



AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

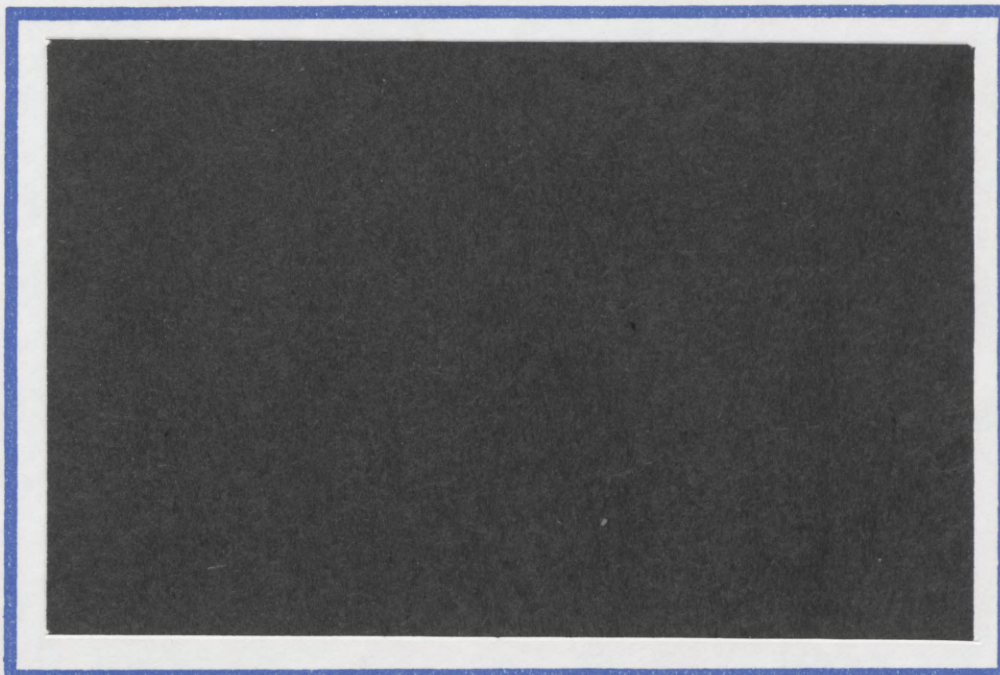
TEL-AVIV

24-84

THE FOERDER INSTITUTE FOR ECONOMIC RESEARCH

TEL-AVIV UNIVERSITY

RAMAT AVIV ISRAEL



GIANNINI FOUNDATION OF
AGRICULTURAL ECONOMICS
LIBRARY

JAN 4 - 1985

מכון למחקר כלכלי ע"ש ד"ר ישעיהו פורדר ז"ל

ע"י אוניברסיטת תל-אביב

BUDGET DEFICITS AND RATES OF INTEREST
IN THE WORLD ECONOMY

By

Jacob Frenkel * and Assaf Razin **

Working Paper No.24-84

July, 1984

* University of Chicago and National Bureau of Economic Research
** Tel-Aviv University

This paper was written while A.Razin was a visiting professor at the Woodrow Wilson School of International Affairs, Princeton University. In revising the paper we have benefitted from useful comments by R.Dornbusch, I.Gilboa, R.E.Lucas, Jr., F.Modigliani, M.Mussa and T.J.Sargent. The research reported here is part of the NBER's research programs in International Studies and Economic Fluctuations. Any opinions expressed are those of the authors and not of the National Bureau of Economic Research.

Financial assistance from the Foerder Institute for Economic Research is gratefully acknowledged.

I. Introduction

This paper deals with international economic interdependence and the international transmission of the effects of budget deficits. The question of the effects of budget deficits on rates of interest, on consumption, and on the nature of the real equilibrium, has been the subject of analysis in both closed and open economy contexts. Much of the traditional analysis demonstrating that budget deficits exert real effects have built on the presumption that government bonds constitute net wealth to individual asset-holders. The key issue of debate has been whether asset-holders take full account of the prospective tax liabilities that are associated with the need to service the debt created by current deficits. Recent research on the effects of government spending has embodied the Ricardian proposition according to which individuals take full account of future tax liabilities and, therefore, as long as government spending remains unchanged, the path of taxes and thereby the path of budget deficits do not affect the real equilibrium of the system. [For expositions see Bailey (1962) and Barro (1974)].

In the international context this perspective implies that world rates of interest and other real variables depend only on total government spending rather than on the time path of the deficit. Thus, in a previous paper (Frenkel and Razin (1984)) we have analysed the effects of fiscal spending on real rates of interest, debt and their international transmission. Our model, which assumed full rationality, perfect markets and absence of distortions, embodied the Ricardian feature and, therefore, in it budget deficits played no role. As is well known there are several ways by which a "fully rational" model can incorporate the feature that budget deficits influence the nature of the real equilibrium. For example, models which allow for distortionary taxes, [e.g. Barro (1979), Kydland and Prescott (1980), Lucas and Stokey (1983), and Razin and Svensson (1983)],

price rigidities or incomplete markets for intergenerational risk sharing [e.g. Stiglitz (1983)], or individuals with finite horizons, [e.g. Diamond (1965) and Samuelson (1958)], will all yield the result that fiscal deficits matter.

