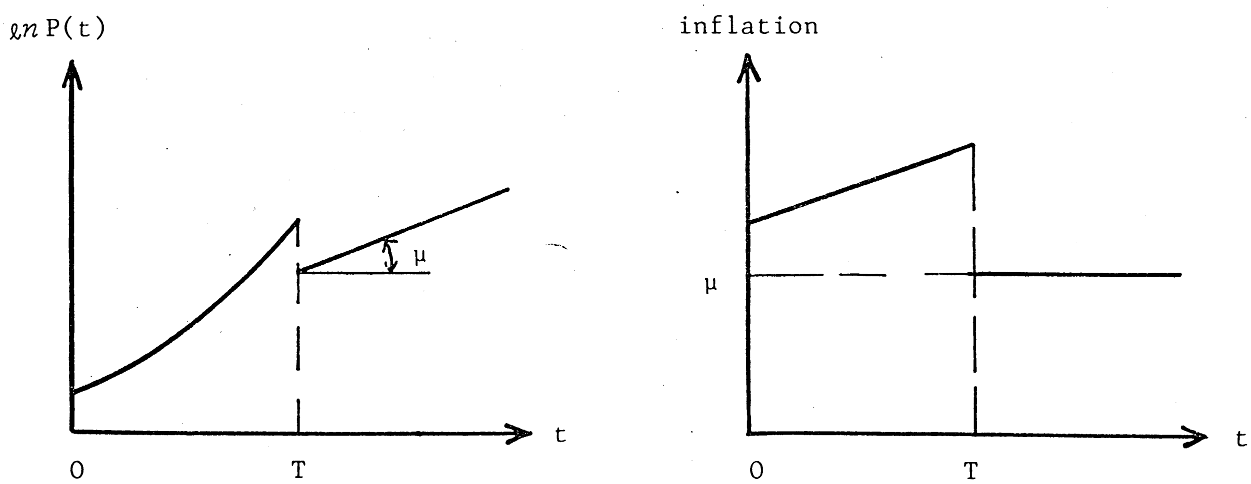
(iii) Real balances are falling over time -- as shown by the downward sloping arrow path in Figure 3 -- until point A_3 is reached at which the policy switch takes place and real balances jump up to point B. In this case the rate of inflation is rising over time before the policy switch and drops to a lower level after the policy switch, as described in panel (iii) of Figure 4.



(i) $v'(m_s)m_s > v'(\bar{m})\bar{m}$



(ii) $v'(m_s)m_s = v'(\bar{m})\bar{m}$



(iii) $v'(m_s)m_s < v'(\bar{m})\bar{m}$

FIGURE 4

The last case is of particular interest, because it produces a path which comes close to observed patterns during hyperinflations that were stopped by fiscal and monetary reforms (see Sargent (1982) and Bental and Eckstein (1985)). However, before we discuss this issue in more detail it will prove useful to discuss the economic parameters that determine the relevant path and the effect that the stabilization's timing has on the nature of the path.

First, observe that when $u(c)$ is linear, so that $\theta = \theta_s$, then $m_s^g(b) \equiv \bar{m}$. In this case there is no price jump at the stabilization instant, real balances are equal to $\bar{m}$ at each point in time, and the rate of inflation is constant and equal to the rate of money growth.

Second, observe that $v'(m)m$ is increasing in real balances when the interest elasticity of demand for real balances is larger than one (in absolute value) and it is declining in real balances when the interest elasticity of demand for real balances is smaller than one. (This is immediately seen by using (8) to write the demand for real balances in implicit form as $v'(m) = \theta i$ or $v'(m_s) = \theta_s i_s$, where i is the nominal interest rate.) Hence, since $m_s > \bar{m}$, the right-hand-side of (18) is larger than one for everywhere interest elastic demand functions for money and smaller than one for everywhere interest inelastic demand functions for money, implying that the time path of inflation and real balances depends on whether the interest elasticity of money demand is larger or smaller than one, but not on the timing of the policy switch. However, the timing of the policy switch does affect the nature of the inflationary process when the interest elasticity of money demand is on both sides of one for real balances above $\bar{m}$. A case in point is the Cagan-type demand function for money which is derivable from

$$v(m) = \begin{cases} \int_m^1 \ln x dx & \text{for } 0 \leq m \leq 1 \\ 0 & \text{for } m \geq 1 \end{cases}$$

(or the demand function used in the example in Section 4). Figure 5 describes $v'(m)m$ for this function. If $\bar{m} < 1/e$, as drawn in the figure, then for $\bar{m} < m_s < m^*$ the right-hand-side of (18) is positive and it is negative for $m^* < m_s < 1$. This results in a curve $m_T(b)$, drawn in Figure 6, that describes the level of $m(T)$ that corresponds to every possible level of b (formally defined by $m_T(b) = m_s^g(b)v'[m_s^g(b)]/v'(\bar{m})$). The larger is T the larger is the debt level at which the policy switch takes place. Therefore, starting with b_0 a relatively early date for a policy switch results in the upward sloping path in Figure 6 -- on which inflation is falling and real balances are rising prior to the policy switch -- while a policy switch at a later date results in rising inflation and declining real balances (described by the declining arrow path in Figure 6). There exists also a single intermediate date for a policy switch that leads to constant inflation and real balances.

Since $(g-\tau)$ is constant prior to the policy switch and debt is rising, the fiscal deficit inclusive of interest payments is rising over time. Therefore it is clear from our discussion that a rising deficit may be associated with either rising or falling inflation when stabilization is to be effected by budget cuts, despite the constancy of the rate of money growth. One should therefore note that a negative or zero correlation between budget deficits and inflation may arise even though it is clear that the deficit is the ultimate cause of inflation.

