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DISCRIMINATING MONOPOLY, FORWARD
MARKETS AND INTERNATIONAL TRADE

by

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INTRODUCTION

Recent empirical studies [see e.g. Kravis and Lipsey (1977, 1978), Isard (1977), Aspe and Giavazi (1982)] demonstrate that there are notable divergencies between domestic and export prices. This empirical evidence suggests that the law of one price is systematically violated. Kravis and Lipsey (1977, p. 155) argue that "many firms involved in international trade, particularly in manufactures, are in the position of a discriminating monopolist faced with separate markets, each characterized by a different demand elasticity". A model which explicitly allows for a price discrimination has been used by several authors. Aspe and Giavazi (1982) used such a model to investigate the behavior of price and output in two industries of the German manufacturing sector. Katz, Paroush and Kahana (hereafter KPK), (1982) used a model of a price discriminating firm which operates under price uncertainty, to investigate its optimal level of output and sales in the two markets. The main assumption made in KPK [1982] is that the firm determines its level of output and the allocation of sales between the two markets before the resolution of uncertainty. Furthermore, no forward markets were available to this firm.

In this paper we analyze a price discriminating firm which sells its produce both in the domestic and on world markets under either exchange rate uncertainty or foreign price uncertainty. This firm is a monopoly in the domestic market but a price-taker on the world market. Our model differs substantially from the KPK model and in some cases conforms better with reality due to the following two assumptions. First, the firm determines its level of output before the resolution of uncertainty but decides about the optimal allocation of sales only after it observes the foreign price

denominated in domestic currency. Secondly, a forward markets to share the exchange-rate risk are available and their impact on the firm's policy is analyzed. Hence, this work integrates two strands of literature. It combines studies on price discriminating firm under price uncertainty (some were mentioned above) and studies on firm behavior when forward markets are available (Danthine [1978], Holthausen (1979), Katz and Paroush (1979), Feder, Just and Schmitz (1980)).

The paper is organized as follows. In Section 2 the model is presented. The effect of uncertainty on production and export when no forward markets exist is investigated in Section 3. It is shown that, under some conditions, the export increases when we introduce uncertainty. This result seems to contradict the prevalent view that in the firm's microeconomic level, export should decline as a result of introducing uncertain exchange rate (see e.g., Clark [1973] and Baron [1976]], Yeager [1976, Ch. 13]). In Section 4 we analyze the impact of introducing forward markets on the firm's output and export. We show that the "separation theorem" holds in this model i.e., the optimal production level does not depend on the utility function and the probability distribution function of the random exchange rate. Moreover, in the presence of forward markets, the total output of the discriminating monopoly increases to that level of the perfectly competitive firm (in both markets) while its export is larger than that of the competitive firm. In Section 5 we discuss the optimal hedging policy of this firm. In Section 6 it is shown that as a result of price discrimination, the domestic price variability is magnified relative to the foreign price variability (denominated in domestic currency).

II. THE MODEL

Consider a firm which produces a homogeneous commodity both for a domestic market where the firm has a monopolistic power and for foreign markets where it faces perfect competition. The assumed separation between domestic and foreign markets which enables this firm to apply price discrimination is due to the existence of artificial and/or natural barriers to trade. The firm faces random exchange rate $\tilde{e}$. Assume that $\tilde{e}$ has a given distribution function on $[e_1, e_2]$, $0 < e_1 < e_2 < \infty$, and that this distribution of $\tilde{e}$ is known to the firm.

We assume that the firm is risk-averse with a von Neumann-Morgenstern utility function U ; U is differentiable, $U' > 0$ and $U'' < 0$. The firm maximizes the expected utility of profits denominated in local currency. The firm's production technology gives rise to a cost function $C(Q)$, where Q is the total output of this commodity (to be allocated for both markets). We assume that $C' > 0$ and $C'' < 0$. Denote by Q^d , the sales in the domestic market where the firm is a monopoly, and by Q^F , the total sales abroad where the firm is a perfect competitor.

Production usually takes time, hence we assume that the firm chooses its optimal level of output before $\tilde{e}$ is known. Since we are interested in the case where the firm exports, we shall assume that the optimal output of the firm is greater or equal to $Q^d(e_1)$. The timing of output allocation between domestic and foreign markets is a crucial assumption in this model. We believe that the assumption made in Katz-Paroush-Kahana [1982], that the firm allocates its output between the domestic and export before the resolution of uncertainty does not characterize many cases. Shifts of produce from one market to another within a short period of time are usually possible.