In this paper we extend our previous analysis in order to deal with the role of budget deficits in effecting world rates of interest, consumption and international indebtedness, and to study the resulting nature of the international transmission mechanism. Our earlier model is modified by the inclusion of individuals with the possibility of finite horizons. We adopt the formulation developed in Blanchard (1984) according to which the finiteness of the horizon is introduced through the assumption that, at each point in time, individuals, who are assumed to have no bequest motive, face a given probability of death. The main attraction of Blanchard's formulation lies in its capacity to generate a relatively simple pattern of aggregate behavior in a general equilibrium framework in which rational individuals respond to government deficits.

Analyses of the nature of the international transmission mechanism and of the impact of fiscal policies on world rates of interest have been central to earlier studies of open-economy macroeconomics. These examinations of the international transmission of fiscal policies have employed variants of the Mundell-Fleming models [Mundell (1968), Fleming (1962)]. These contributions were stimulated by the increased integration of world capital markets. They highlighted the interdependence among the exchange rate regime, the degree of capital mobility and the impact of fiscal policies, and they paid special attention to the conditions determining whether fiscal policies are transmitted positively or negatively to the rest of the world. Typical to this approach has been the lack of a sharp distinction between the impacts of different time patterns of fiscal spending and budget deficits. Our paper attempts to deal with these ques-

tions using an analytical framework within which capital markets are fully integrated, economic agents are rational and the behavior of all agents, including governments, are constrained by an intertemporal solvency requirement.

Section II outlines the building blocks of the model. It contains a derivation of individual consumption, a characterization of aggregate wealth accumulation, and a specification of the government budget constraint. In this section we illustrate the key features of the model by analysing the impact of budget deficits for a small open economy.

The core of the paper is contained in sections III, IV, V and VI where we analyse the interdependence between two open economies. In section III, we characterize the equilibrium conditions for the integrated two-country world economy. The impacts of budget deficits are analysed in section IV. It is shown that a budget deficit raises world rates of interest as well as domestic wealth and consumption while it lowers foreign wealth and consumption. Thus, the deficit is shown to be transmitted negatively to the rest of the world.

The channel of transmission is the world capital market and the negative transmission results from the higher rate of interest. In section V we analyse the impact of future budget deficits on the current values of wealth and on the term structure of interest rates. It is shown that also future deficits are transmitted negatively to the rest of the world in that they raise domestic wealth and consumption while lowering foreign wealth and consumption. On the other hand the impact of future deficits on the current short-term rate of interest depends on whether the domestic economy is a net saver or dissaver in the world economy, i.e., if it runs a current account surplus or deficit.

The analysis of budget deficits is conducted under the assumption that the deficit results from changes in the time path of taxes while holding constant the time path of government spending. In section VI we analyse the impact of changes

in the level of government spending. In order to separate the effects of the level of government spending from those of the budget deficit we assume in this section that changes in fiscal spending are accompanied by corresponding changes in taxes so as to yield a balanced budget. It is shown that a transitory rise in government spending raises interest rates and lowers domestic and foreign wealth. On the other hand, an expected future rise in government spending lowers interest rates and reduces the value of domestic wealth and consumption, while it raises the value of foreign wealth and consumption. We then analyse the effect of a permanent rise in government spending. It is shown that its impact on the rate of interest depends on whether the domestic economy is a net saver or dissaver in the world economy or, equivalently, whether it has a current account surplus or deficit. If the home country runs a current account surplus then a rise in government spending raises world interest rates and lowers domestic and foreign wealth and consumption. On the other hand if the home country runs a current account deficit then a permanent balanced-budget rise in government spending lowers interest rates and domestic wealth and raises foreign wealth. We thus, show the critical importance of specifying the details of fiscal policy in terms of the level and timing of government spending and taxes. Section VII contains some concluding remarks.

II. The Model

Our model has two economies each of which consists of consumers and government. The path of output in each economy is assumed to be given. In this section we outline the key building blocks of the model, starting with the specification of consumption in the home country.

II.1 Consumption

Individuals are assumed to face a given probability of death. We denote the probability that an individual will survive from one period to the next by γ , which is assumed to be independent of the individual's age. Thus, the probability that an individual will survive the next t periods is γ^t .