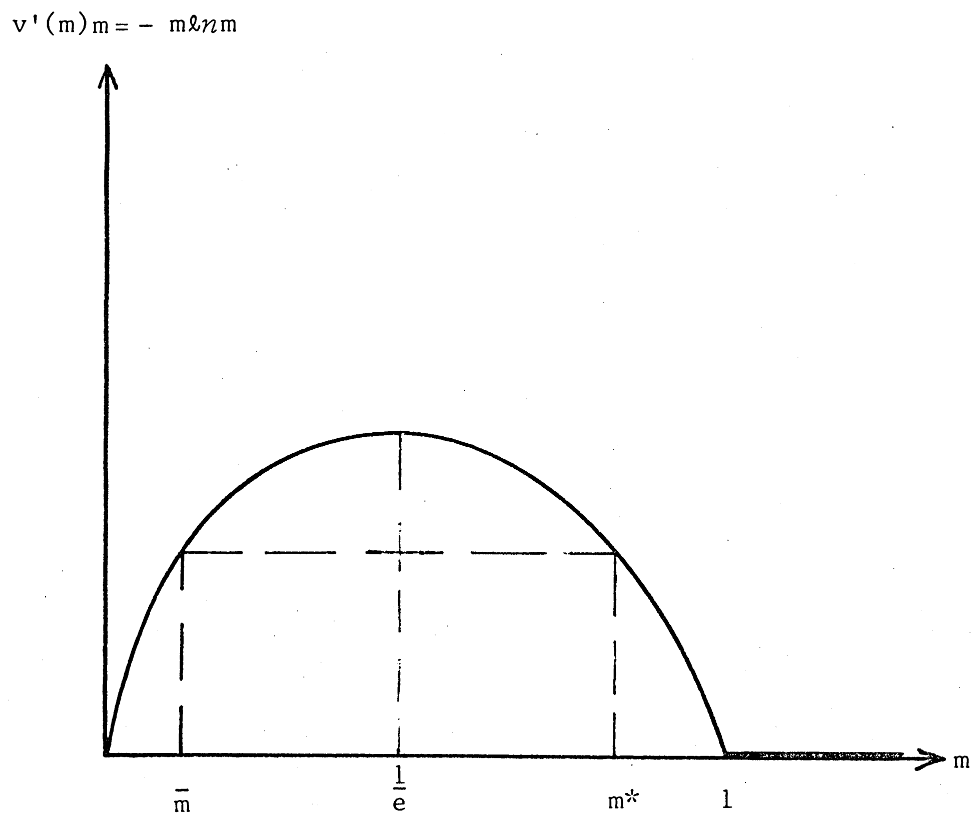


FIGURE 5

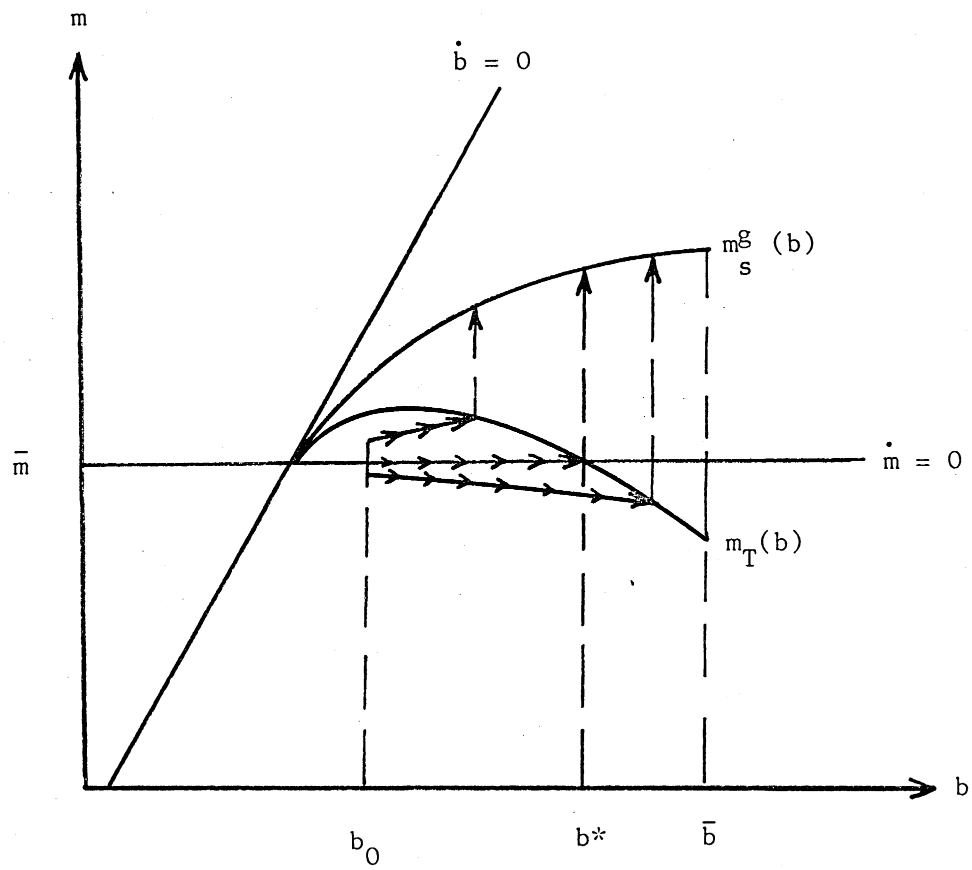


FIGURE 6

Before considering the case of uncertainty about the timing of the policy switch we would like to note that this framework can generate dynamics that mimic the dynamics of real balances and inflation during hyperinflations and the period following their cessation by means of monetary and fiscal policy. It has been noted that during hyperinflations real balances were falling and the rate of inflation was rising prior to stabilization, and that after (and often somewhat before) a fiscal reform real balances rose and the rate of inflation declined rapidly despite the continued growth in the stock of money (e.g. Sargent (1982)). Our model is capable of generating these features. Naturally, there are many aspects of the hyperinflations that have to do with structural characteristics that are absent from our model, such as the behavior of exchange rates and unemployment. Consequently, we do not claim to provide a complete theoretical explanation of these episodes. All we do is to enlighten a particular aspect that turns out to be rather difficult to explain (see Bental and Eckstein (1985)).

Suppose that the stabilization effort takes place in two stages at time T and time $T' > T$. In the first stage government expenditure is reduced while in the second stage the rate of money growth is reduced. For simplicity assume that the budget cut is just sufficient to bring the economy to a steady state when only the first stage of the stabilization effort is effected, and that this would have resulted in dynamics of type (iii) that are described by the downward sloping arrow path in Figure 7. Since this is a dynamic path that results from an interest inelastic money demand function, it is consistent with the empirical evidence. On this path real balances are falling and the rate of inflation is rising, which is consistent with the stylized facts of the hyperinflations (and other inflations as

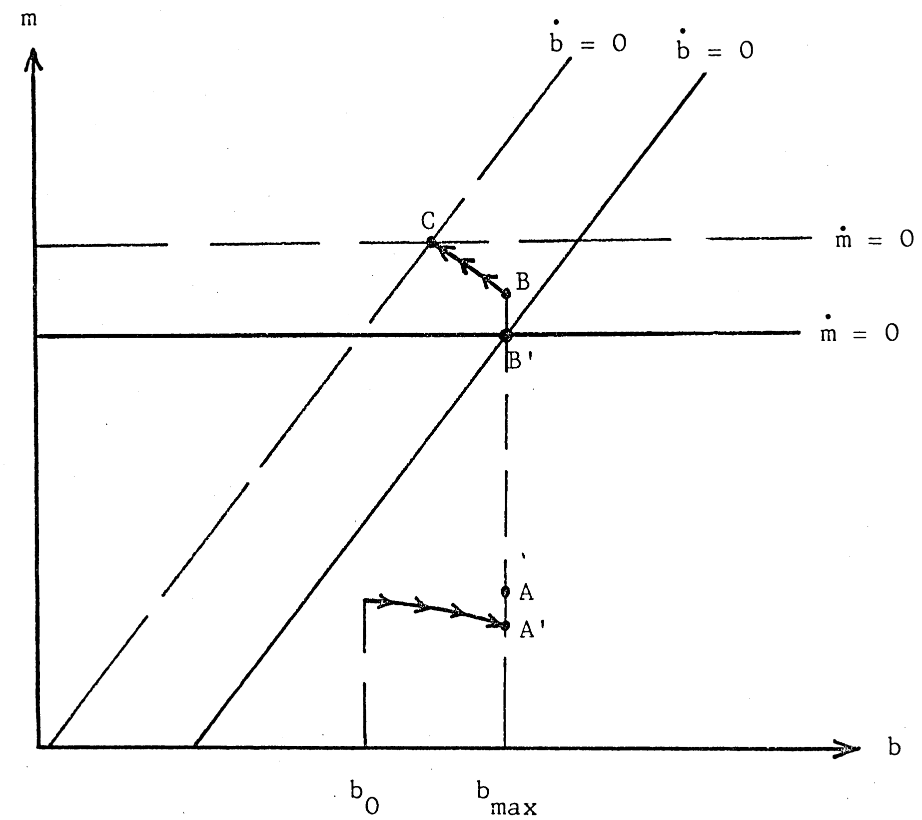


FIGURE 7

well). Now, if at T when $b(T) = b_{max}$ stabilization was effected by a budget cut only, real balances would have jumped from A' to B' and the rate of inflation would have dropped to the rate of money growth, where the solid lines $\dot{b} = 0$ and $\dot{m} = 0$ describe the relevant curves for the post-budget-cut circumstances. However, the jump to B' is not an equilibrium jump when the public expects a reduction in the rate of money growth at time T'. This point can be seen as follows. A reduction in μ shifts upward both the $\dot{b} = 0$ and the $\dot{m} = 0$ loci (see (13) and (14)). Assuming, however, that the initial rate of money growth is smaller than the maximum inflation tax rate of money growth, the $\dot{b} = 0$ locus shifts upwards more than the the $\dot{m}=0$ locus,as shown by the broken lines (this is shown in Drazen and Helpman (1985)). Hence, point C is the steady state that has to be reached following the reduction in the rate of money growth. If the economy is at B' prior to the reduction in the rate of money growth it will never reach C after the reduction in the rate of money growth, because when the broken lines $\dot{b} = 0$ and $\dot{m} = 0$ become relevant point B' brings about rising debt and falling real balances.

We showed in Drazen and Helpman (1985) that an anticipated change in the rate of money growth generates a dynamic path on which the price level is continuous (see also Drazen (1985)). Therefore, if the economy is to reach C at time T' without a price jump at time T' it has to follow the arrow path connecting B with C after the budget cut. This determines the required time interval between the budget cut and the reduction in the rate of money growth. Alternatively, if the time interval T–T' between the two stages is taken as exogeneous, the level of debt at which the budget cut must take place is endogenously determined.It also determines point B as the point to which the economy has to jump following the budget cut. Given B and condition (16)

-- which determines the required jump in real balances as a result of the budget cut -- we find point A as the point at which the economy has to be the instant before the budget cut. If $\bar{m}$ that solves (14) is above A then prior to the budget cut point A is approached by a downward sloping path, just like the path shown in Figure 7 that approaches A'. Under these circumstances the time patterns of real balances and inflation looks like the graphs in Figure 8. Prior to the budget cut real balances are falling and the rate of inflation is rising (with real balances being below $\bar{m}$ and inflation being above the rate of money growth). The budget cut brings about a discontinuous increase in real balance holdings and a drop in the inflation rate below the rate of money growth (because B is above B' at which real balances are above $\bar{m}$). Then, following the budget cut, real balances are rising and the rate of inflation is falling until time T' at which the rate of money growth is reduced to the new steady state level.

