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A RATIONAL EXPECTATIONS KEYNESIAN MODEL WITH PRICE FLEXIBILITY

By

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I. INTRODUCTION

Keynesian models of business fluctuations, in arguing for the existence of low employment equilibria, assign a central role to expectations of income and demand. If firms expect to be unable to sell all they want at market prices, they will cut demand for factors. This spillover from expected product market conditions to actual factor demand in turn affects actual demand for product. A low employment equilibrium results if induced labor demand implies a level of actual product demand consistent with firms' expectations.

The Keynesian framework stressing aggregate product demand has been criticized as being dependent on non-optimizing behavior, on prices being exogeneously fixed at non-market clearing levels, or on arbitrary assumptions about expectations. The purpose of this paper is to present an internally consistent model in the Keynesian tradition not subject to these criticisms. We construct a model where prices are endogeneously determined and where optimizing agents form expectations rationally, but where an unemployment equilibrium with Keynesian features still emerges. Specifically, expectations of low product demand (by firms) or low income (by consumers) may themselves lead to a low-output, low-employment equilibrium where such expectations are fully confirmed. A further result is that in this framework <u>any</u> level of employment (no greater than desired labor supply, of course) is consistent with a rational expectations steady state even when prices are flexible.

The possibility of multiple equilibria suggests unexploited gains from trade when one considers the Pareto-inferior equilibria, a point on which non-Walrasian models have been criticized. This criticism is not quite on the mark. The real question in analyzing an equilibrium is not whether there exists another equilibrium which is preferred by all agents, but whether the market mechanism is such that this preferred equilibrium can be attained by individual optimizing (non-cooperative) behavior. The main point in this paper is that the restrictions which a decentralized market places on individual trading will prevent all potential gains from being realized.

The primary constraint in a decentralized market considered here concerns a seller's ability to effectively indicate his desire to sell more by cutting prices. In the textbook model of atomistic competition, agents who wish to sell more need only cut their price infinitesimally relative to that which other sellers are charging. It is as if information about price cuts is instantaneously and costlessly transmitted to potential buyers. In fact, price cuts do not work instantaneously. Information about price differentials must diffuse; trading partners must be sought out.

In the output market, this will be modelled via slow diffusion of information. Imperfect information on the part of consumers about price differentials means that firms gain (or lose) customers in response to price changes at only a finite rate. The arrival of selling opportunities is sensitive to price cuts, but not "infinitely" sensitive. Atomistic firms thus become price setters (rather than price takers) facing downward sloping demand curves.

In the labor market, one may similarly argue that the willingness of a worker to supply labor at a wage lower than the market wage does not meanhe will find higher employment instantly. An argument analogous to that used in the product market suggests that the arrival of job offers occurs at only a finite rate and that infinitesimal wage cuts will not instantaneously raise to one the probability of finding a vacancy. Since workers must search out job possibilities, rather than advertising reservation wages in strict analogy to a firm advertising prices, the arrival rate of offers may be relatively insensitive to the worker's reservation wage. The unemployed worker in general can indicate his wage demand only after he has found an employer who might potentially be hiring. It would therefore not be unreasonable to analyze the polar case where the arrival rate of job offers is independent of the individual worker's reservation wage.

The case where an individual worker cannot affect the rate at which job offers arrive by varying his wage may be modelled in a non-dynamic framework by imposing the constraint that he cannot bargain individually with a firm to work for a lower wage. Allowing an individual worker to cut his wage relative to other workers and to gain higher employment at their expense with probability one would imply, in the arrival scenario, that wage cuts imply instantaneous arrival of job offers. No individual bargaining would therefore be consistent with our basic aim of examining the implications of non-instantaneous arrival of selling opportunities in response to price cuts. Though the limitation of no individual bargaining by workers appears to have a good deal of empirical validity, my purpose in imposing it (based on the underlying search story) is not to argue that labor markets actually work this way. The above arguments were simply meant to suggest why it is reasonable to consider a model when such a constraint is imposed.

In a model where firms are price setters, the firm's demand for labor, as well as the real wage it is willing to pay, will depend on the amount it thinks it can sell at a given price. Expectations of low product demand will induce low labor demand and possible excess supply of labor at the going real wage. Income paid out by the representative firm and thus actual demand for product will also depend on product demand expectations. Since, in steady state, all revenues from sales are currently paid out as income and all income is currently consumed, there will exist a steady state where demand expectations consistent with less than full employment are self-reproducing. In fact these last conditions ensure that any level of economic activity which is technologically feasible can be a steady state in this model.