Let the utility function at period t of an individual who was born in period s be:

$$(1) \quad U = \sum_{v=t}^{\infty} \delta^{v-t} \log c_{sv}$$

where δ denotes the subjective discount factor and where c_{sv} denotes the rate of consumption in period v of an individual born in period s . It is assumed that the individual maximizes expected utility which is computed on the basis of the probability of survival: The probability, as of period t , that the individual will be alive in period v (and enjoy the utility of level $\log c_{sv}$) is γ^{v-t} . Therefore, expected utility can be written as

$$(2) \quad E_t \sum_{v=t}^{\infty} \delta^{v-t} \log c_{sv} = \sum_{v=t}^{\infty} (\gamma\delta)^{v-t} \log c_{sv} .$$

Equation (2) is the certainty equivalent utility function with an effective discount factor equaling $\gamma\delta$. Thus, the probability of death reduces the effective discount factor.

Following Yaari (1965) and Blanchard (1984) we assume that because of uncertain life time all loans require in addition to regular interest payment a purchase of life insurance. This life insurance guarantees to cover the outstanding debt in case of death. It is assumed that there is a large number of individuals and, therefore, competition among insurance companies implies that the percentage insurance premium equals the probability of death. The present-value factor, which is composed of one-period rates of interest compounded from period zero up to period t , is denoted by α_t and, therefore, α_{t-1}/α_t is the market discount factor that is equal to one plus the market rate of interest in period $t-1$. Analogously, the market risk factor γ^{t-1}/γ^t equals one plus the life-insurance premium. It follows that the effective interest factor faced by individuals is $(\gamma^{t-1}/\gamma^t)(\alpha_{t-1}/\alpha_t)$.

Using these considerations the budget constraint in period $t-1$ for an individual born in periods s is:

$$(3) \quad b_{st} = \frac{\gamma^{t-1} \alpha_{t-1}}{\gamma^t \alpha_t} (b_{st-1} + c_{st-1} - \bar{y}_{t-1})$$

where b_{st} denotes the value of debt at the beginning of period t (which was carried over from the end of period $t-1$), and \bar{y}_{t-1} denotes disposable income which is assumed to be the same across all individuals regardless of age. The budget constraint states that at the beginning of each period the value of debt equals the corresponding value in the previous period plus the excess of consumption over income plus the interest on that quantity. This last term is

reflected in the effective interest factor in equation (3). In addition to the periodical budget constraint, the individual is also subject to a solvency requirement by which the present value of his debt must approach zero in the limit:

$$(4) \quad \lim_{t \rightarrow \infty} \gamma_{\alpha_t}^t b_{st} = 0$$

Consolidation of the periodical budget constraints together with the solvency requirement yields:

$$(5) \quad \sum_{v=t}^{\infty} \frac{\gamma_{\alpha_v}^v}{\gamma_{\alpha_t}^t} e_{sv} = \sum_{v=t}^{\infty} \frac{\gamma_{\alpha_v}^v}{\gamma_{\alpha_t}^t} \tilde{y}_v - b_{st} \equiv w_{st}$$

Equation (5) is the consolidated budget constraint according to which the sum of present values of consumption in all periods must equal the sum of the present values of disposable income minus the debt commitment. This quantity is defined as the value of wealth, w_{st} . As is seen in equation (5) the value of wealth is composed of two terms. The first is the present value of expected life time income, and the second (negative component) is the accumulated financial debt. For further reference it is convenient to follow Blanchard's convention of referring to the first term as human wealth. We assume that except for the uncertain length of life, about which the individual knows the objective probability, individuals possess perfect foresight concerning all other relevant economic magnitudes.

Maximization of the certainty equivalent utility function (2) subject to the consolidated budget constraint (5) yields

$$(6) \quad c_{sv} = (1-\gamma\delta) \frac{\gamma^v \alpha_t}{\gamma^t \alpha_v} w_{st}$$

where c_{sv} denotes planned consumption for period v of an individual born in period s and who makes his plan in period t . Thus, this consumption is contingent on his survival in period v . In the case for which $v=t$ we obtain

$$(6') \quad c_{st} = (1-\gamma\delta) w_{st} .$$

Equation (6') describes the consumption function. It exhibits constant marginal propensity to consume out of wealth. As may be seen this marginal propensity decreases with the probability of survival, γ .

