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PORTFOLIO IMPLICATIONS OF EMPIRICAL REJECTIONS
OF THE EXPECTATIONS HYPOTHESIS*

by

David A.Hsieh** and Leonardo Leiderman***

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** Graduate School of Business, University of Chicago
***Department of Economics, Tel-Aviv University

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FOERDER INSTITUTE FOR ECONOMIC RESEARCH
Faculty of Social Sciences, Tel-Aviv University
Ramat Aviv, I s r a e l.

Portfolio Implications of Empirical Rejections of the Expectations Hypothesis

Abstract

The purpose of this paper is to characterize a portfolio strategy that exploits the information conveyed by empirical rejections of the term-structure Expectations Hypothesis. After providing new evidence on such rejections, the analysis derives optimal portfolio positions across Treasury bills of 1 through 6 months maturities and gives a quantitative assessment of the implied risk/return tradeoffs.

David A. Hsieh
Graduate School of Business
University of Chicago
1101 East 58th Street
Chicago, IL 60637
(312)-962-7418

Leonardo Leiderman
Department of Economics
Tel-Aviv University
Ramat Aviv 69978
Tel-Aviv, ISRAEL
011-972-52-449117

I. Introduction

One version of the popular and controversial Expectations Hypothesis of the term structure of interest rates asserts that expected holding period yields on comparable bonds of different maturities are equal to the corresponding spot interest rates, up to constant term-premia.[1] This and other versions of the hypothesis have been rejected in previous econometric studies.[2] Specifically it has been found that information (e.g. market interest rates) available at the start of each trading period can be used to forecast excess holding period yields over the spot interest rate. Although in principle these rejections could be explained in terms of market inefficiency or irrationality, an economically more appealing explanation is in terms of the existence of time varying risk premiums.

While there seems to be a growing consensus on the lack of empirical validity of the Expectations Hypothesis, the portfolio implications of these findings remain to be explored. One possibility is that these empirical rejections imply negligible risk/return tradeoffs; in which case investors will approximately act as if this econometric evidence was nonexistent. On the other hand portfolio and econometric considerations may coincide in their practical implications, suggesting nonnegligible expected profits to be made by speculating across different maturities.

The purpose of this paper is to characterize a portfolio strategy that exploits the information conveyed by empirical rejections of the Expectations Hypothesis. In particular, the analysis derives optimal portfolio positions across bills of different maturities and provides a quantitative assessment of the implied risk/return tradeoffs.

Section II of the paper reports new evidence resulting in rejection of the Expectations Hypothesis for Treasury bills of 1 through 6 months maturities for the period 1960:2 - 1983:11. Section III uses a mean-variance framework to perform the portfolio analysis, and Section IV concludes the paper.

II. New Empirical Rejections of the Expectations Hypothesis

Let $P(i)_t$ denote the price at time t of a bill (a pure discount bond) that matures and pays $1 for certain at time $t+i$. In the present study, a time period equal to one month is chosen and $i = 1, \ldots, 6$. The one period holding yield is

$$H(i)_{t+1} \equiv \ln[P(i-1)_{t+1}/P(i)_t]; \qquad (1)$$

where $P(0) = 1$. The term premium is here defined as the excess of the holding period yield over the spot rate of interest:

$$T(i)_{t+1} \equiv H(i)_{t+1} - R_t; \qquad (2)$$

where $R_t = \ln[1/P(1)_t]$. Under rational expectations, the term premium can be expressed as the sum of two orthogonal components:

$$T(i)_{t+1} \equiv E_t[H(i)_{t+1} - R_t] + \varepsilon(i)_{t+1}, \qquad (3)$$

where E_t is the expectations operator conditional on information available at time t, and ε is a rational forecast error.

Controversies in term structure theory center around the specification of the expected term premium (i.e., the $E_t[\cdot]$ term appearing in eq. (3)). For the present purposes, it is convenient to test the competing hypotheses within the following specification:

$$T(i)_{t+1} = a(i) + \underline{b}'(i)\underline{x}_t + \varepsilon(i)_{t+1}, \qquad (4)$$

where $\underline{x}_t$ is a vector of information set components available to agents as of time t. The null-hypothesis to be tested is a constant term-premia version of the Expectations Hypothesis that postulates $\underline{b}' = \underline{0}$. The alternative hypothesis gives room for potential time variation in risk premiums, to be captured by nonzero b's.

