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GENERALIZED EXPECTED UTILITY ANALYSIS OF RISK AVERSION WITH STATE DEPENDENT PREFERENCES

By

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ABSTRACT

This paper extends Machina's generalized expected utility theory to include state-dependent preferences. The extended framework is used to define the relation more risk-averse and to conduct comparative static analysis of demand for air travel insurance with respect to variations in risk aversion. We show that when the practice of taking out air travel insurance is inconsistent with the expected utility theory, it is in disagreement with Machina's hypothesis II.

GENERALIZED EXPECTED UTILITY ANALYSIS OF RISK AVERSION WITH STATE-DEPENDENT PREFERENCES

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1. Introduction

At the core of the expected utility theory of individual decision-making under risk is the independence axiom, which implies that the preference functional representing the decision-maker's preferences over lotteries (i.e., objective probability distributions) is linear in the probabilities. This assumption, however, is inconsistent with a substantial body of experimental evidence to the effect that preferences over lotteries are in fact systematically nonlinear in the probabilities.

Motivated by this discrepancy between theory and evidence, Machina [1982] generalized the expected utility theory in a manner which eliminated many of the reported inconsistencies while preserving most of its useful theoretical properties. The essence of this modification consists of the replacement of the independence axiom by the weaker assumption that the decision

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* Some of the results reported in this paper were originally conceived during a conversation with Mark J. Machina. Mark also offered numerous useful comments and suggestions on an earlier version of this paper. For both I am deeply grateful.

maker's preferences over lotteries are representable by a "smooth" (i.e., Fréchet differentiable) preference functional. Preference relations that satisfy this condition can be shown to be "locally expected utility maximizing," in the sense that small changes in the probabilities about a given distribution will be ranked according to the expectation of the "local utility function" at that distribution. Furthermore, except for the independence axiom, if the local utility functions possess the properties that are typically imposed on the von Neumann-Morgenstern utility function, then the general behavioral implications of the expected utility theory are preserved.

Originally developed for univariate state-independent preferences, this analytical framework may be extended to include state-dependent preferences. Besides the pure theoretical interest, such an extension is motivated, by the criticism leveled against the expected utility theory as a model of behavior in circumstances where the state-dependent nature of the decision-maker's preference is a natural element of the decision-problem. This criticism was voiced by Eisner and Strotz [1961], who claim that the observed practice of taking out air-travel insurance is inconsistent with expected utility maximizing behavior. Unlike the experimental evidence against the independence axiom, much of which is based upon responses to hypothetical questions and as a consequence raises serious methodological questions, the purchase of air-travel insurance represents actual decisions, and is therefore more compelling.

Elsewhere (see Karni [1984]) I have shown that the inconsistency pointed out by Eisner and Strotz is valid only under certain air-travel insurance pricing formula, and, therefore, cannot be regarded as conclusive regarding the validity of the expected utility theory. However, even within the context where Eisner and Strotz's criticism is valid taking out flight insurance is not inconsistent with the generalized expected utility framework modified to allow for the dependence of the decision-maker's preferences on the state of nature.

2. Comparative Risk Aversion with State-Dependent Preferences: The Generalized Expected Utility Case

To develop a measure of absolute risk aversion for state-dependent preferences the generalized expected utility framework must first be modified to incorporate such preferences. Thereafter the analysis proceeds along the same lines as in the case of the expected utility theory,

2.1 The Representation of State-Dependent Preferences

Let S be a finite set of states of nature and denote by P the set of all probability distributions on S. Let F be a function from S on to the set of integers $\{1,\ldots,n\}$, thus ordering the set S. Henceforth this order, which is arbitrarily chosen, is maintained. Denote by $D[0,M]$ the set of cumulative probability distribution on the closed interval $[0,M]$, and for each $s \in S$ let $H_s(w) \in D[0,M]$ denote the conditional cumulative distribution of wealth in state s. Denote by B the set of ultimate outcomes, namely $B = \{(w,s) | w \in [0,M], s \in S\}$. Let $L[B]$ be the set of all joint probability distributions on B. For $H \in L[B]$ let $H(y,i) = \Pr\{w \leqq y, s \leqq i\} = \sum_{s=1}^{i} p(s)H_s(y)$, for all $y \in [0,M]$, $i = 1,\ldots,n$.

Decision-makers are assumed to have preference relations on $L[B]$ that are complete, transitive and representable by a continuous real-valued preference function $V(\cdot)$ on $L[B]$. The last restriction requires that the preference relation be continuous, (i.e., for every $H \in L[B]$ the set of elements that are weakly preferred to H and the set of elements to which H is weakly preferred are closed in L[B]). The definition of continuity requires a definition of convergence of a sequence. In the present case a sequence $\{H^n(\cdot,\cdot)\} \subset L[B]$ is said to converge to $H(\cdot,\cdot)$ if and only if for all $s \in S$: $p^n(s) \to p(s)$ and $H_s^n(w) \to H_s(w)$ for each continuity point w of $H_s(w)$.