3. Uncertainty about Timing

The case where the date of a stabilization is uncertain is considerably more complicated. The basic structure is the same as above, except that now the regime switch may occur at any time between 0 and some T_{max}, where the cumulative distribution of a switch occurring until T is F(T). We do not assume that $F(\cdot)$ is necessarily continuously differentiable. Clearly $F(0) = 0$ and $F(T_{max}) = 1$. We consider the case where only one switch takes place.

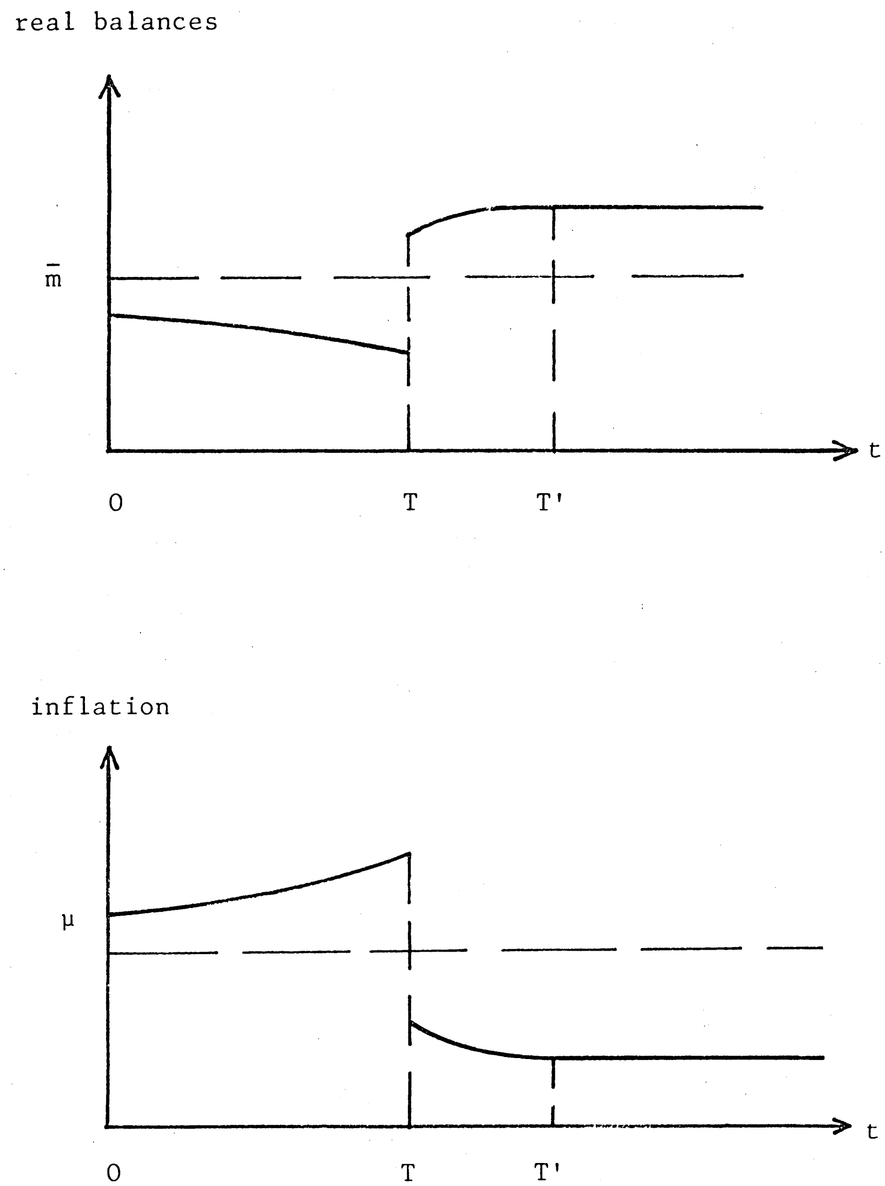


FIGURE 8

The expectation of discounted utility is taken over $dF(T)$. The individual's present discounted utility if a switch occurs at T is given by (2). Expected welfare over all possible realizations of T may then be written

$$(19) \qquad \int_0^{T_{max}} \{\int_0^T e^{-\beta t}[u(c(t)) + v(\frac{M(t)}{P(t)})]dt + e^{-\beta T}V^s(b(T) + \frac{M(T)}{P_s(T)};T)\}dF(T)$$

The individual's problem is therefore to maximize (19) subject to constraints (3) and (4). The derivation of the first-order conditions for this case are presented in Appendix 2, to which the reader may refer.

The necessity of a jump in the real interest rate at the time of stabilization implies that the real interest rate is equal to the discount rate plus a risk premium reflecting the probability of a jump. More specifically, individual maximization with respect to the level of consumption plus market clearing implies (see equation (A2.5) in Appendix 2)

$$(20) \qquad dR = \beta dt + (1 - \frac{\theta_s}{\theta})\frac{dF}{1-F}$$

where θ_s is the marginal utility of consumption after stabilization ($\theta_s = u'(y_0 - g_s)$), while θ is the pre-stabilization marginal utility of consumption. θ_s is falling over time, since the longer that no stabilization has taken place, the higher is the level of debt at the time of stabilization, the lower must be government post stabilization expenditures, and hence the higher is private consumption.

The second term on the right-hand-side is the risk premium built into the real interest rate. It is the product of the instantaneous probability of a regime switch at t conditional on one not having previously taken place $(\frac{dF}{1-F})$ and the percentage fall in marginal utility of consumption which the regime switch induces $(\frac{\theta - \theta_s}{\theta})$. Since typically both of these terms are rising over time until a switch occurs, the real interest rate dR will be rising as well.(The hazard rate may in some circumstances be falling over time).

Individual maximization further implies the equality of the ratio of marginal utilities of real balances and consumption to the nominal interest rate, where the latter includes both a real and a nominal risk premium. Specifically, we obtain (see equation (A2.8) in Appendix 2)

$$\text{(21a)} \qquad \frac{v'(m)}{\theta}dt = \beta dt + \frac{dP}{P} + \frac{dF}{1-F}\left(1 - \frac{\theta_s/P_s}{\theta/P}\right)$$

$$\text{(21b)} \qquad = \beta dt + \frac{dF}{1-F}\left(1 - \frac{\theta_s}{\theta}\right) + \frac{dP}{P} + \frac{dF}{1-F}\left(1 - \frac{P}{P_s}\right)\frac{\theta_s}{\theta}$$

The second right-hand-side term in (21b) is the risk premium associated with jumps in the real interest rate. The last term is a risk premium reflecting changes in the real value of money due to a regime switch. It includes three effects: the hazard rate, the percentage change in the real value of nominal balances from a price jump $(\frac{1/P - 1/P_s}{1/P})$ and the change in the utility value of real balances (θ_s/θ).

Equation (21) allows us to derive the dynamic equations for debt and real balances, precisely as we did in section 1. When F is differentiable we obtain from the government budget constraint (10) and from (11)

(22) $$\dot{b} = (\beta + \frac{F'}{1-F}[1 - \frac{\theta_s}{\theta}])b + g - \tau - \mu m$$

(23) $$\frac{\dot{m}}{m} = \beta + \mu - \frac{v'(m)}{\theta} + \frac{F'}{1-F}[1 - \frac{m_s^g(b)}{m} \frac{\theta_s}{\theta}]$$

These equations may be expressed in a time-autonomous form. Note first that θ_s may be written as a function of b since by (15) $\theta_s = v'(m_s^g(b))/(\beta + \mu)$. The conditional probability of a switch should logically also be a function of debt. Since there is a maximum level of debt consistent with stabilization and since b grows without bound in the absence of stabilization, one must expect that if no regime switch has occurred before b hits some b_{max} (less than or equal to maximum feasible steady state b attained, for example, at $g_s = 0$), then a regime switch must occur at that time. More generally, one may argue that the probability of a regime switch grows as $b(t)$ approaches b_{max}, with a regime switch occurring with certainty sometime between 0 and the time $b(t)$ hits b_{max}. We represent this by writing the conditional density of a switch as a function of b

(24) $$\frac{F'(t)}{1-F(t)} \equiv \phi(b(t)).$$

where $\phi(b) \geq 0$. The restriction that $F(T_{max}) = 1$ will imply that ϕ becomes infinite as b approaches b_{max}, unless the distribution has a mass point at T_{max}.