The view that imperfectly competitive behavior provides a basis for Keynesian results is not new. Many earlier papers, however, often simply assumed a monopolistic market structure so that firms faced downward sloping demand curves (Benassy [1976] or Hart [1982]), or assumed that sellers acted on the basis of perceived or conjectured downward sloping demand curves (Negishi [1974, 1980], Hahn [1978]) which needed to coincide with the true demand curves only at the point of equilibrium (or in a small neighborhood of that point). Here, monopoly power is seen as characteristic of decentralized economies with a large number of sellers because of informational frictions. Moreover, firm's expectations about the entire demand curve they face are correct in steady state, the results not being dependent on a limited notion of rationality.

By considering underlying sources of imperfectly competitive behavior, this paper is closest in spirit to those of Weitzman [1982] and Calvo and Phelps [1983]. Weitzman very convincingly argued that the phenomenon of increasing returns inherent in many facets of a modern economy is intrinsically linked to the existence of Keynesian unemployment equilibrium. Since the specific source of imperfect competition here is slow flow of information about price differentials, this model of the output market is analogous to that of Calvo and Phelps. The results differ in two crucial respects. First, the labor market here may be characterized by excess supply, as opposed to market-clearing in their paper. Second, they find a unique steady-state equilibrium (or, at most, a finite number of such equilibria), whereas here any level of economic activity can be a steady-state.

Weitzman also finds the same continuum result in his increasing returns, endogenous price model. Though the conclusions of Weitzman's paper are similar to those derived here, the models are quite different. He presents what he calls a detailed "example" of Keynesian equilibrium in an atemporal model of locational monopoly (using the device of an "attribute circle" often used in monopolistic competition models). The framework is such that the ultimate source of monopoly power is increasing returns to scale in production. Here, the market set-up is far more standard, the source of imperfect competition is quite different, and the framework is explicitly intertemporal to highlight the importance of expectations of the future.[1]

The organization of this paper is as follows. In the next section the model is set up and optimal firm behavior is discussed. Section three considers the firm in steady state. In section four, optimal consumer behavior is discussed, and section five uses these results to derive the output demand curve in steady state. In section six the two components of the model are brought together and a rational expectations, general equilibrium is derived. The two main conclusions of the paper emerge in this section. In section seven, a mathematical example is presented to help clarify the results, which are summarized and interpreted in section eight.

2. GENERAL SET-UP AND FIRM BEHAVIOR

Consider a discrete time model with two physical commodities, output and labor. The labor market operates at the beginning of the period ("morning"); the goods market at the end ("afternoon"), after output has been produced. There are two types of agents: firms which demand labor and product output; and worker-consumers who supply labor and demand output. Consumers are also owners of the firm.

Two aspects of the transaction's structure should be noted, one concerning the temporal separation of the operation of markets within a period, the second concerning a medium of exchange. Clower's [1965] pioneering work convincingly emphasized the need to distinguish between notional and effective demand in analyzing the Keynesian model. In a decentralized multi-good economy, willingness to supply labor does not constitute effective demand for goods. This observation, it seems to me, is at the heart of Keynesian unemployment equilibria. Since employment of factors does not ensure that the firm will be able to sell the goods produced, expectations of product demand become crucial in determining labor demand and, ultimately, consumer income. To model this crucial uncoupling of trades for labor and goods and still retain the simplicity of a two-commodity model, I assume simply that the two markets operate at different points in time.

A more standard way of treating this problem is to introduce a medium of exchange, such that labor is not traded for goods directly. But as has been argued in detail elsewhere (Drazen [1980]), the constraint that all transactions must use money is simply a convenient way of modelling this uncoupling in a two-good economy, but is not the underlying cause of non-synchnonous trading.

To make clear that money as a constraint on the transactions structure is not the cause of effective demand shortfalls, I allow the firm to issue "checks" in whatever quantities it needs to buy labor. The constraint the firm will face is that it must return its "checking account" to zero at the end of the period. Since it replenishes its checking account with receipts from sales, it is clear that it is expected sales which constitute the true constraint.

Specifically, the firm buys labor in the morning, paying with checks. Labor is the numeraire, so the model determines relative, but not absolute prices. In the afternoon, the firm receives receipts from the sale of output. An amount equal to the wagebill is deposited in the firm's "checking account" to eliminate its overdraft. The excess of receipts over wagebill, meaning profits, are paid out to owners of the firm. Profits paid out at the end of the period are available for spending at the beginning of the following period.

The objective of the infinitely lived, representative firm is to maximize, conditional on its demand expectations, the present discounted value of expected profits by choice of labor input, output price (expressed in labor units), and inventories to be carried over between periods. The expected demand curve is the product of the expected number of customers multiplied by expected demand per customer. "Customers" are two person families consisting of one young worker and one retiree.