Instead of the independence axiom of the expected utility theory the critical restriction imposed on the preference relation is that its functional representation $V(\cdot)$ be Fréchet differentiable. This, in turn, requires the existence of a norm on the space $\Delta L[B] = \{\lambda(H^*-H) | H^*, H \in L[B], \lambda \in R\}$. This norm is given by $||\lambda(H^*-H)|| = |\lambda|\rho(H^*,H)$ where the metric $\rho(\cdot,\cdot)$, is defined as $\rho(H^*,H) \equiv \sum_{s\in S} \int_0^M |H^*(w,s) - H(w,s)|dw$. The functional $V(\cdot)$ is Fréchet differentiable at the point H in L[B] if there exists a continuous linear functional $\psi(\cdot;H)$ on $\Delta L[B]$ such that

$$(1) \qquad V(H^*) - V(H) = \psi(H^*-H;H) + o(||H^*-H||),$$

Since $\Delta L[B]$ is a linear subspace of $L^1[B]$, by the Hahn-Banach Theorem there exists a continuous linear extension of $\psi(\cdot;H)$ to $L^1[B]$. Consequently, by the Riesz representation theorem on $L^1[B]$, there is a function $\xi(\cdot;H) \in L^\infty[B]$ such that for any $H^* \in L[B]$

$$(2) \qquad \psi(H^*-H;H) = \sum_{s\in S} \int_0^M [H^*(w,s) - H(w,s)]\xi(w,s;H)dw.$$

Let

$$(3) \qquad U(w,i;H) = - \sum_{s=1}^{i} \int_0^w \xi(x,s;H)dx.$$

Then integrating (2) by parts we get:

$$(4) \qquad \psi(H^*-H;H) = -\sum_{i=1}^{n}\int_0^M\left[\int_0^w \xi(x,i;H)dx\right]\left[dH^*(w,i) - dH(w,i)\right]$$

$$= -\sum_{i=1}^{n}\int_0^M\left[\int_0^w \xi(x,i;H)dx\right]\sum_{s=1}^{i}\left[p^*(s)dH_s^*(w) - p(s)dH_s(w)\right].$$

Rearranging the summation and using (3) we obtain:

$$(5) \qquad V(H^*) - V(H) = \sum_{i=1}^{n}\int_0^M U(w,i;H)\left[p^*(i)dH_i^*(w) - p(i)dH_i(w)\right]$$

$$+ o(||H^*-H||).$$

Thus, for a differential movement from $H(\cdot)$ to $H^*(\cdot)$, the change in the value of the preference functional $V(\cdot)$ is given by the first term on the right-hand side of (4), which is the difference in the expectation of the random variable $U(\cdot,\cdot;H)$ under $H^*(\cdot,\cdot)$ and $H(\cdot,\cdot)$ respectively. $U(\cdot,\cdot;H)$ may be interpreted as a "local state-dependent utility function."

2.2 Conditional Reference Sets and Comparability of Risk Aversion

As in the case of the expected utility theory, interpersonal comparison of risk aversion in the framework of the generalized expected utility requires that the individuals being compared have identical reference sets. In the latter case, however, the utility functions are local. Consequently, the reference sets are local in the same sense, namely, they are conditional on the elements of $L[B]$ for which the relevant state-dependent local utility functions are defined. To define the conditional reference set we begin by considering risky prospects such that the level of wealth in each state of nature is given with certainty. Formally, given an n-dimensional vector $(w_1,\ldots,w_n) \equiv \underset{\sim}{w} \in [0,M]^n$ and $\underset{\sim}{p} \in P$ such that $p(s) > 0$, $s = 1,\ldots,n$, let $I(\underset{\sim}{p},\underset{\sim}{w}) \in L[B]$ be such that for all $s \in S$, $I_s(w_s') = 1$ for $w_s' \leqq w_s$ and $I_s(w_s') = 0$ otherwise.

In other words, given $I(\cdot,\cdot)$ the conditional probability of obtaining w_s in state s is 1, and the probability of realizing the ultimate outcome $\langle w,s\rangle \in B$ is $p(s)$ if $w = w_s$ and zero otherwise. Thus, the unconditional mean wealth corresponding to $I(\tilde{p},\tilde{w})$ is $\sum_{s\in S} p(s)w_s$. Given $\tilde{p}^0 \in P$ denote by $K(\tilde{p}^0,c)$ the subset of $L[B]$ which consists of all $I(\tilde{p}^0,\tilde{w})$ such that $\sum_{s\in S} p^0(s)w_s = c$. Thus, $K(\tilde{p}^0,c)$ is the subset of $L[B]$ consisting of all the elements whose conditional distributions on $[0,M]$ are degenerate and which yield a given constant unconditional actuarial wealth.