We now turn to the derivation of aggregate consumption function. Population is normalized so that every cohort born in period s starts with one individual. Due to death, its size in period t becomes γ^{t-s} . The equality between the probability of survival of a given cohort and its frequency relative to its initial size stems from the law of large numbers. Since at each period t there are γ^{t-s} members of each cohort, the (constant) aggregate size of population is:

$$\sum_{s=-\infty}^t \gamma^{t-s} = \frac{1}{1-\gamma} .$$

Aggregate consumption in period t is the sum of consumption of individuals from all cohorts. Since consumption of the s cohort is $\gamma^{t-s} c_{st}$, aggregate consumption, c_t , is

$$(7) \quad c_t = \sum_{s=-\infty}^t \gamma^{t-s} c_{st} .$$

Using the individual consumption function (6') and the definition of wealth from equation (5) yields

$$(8) \quad c_t = (1-\gamma\delta)(H_t - B_{pt})$$

where

$$H_t = \sum_{s=-\infty}^t \gamma^{t-s} \sum_{v=t}^{\infty} \frac{\gamma^v \alpha_v}{\gamma^t \alpha_t} \tilde{y}_v \quad \text{and}$$

$$B_{pt} = \sum_{s=-\infty}^t \frac{\gamma^t}{\gamma^s} b_{st}$$

Equation (8) is the aggregate consumption function in which H_t denotes aggregate human wealth, B_{pt} denotes aggregate private sector indebtedness and $(H_t - B_{pt})$ denotes aggregate wealth, W_t . In equation (8) the aggregate marginal propensity to consume out of wealth is constant. This reflects the specification of the utility function (which yields a unit elastic consumption function) and the assumption that the probability of death is equal across cohorts.

II.2. Dynamics of Aggregate Wealth

The analysis of aggregate consumption specified the behavior of the economy at a point in time as a function of aggregate wealth. In order to characterize the behavior of the economy over time so as to be able to deal with the dynamic effects of budget deficit, we need to determine the evolution of aggregate wealth through time. Since, as will be shown later, the two components of wealth are governed by different laws of motion, we need to study them separately. We start with an analysis of the dynamics of human wealth.

Aggregate human wealth in period t (which was defined in equation (8)), can be expressed as the product of the size of the population $(1/1-\gamma)$ and the (cohort-independent) individual human wealth. Thus,

$$(9) \quad H_t = \sum_{v=t}^{\infty} \frac{\gamma^v \alpha_v}{\gamma^t \alpha_t} y_v$$

where

$$y_v = \frac{1}{1-\gamma} \tilde{y}_v$$

Lagging equation (9) by one period yields an analogous expression for H_{t-1} which, together with equation (9), yields¹

$$(10) \quad H_t = \frac{\gamma^{t-1} \alpha_{t-1}}{\gamma^t \alpha_t} (H_{t-1} - y_{t-1})$$

Equation (10) describes the evolution of aggregate human wealth as a function of the difference between its own lagged value and the lagged value of aggregate income y_{t-1} . Equation (9) expresses aggregate human wealth in period t as the sum of the present values (as of period t) of aggregate income. It is important to note that both, the definition of aggregate human wealth (in equation (9)) and its law of motion (in equation (10)) use the effective interest factor which includes the life-insurance premium associated with the probability

¹In deriving equation (10) we first write H_{t-1} as

$$H_{t-1} = \frac{1}{1-\gamma} \sum_{v=t-1}^{\infty} \frac{\gamma^v \alpha_v}{\gamma^{t-1} \alpha_{t-1}} \tilde{y}_v = \frac{1}{1-\gamma} (\tilde{y}_{t-1} + \sum_{v=t}^{\infty} \frac{\gamma^v \alpha_v}{\gamma^{t-1} \alpha_{t-1}} \tilde{y}_v)$$

Subtracting y_{t-1} from H_{t-1} , multiplying by $(\gamma^{t-1} \alpha_{t-1} / \gamma^t \alpha_t)$ and using equation (9) yields equation (10) in the text.

of death. This feature is specific to the human wealth component and will not play a role in the computation of the other component of wealth to which we turn next.

The evolution of aggregate private debt, B_{pt} , can be obtained by substituting equation (3) into the definition of debt in equation (8) along with the definitions of lagged aggregate consumption, income and debt:

$$(11) \quad B_{pt} = \frac{\alpha_{t-1}}{\alpha_t} (B_{pt-1} + c_{t-1} - y_{t-1})$$

In contrast with the law of motion governing the accumulation of human wealth (in equation (10)) and individual debt (in equation (3)), the accumulation of aggregate private debt is governed by the market interest factor (α_{t-1}/α_t) rather than the effective interest factor. The absence of the life-insurance premium from the law of motion governing aggregate debt accumulation in equation (11) stems from the fact that from the perspective of the society at large, the life-insurance premia represents transfers within the society which do not alter the social rates of return.