We assume that all customers are identical, and that each customer buys from only one firm at a point in time. The number of customers which the firm has, denoted x_t, is assumed to change depending on the price it charges relative to the market price. A firm charging a relatively low price will gain customers, but only at a finite rate. The basis for this crucial assumption is that information about price differentials diffuses only slowly. Using a basic model of slow information diffusion, Phelps and Winter [1970] derived a customer arrival function depending on firms price relative to market price with the above characteristic.[2] A discrete time version of such an equation is

$$x_{t+1} = (1 + \mu(p_{t+1}, \bar{p}_{t+1}))x_t \tag{1}$$

where p_{t+1} is the price the firm charges at time $t+1$, and $\bar{p}_{t+1}$ is the average market price (defined as the average of other firms' prices weighted by their market share), and where μ is decreasing in $p(\frac{\partial\mu}{\partial p} \equiv \mu' < 0)$, homogeneous in p and $\bar{p}$, and of the same sign as $\bar{p}_t - p_t$.[3]

Product demand of an individual customer depends on the price quoted by the firm from which he buys. (As will be clear below when the actual demand curve is derived, it will be affected by expected future prices but not by $\bar{p}$, once the consumer has chosen where to buy). The individual demand curve expected by the firm may be written $\hat{d}(p_t)$, where we suppress for now the functional dependence of demand on income and consumer expectations. We assume that the firm has point expectations of demand at each price (the circumflex over d representing expectations) and that the expected demand <u>curve</u> is assumed to remain constant in the future (as indicated by no time subscript on $\hat{d}$). The latter assumption of convenience is consistent with our ultimate concentration on steady states.

Inventories i are expected to evolve according to

$$i_{t+1} = i_t + f(N_t) - x_t\hat{d}(p_t) \tag{2}$$

where output produced at t is a function $f(\quad)$ of total labor input N_t.

The firm's objective of maximizing present discounted profits may then be characterized as choosing an infinite sequence of p_t and N_t to maximize the present discounted value expression

$$\sum_{t=o}^{\infty}\left(\frac{1}{1+\rho}\right)^{t}[p_t x_t \hat{d}(p_t) - N_t + \lambda_t i_t + \alpha_t x_t] \tag{3}$$

(where λ_t and α_t are non-negativity multipliers on the state variables and ρ is the discount rate) subject to the evolution of x_t and i_t, as given by (1) and (2), and the initial conditions x_o and i_o. We assume, for simplicity, there is neither population growth nor technical progress. Note that since labor is the numeraire, the wage bill is simply N_t.

Taking p and N as control variables and x and i as state variables, we define a state function V(x,i) and write the problem as

$$V(x_t,i_t) = \underset{p_t,N_t}{\text{Max}}\ \{p_t x_t \hat{d}(p_t) - N_t + \lambda_t i_t + \alpha_t x_t + \frac{1}{1+\rho}V(x_{t+i},i_{t+1})\} \tag{4}$$

subject to (1), (3), the initial conditions on x and i, and the firm's expectations about $\bar{p}_t$ and $\hat{d}(\ )$. Necessary conditions for a maximum are

$$x_t\hat{d}(p_t) + p_t x_t \hat{d}' + \frac{1}{1+\rho}V_x[t+1]\mu' x_t - \frac{1}{1+\rho}V_i[t+1]x_t\hat{d} = 0 \tag{5}$$

$$-1 + \frac{1}{1+\rho}V_i[t+1]f' = 0 \tag{6}$$

where

$$V_x[t] = p_t\hat{d}(p_t) + \frac{1}{1+\rho}V_x[t+1](t+\mu) - \frac{1}{1+\rho}V_i[t+1]\hat{d}(p_t) + \alpha_t \tag{7a}$$

$$\alpha_t x_t = 0 \tag{7b}$$

$$V[t] = \frac{1}{1+\rho}V_i[t+1] + \lambda_t \tag{8a}$$

$$\lambda_t i_t = 0 \tag{8b}$$

These conditions are also sufficient when the firm's profit function in each period is concave. Equations (1), (2) and (5) through (8) could then be solved for optimal time paths for p_t and N_t, conditional on the firm's expectations and initial conditions.

3. THE FIRM IN STEADY STATE

We now solve for a rest point of the firm's problem where we assume that firm expectations about the demand curve are correct. Therefore, all plans are assumed to be carried out and expected i and planned p are equal to actual values.