For arbitrary $H \in L[B]$, $I(\tilde{p}^0,\tilde{w}) \in K(\tilde{p}^0,c)$ and $q \in [0,1]$ we have, from equation (5),

$$(6) \qquad \frac{d}{dq}V[(1-q)H + qI(\tilde{p}^0,\tilde{w})]\Big|_{q=0}$$

$$= -\sum_{s\in S}\int_0^M U(w,s;H)p(s)dH_s(w) + \sum_{s\in S} p^0(s)U(w_s,s;H).$$

Equation (6) represents the change in the value of the preference functional resulting from a differential shift in the probability distribution H in the direction $I(\tilde{p}^0,\tilde{w})-H$. To obtain a conditional reference point we maximize (6) on $K(\tilde{p}^0,c)$, i.e., given H we solve for the distribution $I(\cdot,\cdot)$ that results in the most preferred differential shift among the distributions $I(\cdot,\cdot)$ with the same unconditional actuarial wealth. Formally this is equivalent to maximizing

$$(7) \qquad \sum_{s\in S} p^0(s)U(w_s,s;H)$$

on $[0,M]^n$ subject to the constraint

$$(8) \qquad \sum_{s\in S} p^0(s)w_s = c.$$

Assuming risk aversion so that the local utility function $U(\cdot,\cdot;H)$ is concave in w for all s and an internal solution, the necessary and sufficient conditions for maximum are given by (8) and

$$(9) \qquad U_w(w_s,s;H) = \gamma \qquad\qquad s = 1,\dots,n$$

where γ is the Lagrange multiplier corresponding to the constraint (8) and $U_w(\cdot,\cdot;H)$ denotes the partial derivative of U with respect to w. Denote the solution by $\underset{\sim}{w}^*(c,H)$, then this solution represents a <u>conditional reference point</u>. Given $H \in L[B]$ the set of all conditional reference points obtained for all $c \geqq 0$ is the <u>conditional reference set</u>, $RS_V(H)$. Formally,

> <u>Definition 1</u>: Let $\underset{\sim}{w}^*(c,H)$ be the solution to (7)-(8), then the conditional reference set is $RS_V(H) = \{\underset{\sim}{w}^*(c,H) \in [0,M]^n | c \geqq 0\}$.

The conditional reference set is a local property of the non-linear preference functional $V(\cdot)$. Notice also that given $H \in L[B]$ the conditional reference set can be represented as an n-dimensional vector of monotonic increasing and differentiable functions $f_i(\cdot|H) : [0,M] \to [0,M]$, where $f_1(w|H) = w$.

The comparison of two individuals with state-dependent preferences in terms of their risk aversion requires that their conditional reference sets be identical. This is stated formally in:

> <u>Definition 2</u>: Two preference functionals $V(\cdot)$ and $V^*(\cdot)$ representing state-dependent preferences are said to be <u>locally comparable at $H \in L[B]$</u> if and only if $RS_V(H) = RS_{V*}(H)$. They are said to be <u>globally comparable</u> if they are locally comparable at all $H \in L[B]$.

Notice that if the preference functionals are linear in the probabilities, namely, the local utility functions are independent of H, then we are in the expected utility framework. The local and global comparability in this case amount to the same condition.

2.3 Conditional Risk Premium

In Karni [1983] I defined the concept of risk premium for state-dependent preferences as the difference between the actuarial value of a risky prospect and the actuarial value of a reference point yielding the same level of expected utility. For lack of better terminology I refer to the reference point described above as the reference equivalent. Extending this notion to the generalized expected utility framework I introduce the notion of conditional reference equivalent, which is defined as:

Definition 3: For any $H \in L[B]$, $I(\tilde{p},\tilde{w}) \in K(\tilde{p},c)$ and $q \in [0,1]$ the conditional reference equivalent of $I(\tilde{p},\tilde{w})$, according to the preference functional $V(\cdot)$, is $I(\tilde{p},\tilde{w}')$ where $\tilde{w}' \in RS_V(H)$ is defined by the equation

$$\frac{d}{dq}V[(1-q)H + qI(\tilde{p},\tilde{w})]\bigg|_{q=0} = \frac{d}{dq}V[(1-q)H + qI(\tilde{p},\tilde{w}')]\bigg|_{q=0}$$

Thus the conditional reference equivalent is the element of the conditional reference set which is indifferent according to the local utility function $U(\cdot;H)$ to the initial risky prospect. Using definition 3 we define the conditional risk premium, $\pi_V(I(\tilde{p},\tilde{w});H)$ as follows:

$$\pi_V(I(\tilde{p},\tilde{w});H) = \sum_{s\in S} p(s)[w_s - w'_s]. \qquad (10)$$

Thus, the conditional risk premium is the difference between the actuarial value of the conditional risky prospect $I(\tilde{p},\tilde{w})$ and the actuarial value of its conditional reference equivalent $I(\tilde{p},\tilde{w}')$. The conditional risk premium is a measure of the decision-maker's risk aversion in the neighborhood of H, i.e., it is a local measure in L[B]. Utilizing this measure the relation "more risk averse," in a local and a global sense, are defined in:

Definition 4: Let $V(\cdot)$ and $V^*(\cdot)$ be non-linear functional representation of state-dependent preferences: (a) If $V(\cdot)$ and $V^*(\cdot)$ are comparable at $H \in L[B]$, $V(\cdot)$ is said to be more risk averse than $V^*(\cdot)$ at H if and only if $\pi_V(I(\tilde{p},\tilde{w});H) \geqq \pi_{V^*}(I(\tilde{p},\tilde{w});H)$ for all $I(\tilde{p},\tilde{w})$ and $\tilde{p} \in P$. (b) If $V(\cdot)$ and $V^*(\cdot)$ are globally comparable and the above inequality holds for all $H \in L[B]$ then $V(\cdot)$ said to be globally more risk averse than $V^*(\cdot)$.