Using the aggregate consumption function from equation (8), together with the law of motion of human wealth from equation (10) and iterating yields

$$(12) \quad B_{pt} = \frac{(\gamma\delta)^t}{\alpha_t} [(B_{po} - H_o) - (1-\gamma) \sum_{v=1}^{t-1} (\gamma\delta)^{-v} \alpha_v H_v] + \gamma H_t$$

where H_v is defined by equation (9) and where B_{po} is the initially given level of debt. Equation (12) characterizes the evolution of debt through time. This evolution together with the evolution of human wealth (from equation (10)),

determines the path of total wealth and, thereby, the path of consumption. The path of total wealth W_t can be obtained by subtracting equation (12) from H_t :

$$(13) \quad W_t = \frac{1}{\alpha_t} [(\gamma\delta)^t W_0 + (1-\gamma) \sum_{v=1}^t (\gamma\delta)^{t-v} \alpha_v H_v] .$$

Equation (13) expresses the value of wealth at each period of time in terms of its initial value W_0 as well as terms involving human wealth. The dependence of the path of wealth on its composition reflects the asymmetry between human and non-human wealth which arises from the uncertainty concerning the length of life. This asymmetry disappears when the probability of survival is unity. Thus, with $\gamma=1$, the value of wealth becomes

$$(13') \quad W_t = \frac{\delta^t}{\alpha_t} W_0 .$$

In this case, which corresponds to the one analyzed in Frenkel and Razin (1984), the evolution of wealth depends on its aggregate value and not on its composition.

In order to examine the effects of the uncertain length of life on the evolution of aggregate consumption we express c_t in terms of c_{t-1} using equation (13) in the aggregate consumption function (8):

$$(14) \quad c_t = \frac{\alpha_{t-1} [(\gamma\delta)^t W_0 + (1-\gamma) \sum_{v=1}^t (\gamma\delta)^{t-v} \alpha_v H_v]}{\alpha_t [(\gamma\delta)^{t-1} W_0 + (1-\gamma) \sum_{v=1}^{t-1} (\gamma\delta)^{t-1-v} \alpha_v H_v]} c_{t-1} ,$$

and in the special case with $\gamma=1$, equation (14) reduces to

$$(14') \quad c_t = \frac{\alpha_{t-1}^\delta}{\alpha_t} c_{t-1} .$$

The comparison between (14') and (14) reveals the role of uncertainty concerning the length of life. As seen in (14'), given the rates of interest, the only variable relevant for predicting future consumption is current consumption; in particular, once current consumption is known, knowledge of wealth is not required for the prediction of future consumption, as demonstrated in Hall (1978). In the general case, however, as seen from equation (14), once $\gamma < 1$ knowledge of current consumption is not sufficient for the prediction of future consumption, and one needs to know the detailed path of the composition of wealth.

II.3 Government

Government spending can be financed by taxes or by debt issue. The requirement of intertemporal solvency implies that the sum of the present values of government spending and debt retirement equals the sum of the present values of taxes. Alternatively this solvency requirement can be expressed as

$$(15) \quad \sum_{v=t}^{\infty} \frac{\alpha_v}{\alpha_t} (T_v - G_v) = B_{gt}$$

where T_v, G_v denote taxes and government spending in period v and where B_{gt} denotes the value of government debt at the beginning of period t . Equation (15) expresses the solvency requirements by stating that the value of government debt in period t must equal the sums of the present values of current and future budget surpluses.

Equations (15) implies that government debt in the beginning of period t is

$$(16) \quad B_{gt} = \frac{\alpha_{t-1}}{\alpha_t} (B_{gt-1} + G_{t-1} - T_{t-1}) .$$

As is evident from equation (16), in analogy with the evolution of aggregate private debt in equation (11), the law of motion governing the accumulation of government debt depends on the market interest factor.

The sum of private debt, B_p , and government debt B_g equals the value of the economy's external debt, B . Thus, by adding equations (11) and (16) we obtain

$$(17) \quad B_t = \frac{\alpha_{t-1}}{\alpha_t} [B_{t-1} + (c_{t-1} + G_{t-1}) - Y_{t-1}]$$

where $Y_{t-1} = y_{t-1} + T_{t-1}$ is gross domestic product.

II.4 Budget Deficits in a Small Open Economy

Prior to incorporating the foreign economy and determining the characteristics of world equilibrium we illustrate in this section one of the key implications of the model. For this purpose suppose that the economy that was described in the previous sections is small in the sense that it is facing an exogenously given path of world rates of interest. In order to examine the role of government budget deficits, suppose that the government changes the time pattern of taxes and debt issue while holding the path of spending unchanged. Specifically, consider the situation in which current taxes are reduced while

taxes in a future period, $t=s$, are raised so as to satisfy the government intertemporal solvency constraint.