In order to conduct these tests, a specification of the information set is required. The following variables were included in the information set used in this investigation: (i) short and long spot interest rates, denoted by R_t and RB_t; (ii) forward interest rates that are implicit in Treasury bill prices, given by $F(i)_t \equiv \ell n[P(i-1)_t/P(i)_t]$ for $i = 2, \ldots, 6$; (iii) an index of stock returns over the previous month, denoted by RS_t; and (iv) proxies for real and monetary shocks, denoted by Y_{t-1} and M_{t-1} respectively. This specification of the information set differs from those used in previous work in two main respects: first, the entire list of implicit forward interest rates (for the pertinent 1-6 months bills term structure) is included;[3] second, proxies for real and monetary shocks, typically present in theoretical term structure analysis, are also included.

The data base for 1-6 months Treasury bills consists of averages of end of month bid and asked prices available on the monthly U.S. Government Bond File of the Center for Research in Security Prices (CRSP) of The University of Chicago. Prices of bills with 1 to 6 months maturity are obtained as follows. On the last trading day of each month the bill that has a maturity closest to 6 months is chosen; this same bill is chosen at the end of the next month as the 5-months bills, and so on.[4] RB_t is the yield on Treasury securities at constant maturity of 20 years; Y_{t-1} is the lagged rate of change, from $t-13$ to $t-1$, of the industrial production index and M_{t-1} is the lagged rate of change, from $t-13$ to $t-1$ of M1. The data sources for these three variables are the DRI tape updated with recent issues of the Federal Reserve Bulletin, Survey of Current Business, and the H.6 release by the Federal Reserve. Y and M are not seasonally adjusted. RS_t is the value-weighted New York Stock Exchange Returns Index, from t-1 to t, from CRSP. The sample period is 1960:2 - 1983:11. Sample means for the $H(i)_{t-1}$ variables (i=1, ..., 6) are: (.00475, .00516, .00534, .00531, 00556, .00562); and those for the $T(i)_{t+1}$ variables (i=2, ..., 6) are: (.00041, .00059, .00056, .00081, .00087).

Table 1 reports the results of estimating eq. (4) by OLS. Three test-statistics are given in the table. $\chi^2(11)$ is the statistic for testing the restriction $\underline{b}'(i) = \underline{0}$, that is implied by the present version of the Expectations Hypothesis. STAB1 and STAB2 are $\chi^2(12)$ statistics for testing the hypothesis that the coefficients are stable. For STAB1, the entire sample is divided into two halves: 60:2-71:12 and 72:1-83:11. STAB2 uses 76:1 as the breakpoint for testing stability because this date marks the start of trading in T-bill futures, an event that may have affected the coefficients.[5] The reported standard errors and test statistics are robust to heteroskedasticity.[6]

The main features of the results in table 1 are as follows (significance is measured at the 5% level). First, all the estimated constant terms are insignificantly different from zero. Second, for each maturity i, the "own" forward interest rate F(i) has a positive and significant coefficient; the only exception being i=5 months. For i=4 months, however, the F(5) forward rate enters significantly. Third, all the coefficients on R_t have negative signs, yet they do not differ significantly from zero. The absolute size of the coefficients increases with the length to maturity. Fourth, some of the reported coefficients on M_{t-1} and RS_t are significantly different from zero. The coefficients on RB_t and Y_{t-1}, however, are insignificant in all cases. Fifth, while STAB1 indicates that the hypothesis of coefficients' stability pre- and post-72:1 cannot be rejected, STAB2 indicates that some significant parameter shifts occurred post-76:1.[7] Last, although the reported $\bar{R}^2$ suggest relatively poor forecastability of term premiums, the $\chi^2(11)$ statistic indicates that the $\underline{b}'(i) = \underline{0}$ hypothesis is rejected in all cases.

III. Portfolio Implications

The purpose of this section is to use the mean-variance portfolio technique to determine optimal positions within the set of 1 through 6 months bills and to quantitatively assess the risk/return tradeoffs implied by the foregoing empirical rejection of the constant term premia version of the Expectations Hypothesis.

Let $H_{t+1} = [R_t, H(2)_{t+1}, \ldots, H(6)_{t+1}]$ denote the vector of returns, from t to t+1, on the six bills considered. H_{t+1} is unknown at time t. Denote by $h_{t+1}(=E_t H_{t+1})$ and Ω_t the conditional mean and covariance matrix of H_{t+1}. A portfolio at time t is given by a vector of shares $s_t = [s(1)_t, \ldots, s(6)_t]$, where $s_t'L = 1$ and $L = [1, \ldots, 1]'$. The portfolio s_t has a conditional mean $m_t = s_t'h_{t+1}$ and conditional variance $v^2_t = s_t'\Omega_t s_t$.