Thus, in spirit if not in detail, the relation "more risk averse" is defined according to the original notion of Pratt [1964].

2.3 Equivalent Characterizations

The partial ordering on the class of comparable non-linear preference functionals introduced in definition 4 has equivalent characterizations in terms of properties of the local utility functions. These are summarized in Theorem 1 below and should be recognized as the analogue of the characterizations of the relation "more risk averse" for linear preference functionals,

THEOREM 1: Let $V(\cdot)$ and $V^*(\cdot)$ be globally comparable, state-dependent Fréchet differential preference functionals on L[B] and denote by $U(w,s;H)$ and $U^*(w,s;H)$ their respective local utility functions. Suppose that the local utility functions are differentiable then the following conditions are equivalent.

(i) $\dfrac{-U_{ww}(w,s;H)}{U_w(w,s;H)} \geqq -\dfrac{U^*_{ww}(w,s;H)}{U^*_w(w,s;H)}$ for all $H \in L[B]$ and all $s \in S$ and $w \in [0,M]$.

(ii) For each $H \in L[B]$ and $\underset{\sim}{p} \in P$ there exists monotonic increasing and concave transformation $T_H[\cdot]$ such that:

$$\sum_{s\in S} p(s)U(f_s(w|H),s;H) = T_H[\sum_{s\in S} p(s)U^*(f_s(w|H),s;H)]$$

(iii) $\pi_V(I(\underset{\sim}{p},\underset{\sim}{w});H) \geqq \pi_{V^*}(I(\underset{\sim}{p},\underset{\sim}{w});H)$ for all H and $I(\underset{\sim}{p},\underset{\sim}{w})$ in $L[B]$.

Clearly if $V(\cdot)$ and $V^*(\cdot)$ are not globally comparable but are locally comparable at H then Theorem 1 holds locally at H. The proof of Theorem 1 is analogous to that of Karni [1983] Theorem 1 and is not provided here.

3. Some Economic Implications

With appropriate restrictions on the properties of the local utility functions the analytical framework presented in subsection 2.1 preserves the economic implications of the expected utility theory. In addition, however, it is not contradicted by an empirical observation that may be inconsistent with the expected utility theory. These points are illustrated by the following discussion.

3.1 Risk Aversion and Air-Travel Insurance

Consider the problem presented originally by Eisner and Strotz [1961] of selecting the optimal air travel insurance coverage. Suppose that rather than being an expected utility maximizer, the passenger's preferences are represented by a Fréchet-differentiable, state-dependent non-linear preference functional. Let p and (1-p) denote the probabilities of states 1 and 2, respectively, where state of nature 1 represents life or a safe trip while state of nature 2 represents death or a plane crash. Assume that the insurance company is risk neutral. Denoting by C the level of flight insurance coverage, the insurance premium $\ell(C)$ is given by $(1-p+b)C$, where $b \in [0,p)$. Thus, if we denote by w_1^o and w_2^o the passenger's initial wealth in states 1 and 2 then upon taking out air-travel insurance policy his terminal wealth in the two states of nature respectively are:

$$w_1(C) = w_1^o - (1-p+b)C \text{ and } w_2(C) = w_2^o + (p-b)C.$$

Given the initial wealth distribution $I[\tilde{p},\tilde{w}^o]$, where $\tilde{w}^o = \langle w_1^o, w_2^o \rangle$ and $\tilde{p} = \langle p, 1-p \rangle$ the traveler's objective is to choose a level of insurance coverage C so as to maximize

$$V(I[\tilde{p},\tilde{w}(C)]) \qquad \text{subject to} \quad C \geqq 0,$$

where $\tilde{w}(C) = \langle w_1(C), w_2(C) \rangle$.

Theorem 2 below asserts that a more risk averse decision-maker in the sense of definition .4 will take out more insurance coverage than a comparable less risk averse one.

<u>THEOREM 2</u>: Let $V(\cdot)$ and $V^*(\cdot)$ be globally comparable, risk averse, state-dependent, Fréchet differentiable preference functionals on $L[B]$, whose local utility functions are denoted $U(w,s;H)$ and $U^*(w,s;H)$

respectively. If $V(\cdot)$ is more risk averse than $V^*(\cdot)$ than $V(\cdot)$ chooses larger flight insurance coverage than $V^*(\cdot)$ for all $I[\tilde{p},\tilde{w}^o]$ and $b \in [0,p)$.

Theorem 2 is analogous to Theorem 2 in Karni [1983]. It illustrates the fact that by redefining the analytical concepts that were developed within the framework of the expected utility theory in terms of the state-dependent local utility functions the behavioral implications of the expected utility theory can be obtained within the framework of Machina's generalized expected utility analysis. The proof of Theorem 2 appears in Section 4.