No inventories will be carried in steady-state, which follows formally from noting that when x and i are constant, so that $V_i(t) = V_i(t+1)$, (8a) implies that λ must be strictly positive. From (8b), we then obtain $i = 0$. All firms will have the same number of customers equal to $\bar{x}$, the exogeneously determined ratio of total customers (that is, two-person families) to total firms. Since $\bar{x}$ is also the number of workers hired by the representative firm, we may express labor input per representative worker, n, as the ratio $N/\bar{x}$. We can then substitute $\bar{x}n$ for N in the above equations and solve for n rather than N.

Each firm charges the same price in steady state, so that $\bar{p}$ equals p and, therefore, $\mu = 0$. To solve for rest-point p and n we note that $\alpha = 0$ from (7b). Since α and μ both vanish, and since (6) implies that $V_i = \frac{1+\rho}{f'}$, (7a) becomes

$$V_x = \frac{1+\rho}{\rho}(p - \frac{1}{f'}). \tag{9}$$

Substituting V_x and V_i into (5) and dividing through by $x\hat{d}$, we obtain

$$1 + \frac{p\hat{d}'}{\hat{d}} + \frac{\mu'}{\rho}(p - \frac{1}{f'}) - \frac{\hat{d}'}{\hat{d}f'} = 0$$

Rearranging and expressing this in elasticities we obtain

$$p = \frac{1}{f'(\bar{x}n)}\,(1 - (\hat{\varepsilon}(\cdot) + \sigma(\cdot))^{-1})^{-1} \qquad (10)$$

where $\hat{\varepsilon} = \frac{-p\hat{d}'}{\hat{d}}$ is the regular elasticity of the demand curve at p and $\sigma = \frac{-\mu' p}{\rho}$ is the non-instantaneous component of long-run elasticity, due to finite customer arrival rates.[4] (Both $\hat{\varepsilon}$ and σ are functions).

It is this second factor which allows atomistic firms to act like price setters in the output market. When $\mu' = 0$ (firms believe that market share is fixed with respect to price differentials) we get the simple monopoly solution while when μ' approaches $-\infty$ (instantaneous response to price differentials), we approach the pure competitive solution. (When $\mu' = -\infty$, the firm does not solve the above maximization problem, since (1) is not a constraint. Therefore, the results below do not apply to the competitive model.) As long as μ' is finite and the discount rate is positive, the ratio of price to marginal cost in steady state will be between $\frac{\hat{\varepsilon}}{\hat{\varepsilon}-1}$ and 1. Hence, when atomistic firms face slow customer arrival such that they become price setters, it is possible to have a steady state with price above marginal cost.[5]

The inventory equation (2) becomes

$$f(\bar{x}n) = \bar{x}\hat{d}(p) \qquad (11)$$

These two equations (10) and (11) determine steady-state p and n conditional on the demand curve that firms (correctly) anticipate.

This result is worth stressing. When firms are price setters based on the demand curve for output they expect to face, the level of the curve determines employment and the relative price of labor to goods. As long as quantity of labor demanded is less than the desired supply at real wage $1/p$, employment will be equal to the value of n derived from $\hat{d}(p)$, independent of the size of excess labor supply. Conceptually, this is the Keynesian spillover effect of product market conditions onto the labor market. Desire to supply more labor at the going real wage does not affect employment directly; it can do so only indirectly by affecting effective demand for output. For future reference, we note the relation between the chosen value of n and expectations about demand implied by (11). Holding expectations about the shape of the demand curve given, n may be thought of as indexing expectations of the level of demand. (Since the model determines only real relative prices, an all-round deflation, were it possible, would have no real effect. Or, if one could imagine a general nominal wage cut, firms would correctly anticiptate the nominal price they would receive at the end of the period would be proportionally lower, so nothing real will have changed).

4. CONSUMER BEHAVIOR

Consumers live two periods, supplying labor in the first period of their lives. Young consumers are willing to supply any amount of labor less than or equal to some maximum $\bar{n}$. (The results are robust to more standard specifications of labor supply, as discussed below). Since individual workers do not bargain with firms, n is exogenous to the consumer if labor demanded per worker is less than $\bar{n}$.