Given these specifications of θ_s and $\frac{F'}{1-F}$, (22) and (23) may be written

(25) $$\dot{b} = (\beta + \phi(b)[1 - \frac{\theta_s(b)}{\theta}])b + g - \tau - \mu m$$

(26) $$\frac{\dot{m}}{m} = \beta + \mu - \frac{v'(m)}{\theta} + \phi(b)[1 - \frac{m_s^g(b)}{m} \frac{\theta_s(b)}{\theta}]$$

These equations form an autonomous system of two equations in m and b which yield equilibrium time paths of these variables. It is the existence of both real and nominal risk premia which complicate these equations.

As in the certainty case, we may derive phase diagrams in $m - b$ space. The equation for the $\dot{b} = 0$ locus may be written

$$\beta b + \phi(b)(1 - \frac{\theta_s(b)}{\theta})b + g - \tau = \mu m \tag{27}$$

Assuming that $\phi(\cdot)$ is rising in b the second left-hand-side term is unambiguously increasing in b. Hence, the $\dot{b} = 0$ locus is upward sloping with a slope greater than in the certainty case. b is rising to the right of this locus, falling to the left of it.

The $\dot{m} = 0$ locus is defined by

$$\frac{v'(m)}{\theta} - (\beta + \mu) = \phi(b)[1 - \frac{\theta_s(b)m_s^g(b)}{\theta m}] \tag{28}$$

As steady state b rises, θ_s falls and m_s rises. The derivative of the right-hand-side of (28) is therefore ambiguous with respect to b. Therefore the $\dot{m} = 0$ locus may either rise or fall in m-b space and in fact need not be single signed. Real balances will be falling below the locus, rising above it. The locus will lie below $m_s^g(b)$, since along $m_s^g(b)$ the probability ϕ of a regime switch is zero, so that $\dot{m} > 0$ from (26). The phase diagram may be represented as in Figure 9, where we show two possible loci for $\dot{m} = 0$.

We may now derive the dynamic path. The ambiguity about the possible slope of the $\dot{m} = 0$ locus (discussed in the previous paragraph) combined with the jump in real balances at T_{max} (due to a price level jump at this point) will lead to a variety of possible paths. The jump in real balances at T_{max} arises (as in the certainty case) because even when the switch date is

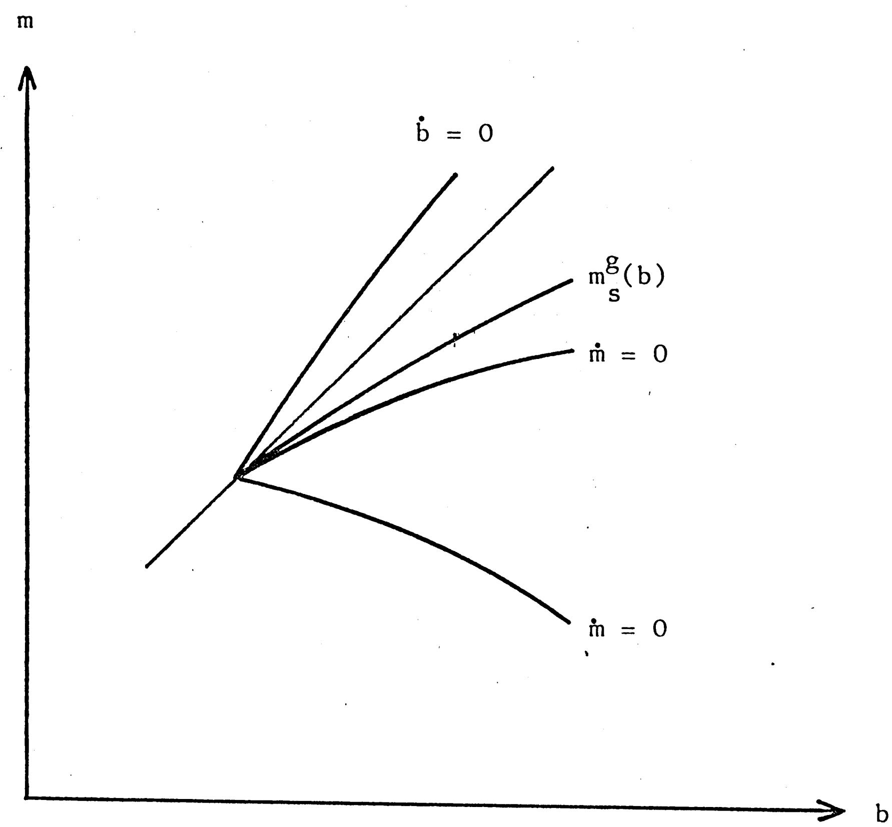


FIGURE 9

known, the jump in the marginal utility of consumption at the switch date induces a jump in the price level. Specifically, at T_{max} we have (see equation A2.7 in Appendix 2)

$$(29) \qquad \frac{\theta}{\theta_s(b_{max})} = \frac{P(T_{max})}{P_s(T_{max})} = \frac{m_s^g(b_{max})}{m_{max}}$$

where m_{max} denotes the pre-switch level of real balances at T_{max} (analogous to $m(T)$ in the certainty case). Since $\theta > \theta_s(b_{max})$ (as government spending is lower and therefore consumption higher after a stabilization), m_{max} must lie below $m_s^g(b_{max})$.

The key to the behavior of the dynamic path is the location of m_{max} relative to $\dot{m} = 0$ locus. If m_{max} lies above this locus, the dynamic path will be rising as we approach T_{max}; if it lies below, it will be falling. As demonstrated in Appendix 3, where possible paths are discussed in detail, the location of m_{max} relative to the $\dot{m} = 0$ locus depends on whether it is greater or lesser than $\bar{m}$, the value of real balances for which $\dot{m} = 0$ in the certainty case (see the discussion preceding equation (18)). m_{max} will lie below the $\dot{m} = 0$ locus under uncertainty when m_{max} is less than $\bar{m}$ and will lie above when it is greater.

The other crucial characteristic is whether the dynamic path crosses the $\dot{m} = 0$ locus. If it does, real balances will not move monotonically along the path. If it does not cross, they will. (Debt is monotonically rising along every path). Of course, whether or not the dynamic path crosses the $\dot{m} = 0$ locus depends on the characteristics of that locus. Leaving a more detailed discussion of the paths to be presented in Appendix 3, the two possibilities about the location of m_{max} and the two possibilities about crossing $\dot{m} = 0$ yield four general types of paths for real balances: they may rise monotonically; they may rise near T_{max}, falling or oscillating beforehand; they may fall monotonically; and, they may fall near T_{max}, rising or oscillating beforehand. The conditions for each case are summarized in Table 1, and the cases are illustrated in the four panels of Figure 10. One should, of course, remember that these are paths only until a stabilization takes place. When a stabilization takes place at some t before T_{max}, the value of real balances jumps to $m_s^g(b(t))$, due to a price level jump.