3.2 Flight Insurance and the Expected Utility Hypothesis

Much of the experimental evidence which challenges the independence axiom of the von Neumann-Morgenstern expected utility theory is subject to the methodological criticism of not representing actual decisions. If one takes the attitude that decision-making is an exerting mental process, this evidence must be taken with a great deal of skepticism since there is no reason to suppose that rational decision-makers exert themselves to make good decisions in hypothetical situations. Evidence recording actual decisions is more compelling. One such piece of evidence which may contradict the expected utility hypothesis was suggested by Eisner and Strotz [1961].

The essence of Eisner and Strotz's argument is that if, prior to embarking on a flight, one is already in possession of an optimal amount of term life-insurance coverage then the increase in the total probability of death resulting from taking a plane trip makes it optimal to gamble on one's life rather than take out additional life insurance coverage. Barring gambling on one's life the optimal solution is not to buy any air-travel insurance. Hence the observed practice of taking out air-travel insurance policies contradicts the expected utility hypothesis.

This conclusion depends critically on the insurance pricing formula used by Eisner and Strotz. In particular, it can be shown (see Karni [1984]) that if air-travel insurance is offered according to the terms specified in the preceding sub-section, then the aforementioned inconsistency may be eliminated. Thus, while their sweeping conclusions are not warranted by their argument it is nevertheless interesting to see that the purchase of air-travel insurance is not inconsistent with the generalized expected utility theory even within the model of Eisner and Strotz.

Suppose that insurers set the premium, $\ell(\cdot)$, of term life-insurance policies according to the formula:

$$\ell(c) = mpC \qquad m \geq 1 \tag{11}$$

where, as before, C denotes the insurance coverage and p denotes the probability of death during the term covered by the policy. Consider an individual whose pre-insurance wealth in the states 1 and 2 is given by $\langle w_1^0, w_2^0 \rangle$, where the states 1 and 2 represent life and death during the period covered by the insurance, respectively. Suppose that the probability of the individual dying during the term, say a day, of the policy is p' and that he can purchase insurance according to the terms specified in (11) with $p = p'$. The decision-maker will choose his insurance coverage, C, so as to maximize:

(12) $$V(I[\underset{\sim}{p}',\underset{\sim}{w}(C)]) \qquad \text{subject to} \quad C \geq 0$$

where $\underset{\sim}{p}' = (1-p',p')$ and $\underset{\sim}{w}(C) = \langle w_1 - mp'C, w_2 - (1-mp')C \rangle$. Let C^* denote the optimal coverage then differentiating $V(\cdot)$ with respect to C and evaluating at C^* we obtain:

(13) $$\frac{d}{dC}V(I[\underset{\sim}{p},\underset{\sim}{w}(C)])\Big|_{C^*}$$

$$= \frac{d}{dC}[(1-p')U(w_1(C),1;I^*) + p'U(w_2(C),2;I^*)]\Big|_{C^*}$$

$$= -mp'(1-p')U_w(w_1(C^*),1;I^*) + (1-mp')p'U_w(w_2(C^*),2;I^*)$$

$$= 0$$

where $U(\cdot,\cdot;I)$ is the local utility function at $I^* \equiv I[\underset{\sim}{p},\underset{\sim}{w}(C^*)]$. Thus,

(14) $$\frac{U_w(w_2(C^*),2;I)}{U_w(w_1(C^*),1;I)} = \frac{m(1-p')}{1-mp'}.$$

The expression on the left-hand side of equation (14) is the subjective rate of substitution between wealth in the two states. The expression on the right hand side is the market rate of substitution. If we think of wealth in a given state as a contingent commodity then the expression on the right hand side of (14) is the ratio of the wealth given up in state 1 per dollar worth of wealth received in state 2. As a ratio between two quantities of contingent goods this expression is the price of 1 dollar worth of insurance coverage.

Suppose that upon taking out one day term life insurance policy in the amount C^* our decision-maker learns that on the very same day he must take a plane trip. Suppose further that the probability of his dying in a plane crash is $\tilde{p}$ and that the probability of his dying from all other causes during that day is now p", where $\tilde{p} + p'' > p'$. In other words, since a part of the day will be spent in flight, the probability of dying as a result of say a car accident is reduced. The total probability of dying on the given day is, however, increased as a result of taking the trip.

Before boarding the flight the decision-maker is offered air-travel insurance according to the terms specified in (11), namely, if he takes out flight insurance coverage in the amount $\hat{C}$ then he must pay the premium $\ell(\hat{C}) = m\tilde{p}\hat{C}$. Upon taking out flight insurance the original probability distribution $\underset{\sim}{p}$ of wealth across states which is described in the following table:

	Probability	Wealth
State 1	$1 - p'$	$w_1(C^*)$
State 2	p'	$w_2(C^*)$

becomes, $\underset{\sim}{\hat{p}}$ and is described below:

		Probability	Wealth
State 1		$1 - \tilde{p} - p''$	$w_1(C^*) - m\tilde{p}\hat{C}$
State 2	plane crash	$\tilde{p}$	$w_2(C^*) + (1-m\tilde{p})\hat{C}$
	safe trip	p''	$w_2(C^*) - m\tilde{p}\hat{C}$.