Therefore we assume that the allocation of output between the two markets is done after the observation of $\tilde{e}$. Thus the firm acts as a discriminating monopoly and equates its marginal revenue in the domestic market to the marginal revenue from export (denominated in domestic currency), i.e. currency), i.e.

$$(1) \quad p^d(Q^d) (1 - 1/\eta) = \tilde{e}P^*$$

in each state of nature, where $p^d(Q^d)$ is the domestic inverse demand function for this commodity, η is the price elasticity of demand and P^* is the foreign price of the commodity.

Equation (1) defines, implicitly, the amount of commodity Q^d sold domestically as a function of the random variable $\tilde{e}$, given the domestic demand function and that the firm's output is at least as much as $Q^d(e_1)$. Assuming that the marginal revenue is decreasing it is clear from (1) that $Q^d(\tilde{e})$ increases as e declines. Define $f(\tilde{e})$ by

$$(2) \quad Q^d(\tilde{e}) = f(\tilde{e}) \text{ where } Q^d(\tilde{e}) \text{ satisfies (1) for each } \tilde{e}.$$

The level of output, which is to be determined before the realization of $\tilde{e}$ is attained by maximizing $EU(\tilde{\pi})$ where $\tilde{\pi}$ are the (random) profits denominated in domestic currency. In the presence of currency forward market with a forward exchange rate e_f , we denote by X the amount of foreign currency that the firm sells forward, π is given by

$$(3) \quad \tilde{\pi} = \tilde{e}P^*(Q - f(\tilde{e})) + p^d(f(\tilde{e}))f(\tilde{e}) + X(e_f - \tilde{e}) - C(Q)$$

When no forward transaction of foreign exchange is available we take $X \equiv 0$. Now consider

$$(4) \quad \max_{Q, X} EU(\pi(\tilde{e}))$$

The first-order conditions are:

$$(5) \quad E[\tilde{e}P^* - C'(Q)]U'(\hat{\pi}) = 0$$

$$(6) \quad E(e_f - \tilde{e})U'(\hat{\pi}) = 0$$

By our assumptions about $U(\cdot)$ and $C(\cdot)$, (5) and (6) are necessary and sufficient conditions for optimality. We denote by $\bar{Q}$ the optimal output when $X \equiv 0$ (i.e., the solution to (5) for this case) and by Q^* and X^* the optimal solution when forward market exists. In the next section we analyze the price discriminating firm's output and sales when $X \equiv 0$.

III. THE EFFECT OF UNCERTAINTY ON OUTPUT AND EXPORT

Consider the case where no forward market for foreign exchange is available. It is traditional to compare the firm's optimal output under uncertainty with the "certainty equivalent" level, i.e., with the case where $\tilde{e}$ is replaced by $E\tilde{e} = \bar{e}$ (see for example Sandmo [1971] and Leland [1972]).

PROPOSITION 1: The optimal output of the price discriminating monopoly under the certain exchange rate $\bar{e} = E\tilde{e}$ is greater than its optimal output under the random exchange rate $\tilde{e}$.

PROOF: Let us first show that when $X \equiv 0$ the profit $\pi(\tilde{e})$ is an increasing function in e . Let $\theta, \bar{\theta} \in [e_1, e_2]$ and $\theta < \bar{\theta}$. Even if the firm does not reallocate its output when $\tilde{e} = \theta$ between the two markets, in the case where $\tilde{e} = \bar{\theta}$, its profits satisfy:

$$\pi(\bar{\theta}) \geq \bar{\theta}P^*(\bar{Q} - f(\theta)) + P(f(\theta))f(\theta) - C(\bar{Q}) > \theta P^*(\bar{Q} - f(\theta)) + P(f(\theta))f(\theta) - C(\bar{Q}) = \pi(\theta)$$

Since a shift in the domestic sale from $f(\theta)$ to $f(\bar{\theta})$ may only increase its profits, then profits are increasing with the exchange rate. Denote by Q^C the output when $\bar{e} = E\tilde{e}$ prevails. Let A be the set of all states of the world where $\tilde{e} \geq (C'(\bar{Q})/P^*)$ then (5) can be rewritten as

$$(7) \quad \int_A \left(\tilde{e} - \frac{C'(\bar{Q})}{P^*} \right) U'(\tilde{\pi}) = \int_{\sim A} \left(\frac{C'(\bar{Q})}{P^*} - \tilde{e} \right) U'(\tilde{\pi})$$

Since U' is monotone decreasing and $\pi(e)$ increases in e ,

$$(8) \quad \sup_A U'(\tilde{\pi}(e)) \leq \inf_{\sim A} U'(\tilde{\pi}(e))$$

Combining (7) and (8) yields

$$\int_A \left(\tilde{e} - \frac{C'(\bar{Q})}{P^*} \right) > \int_{\sim A} \left(\frac{C'(\bar{Q})}{P^*} - \tilde{e} \right) \quad \text{or}$$

$$(9) \quad E(\tilde{e} - C'(\bar{Q})/P^*) > 0 \quad \text{or} \quad P^*E\tilde{e} > C'(\bar{Q}),$$

But under certainty the firm equates the price and the marginal cost, i.e., $C'(Q^C) = \bar{e}P^*$. Since $C'' > 0$ we proved from (9) that $\bar{Q} < Q^C$. Q.E.D.