TABLE 1

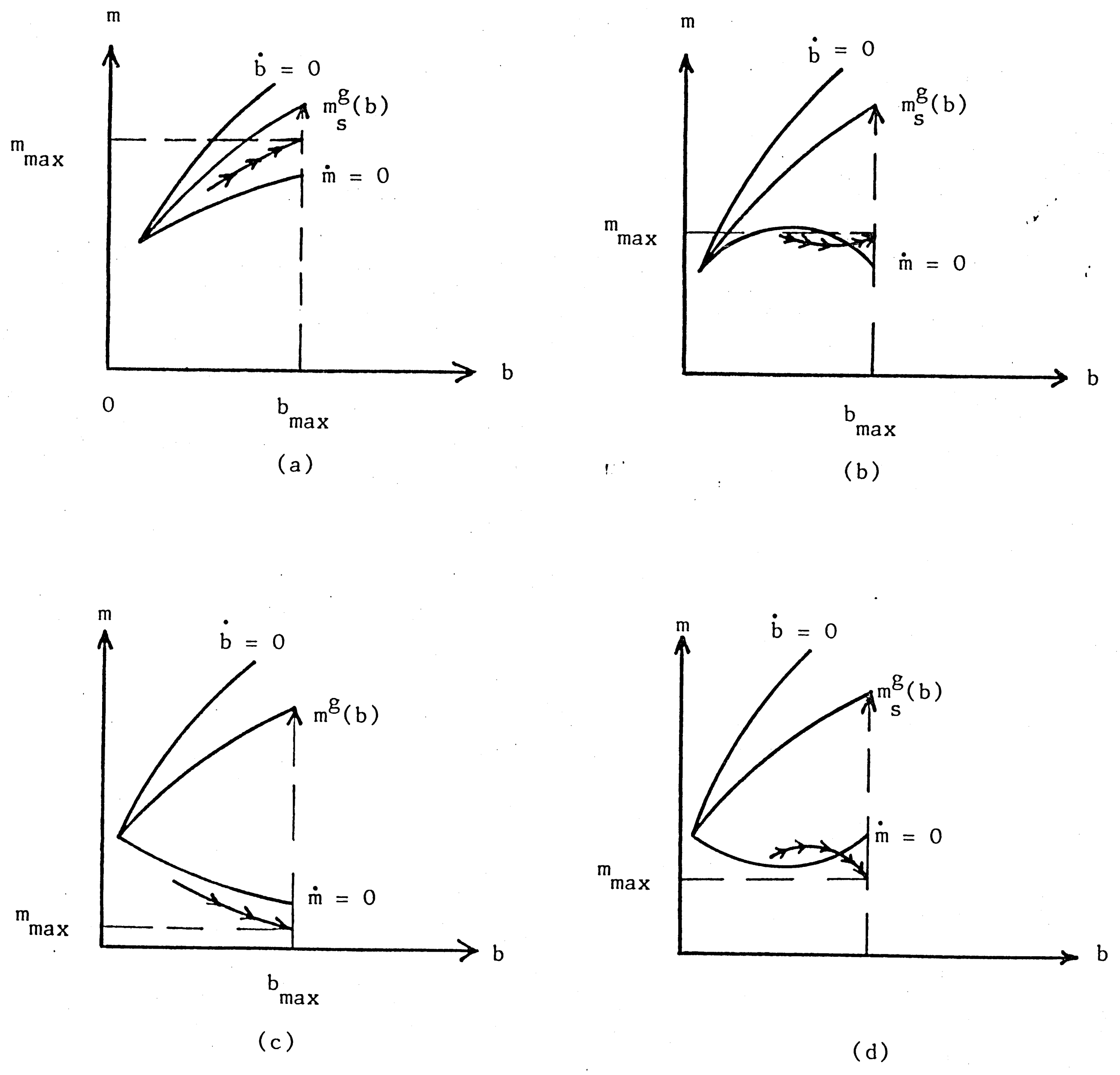
POSSIBLE DYNAMIC PATHS

	Path does not cross $\dot{m} = 0$ locus[1]	Path crosses $\dot{m} = 0$ locus
$m_{max} > \bar{m}$ (m_{max} above $\dot{m}=0$ locus)	Real balances rise monotonically (Figure 10a)	Real balances fall, then rise[2], or, oscillate and then rise (Figure 10b)
$m_{max} < \bar{m}$ (m_{max} below $\dot{m}=0$ locus)	Real balances fall monotonically (Figure 10c)	Real balances rise, and then fall[3], or, oscillate and then fall (Figure 10d)

1. This is consistent with the $\dot{m} = 0$ being either positively or negatively sloped.

2. This is consistent with the $\dot{m} = 0$ locus rising and then falling, or falling throughout.

3. This is consistent with the $\dot{m} = 0$ locus falling and then rising, or rising throughout.



m
ḃ = 0
$m_s^g(b)$
m_{max}
ṁ = 0
b
0
b_{max}
(a)
m
ḃ = 0
$m_s^g(b)$
m_{max}
ṁ = 0
b
b_{max}
(b)
m
ḃ = 0
$m^g(b)$
ṁ = 0
m_{max}
b
b_{max}
(c)
m
ḃ = 0
$m_s^g(b)$
ṁ = 0
m_{max}
b
(d)

FIGURE 10

The behavior of inflation along any path depends not only on the behavior of real balances, but also on the existence of the risk premia, which reflect the possiblity that at any instant a regime change may take place which would induce a jump in both the price level and the real interest rate. The first-order condition (21) equates the ratio of marginal utilities of real balances and consumption to the nominal interest rate, which reflects risk premia which may fluctuate. Solving for the inflation rate, we obtain (where $F(\cdot)$ is differentiable)

$$\begin{aligned}\pi(t) &= -\beta + \frac{v'(m(t))}{\theta} - \phi(b(t))\left(1 - \frac{\theta_s(b(t))}{\theta}\right) - \phi(b(t))\left(1 - \frac{P(t)}{P_s(t)}\right)\frac{\theta_s(b(t))}{\theta} \\ &= -\beta + \frac{v'(m(t))}{\theta} - \phi(b(t))\left[1 - \frac{\theta_s(b(t))}{\theta}\frac{m_s^g(b(t))}{m(t)}\right]\end{aligned} \tag{30}$$

Differentiating with respect to time we obtain

$$\begin{aligned}\dot{\pi} &= \frac{\dot{m}}{m}\left[\frac{mv''}{\theta} - \phi\frac{\theta_s}{\theta}\frac{m_s^g}{m}\right] \\ &\quad + \dot{b}\left[-\phi'\left(1 - \frac{\theta_s}{\theta}\frac{m_s^g}{m}\right) + \phi\left(\frac{m_s^g}{m} \cdot \frac{d\theta_s/db}{\theta} + \frac{\theta_s}{\theta}\frac{dm_s^g/db}{m}\right)\right]\end{aligned} \tag{31}$$

The term multiplying $\dot{m}/m$ is unambiguously negative, so that the first term is unambiguously positive along the dynamic path, when real balances are falling, negative if they are rising. The term in the second brackets is of ambiguous sign, so that $d\pi/dt$ may be of either sign and can change sign. Therefore, inflation and real balances need not move in opposite directions.

4. AN APPLICATION TO HYPERINFLATION

The dynamic path illustrated in panel b of Figure 10, in which real balances rise as we approch T_{max}, but fall (or oscillate) beforehand is especially interesting because it is broadly consistent with the behavior of key variables during several of the European hyperinflations of the 1920's. These inflations were ended by fiscal reforms which included sharp cuts in government expenditures, reforms whose timing was uncertain before they were actually enacted. It is interesting that a model of probabilistic expenditure cuts can replicate the broad facts without unrealistic assumptions about money demand functions or expectations.

The broad facts to be explained were set out above. The accelerating inflation and the sharp drop in real balances which characterize hyperinflations were reversed before the fiscal reforms which lead to a lower growth rate of money was enacted. (Austria and Hungary in the 1920's provide especially clear examples of this phenomenon).

A standard explanation for the reduction in inflation prior to an actual fiscal reform is that it was expected that the unavoidable stabilization was imminent. This explanation, however, raises the question of why people believed a stabilization was imminent at this point in time, but not at some earlier point. Our model can yield a dynamic path on which real balances fall and then rise before a budget cut and inflation moves in the opposite direction without imposing arbitrary assumptions about expectations. We illustrate this possibility by means of an example, which is kept drastically simple in order to calculate a solution.

Let the utility function be of the form

$$u(c) + v(m) = -e^{-c} - e^{-km} \qquad (32)$$

where k is a constant. For simplicity, we assume that k is related to the discount rate via

$$(33) \qquad \beta = ke^{1/2}.$$

We further specify output, taxes, and government spending so that

$$(34) \qquad y_0 - \tau = 1$$

$$(35) \qquad g - \tau = 0$$

The last assumption is consistent with a growing deficit if we assume that initial debt is positive, so that debt and debt service are always positive as long as (35) holds. For simplicity, we further assume that $\mu = 0$. Finally, we assume that the probability of a regime switch follows a Poisson process with parameter $\lambda = 1$, so that the instantaneous probability of a switch $\phi(b)$ is constant over time at a level of unity with a mass point at T_{max}. (This assumption is made both for simplicity and to address the issue of expectations raised above.)

With these assumptions, we may derive specific dynamic equations for real balances and debt. The equation for the terminal surface $m_s^g(b)$, given in equation (16), becomes

$$(36) \qquad \frac{ke^{-km_s^g}}{e^{-(1+\beta b)}} = \beta$$

which using (33) implies

$$(37) \qquad m_s^g(b) = \frac{1}{2k} + e^{1/2}b$$

Using (36) and the definition $\theta_s = v'(m_s^g)/(\beta+\mu)$, we obtain

$$(38) \qquad \theta_s = \frac{ke^{-km_s^g}}{\beta} = e^{-(1+\beta b)}$$

The two dynamic equations become

$$(39) \qquad \frac{\dot{b}}{b} = \beta + 1 - e^{-\beta b}$$

$$(40) \qquad \frac{\dot{m}}{m} = \beta + 1 - ke^{1-km} - \frac{m_s^g}{m} e^{\frac{1}{2} - km_s^g}$$

where θ, the marginal utility of consumption before a regime switch, has been set equal to e^{-1}, since (34) and (35) imply that $c = 1$ before a stabilization. We, of course, assume $b_0 > 0$ to yield a rising deficit.