State 2, namely the decision-maker dies during the day, is now refined to distinguish the cause of death since the total compensation depends on whether he dies as a result of a plane crash or from other causes.

To determine whether the decision-maker should take out a positive amount of air-travel insurance consider the path $I[\underset{\sim}{p}(\alpha),\underset{\sim}{w}], \alpha \in \{0,1\}$ where $\underset{\sim}{p}(0) = \underset{\sim}{p}$ and $\underset{\sim}{p}(1) = \underset{\sim}{\hat{p}}$ (i.e, $\tilde{p}(0) = 0$, $p''(0) = p'$ and $\tilde{p}(1) = \tilde{p}$, $p''(1) = p''$).

Differentiating $V(\cdot)$ with respect to $\hat{C}$ and evaluating at $\hat{C} = 0$, $\alpha = 1$ we get:

$$(15) \qquad \frac{d}{d\hat{C}}V(I[\underset{\sim}{p}(1),\underset{\sim}{w}(\hat{C})])\Big|_{\hat{C}=0,}$$

$$= \frac{d}{d\hat{C}}[(1-\tilde{p}(1)-p''(1))U(w_1(C^*)-m\tilde{p}(1)\hat{C},1;I(1))$$

$$+ \tilde{p}(1)U(w_2(C^*) + (1-m\tilde{p}(1))\hat{C},2;I(1))$$

$$+ p''(1)U(w_2(C^*) - m\tilde{p}(1)\hat{C},2;I(1))]\Big|_{\hat{C}=0}$$

$$= \{-(1-\tilde{p}(1)-p''(1))m\tilde{p}(1)U_w(w_1(C^*),1;I(1))$$

$$+ [\tilde{p}(1)(1-m\tilde{p}(1)) - p''(1)m\tilde{p}(1)]U_w(w_2(C^*),2;I(1))\}$$

where $\qquad I(1) = I[\underset{\sim}{p}(1),\underset{\sim}{w}(C^*)].$

Thus,

$$(16) \qquad \{\frac{U_w(w_2(C^*),2;I(1))}{U_w(w_1(C^*),1;I(1))} - \frac{m(1-\tilde{p}(1)-p''(1))}{1-m[\tilde{p}(1)+p''(1)]}\}\tilde{p}(1)(1-m[\tilde{p}(1)+p''(1)])$$

$$\cdot\, U_w(w_1(C^*),1;I(1)).$$

If the preference functional $V(\cdot)$ is linear in the probabilities, and consequently $U_w(w_i(C^*),i;I[\cdot;\cdot]) = U_w(w_i(C^*),i)$ for all $I[\cdot;\cdot]$ and $i = 1,2$, then from (16) and since, by definition $\tilde{p}(0) = 0$, $p''(0) = p'$, we have:

$$(17) \qquad \frac{U_w(w_2(C^*),2)}{U_w(w_1(C^*),1)} = \frac{m[1-\tilde{p}(0)-p''(0)]}{1-m[\tilde{p}(0)+p''(0)]} = \frac{m(1-p')}{1-mp'}.$$

But as we move from $\alpha = 0$ to $\alpha = 1$ the market rate of substitution or the price of insurance $\frac{m[1-\tilde{p}(\alpha)-p''(\alpha)]}{1-m[\tilde{p}(\alpha)+p''(\alpha)]}$ increases. Therefore, the expression in (17) must be negative and consequently the optimal level of $\hat{C}$ must be negative. If $\hat{C} < 0$ is institutionally prohibited, as is actually the case with flight insurance, then the optimal level of $\hat{C}$ is zero. Thus the purchase of a positive amount of air-travel insurance coverage is inconsistent with the linearity of the preference functional. This is the Eisner and Strotz's claim.

According to the generalized expected utility analysis the subjective rate of substitution given by the ratio of the marginal utility of wealth in the two states may change with $I[\cdot;\cdot]$. In particular, if this ratio increases by more than the increase in the market rate of substitution then the expression in (16) is positive, and the purchase of air-travel insurance is not inconsistent with the postulated structure of the decision-maker's preferences. The interpretation of this sufficient condition is as follows: As the probability of death increases the marginal utility of wealth in the event that the individual dies increases relative to the marginal utility of wealth in the event that he survives by more than the increase in the price of the insurance. In particular, if m is close to 1, as indeed seems to be the case, then the sufficient condition for the purchase of air travel insurance is that as the probability of death increases the marginal utility of wealth in case of death increases relative to that in the event that the individual stays alive. This condition is proposed as an hypothesis about the structure of the individual preferences. Henceforth we shall refer to it as the flight insurance hypothesis.

3.3 Further Discussion of the Flight Insurance Hypothesis

Though formally not comparable the behavioral interpretation of the flight insurance hypothesis is in disagreement with hypothesis II of Machina [1982]. In view of the central importance of hypothesis II in explaining the observed violations of the independence axiom this conclusion merits further elaboration.