Agents are assumed to allocate \$1 per period into the $s(i)_t$ shares so as to maximize $[m_t - 0.5\delta v^2_t - c_t]$, where δ reflects their degree of risk aversion,[8] and c_t is the transactions cost. This optimization problem is solved here under the constraint that short positions in T-bills are not allowed. Before discussing the solution, it is well to explain how m_t, v^2_t, and c_t are formed.

In order to account for possible time variation of the coefficients in the forecasting equation we use a Kalman filter methodology, and the forecasts are based on data up to the point when the portfolio is formed. Specifically, it is postulated that

$$T_{t+1} = Z_t\lambda_t + U_{t+1},$$

where $T_{t+1} = [T(2)_{t+1}, \ldots, T(6)_{t+1}]'$. Z_t is a 5 x 12 matrix of information set components (see the first column of Table 1 for the complete list), λ_t is the unobserved 12 x 1 vector of coefficients at t, and U_{t+1} is an unobservable vector white noise process with zero mean and constant covariance matrix R. To allow for time varying parameters, we use the specification:

$$\lambda_t = \lambda_{t-1} + n_t,$$

where n_t is an unobservable vector white noise process with mean zero and constant variance Q. Our calculations set $Q = \alpha I$. If $\alpha = 0$, this is a rolling regression. If $\alpha > 0$, then past observations are weighted less than recent observations. In what follows, the value $\alpha = 0.1$ is used; runs using other values of α were also conducted (for $0 \leq \alpha \leq 1.0$) and the main features of the portfolio results are not sensitive to the specific value used. Using these specifications, we compute for each t the conditional mean h_{t+1} and conditional covariance Ω_t that are in turn used to calculate the optimal shares s_t.

Transaction costs are taken into account in the following way. At the beginning of period t there is $\$s(i+1)_{t-1}$ of T-bills with i months to maturity in the portfolio. The desired amount is $\$s(i)_t$, and the transaction cost for reaching this position is assumed to be $\gamma|s(1)_t - s(2)(i+1)_{t-1}|$. For the entire portfolio the total transaction cost is:

$$c_t = \gamma|s(1)_t - s(2)_{t-1}| + \ldots. + \gamma|s(5)_t - s(6)_{t-1}| + \gamma s(6)_t.$$

The optimal portfolios discussed below are derived under the assumption that $\gamma = .05\%$.

To solve for the optimal portfolio shares we perform a change of variable so as to impose the no short sales and summing up constraints. Define s_t in terms of new variables $q_t = [q_{t1}, \ldots, q_{t5}]'$;

$$s(1)_t = 1/[1 + \sum_{j=1}^{5} \exp(q_{tj})],$$

$$s(i)_t = \exp(q_{ti-1})/[1 + \sum_{j=1}^{5} \exp(q_{tj})], \quad i = 2, \ldots, 6.$$

For convenience write $s_t = s(q_t)$ and $c_t = c(q_t)$. A Gauss-Newton method is then used to solve for q_t that maximizes $s(q_t)'h_{t+1} - 0.5\delta s(q_t)'\Omega_t s(q_t) - c(q_t)$.

Table 2 reports the results on the optimal portfolios for different subsamples. Preliminary work indicated that the portfolio results are generally invariant with respect to the choice of δ; the figures reported in the table assume $\delta = 4.0$.[9] The results for the average portfolio positions, given in row 8, indicate that 1 and 2 month bills generally obtain the largest weights. When combined, these bills have a share greater than 50 percent of the portfolio for all subsamples. In two of the subsamples, 3- and 4-months bills have a combined share of above 30 percent. These figures are consistent with diversification across 1 through 6 months bills.

The mean return on the portfolio is reported in row 2. Comparing this return to the riskless return given in row 1 suggests that the gains from diversification are not large, ranging from .06 percent per month to .09 percent per month.

To quantitatively assess the risk/return tradeoff, we calculate two measures of the average slope of this tradeoff. The first is the average standardized expected excess return (ASEER), defined as $\{(1/T)\Sigma_t [(\hat{m}_t - \hat{R}_t)/v_t]\}$.

The second is the average standardized actual excess return (ASAER) defined as $\{(1/T)\Sigma_t[(s_t^{\prime}\hat{H}_{t+1} - \hat{R}_t)/v_t]\}$, where $\hat{m}_t = m_t - c_t$, $\hat{R}_t = R_t - \gamma$, and $s_t^{\prime}\hat{H}_{t+1} = s_t^{\prime}H_{t+1} - c_t$. These two measures are given in rows 4 and 5 of Table 2. According to these measures the most favorable risk/return tradeoff obtained in the 1971-75 period, and the most unfavorable one in 1963-65.