Now let us show that under certain assumption about the demand curve the impact of uncertainty in $\tilde{e}$ about domestic sales and the exported quantity, in some cases, can be determined.

PROPOSITION 2: Assume that the domestic marginal revenue function is

strictly convex. Let the exchange rate be fixed initially at

$\bar{e}$. Introducing uncertainty about $\tilde{e}$ will

- (i) Reduce the expected domestic sales,
- (ii) For a given demand curve (satisfying the above assumption) and variable $\tilde{e}$ for some risk-averse firms the expected export under uncertainty is larger than the export under the fixed exchange rate $\bar{e} = E\tilde{e}$.

PROOF: By definition of $f(e)$ we have $MR^d(f(\tilde{e})) = \tilde{e}P^*$ for all states of nature. This implies that when $MR^d(\cdot)$ is strictly convex then $f(\tilde{e})$ is strictly concave. Therefore $Ef(\tilde{e}) < f(\bar{e})$ which proves (i). To establish (ii) it is enough to show that for a firm which is "almost risk neutral" we have $Q^C - \bar{Q} < f(\bar{e}) - Ef(\tilde{e})$. When U' is "almost constant" we find from (5) that $E\tilde{e}P^* - C'(\bar{Q})$ is close to 0; therefore, since $C'(Q^C) = \bar{e}P^*$ we obtain that $Q^C - \bar{Q}$ is sufficiently small (assuming that C'' is bounded away from 0 on $(Q^C/2, \infty)$). Since the choice of utility function and cost function has no effect upon the difference $f(\bar{e}) - Ef(\tilde{e})$ our argument proves (ii). Q.E.D.

Now let us study the impact of increasing risk aversion upon the level of output and export.

PROPOSITION 3: As the firm's risk aversion increases its optimal output and export decreases.

PROOF: Consider two risk averse firms with utility functions U_A and U_B . Assume that firm A is more risk averse than B, thus $U_A(\cdot) = F(U_B(\cdot))$, where $F' > 0$ $F'' < 0$. Denote by Q_A and Q_B the optimal production of these firms. The first order condition for A can be written as

$$(10) \quad E\{F'[U_B(\tilde{x}_A)]U'_B(\tilde{x}_A)(\tilde{e} - C'(Q_A)/P^*)\} = 0$$

Using the monotonicity of $\tilde{x}_A$ and F' , by a similar argument to the one brought in the proof of Proposition 1, we obtain from (10) that

$$(11) \quad EU'_B(\pi_A)(\xi - C'(Q_A)/P^*) > 0$$

Since $EU_B(\pi)$ is concave in Q and its derivative at $Q = Q_B$ is 0 we obtained from (11) that $Q_A < Q_B$.

To establish that the export of firm A is smaller than that of firm B (the two firms face the same domestic demand curve), we note that their local sales are the same ($f(e^*)$), and that $Q_B - Q_A > 0$. Q.E.D.

IV. THE IMPACT OF FORWARD MARKETS ON THE FIRM'S OUTPUT AND EXPORT

Now we assume that forward markets for foreign exchange are available. Even though our firm has monopolistic power in the domestic market the next proposition shows that the "separation theorem" holds; i.e., its optimal production level (and hence its exported quantities) does not depend on its utility function or on the particular distribution of ξ . Similar result for competitive firms were obtained by Ethier [1973], Baron [1976], Danthine [1978], Holthausen [1979], Katz and Paroush [1979], Feder Just and Schmitz [1980] and others.

PROPOSITION 4 (Separation Theorem). When forward markets for foreign exchange are available the firm's optimal production is given by

$$(12) \quad C'(Q^*) = e_f P^*.$$

Let us make few comments about this result before we bring its proof: (a) from (12) we see that Q^* does not depend upon the utility function or on the probability distribution of the random exchange rate.

(b) The output of this discriminating monopoly is identical to that of a competitive firm (in the presence of forward market).

(c) Since for each realization of $\tilde{\epsilon}$ the discriminating monopolist's domestic sales are less than those of a price-taking firm (in the local market), and in view of (b), the monopolist firm's export is larger than that of a competitive firm. Thus, abolishing the separation between domestic and foreign markets by reducing tariffs and/or transport costs may increase local sales and decrease firm's export.