From equation (15) the solvency requirement as of period $t=0$ is:

$$(15') \quad \sum_{v=0}^{\infty} \alpha_v (T_v - G_v) = B_{go} .$$

In order to find the effect of the current budget surplus on current consumption we need to determine the change in current wealth, W_0 . Since the only component of wealth that is affected is the current value of human wealth, H_0 , it is sufficient to determine the effect of the change in the timing of taxes on current human wealth. Using equation (9) current human wealth is

$$(9') \quad H_0 = \sum_{v=0}^{\infty} \gamma^v \alpha_v (Y_v - T_v) ,$$

and the change in current human wealth is

$$dH_0 = -dT_0 - \gamma^s \alpha_s dT_s .$$

From the government solvency requirement (15') we note that

$$dT_0 = -\alpha_s dT_s$$

substituting into the above equation, which expresses the change in current human wealth, we obtain

$$dH_0 = - (1-\gamma^s) dT_0 .$$

Thus, as shown by Blanchard (1984), a current budget deficit ($dT_0 < 0$) raises current wealth and current consumption even though individuals know that future taxes will be raised by an equal amount in terms of present value. This positive wealth effect diminishes the higher is the value of γ^S . That is, the higher the probability of survival and the closer the date of offsetting future tax changes, the weaker is the impact of current budget deficits on current wealth and consumption. In the limit, when $\gamma=1$, the Ricardian proposition re-emerges and, once government spending are given, the time pattern of taxes and government debt issue are irrelevant.

The explanation for this result can be given as follows. If the probability of survival was unity, then the rise in future taxes which is equal in present value to the reduction in current taxes would leave wealth unchanged. The same change in the pattern of taxes raises wealth if each individual knows that there is a probability of $(1-\gamma^S)$ that he will not survive to pay these higher future taxes. Therefore, the current reduction in taxes constitutes net wealth. Equivalently, the explanation can be stated in terms of the difference between the market and the effective interest factors. While the government solvency requirement implies that changes in current taxes must be made up for by α_s times the offsetting change in future taxes, individuals discount these future taxes by $\gamma^S \alpha_s$. Therefore, as long as $\gamma < 1$, the current budget deficit raises human wealth.²

This discussion illustrates that the model developed in the previous sections contains the key ingredient necessary for a meaningful analysis of the

²Yet another interpretation may be given in terms of a "transfer problem criterion" familiar from the theory of international transfers. Accordingly, the budget deficit exerts real effects because it redistributes wealth from those who have not yet been born, and whose marginal propensity to consume current goods is obviously zero, to those who are currently alive, and whose marginal propensity to consume current goods is positive. The clear presumption concerning the results of this redistribution is analogous to the presumption concerning the effects of an international transfer on relative prices in the presence of non-traded goods.

effects of budget deficits. The illustration was provided, however, within a limited set up in which the foreign economy was ignored and the path of the rates of interest was given exogenously. Since our prime interest is in the impact of budget deficits on the path of world rates of interest, on the international transmission mechanism, on external debt, and on the possible policy responses of the interdependent economies, we turn next to the specification of the foreign economy and to the analysis of world equilibrium.

II.5 The Foreign Economy

In this section we specify the behavioral equations for the foreign economy. The foreign economy is assumed to be characterized by an economic structure similar to that of the home country. We denote variables pertaining to the foreign economy by an asterisk (*). Analogously to the home country's behavior, the aggregate consumption function is

$$(18) \quad c_t^* = (1 - \gamma^* \delta^*) (H_t^* - B_{pt}^*)$$

where H_t^* is foreign human wealth,

$$(19) \quad H_t^* = \sum_{v=t}^{\infty} \frac{\gamma^{*v} \alpha_v^*}{\gamma^{*t} \alpha_t^*} y_v^* ,$$

and where B_{pt}^* is foreign private debt which can be expressed as in equation (12) by adding an asterisk to all the country specific variables. Since we assume that capital markets are integrated internationally, the market interest factors are the same for both countries. It is noteworthy, however, that if the probability of survival differs across countries, i.e., if γ differs from γ^* , then the effective interest factors will differ across countries.

III. World Equilibrium

Our previous discussion focused on the behavior of a small open economy that takes the path of world rates of interest as given. In this section we turn to the determination of the equilibrium path of these world rates of interest in a two-country world economy. We will continue to assume that world capital markets are fully integrated and, therefore, that individuals and governments in both countries face the same market rates of interest. This feature provides for the key channel through which policies undertaken in one country impact on economic conditions in the rest of the world. The structure of the model embodies the assumptions that the behavior of individuals is rational, that it is based upon self fulfilling expectations, and that governments and individuals are constrained by intertemporal budget constraints. These assumptions imply that economic policies have an impact on the entire path of interest rates and, thereby, on the paths of the key economic variables relevant for current and future generations in both countries.

III.1 The Formal Equilibrium Conditions

World equilibrium requires that in each period the given supply of world output equals world demand. In what follows we discuss the various equilibrium conditions. Using the consumption functions (8) and (18), equilibrium in period $t=0$ requires that world private spending equals world output net of world governments' spending:

$$(20) \quad (1-\gamma\delta)(H_0 + B_{go} - B_0) + (1-\gamma^* \delta^*)(H_0^* + B_{go}^* + B_0^*) = \bar{Y}_0 - G_0 - G_0^*$$

where \bar{Y} denotes world output. In equation (20) we have substituted the difference between external and government debts for the initially given values of

private debts. It is also noteworthy that equation (20) embodies the requirement that the sum of the balance of external indebtedness of both countries must be zero and, therefore, $B_0 = -B_0^*$.