Are the reported slopes of the tradeoff significantly different from zero? To answer this question, we define $k_{1t} \equiv (\hat{m}_t - \hat{R}_t)/v_t$, and $k_{2t} \equiv (s_t^{\prime}\hat{H}_{t+1} - \hat{R}_t)/v_t$, and calculate the following t-statistics: $t_j = [(1/T)\Sigma k_{jt}]/\sqrt{(1/T)\Sigma(k_{jt} - \bar{k}_j)^2}$, where $\bar{k}_j$ is the mean and $j = 1$ and 2. These t-statistics are given in rows 6 and 7. They indicate that the slopes reported in rows 4 and 5 are significantly different from zero. However, as discussed above, the implied excess return on the portfolio over the 1-month T-bill is of a small order of magnitude.

IV. Conclusions

This paper investigated the properties of optimal portfolios, across Treasury bills of 1-6 months maturities, constructed using information conveyed by empirical rejections of the Expectations Hypothesis. These portfolios exhibit diversification across the maturities considered. Moreover, measures of standardized excess returns are shown to be statistically different from zero. Yet in our view, the excess profits on the portfolios over the pertinent risk-free rates are of a quantitatively small order of magnitude.[10]

In a recent contribution, Shiller, Campbell, and Schoenholtz (1983, pp. 174-5) point to an apparent puzzle:

"The simple expectations theory, ..., has been rejected many times in careful econometric studies. But the theory seems to reappear perennially in policy discussions as if nothing had happened to it. It is uncanny how resistant superficially appealing theories in economics are to contrary evidence."

If robust in several dimensions,[11] the findings of this study offer at least a partial resolution of this puzzle. Under our interpretation, the results above represent a case in which econometric rejections of the Expectations Hypothesis seem to have negligible practical implications for economic agents; so that the latter would act, approximately, "as if" these rejections were nonexistent.

FOOTNOTES

1 See the discussion in Cox, Ingersoll, and Ross (1981) and Kane (1983), as well as references therein.

2 See for example Hamburger and Platt (1975), McCulloch (1975), Pesando (1975), Friedman (1979), Shiller (1979), Fama (1984), Jones and Roley (1983), Kane (1983), Shiller, Campbell, and Schoenholtz (1983), and Mankiw and Summers (1984).

3 Fama (1984), for example, includes only the F(i) and R_t rates in the T(i) regression.

4 We thank E.Fama for providing us an updated version of the data used in his 1984 paper.

5 This date for splitting up the sample was suggested to us by the Editor.

6 The equations are estimated by OLS and the standard errors and test statistics are corrected using the methods suggested by Hansen (1982), Hsieh (1983), and others.

7 In future work, it would be useful to conduct further stability tests so as to identify the number and location of additional potential breakpoints, and to determine whether the transitions among regimes are abrupt (as assumed here) or gradual.

8 Given the present portfolio strategy of investing $1 per period, δ measures constant and relative risk aversion.

9 This number is familiar from Grossman and Shiller's (1981) investigation.

10 This statement should be qualified in that when multiplied by a large portfolio, a small differential in returns can lead to large differences in dollar earnings.

11 For example, with respect to including other assets in the portfolio and to using a different objective function and/or investment horizon.