First-period income n is divided between current consumption c_t^1 and saving to finance second period consumption c_{t+1}^2. There are no bequests. Saving takes the form of purchases of shares in firms. The total number of shares is fixed (no new shares are issued), so young consumers must buy shares from the current old. The expected return to shares is the sum of profits expected to be paid out by firms, $\hat{\pi}$, and expected capital gains. Profits of firms earned in t are paid out at the beginning of t+1 before shares bought in t are sold to the next generation. Shares bought at price q_t in t are expected to be sold at price $\hat{q}_{t+1}$. (A circumflex over a variable represents expected value, where all expectations are, as before, point expectations). Denoting by s^d the desired number of shares purchased, the budget constraints of a young consumer born at t are

$$p_t c_t^1 + q_t s_t^d = n_t \qquad (12)$$

$$s_t^d(\hat{q}_{t+1} + \hat{\pi}_t) = \hat{p}_{t+1} c_t^2 \qquad (13)$$

where the t subscript on $\hat{\pi}$ serves as a reminder that they are earned at t. n_t is taken as given as the minimum of quantity of labor demanded and $\bar{n}$.

The consumer's objective is to maximize expected utility, where preferences are represented by a utility function $U(c_t^1, c_{t+1}^2)$, subject to his budget constraints. His choice variables are c_t^1, c_{t+1}^2 and s_t^d. Given his point expectations, this yields a first-order condition

$$U_1/U_2 = \frac{p_t}{\hat{p}_{t+1}/(1+\hat{r}_t)} \qquad (14)$$

(where $U_i = \frac{\partial U}{\partial c^i}$ and where $\hat{r}_t$ is the expected rate of return to shares, namely $\hat{r}_t = \frac{\hat{q}_{t+1} - q_t + \hat{\pi}_t}{q_t}$), as well as equations (12) and (13).

These could be solved to yield c_t^1, c_{t+1}^2, and s_t^d for given n_t, current prices p_t and q_t, and the expectational variables $\hat{p}_{t+1}$, $\hat{q}_{t+1}$, and $\hat{\pi}_t$.

5. OUTPUT DEMAND IN STEADY STATE

In steady state, where the consumer has unchanging expectations about the future, c_t^1 and c_t^2 are constant and are equal to those values planned by a representative young consumer. These are implicitly determined by the first order condition (14) together with the lifetime budget constraint. In steady state the former may be written

$$U_1/U_2 = 1 + \hat{\pi}/q \tag{15}$$

Share price q will be determined by market clearing in the share market, share demand per consumer being equal to <u>fixed</u> share supply per consumer. Denoting the latter by $\bar{s}$ and using (12) to express demand we may write

$$\bar{s} = \frac{n_t - p_t c^1}{q_t}$$

or

$$q = \frac{n - pc^1}{\bar{s}} \tag{16}$$

The quantities c^1, c^2 and q in steady state would then be determined (as functions of p and $\hat{\Pi}$) by (15), (16), and the steady-state version of (13), namely

$$\bar{s}(q + \hat{\Pi}) = pc^2 . \tag{17}$$

It will be useful to derive a lifetime budget constraint by substituting (16) into (17) to obtain

$$p(c^1 + c^2) = s\hat{\Pi} + n \tag{18}$$

Steady-state c^1, c^2, and q would then be determined for given p, n, and $\hat{\Pi}$, by (15), (16) and (18).

If we temporarily consider (15) and (18) as defining first and second period consumption as <u>functions</u> of the variables exogenous to the consumer, then the actual output demand curve will be <u>defined</u> as the sum of these two functions. In steady-state equilibrium the actual and expected demand curves (the latter is the $\hat{d}(\cdot)$ function) must, by definition of the equilibrium, be identical. The actual demand function then allows us to express the elasticity of demand ε as a function of p, as well as the other variables exogenous to the consumer. We write this as $\varepsilon(\cdot)$.

6. RATIONAL EXPECTATIONS STEADY STATES

In a rational expectations steady state, all expectations are correct and all real variables are constant over time. We combine the solutions to the firm's and consumer problem's to obtain a general equilibrium. Since expectations are correct we replace $\hat{\pi}$, expected profits per share on the part of consumers, by actual profits per share. Profits per share are the representative firm's profits per family divided by $\bar{s}$, shares per family (in equilibrium). Remember that the number of families or customers per firm is identical to workers per firm. Profits per family are then sales per customer minus income per worker, expressed in labor units. Actual profits per share may then be written

$$\pi = \frac{p(c_1 + c_2) - n}{\bar{s}} \tag{19}$$

The firm's expectations are also correct. Actual output demand may be substituted into the firm's first-order conditions (10) and (11). We will then have six equations in the six unknowns c^1, c^2, p, q, n, and π. The six steady-state rational expectations equations are

$$p = \frac{1}{f'(\bar{x}n)} \; (1 - (\varepsilon(\cdot) - \sigma(\cdot))^{-1})^{-1} \tag{10}$$

$$f(\bar{x}n) = \bar{x}(c^1 + c^2) \tag{11'}$$

from the firm's optimum;

$$U_1/U_2 = 1 + \pi/q \tag{15'}$$

$$p(c^1 + c^2) = \bar{s}\pi + n \tag{18'}$$

from the consumers optimum;

$$q = \frac{n - pc^1}{\bar{s}} \tag{16}$$

from equilibrium in the share market;

$$\pi = \frac{p(c^1 + c^2) - n}{\bar{s}} \tag{19}$$

from the definition of actual profits.