For all future periods ($t > 0$), the equilibrium condition can be written as

$$(21) \quad (1-\gamma\delta)[(\gamma\delta)^t(H_0+B_{g0}-B_0) + (1-\gamma) \sum_{v=1}^t (\gamma\delta)^{t-v} \alpha_v H_v] + \\ (1-\gamma^*\delta^*)[(\gamma^*\delta^*)^t(H_0^*+B_{g0}^*+B_0) + (1-\gamma^*) \sum_{v=1}^t (\gamma^*\delta^*)^{t-v} \alpha_v H_v^*] = \\ \alpha_t (\bar{Y}_t - G_t - G_t^*) .$$

In equation (21) we have substituted equation (13) for the value of wealth in the home country's consumption function (8) while noting that $W_t = H_t - B_{pt} = H_t + B_{gt} - B_t$; the expressions pertaining to the foreign country's wealth were obtained by adding an asterisk to the relevant foreign variables and by noting that $B_0 = -B_0^*$.

The next set of conditions state that the values of human wealth in each period must equal the sum of the present values of disposable incomes where, as shown in equation (9), the discounting employs the effective interest factors. Thus, for period $t=0$, this requirement is:

$$(22) \quad H_0 = \sum_{v=0}^{\infty} \gamma^v \alpha_v (Y_v - T_v) ,$$

and for all other periods ($t > 0$) we have

$$(23) \quad \alpha_t H_t = \frac{1}{\gamma^t} \sum_{v=t}^{\infty} \gamma^v \alpha_v (Y_v - T_v)$$

$$(24) \quad \alpha_t H_t^* = \frac{1}{\gamma^{*t}} \sum_{v=t}^{\infty} \gamma^{*v} \alpha_v^* (Y_v^* - T_v^*)$$

Finally, the system is closed by the requirements that both governments are intertemporally solvent. Using equation (15) and its foreign-country analogue, we obtain

$$(25) \quad \sum_{t=0}^{\infty} \alpha_t (T_t - G_t) = B_{go}$$

$$(26) \quad \sum_{t=0}^{\infty} \alpha_t (T_t^* - G_t^*) = B_{go}^*$$

For any given initial values of the external debt, governments' debt and the paths of governments' spending, the system of equations (20)-(26) solves for the initial period's values of H_0 and H_0^* as well as for α_t , $\alpha_t H_t$ and $\alpha_t H_t^*$ for all other periods ($t > 0$). In addition, the system also solves for the sum of the present values of taxes in each country. Finally, it is relevant to note that in specifying the above system of equations we have made use of Walras Law. Therefore, we have excluded the condition analogous to equation (22) according to which the initial value of foreign human wealth, H_0^* , equals the sum of the present values of foreign disposable incomes.

As is evident from the multiplicity of the equilibrium conditions and the dependence of the paths of consumption on the compositions of wealth, an analytical solution of the system of equations (20)-(26) is extremely complex. In order to obtain a more tractable system, we turn now to an alternative representation of the equilibrium conditions.³

³The explicit solution of the system for the case $\gamma=1$ is given in Appendix A.

III.2 An Alternative Representation of the Equilibrium Conditions

One of the sources for complexities of the formal system presented in section III.1 stems from the multiplicity of market clearing conditions which are interrelated. Specifically, as seen in the set of equations (21), as long as γ and γ^* are less than unity, the present-value factors, α_t , depend on the entire path of past and future values of these factors. In order to simplify the analysis we provide an alternative representation of the equilibrium conditions. The key feature of the alternative representation is an aggregation of the entire set of the future equilibrium conditions into an equivalent single condition.

To facilitate the exposition we divide the horizon into two periods: the present, which is denoted by $t=0$, and the future ($t=1,2,\dots$), and we suppose that outputs, government spendings and taxes, do not vary across future periods ($t=1,2,\dots$). In aggregating the future into an equivalent single period we need to compute the present values of the various flows. For that purpose we define an average-interest factor

$$R = \frac{1}{1+r}$$

where r denotes the rate of interest. This average-interest factor represents the entire path of the rates of interest that actually do change over time. For further reference R may be termed as "constancy equivalent" interest factor.