The value of m_{max} may be found from equation (29) yielding

$$(41a) \qquad m_{max} = m_s^g e^{\frac{1}{2} - km_s^g}$$

Combined with the solution for $m_s^g(b)$ at b_{max}, this yields

$$(41b) \qquad m_{max} = \frac{1+\beta b_{max}}{\beta} e^{\frac{1}{2} - \beta b_{max}} .$$

This gives the endogeneous m_{max} for every value of the exogenous b_{max}.

We now ask what are the characteristics of these curves. The $\dot{b} = 0$ locus is clearly a vertical line at the value of b which solves

(42) $\beta + 1 = e^{-\beta b}$.

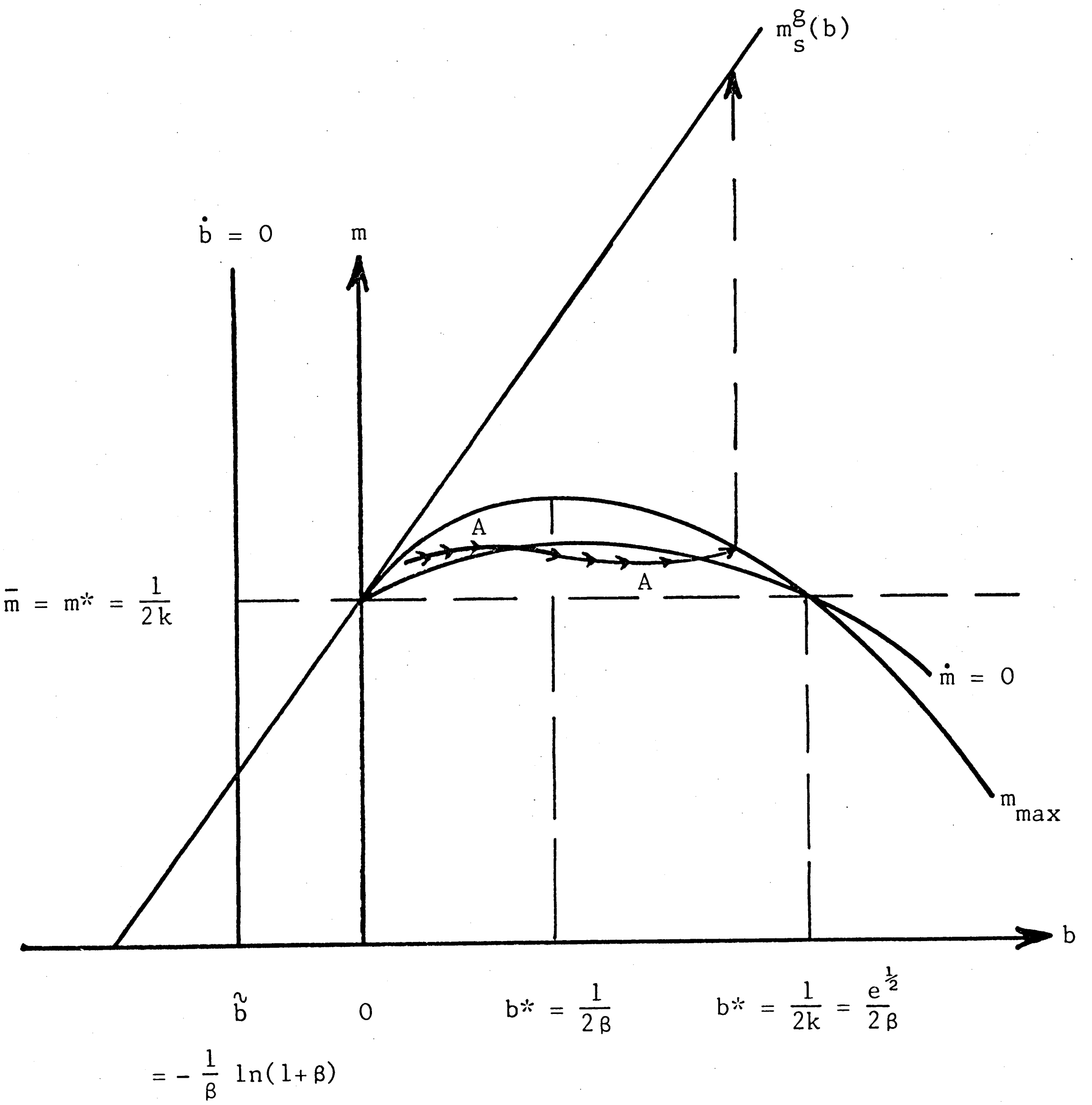
To evaluate m_{max} we note from (41a) that m_{max} is at a maximum when $km_s^g = 1$. From (37) and (33) we thus obtain that m_{max} reaches its maximum when debt is at a level $b^* = 1/2\beta$. m_{max} is rising to the left of this point and falling to the right, approaching zero as b_{max} approaches infinity. We represent this by the appropriately labelled curve in Figure 11.

From equation (40) and (41a) the $\dot{m} = 0$ curve may be written

(43) $$m(1+\beta - ke^{1-km}) = m_s^g e^{\frac{1}{2} - km_s^g} = m_{max}$$

Since the left-hand-side of (43) is monotonically rising in m, m reaches its maximum along $\dot{m} = 0$ when the right-hand-side is maximized using (37), namely at $b^*=1/2\beta$ where m_{max} also reaches its maximum.The two loci cross at two points. From (43) we see that m along the $\dot{m} = 0$ locus equals m_{max} when $\beta = ke^{1-km}$, or using (33), when (denoting the crossing point by m^{**}) $m^{**} = \frac{1}{2k}$. This will be true at two points, one to the left of b^*, the other to the right. Let us denote the value of debt at the second crossing point by b^{**}. The first crossing point is at $b = 0$, which may be seen by noting that from (37) and (41a) m_{max} equals $1/2k$ at $b = 0$. Real balances along $m_s^g(b)$ also equal $1/2k$ when debt equals 0.

As was indicated in the general discussion of the uncertainty case, m_{max} will lie above or below the $\dot{m} = 0$ locus depending on whether m_{max} is above or below $\bar{m}$, the value of real balances for which $\dot{m} = 0$ in the certainty case. In our example, this corresponds to a value $\bar{m} = 1/2k$. Therefore m_{max} lies above the $\dot{m}=0$ locus between debt equal to zero and b^{**}. We represent the curves as in Figure 11.



$m_s^g(b)$
$\dot{b} = 0$
m
A
A
$\bar{m} = m^* = \frac{1}{2k}$
$\dot{m} = 0$
m_{max}
b
$\tilde{b}$
$= -\frac{1}{\beta} \ln(1+\beta)$
0
$b^* = \frac{1}{2\beta}$
$b^* = \frac{1}{2k} = \frac{e^{\frac{1}{2}}}{2\beta}$

FIGURE 11

The change in sign of the $\dot{m} = 0$ locus opens up the possibility that the dynamic path for real balances will also change sign. Real balances will fall before rising (as in path AA in Figure 11 where in fact m rises and falls before rising towards m_{max}) as long as b_{max} is sufficiently above b^*, but below b^{**}. (In terms of real balances, m_{max} must be sufficiently below the maximum value of m along the $\dot{m} = 0$ locus, but above $\bar{m}$).

The inflation rate also oscillates, falling at the end of the path, but rising earlier on. The value of the inflation rate may be found from (21a), namely,

$$\Pi \equiv \frac{\dot{P}}{P} = \frac{v'(m)}{\theta} - \beta - \mu - \phi(b)[1 - \frac{\theta_s(b)m_s^g(b)}{\theta m}]$$

which in our example becomes

$$(44) \qquad \Pi = -\beta - 1 + ke^{1-km} + \frac{m_s^g(b)}{m} e^{\frac{1}{2} - km_s^g(b)}$$

where $m_s^g(b)$ is defined by (37).

The change in the inflation rate over time is therefore

$$(45) \qquad \dot{\Pi} = -\left(k^2 e^{1-km} + \frac{m_s^g}{m^2} e^{\frac{1}{2} - km_s^g}\right)\dot{m} + \frac{e^{1/2}}{m}\left(\frac{d}{db}[m_s^g e^{-km_s^g}]\right)\dot{b}$$

The term in the first parentheses is positive so the term multiplying $\dot{m}$ is always negative. The term in the second parentheses (and hence the term multiplying $\dot{b}$) is positive until $b = b^*$, negative thereafter. Therefore, along paths such as AA in Figure 11, there are four regions of interest, which are noted in Figure 12 . In region I, where the path is above the $\dot{m} = 0$ locus, real balances are rising, so that the first term in (45) is negative. The second term is positive, since $b < b^*$, so the sign of $\dot{\pi}$ is ambiguous. In region II, real balances are now falling, and debt is less than b^*, so that both terms of (45) are positive and the inflation rate is unambiguously rising. In region III, real balances are still falling but $b > b^*$ so that the two terms in (45) are of opposite sign, and inflation may be either rising or falling. Finally, in region IV, where real balances are rising, both terms in (45) are positive and inflation is unambiguously falling.