Table 1 - Estimated Monthly Term Premium Equations - 1960:2 - 1983:11

	Dependent Variable				
Regressors	$T(2)_{t+1}$	$T(3)_{t+1}$	$T(4)_{t+1}$	$T(5)_{t+1}$	$T(6)_{t+1}$
Constant	0.0001	0.0000	-0.0000	0.0000	0.0000
	(0.0001)	(0.0002)	(0.0003)	(0.0004)	(0.0005)
R_t	-0.1355	-0.2970	-0.4825	-0.5796	-0.6664
	(0.1640)	(0.2881)	(0.4221)	(0.5482)	(0.6380)
$F(2)_t$	0.3728	-0.3030	-0.3346	-0.5028	-0.6040
	(0.1343)	(0.2406)	(0.3534)	(0.4674)	(0.5697)
$F(3)_t$	-0.1184	0.4448	0.4435	0.1860	-0.0790
	(0.1094)	(0.1886)	(0.2653)	(0.3517)	(0.4376)
$F(4)_t$	-0.0511	-0.0633	0.5329	0.6076	0.6438
	(0.1045)	(0.1935)	(0.2714)	(0.3271)	(0.3875)
$F(5)_t$	0.0368	0.0462	-0.0766	0.3780	0.4317
	(0.0677)	(0.1241)	(0.1895)	(0.2446)	(0.2722)
$F(6)_t$	0.0067	0.0638	0.0797	0.1653	0.6552
	(0.0501)	(0.1016)	(0.1566)	(0.2061)	(0.2805)
RB_t	-0.0000	-0.0001	-0.0000	-0.0001	-0.0002
	(0.0001)	(0.0001)	(0.0001)	(0.0002)	(0.0002)
RS_t	-0.0014	-0.0032	-0.0056	-0.0079	-0.0100
	(0.0012)	(0.0020)	(0.0031)	(0.0041)	(0.0049)
Y_{t-1}	-0.0007	-0.0009	-0.0012	-0.0013	-0.0015
	(0.0006)	(0.0013)	(0.0020)	(0.0027)	(0.0034)
M_{t-1}	-0.0060	-0.0078	-0.0103	-0.0138	-0.0152
	(0.0026)	(0.0049)	(0.0074)	(0.0099)	(0.0118)
$T(i)_t$	0.1914	0.1637	0.1428	0.1760	0.1648
	(0.1305)	(0.1284)	(0.1283)	(0.1323)	(0.1205)
$\bar{R}^2$	0.2868	0.1622	0.1192	0.1293	0.1255
SE	0.0006	0.0012	0.0017	0.0022	0.0027
$\chi^2(11)$	53.2784	26.6689	24.7234	21.2635	22.6369
	(0.00)	(0.00)	(0.01)	(0.03)	(0.02)
STAB1	10.9287	11.2581	15.3289	19.9218	16.8507
	(0.54)	(0.51)	(0.22)	(0.07)	(0.16)
STAB2	11.0820	17.6920	23.6675	24.2867	20.0379
	(0.52)	(0.13)	(0.02)	(0.02)	(0.07)

<u>Note</u>: All variables have been defined in the text. $T(i)_t$ is the lagged dependent variable. Numbers in parentheses beneath coefficients are standard errors. $\chi^2(11)$ is the chi-square statistic for testing the hypothesis that all coefficients other than the constant are equal to zero. STAB1 adn STAB2 are $\chi^2(12)$ statistics for testing the hypothesis that the estimated coefficients are stable. For STAB1, the subsamples used are 60:2-71:12 and 72:1-83:11. For STAB2, they are 60:2-75:12 and 76:1-83:11. Numbers in parentheses below χ^2's are marginal significance levels. Standard errors and test statistics are robust to heteroskedasticity.

Table 2 - Optimal Portfolios of 1-6 Months Treasury Bills (Monthly Data)

	Sample[a]				
	1963-65	1965-70	1971-75	1976-79	1979-83
1. Mean Return on 1-Month Bill[b]	0.00234	0.00381	0.00408	0.00501	0.00807
2. Mean Return on the Portfolio	0.00300	0.00471	0.00481	0.00561	0.00888
3. Standard Deviation on Portfolio	0.00032	0.00101	0.00148	0.00177	0.00231
4. ASEER[c]	0.634	0.663	0.838	0.756	0.660
5. ASAER[d]	0.577	0.639	0.840	0.733	0.648
6. t_1[e]	7.895	13.649	7.229	18.573	12.002
7. t_2[e]	7.642	11.500	6.977	15.882	8.805
8. Average shares:					
1 month	0.13269	0.31491	0.70964	0.69631	0.73974
2 months	0.37723	0.27276	0.16053	0.22687	0.24862
3 months	0.22435	0.18377	0.05990	0.06633	0.00519
4 months	0.15125	0.11874	0.02850	0.00845	0.00321
5 months	0.11331	0.08134	0.02322	0.00142	0.00314
6 months	0.00116	0.02848	0.01821	0.00062	0.00011

Notes:

a The exact sample periods are as follows;

2/63-12/64; 1/65-12/70; 1/71-12/75; 1/76-11/79; and 12/79-12/83.

b The means given in lines 1 and 2 correspond to returns after taking into account transaction costs.

c ASEER $\equiv (1/T)\ \Sigma_t\ [(\hat{m}_t - \hat{R}_t)/v_t]$.

d ASAER $\equiv (1/T)\Sigma_t[(\hat{s}_t' H_{t+1} - \hat{R}_t)/v_t]$.

e These t-statistics are explained in the text.

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