According to Machina's hypothesis II the decision-makers local absolute risk aversion of the local utility functions is non-decreasing with shift towards stochastically dominating probability distributions. One way of stating this result formally is to say that for any $x_1, x_2 \in [0,M]$ such that $x_1 > x_2$, $U_x(x_1;F)/U_x(x_2;F) \geq U_x(x_1;F^*)/U_x(x_2;F^*)$ for all cumulative distribution functions F, F^* on $[0,M]$ such that F^* stochastically dominates F. In particular if the entire probability mass is concentrated on x_1 and x_2 a shift of probability mass from x_2 to x_1 must reduce the ratio of the marginal utility of x_1 and x_2. This implication is is illustrated in Figure 1a below where the indifference curves $U(F)$ and $U(F^*)$ represent the expected local utility functions under F and F^* respectively.

The requirement that (16) be positive implies that, as the probability mass is shifted from state 2 to state 1, the local marginal utility of wealth in state 2 increases relative to that of state 1. This is illustrated in panel (b) of Figure 1 where the indifference curves $U(I(2))$ and $U(I(1))$ represent the expectations of the local utility functions $U(\cdot,\cdot,I)$ under $I(2)$ and $I(1)$ respectively. Notice that the location of the points $(w_1(C^*),\ w_2(C^*))$ below the reference set is an implication of $m > 1$. The shift of probability from $I(2)$ to $I(1)$ does not have an immediate interpretation in terms of stochastic dominance. However, it is quite conceivable that at any level of wealth every decision-maker prefers state 1 (life) over state 2 (death). It is therefore appropriate to regard the shift of probability mass from state 2 to state 1 as a move towards stochastically dominating distribution. Yet as is shown in Figure 1, the effect of such a shift increases rather than reduces the marginal rate of substitution. It is in this sense that the flight insurance hypothesis disagrees with Machina's hypothesis II. In a formal sense this disagreement does not represent a logical contradiction as the two hypotheses are embedded in distinct models of behavior.

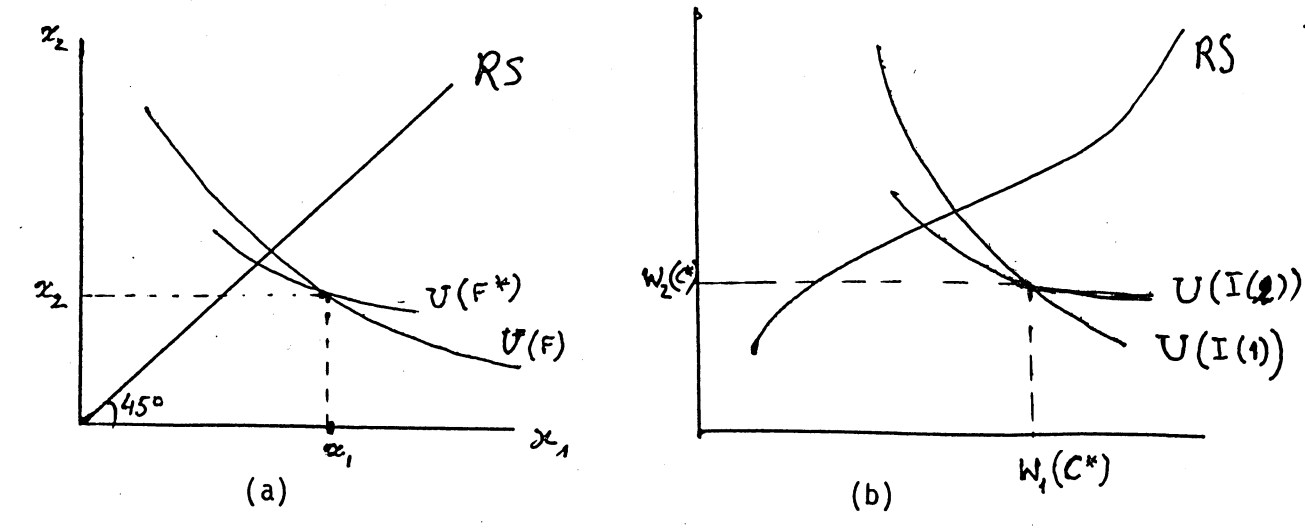

FIGURE 1 - The effects of a shift towards a stochastically dominating distributions: (a) according to Machina's hypothesis II, (b) according to the flight insurance hypothesis

4. Proofs

Theorem 2 is a corollary of the following:

Lemma 1: Given the hypothesis of Theorem 2 for any distribution $\bar{H} \in L[B]$, positive probability q, and $\underset{\sim}{w}(C) = \langle w_1(C), w_2(C) \rangle$ if $\hat{C}$ and $\hat{C}^*$ yield the most preferred distribution of the form $(1-q)\bar{H} + qI[\underset{\sim}{p},\underset{\sim}{w}(C)]$ for $V(\cdot)$ and $V^*(\cdot)$ respectively where $V(\cdot)$ and $V^*(\cdot)$ are strictly quasiconcave in C, over the set of distributions $\{(1-q)\bar{H} + qI[\underset{\sim}{p},\underset{\sim}{w}(C)]\}$, then $\hat{C} \geqq \hat{C}^*$, i.e., the conditional demand for flight insurance coverage is smaller for the less risk averse individual than it is for the more risk averse individual.