These six equations fully characterize a steady-state general equilibrium, meaning optima for both firm and consumer as well as the connections between the two sectors via endogeneously determined prices and profits paid out. One immediately notices that since (18') and (19) are identical, there are only five independent equations in the six unknowns. Therefore, one variable may be specified independently and the system may then be solved for the other variables consistent with it. Suppose n is specified independently. The result says that any positive value of n less than or equal to $\bar{n}$ can be a solution to the system.[6]

This result should be stressed, since it gives us the central conclusions of the paper. First, in a framework where behavior is derived from optimization, where prices are endogenously determined, and where expectations are rational, a low employment general equilibrium ($n < \bar{n}$) is possible. Second, <u>any</u> level of employment or economic activity can be a rational expectations steady state. Returning to the model of firm behavior, consider the relation between n and the firm's expectations about demand, $\hat{d}(p)$. Our results say that any level of firm's expectations consistent with positive n and p can be self-fulfilling. Low expected demand induces low n and, hence, low actual demand. Therefore, the Keynesian result that pessimistic expectations can induce a low employment equilibrium is consistent with expectations being rational and optimizing behavior.

These results are dependent on the limitation on actions we allow individual workers to take. Expanding the set of strategies available to workers will, of course, change the nature of the equilibrium. If we allowed individual workers to bargain with firms, equilibrium would require that n equal $\bar{n}$. But, that is a different model where, as indicated in the introduction, this is assuming that infinitesimal wage cuts are instantaneously effective . (Formally, the limitation on individual worker behavior means that there is no mechanism which ensures the equality of labor demand to desired labor supply.)

Our point here is that under certain reasonable limitations, the resulting optimal, endogenous-price equilibrium displays several Keynesian features, including the dependence of the actual level of economic activity on demand expectations. Firms choose both p and n depending on their expectations of product demand. As long as the n chosen by firms is less than desired supply at equilibrium p, a low employment equilibrium will result. (If the utility function were such that labor supply depended on p, the results would not change. The structure of the model would still imply that firms choose both p and n. As long as the n chosen by firms is less than desired supply at equilibrium, p, a low employment equilibrium will result. The labor supply curve would define the quantity of unemployment.) The model was constructed as it was not simply because the limitation on worker behavior may have some empirical validity, but also to underline the view that under certain reasonable conditions, employment will depend on what happens in the product market, not in the labor market. We will discuss this point more fully below, after presenting an example.

7. AN EXAMPLE

We here present a simple example to illustrate the continuum of possible steady states. The level of economic activity will depend on expectations, but expectations will be self-reproducing. We begin with the representative firm's expectations of the demand curve it will face in steady state and show that any level of expectations consistent with employment per worker n between 0 and $\bar{n}$ (inclusive) is a possible rational expectations equilibrium.

Consider a firm which believes it will face a steady state demand curve $\frac{\bar{x}y}{p}$ where y is a scalar and $\bar{x}$ is the number of customers. y/p is the demand curve of the family. The firm's beliefs about the level of demand are given by y. One should note that y is not an extraneous state variable. It will turn out to be lifetime income expressed in labor units, but this is irrelevant to the individual firm's decision.

The firm's production function is

$$f(N) = AN^{1/2} \tag{20}$$

and the arrival function is assumed of the form

$$\mu(p,\bar{p}) = \ell n(p/\bar{p}) \tag{21}$$

The elasticity of demand is 1 while $\sigma = (\frac{-\mu' p}{\rho})$ equals $\frac{1}{\rho}$ when evaluated at $p = \bar{p}$. The first-order conditions (10) and (11) describing firm behavior are then

$$p = \frac{1}{1/2\ AN^{-1/2}}(1+\rho) \tag{22}$$

$$AN^{1/2} = \frac{\bar{x}y}{p} \tag{23}$$

In steady state $N = \bar{x}n$. We then substitute and obtain

$$p = \left(\frac{2y\bar{x}(1+\rho)}{A}\right)^{1/2} \tag{24}$$

$$n = \frac{y}{2(1+\rho)} \tag{25}$$

Since n must be between 0 and $\bar{n}$, y must be between 0 and $2\bar{n}(1+\rho)$. Equations (24) and (25) indicate that any level of expectations in this range will yield an equilibrium from the firm's point of view. The price of output is endogeneously determined to support this equilibrium.