One sees from paths such as AA that we can reproduce the broad facts of hyperinflation. If the stabilization does not come until the level of debt is close to b_{max}, real balances will be falling over a large part of the path, but will start rising before the stabilization actually takes place. Inflation will be rising over a large part of the path, but will start to fall before the actual stabilization. Inflation and real balances are rising at the end of region I and both are falling at the end of region III. The stabilization will be marked by a jump up to the terminal surface $m_s^g(b)$, that is, by a downward jump in prices and an upward jump in real balances, as was sometimes observed during hyperinflations.

One should note that we have been able to replicate the turnaround in real balances and inflation without assuming that expectations of a stabilization become extremely high at about this point. We have purposely avoided such an assumption, since it would build in exogeneously a characteristic of the path we would like to be endogeneous. Of course, if we were to assume, as we did in the general case, that the instantaneous probability of a regime switch rose with the passage of time (as the level of debt approached b_{max}), the change in real balances and inflation would be

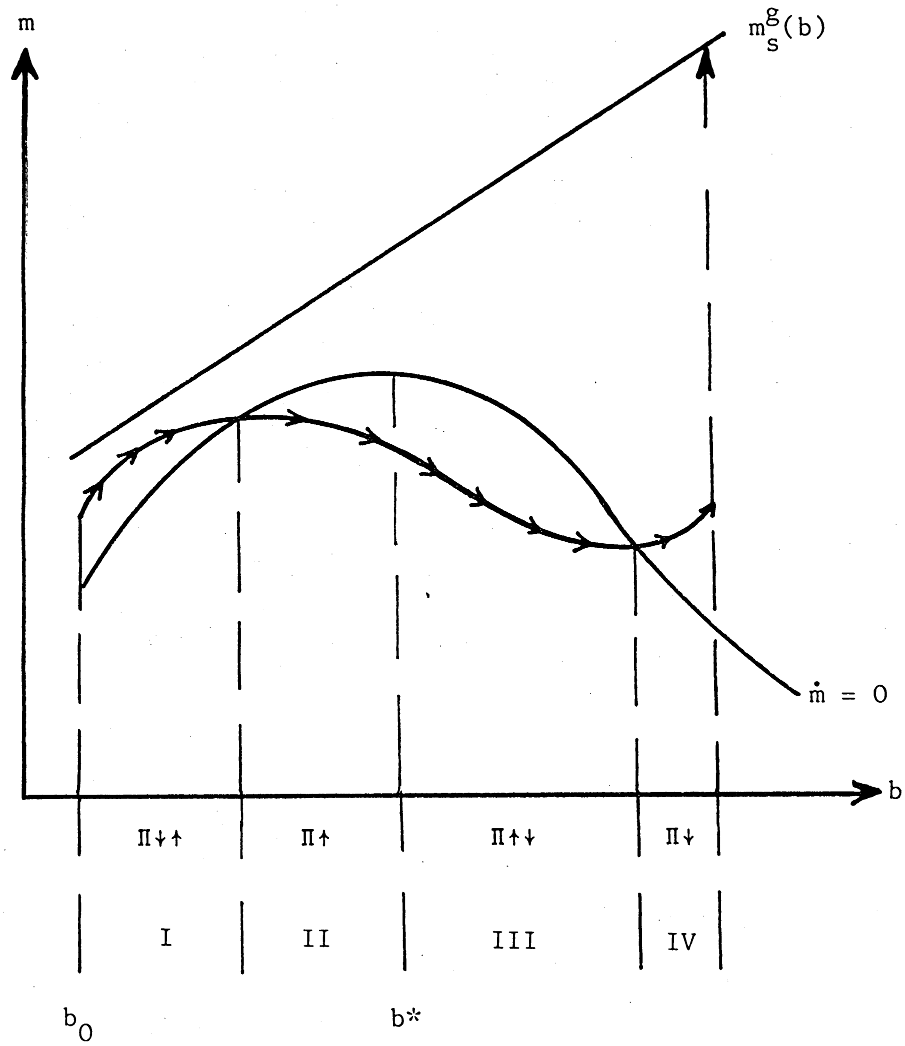


FIGURE 12

extremely rapid as we approach T_{max}, giving an even better replication of hyperinflationary time paths.

A final comment about the behavior of seignorage should be made. Some of the European hyperinflations were characterized by a fairly constant level of seignorage throughout. Our example has this characteristic, but only because the rate of money growth is zero. A constant positive rate of money growth would not yield constant seignorage. However, if we enriched our model to allow the rate of money growth to change over time, it should be possible to find examples with fixed positive seignorage and, more generally, to generate even richer time paths.

5. SUMMARY AND CONCLUSIONS

The purpose of this paper was to consider situations in which current macroeconomic policies are known to be infeasible, implying an eventual regime switch, and where the form of the stabilization is known to be a cut in government expenditures to a level consistent with no further growth in government debt. We considered both the case of a known stabilization date and the case where the date of stabilization is not known. Our goal was to consider the effect of stabilization via budget cuts on the time paths of macroeconomic variables before a stabilization.

When the date of a stabilization is known in advance, there will be a jump in the price level and a "blip" in the real interest rate at the date of stabilizations. A jump occurs even in the full certainty case because when total output is fixed, a cut in government spending implies an upward jump in private consumption. The price level and real interest rate must jump to ensure market clearing at every point in time. Real balances may rise, stay constant, or fall before a stabilization, with inflation moving in the opposite direction.

When the date of stabilization is uncertain, the real interest rate will be rising over time before a stabilization. Real balances may rise monotonically, fall monotonically, or may oscillate. The rate of inflation may similarly exhibit a wide range of possible paths. One such path, where real balances first fall and then begin to rise before a stabilization, and where inflation moves in the opposite direction is broadly consistent with the empirical behavior of key variables during hyperinflations.

What emerges even more strongly than in our previous paper is the lack of any necessary contemporaneous correlation between budget deficits and the rate of monetary growth on the one hand and the rate of inflation on the other. A constant rate of monetary growth and a constant deficit (exclusive of debt service) may be associated with a rate of inflation which rises, falls, or oscillates. This lack of correlation arises even though it is clear that the budget deficit is the ultimate cause of inflation.

APPENDIX I - KNOWN SWITCH DATES

In this appendix we derive the first-order conditions when the switch date is known. These are necessary and sufficient conditions. The problem of maximizing present discounted utility (2) subject to constraints (3) and (4) may be written

$$\max_{\{c(t),z(t),M(t)\}} \int_0^T e^{-\beta t}[u(c(t)) + v(\tfrac{M(t)}{P(t)})]dt$$

$$(A1.1) \quad + e^{-\beta T}V^s[e^{R(T)}w(0) - e^{R(T)}\int_0^T e^{-R(t)}(c(t)+\frac{z(t)}{P(t)} - y)dt + \frac{M(0)+\int_0^T z(x)dx}{P_s(T)}]$$

$$+ \int_0^T \gamma(t)[M(0) + \int_0^t z(x)dx - M(t)]dt$$

where $\gamma(t)$ is the multiplier on constraint (4) in the text. Maximization of (A1.1) with respect to each of the $c(t)$, $z(t)$ and $M(t)$ yields, respectively, (where V^s_w is the derivative of $V^s(\cdot)$ with respect to wealth and it equals $u'(y_0-g_s)$):

$$(A1.2) \qquad e^{-\beta t}u'(y_0-g) = e^{-\beta T}u'(y_0-g_s)e^{R(T)-R(t)}$$

$$(A1.3) \qquad \int_t^T \gamma(x)dx = e^{-\beta T}u'(y_0-g_s)[e^{R(T)-R(t)}\frac{1}{P(t)} - \frac{1}{P_s(T)}]$$

$$(A1.4) \qquad e^{-\beta t}v'(\frac{M(t)}{P(t)})\cdot\frac{1}{P(t)} = \gamma(t)$$