Proof of Lemma 1

Assume by way of negation, that for some $\bar{H} \in L[B]$, and $q \in (0,1)$, $\hat{C}^* > \hat{C}$. Then there exists $\bar{C} \in (\hat{C},\hat{C}^*)$ such that:

$$\frac{d}{dC}\{V^*((1-q)\bar{H} + qI[\underset{\sim}{p},\underset{\sim}{w}(C)])\}\Big|_{\bar{C}} > 0$$

$$> \frac{d}{dC}\{V((1-q)\bar{H} + qI[\underset{\sim}{p},\underset{\sim}{w}(C)])\}\Big|_{\bar{C}}$$

Let $H = (1-q)\bar{H} + qI[\underset{\sim}{p},\underset{\sim}{w}(\bar{C})]$, then $H \in L[B]$ and

$$0 < \frac{d}{dC}\{V^*((1-q)\bar{H} + qI[\underset{\sim}{p},\underset{\sim}{w}(C)])\}\Big|_{\bar{C}}$$

$$= \frac{d}{dC}\{\sum_{i=1}^{2}\int_0^M U^*(w_i,i;H)((1-q)d\bar{H} + qdI[\underset{\sim}{p},\underset{\sim}{w}(C)])\}\Big|_{\bar{C}}$$

$$= q\,\frac{d}{dC}\sum_{i=1}^{2} p_i U^*(w_i(C),i;H)\Big|_{\bar{C}}$$

$$= q[-pU_w^*(w_1(\bar{C}),1;H)(1-p+b) + (1-p)U_w^*(w_2(\bar{C}),2;H)(p-b)]$$

$$= q\{p(1-p)[U_w^*(w_2(\bar{C}),2;H) - U_w^*(w_1(\bar{C}),1;H)]$$

$$-b[pU_w^*(w_1(\bar{C}),1;H) + (1-p)U_w^*(w_2(\bar{C}),2;H)]\}.$$

But $$-b[pU_w^*(w_1(\bar{C}),1;H) + (1-p)U_w^*(w_2(\bar{C}),2;H)] < 0.$$

Hence $U_w^*(w_2(\bar{C}),2;H) > U_w^*(w_1(\bar{C}),1;H)$. Let $< \hat{w}_1, w_2(\bar{C}) > \in RS(H)$. Since $U^*(\cdot,1;H)$ is concave it follows that $\hat{w}_1 < w_1(\bar{C})$. For $H \in L[B]$, $w \in [0,M]$ and $i = 1,\dots,n$, define $T_{iH}[\cdot]$ by $U(w,i;H) = T_{iH}[U^*(w,i;H)]$, then

$$T'_{iH}[U^*(f_i(w|H),i;H)] = T'_H[\sum_{s\in S} p(s)U^*(f_s(w|H),s;H)] \text{ for all } i.$$

Then by the same argument as above,

$$\frac{d}{dC}\{V((1-q)A + qI[\underset{\sim}{p},\underset{\sim}{w}(C)])\}\Big|_{\bar{C}}$$

$$= q\{p(1-p)[U_w(w_2(\bar{C}),2;H) - U_w(w_1(\bar{C}),1;H)]$$

$$- b[pU_w(w_1(\bar{C}),1;H) + (1-p)U_w(w_2(\bar{C}),2;H)]\}$$

$$= q\{p(1-p)[T'_{2H}[U^*(w_2(\bar{C}),2;H)]U_w^*(w_2(\bar{C}),2;H)$$

$$- T'_{1H}[U^*(w_1(\bar{C}),1;H)]U_w^*(w_1(\bar{C}),1;H)]$$

$$- b[pT'_{1H}[U^*(w_1(\bar{C}),1;H)]U_w^*(w_1(\bar{C}),1;H)$$

$$+ (1-p)T'_{2H}[U^*(w_2(\bar{C}),2;H)]U_w^*(w_2(\bar{C}),2;H)]\}$$

But $\hat{w}_1 < w_1(\bar{C})$ and $T_{1H}[\cdot]$ is concave, hence $T'_{1H}[U^*(w_1(\bar{C}),1;H)] \leq T'_{1H}[U^*(\hat{w}_1,1;H)] = T'_{2H}[U^*(w_2(\bar{C}),2;H)]$. Hence, factoring out $T'_{2H}[U^*(w_2(\bar{C}),2;H)]$ we get,

$$\frac{d}{dC}\{V((1-q)A + qI[\underset{\sim}{p},\underset{\sim}{w}(C)])\}\Big|_{\bar{C}}$$

$$\geq qT'_{2H}[U^*(w_2(\bar{C}),2;H)]\frac{d}{dC}\{V^*((1-q)\bar{H} + qI[\underset{\sim}{p},\underset{\sim}{w}(C)])\}\Big|_{\bar{C}} > 0,$$

since $T'_{2H}[\cdot] > 0$ and $\frac{d}{dC}\{V^*(\cdot)\}\Big|_{\bar{C}} > 0$. This is a contradiction. Hence for all $\bar{H} \in L[B]$ and $q \in (0,1)$, $\hat{C}^* \leqq \hat{C}$. Q.E.D.

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