Profits per share are, via (19), $(p\cdot\frac{y}{p} - n)/\bar{s}$, which here becomes

$$\pi = (\frac{y}{s}(1 - \frac{1}{2(1+\rho)}) \cdot \qquad (26)$$

The consumer's utility function is assumed to be

$$U(c^1, c^2) = \ln c^1 + \beta \ln c^2 \qquad (27)$$

(Remember that n is exogenous to the consumer).

The young consumer's problem in steady state is to maximize (27) subject to his budget constraints, which in steady state may be combined into

$$pc^1 + \frac{pc^2}{1+\pi/q} = n \qquad (28)$$

This yields a first-order condition

$$\frac{c^2}{\beta c^1} = 1 + \pi/q \qquad (29)$$

where we have assumed that expected profits π are equal to actual profits. Equations (28) and (29) yield

$$c^1 = \frac{n}{(1+\beta)p} = \frac{y}{2(1+\rho)(1+\beta)p} \qquad (30)$$

$$c^2 = \beta c^1(1 + \pi/q) = \frac{\beta y}{2(1+\rho)(1+\beta)p} \cdot (1 + \pi/q) \qquad (33)$$

To evaluate (31), we must find equilibrium q. Equilibrium in the share market yields

$$q = \frac{n - pc^1}{\bar{s}}$$

$$= \frac{y}{2(1+\rho)} \frac{\beta}{1+\beta} \frac{1}{\bar{s}} \tag{32}$$

This, in turn, yields

$$1 + \frac{\Pi}{q} = 1 + \frac{(y/\bar{s})(1 - \frac{1}{2(1+\rho)})}{(y/\bar{s})(\beta/2(1+\rho)(1+\beta))} \tag{33}$$

$$= 1 + \frac{(1+\beta)(1+2\rho)}{\beta}$$

Second-period consumption can then be written:

$$c^2 = \frac{y}{p}(\frac{\beta + (1+\beta)(1+2\rho)}{2(1+\beta)(1+\rho)}) \tag{34}$$

In a rational expectations steady state, retirement consumption planned by a young consumer is equal to actual consumption by current old. Total consumption by a representative family is then

$$c^1 + c^2 = \frac{y}{p} \cdot \frac{1}{2(1+\rho)()1+\beta)} + \frac{y}{p} \cdot \frac{\beta + (1+\beta)(2\rho+1)}{2(1+\rho)(1+\beta)}$$

$$= \frac{y}{p}(\frac{1+\beta+(1+\beta)(2\rho+1)}{2(1+\rho)(1+\beta)}) \tag{35}$$

$$= \frac{y}{p}$$

This completes the example. As (35) makes clear, any value of y anticipated by the firm will in a rational expectations equilibrium, be self-reproducing. Comparison of (18') and (35) also makes clear that y is not an exogenous state variable. It is lifetime income of the representative consumer. In other words, expectations by firms of consumer "purchasing power" are self-confirming. All other variables can be expressed as functions of y and exogenous parameters. Any level of employment per worker, n, between 0 and $\bar{n}$ is a possible equilibrium, expectations determining the actual level of economic activity.

8. INTERPRETATION AND SUMMARY

Two basic results emerge in this paper. The first is the demonstration that, contrary to what is often argued, the existence of a Keynesian, low-employment equilibrium need depend neither on exogeneously rigid prices, nor on irrational behavior or mistaken expectations on the part of agents. The second result is that any level of economic activity below full employment is a possible steady state. What characteristics of an economy would explain these results?

One central characteristic of the economy which we tried to capture is the nature of price setting. Because of the informational structure (which is taken to characterize decentralized markets), price setting atomistic firms cannot act as if they can sell all they like at the going price. In their demand for labor, firms must consider not simply the real wage, but also the quantity they expect to sell at the price they charge. Since expected demand is central to Keynesian explanations of unemployment, one should note that quantity expectations enter not simply when prices are fixed, but whenever price elasticity is less than infinite. The textbook Keynesian quantity spillover will be characteristic of any macroeconomic model where price incentives work at only a finite rate. Similarly, in the labor market, behavior of workers was based on the limited effectiveness of wage cutting.