PROOF: From (5) and (6) we obtain

$$(13) \quad E[e_f P^* - C'(Q^*)]U'(\tilde{\pi}) = 0$$

Which clearly implies (12). Q.E.D.

COROLLARY: If the forward price incorporates nonpositive risk premium, i.e.

$e_f \geq E\tilde{\epsilon}$, then the firm's optimal output and expected export are larger in the presence of such forward market.

The proof of the Corollary is straightforward from Proposition 1 and the equations $C'(\bar{Q}) = \bar{e}P^*$, $C'(Q^*) = e_f P^*$.

V. OPTIMAL HEDGING POLICIES: COMPETITION VS. MONOPOLY

The optimal hedge X^* and the optimal production level Q^* of the price discriminating monopoly are determined by equations (5) and (12) and are known to the firm before the resolution of the uncertainty. It is usually difficult to solve explicitly for X^* , since the levels of export depend upon the shape of the domestic demand curve. In the case where the currency forward market is unbiased and the firm is a price taker in the domestic market where

$p^d = eP^*$, the optimal hedge x^c is equal to P^*Q^* (see for example Benninga, Eldor and Zilcha [1985]); Namely, when the forward market is unbiased the competitive firm fully hedges its foreign currency proceeds and gets rid of uncertainty altogether. This is also the case when the quantities exported are known ex ante and the forward market is unbiased. Now we show that in our model the optimal hedge of the firm is lower than that of a competitive firm.

PROPOSITION 4: Assume that the currency forward market is unbiased. The forward hedge of the price discriminating monopoly is lower than that of a competitive firm, i.e., $x^* < P^*Q^*$.

Note that even though in both cases the firm produces the same Q^* the hedging policy differs.

PROOF: When $e_f = E\tilde{e} = \bar{e}$ we derive from the first-order conditions (5) and (6) that the firm chooses x^* such that $\text{Cov}(\tilde{e}, U'(\tilde{\pi})) = 0$. If $\pi(e)$ is monotone in e we must have that $\text{Cov}(\tilde{e}, U'(\pi(\tilde{e}))) \neq 0$. The proof of this fact is similar to that of Proposition 1 using the monotonicity of U' . Let $P^* = 1$ then for some constant k ,

$$\pi(\tilde{e}) = [Q^* - x^* - f(\tilde{e})]\tilde{e} + p^d(f(\tilde{e}))f(\tilde{e}) + k,$$

But

$$\begin{aligned} \frac{\partial}{\partial e}[p^d(f(e))f(e)] &= \frac{dp^d}{dq^d} f'(e)f(e) + p^d(q^d)f'(e) = \\ &= f'(e)p^d(q^d)[1 - 1/\eta] = f'(e)e \end{aligned}$$

Thus

$$\frac{\partial}{\partial e}[\pi(\tilde{e})] = Q^* - X^* - f(\tilde{e}) - f'(\tilde{e})\tilde{e} + f'(\tilde{e})\tilde{e} = Q^* - X^* - f(\tilde{e})$$

Since $\pi(e)$ cannot be a monotone function of e this implies that $X^* < Q^*$. Q.E.D.

VI. PRICE DISCRIMINATING MONOPOLY AND DOMESTIC PRICE VARIABILITY

In the classic small country models with perfect competition, domestic price variability is precisely the same as the variability of the foreign price denominated in domestic currency. The reason is that in each state of nature $p^d = \tilde{e}P^*$ (PPP holds), thus we have a perfect correlation. However, in our price discriminating monopoly case, we equate marginal revenues in each state of nature, thus

$$p^d(0^d)(1 - \frac{1}{\eta}) = \tilde{e}P^*$$

Where the monopoly always sells in that portion of the domestic demand curve that the price elasticity of demand $\eta > 1$. Since in each state of nature $(1 - 1/\eta) < 1$, p^d will be more variable than $\tilde{e}P^*$, namely we observe here a magnification effect. This conclusion holds also in the case of an importing firm with a monopoly power in the domestic market (due to some licensing regulations for import for example).

VII. CONCLUDING REMARKS

In this model we considered a price discriminating firm that has a monopoly power in the domestic market and it is a perfect competitor in the foreign market. In our view this model of price discrimination approximates reality in many cases and can be extended to the case where the firm obtains

some monopolistic power in the foreign market as well. Clearly, in such a case some of the results may fail to hold, the separation theorem for example.

Although we have considered uncertain exchange rate, we could have considered, alternatively, uncertainty about the foreign price while the exchange rate is fixed, without any significant differences in our results. Also, one can consider this model where the firm faces both foreign price and exchange rate uncertainty (see e.g. Benninga, Eldor and Zilcha [1985] in the perfect competition case). If forward markets for the exchange rate and the commodity are available, the separation theorem will still hold. However, this model is beyond the scope of this paper.

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