Condition (A1.2) implies that (using $R(0) = 0$)

$$\text{(A1.5)}\quad R(t) = \begin{cases} \beta t & \text{for } 0 \leq t < T \\ \beta T + \ln\dfrac{u'(y_0-g)}{u'(y_0-g_s)} & \text{for } t = T \end{cases} \Rightarrow dR(t) = \begin{cases} \beta dt & \text{for } 0 \leq t < T \\ \beta dt + \ln\dfrac{u'(y_0-g)}{u'(y_0-g_s)} & \text{for } t = T \end{cases}$$

The conditions (A1.3) through (A1.5) also imply

$$\text{(A1.6)}\quad \frac{1}{P(t)} = \int_t^T e^{-\beta(x-t)} \frac{v'(m(x))}{\theta} \frac{1}{P(x)} dx + e^{-(R(T)-R(t))} \frac{1}{P_s(T)} \quad , \quad 0 \leq t < T$$

where $\theta = u'(y_0 - g)$. This is a standard asset pricing equation. (A1.6) implies that at T there is a price jump since we have that

$$\frac{P(T)}{P_s(T)} = e^{R(T) - R(T^-)}$$

which from (A1.5) yields

$$\text{(A1.7)}\quad \frac{P(T)}{P_s(T)} = \frac{u'(y_0-g)}{u'(y_0-g_s)} .$$

Differentiating (A1.6) we obtain

$$\text{(A1.8)}\quad \frac{v'(m)}{\theta} dt = dR + \frac{dP}{P} \quad , \quad 0 \leq t < T$$

APPENDIX 2 - UNCERTAIN DATE OF A REGIME SWITCH

In this appendix we derive the first-order conditions when the date T of a switch is unknown. When the cumulative distribution of a switch occurring until T is $F(T)$, the individual maximizes (19) in the text, subject to (3) and (4). The choice problem may be written

$$\max_{\{c(t),z(t),M(t)\}} \int_0^{T_{max}} \{\int_0^T e^{-\beta T}[c(t) + v(\frac{M(t)}{P(t)})]dt$$

$$(A2.1) \qquad + e^{-\beta T}V^s[e^{R(T)}w(0) - \int_0^T e^{R(T)-R(t)}(c(t) + \frac{z(t)}{P(t)} - y)dt$$

$$+ \frac{M(0) + \int_0^T z(x)dx}{P_s(T)}] + \int_0^T \gamma(t)[M(0) + \int_0^t z(x)dx - M(t)]dt\}dF(T)$$

where $\gamma(t)$ is the multiplier on the constraint (4) in the text. Maximization of (A2.1) with respect to each of the $c(t)$, $z(t)$, and $M(t)$ yields, respectively, where $\theta_s = u'(y_0-g_s)$ is the derivative of $V^s(\cdot)$ with respect to wealth and where $\theta = u'(y_0-g)$

$$(A2.2) \qquad [1 - F(t)]e^{-\beta t}\theta = \int_t^{T_{max}} e^{-\beta T}e^{R(T)-R(t)}\theta_s(T)dF(T)$$

$$(A2.3) \qquad \int_t^{T_{max}} \{-\theta_s(T)e^{-\beta T}e^{R(T)-R(t)} \frac{1}{P(t)} + \frac{\theta_s(T)}{P_s(T)}e^{-\beta T} + \int_t^T \gamma(x)dx\}dF(T) = 0$$

$$(A2.4) \qquad e^{-\beta T}\frac{1}{P(t)} v'(m(t)) = \gamma(t)$$

Differentiating (A2.2) with respect to t we obtain

(A2.5) $$dR = \beta dt + (1 - \frac{\theta_s}{\theta})\frac{dF}{1-F}$$

We may rewrite (A2.3) as

(A2.6) $$e^{-R(t)} \frac{1}{P(T)} \int_t^{T_{max}} \theta_s(T) e^{R(T)-\beta T} dF(T) = \int_t^{T_{max}} \int_t^{T} e^{-\beta x} \frac{v'(m(x))}{P(t)} dx dF(T) + $$
$$+ \int_t^{T_{max}} e^{-\beta T} \frac{\theta_s(T)}{P_s(T)} dF(T)$$

where by (A2.2) the left-hand-side of (A2.6) is simply $[1-F(t)]\theta\frac{e^{-\beta t}}{P(t)}$. Upon substitution of the expression on the left-hand-side of (A2.6) we obtain the asset pricing equation for money balances.

As t approaches T_{max} we therefore have

(A2.7) $$\frac{\theta}{P(T_{max})} = \frac{\theta_s(T_{max})}{P_s(T_{max})}$$

Differentiating (A2.6) with respect to t (after the substitution of (A2.2)) we obtain

$$-dF\theta\frac{e^{-\beta t}}{P} - \beta(1-F)\theta\frac{e^{-\beta t}}{P}dt - (1-F)\theta\frac{e^{-\beta t}}{P}\frac{dP}{P}$$
$$= -(1-F)e^{-\beta t}v'(m)\frac{1}{P}dt - e^{-\beta t}\frac{\theta_s}{P_s}dF$$

which, upon rearranging becomes

(A2.8) $$\frac{v'(m)}{\theta}dt = \beta + \frac{dP}{P} + \frac{dF}{1-F}(1 - \frac{\theta_s/P_s}{\theta/P})$$

APPENDIX 3

POSSIBLE DYNAMIC PATHS UNDER UNCERTAINTY

In this appendix we consider in detail possible dynamic paths under uncertainty about the timing of the regime switch. We will derive four general types of paths which may obtain if no switch occurs before T_{max}. Two sets of characteristics are key: first, whether the value of real balances at T_{max} prior to a switch, denoted m_{max}, lies above or below the $\dot{m} = 0$ locus; second, whether the dynamic path crosses the $\dot{m} = 0$ locus.

On the first, if we denote by $\bar{m}$ the value of real balances when $\dot{m} = 0$ in the certainty case, m_{max} will lie below the $\dot{m} = 0$ locus under uncertainty when m_{max} is less than $\bar{m}$, and will lie above it when it is greater. This follows from noting that since $\frac{v'(\bar{m})}{\theta} = \beta + \mu$ we have (using (29) in the text) that

$$\frac{\dot{m}}{m}(m = m_{max}) = \beta + \mu - \frac{v'(m_{max})}{\theta} \tag{A3.1}$$

$$\gtrless 0 \quad \text{as} \quad m_{max} \gtrless \bar{m}$$

Whether or not the dynamic path crosses the $\dot{m} = 0$ locus will clearly depend on the shape of the locus and the relative position of m_{max}. If the locus is monotonically upward sloping, the dynamic path cannot cross if m_{max} lies above the locus. This follows from noting that the dynamic path must end up above the $\dot{m} = 0$ locus and can end up above the locus in the case where it is always rising only if it is everywhgere above the locus. In this case the dynamic path itself is monotonically rising (Figure 10a). One may note that even if the locus is downward sloping, the dynamic path may still be everywhere rising, though a negatively sloped $\dot{m} = 0$ combined with $m_{max} > \bar{m}$ is not sufficient for a rising dynamic path as was true in the first case

Analogously, if m_{max} lies below the $\dot{m} = 0$ locus ($m_{max} < \bar{m}$), the dynamic path must be monotonically falling if $\dot{m} = 0$ is monotonically falling (Figure 10c). When $m_{max} < \bar{m}$, a positively sloped $\dot{m} = 0$ locus can, but need not yield a monotonically falling dynamic path.

In other cases, especially those where the $\dot{m} = 0$ locus itself changes sign, the dynamic path may change sign, or equivalently, the dynamic path will cross the $\dot{m} = 0$ locus. For example, when m_{max} lies above the $\dot{m} = 0$ locus, a path along which real balances first fall and then rise is consistent either with $\dot{m} = 0$ first rising and then falling, or with the locus everywhere falling (Figure 10b). When m_{max} lies below the $\dot{m} = 0$ locus, real balances first rising and then falling is consistent either with $\dot{m} = 0$ first falling and then rising, or with it rising throughout (Figure 10d).

If the dynamic path crosses the $\dot{m} = 0$ locus more than once, real balances will oscillate, the number of sign changes obviously being equal to the number of crossings. Multiple crossings may occur even when the slope of the $\dot{m} = 0$ locus changes sign only once.

All of these possibilities are summarized in Table 1.

REFERENCES

Bental, B. and Z.Eckstein, Private Notes, 1985.

Drazen, A., "Tight Money and Inflation: Further Results" Journal of Monetary Economics 15 (1985), 113-20.

Drazen, A. and E.Helpman, "Inflationary Consequences of Anticipated Macroeconomic Policies. Part I: Taxes and Money Financing." Mimeo., September, 1985.

Sargent, T. "The Ends of Four Big Inflations," in Robert Hall, editor, Inflation: Causes and Cures, (Chicago: University of Chicago Press), 1982.