The importance of the slow arrival of sales opportunities suggests that the next step is to model more explicitly the search for limited trading opportunities in general equilibrium. Peter Diamond has started this line of research in a series of excellent papers (Diamond [1982], Diamond [1984], among others). The faster potential traders are expected to arrive, the greater the incentive to undertake production. The higher the level of production undertaken, the more traders there will be in the market. These two relations imply multiple equilibria, where low activity equilibria may be characterized by individuals failing to take account of the effect of their decision to buy on the ease with which other agents can sell. (A similar externality is present here. A firm's demand for labor, by affecting consumer income, affects the possibilities of other firms to sell output). Diamond's models are quite stylized, but capture the essence of aggregate demand failures. To the best of my knowledge, the only attempts to apply this view to the labor markets in an explicit search framework has been in some very interesting papers by Howitt and McAfee (Howitt and McAfee [1984], Howitt [1984]).

The role of sales expectations in a non-Walrasian price setting model makes clear why there can be excess notional supply of labor in equilibrium when expectations are self-confirming. The nature of the model implies that there is no effective direct mechanism in the labor market in this model for workers to signal their desire to work more. This feature simply highlights the importance of quantity signals to the firm. For employment to be increased, firms must be convinced they can sell more output at the going real wage (i.e., inverse of the price of output in labor units). In order to gain more employment, workers must indicate their desires in the output market. No individual worker can affect hiring decisions if he bought more output. If enough workers acted in concert to buy more output, derived demanded for labor would be significantly affected. But this is a strategy which would not appear to describe the options actually available to workers.

More generally, agents are constrained by the market to send only price signals to indicate they would like to work more. But the signals that "work" are quantity signals which are conditional in the following sense. Firms would like to indicate that if consumers buy more output, they will hire more labor. Analogously, workers would offer to buy more output at the going price if they could sell more labor at the going wage. Clearly, if these signals were coordinated, the economy could break out of a low employment equilibrium. A decentralized market using only price signals cannot do this.

The result that any level of economic activity is a possible equilibrium also follows from the role of quantity expectations. In steady state, all revenues from sales are currently paid to consumers and all income paid out is spent. Given the full payout and the unitary propensity to consume out of lifetime income, any feasible level of quantity demanded can clearly be self-reproducing.[7] The nature of price setting means that an equilibrium price can always be found to support the equilibrium quantities.

This model suggests one way (though certainly not the only way) to move beyond simple fixed price models in studying quantity-constrained macroeconomic equilibrium. I do not want to argue that this framework is "better" than the Lucas-Sargent equilibrium paradigm. It represents a step in trying to make basic Keynesian models more rigorous. And this is a step adherents of both approaches agree is necessary.

FOOTNOTES

This is a significantly changed version of a paper of the same title dated August 1982. I have benefitted from suggestions and conversations with numerous people, too many to name here. Special thanks go to Ben Eden, Robert Lucas, and Michael Mussa. I have also benefitted from comments on a much earlier version of this paper by workshop participants at Chicago, Cornell, Iowa, Michigan, Northwestern, Pennsylvania, Toronto and Washington Universities. Financial support from the National Science Foundation (SES 79-16527) and funds granted to the Foerder Institute for Economic Research by the Nur Moshe Fund are acknowledged with thanks. Any errors are, of course, my own.

1. Neary and Stiglitz (1983) also assign a central role to expectations.
2. Search behavior of customers in buying from what they believe to be the lowest price firm is embedded therefore in equation (1).
3. Woglom (1982) considers a model where the demand curve faced by an individual firm depends on how information diffuses about the price the firm is charging relative to other firms. Since price increases drive existing customers away, but price cuts do not attract new customers with the same effectiveness, the demand curve has a kink. Hence even when firms set prices optimally, prices may not move in response to a fall in demand and a low activity equilibrium may be established though agents have rational expectations. Though the model presented here is very similar in spirit to Woglom's, we do not rely on a kink in the demand curve. Also, consumer behavior is much more fully specified here, leading to the continuum result.

4. A similar equation may be found in Phelps and Winter (1970).

5. Since new entrants gain customers only over time, free entry in this model would not eliminate this divergence. That is, unless a new entrant started with a sufficiently large number of customers (which would contradict the notion of "new" entrant), the customer arrival function would imply that entry would not be profitable.

6. Non-negativity of profits for all n is guaranteed by diminishing marginal product of labor throughout (so that average product exceeds marginal product) and by price exceeding marginal cost according to (10).

7. The possible infinity of equilibria under rational expectations should be distinguished from the results of Taylor (1977), Blanchard (1979) and others. They find that since equilibrium conditions yield a relation between current and expected future prices, there may be an infinite number of equilibrium price functions. Here, the infinity of equilibria concerns quantities (for each equilibrium set of quantities, there is a unique price) due to payout ratios and propensity to consume.

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