Substituting R^v for α_v in equations (23)-(24) and substituting the resulting values of $\alpha_v H_v$ and $\alpha_v H_v^*$ into the left hand side of equation (21) yields:

$$(1-\gamma\delta)(\gamma\delta)^t [W_0 + (1-\gamma) \frac{R(Y-T)}{(1-\gamma R)(\gamma\delta-R)} (1-(R/\gamma\delta)^t)] +$$

$$(1-\gamma^*\delta^*)(\gamma^*\delta^*)^t [W_0^* + (1-\gamma) \frac{R(Y^*-T^*)}{(1-\gamma^*R)(\gamma^*\delta^*-R)} (1-(R/\gamma^*\delta^*)^t)] \quad ,$$

where $Y-T$ and Y^*-T^* denote the time invariant domestic and foreign disposable incomes for all future periods ($t > 0$). Analogously, the right hand side of equation (21) can be written as

$$R^t [(Y-G) + (Y^*-G^*)]$$

Summing these expressions for all future periods ($t=1,2,\dots$), yields equation (27) as the composite present value equivalent to equations (21):

$$(27) \quad \gamma \delta W_0 + (1-\gamma) \frac{R(Y-T)}{(1-R)(1-\gamma R)} + \gamma^* \delta^* W_0^* + (1-\gamma^*) \frac{R(Y^*-T^*)}{(1-R)(1-\gamma^* R)} = \frac{R}{(1-R)} [(Y-G) + (Y^*-G^*)]$$

Equation (27) states that the present values of world output net of government spending as of period $t=1$ must equal the present value of world private sector consumption which in turn equals the value of world wealth. As is evident the value of each country's wealth is expressed in terms of its initial wealth, as well as in terms of its human wealth. In that formulation the present values are computed by using the constancy-equivalent interest rates.

Substituting R^V for α_v in equation (22) yields:

$$(28) \quad W_0 = (Y_0 - T_0) + \frac{\gamma R}{1-\gamma R} (Y-T) + B_{go} - B_0$$

This equation defines the value of wealth in period $t=0$ in terms of R .

Finally, condition (20) specifying the equality of world demands with world output in period $t=0$ is rewritten as

$$(29) \quad (1-\gamma\delta)W_0 + (1-\gamma^*\delta^*)W_0^* = (Y_0 - G_0) + (Y_0^* - G_0^*)$$

The system of equations (27)-(29) can be solved for the values of W_0 , W_0^* and R for any given value of the parameters. The solutions obtained for the initial values of wealth W_0 and W_0^* , are the same as those which may be obtained from the complete system outlined in section III.1.⁴ While the use of the constancy-equivalent interest factor simplifies the analysis considerably, it does not permit a detailed examination of the impacts of policies on the patterns of consumption and rates of interest within the composite period comprising the "future". However, it provides complete information about the impact of policies on the precise current values of all key variables including wealth, consumption, and debt accumulation, as well as on the average value of the rate of interest.

IV. The Impact of Budget Deficits

In this section we analyse the impacts of budget deficits on world rates of interest and on the levels of domestic and foreign wealth and consumption. The analysis of the effects of budget deficits on the levels of foreign wealth and consumption serves to clarify the nature of the international transmission mechanism of fiscal policies. In order to focus on the impact of deficits rather than the impact of government spending, we will assume that changes in budget deficits are brought about through changes in taxes for a given path of government spending. Since government spending remains unchanged, solvency requires that current changes in taxes be accompanied by offsetting changes in future taxes. The present values of these tax changes must equal to each other. Thus, a change in current taxes, dT_0 , must be related to the future change, dT , according to

⁴ These solutions are obtained with the aid of a shortcut which transformed the entire set of equations for $t=1,2,\dots$, into its composite present-value equivalent. To verify that these solutions are identical to those obtained from the complete system, we note that the complete system with time-varying interest factors could be reduced to an analogous composite system which provides a unique link among W_0 , W_0^* and a composite function of the entire path of the time-varying interest factors $(\alpha_1, \alpha_2, \dots)$. Thus, the equilibrium values of W_0 and W_0^* remain unchanged as long as the value of the composite function of the interest factors is unchanged. Our shortcut replaces the composite function of the time-varying interest factors with its constancy equivalent, and its results are exact for $\gamma=\gamma^*$.

$$(30) \quad dT_o = - \frac{R}{1-R} dT$$

In equation (30) the term $R/(1-R)$ equals $1/r$ which is, in turn, the annuity value of a unit tax change.

Throughout the analysis in this section we assume that the foreign government follows a balanced-budget policy. This assumption ensures that changes in world rates of interest which result from domestic fiscal deficits, do not impact on the solvency of the foreign government and, therefore, do not necessitate secondary changes in foreign fiscal policies.

Using equations (27)-(29) the impacts of changes in domestic taxes on R , W_o , and W_o^* (evaluated around an initial balanced budget) are:

$$(31) \quad \frac{dR}{dT_o} = \frac{(1-\gamma)}{\Delta} \frac{(1-R)(1-\gamma\delta)}{R(1-\gamma R)} > 0$$

$$(32) \quad \frac{dW_o}{dT_o} = - \frac{(1-\gamma)}{\Delta} \frac{(1-\gamma\delta^{**})}{R(1-R)(1-\gamma R)} [\theta(Y-G) + \omega(Y^*-G^*)] < 0$$

$$(33) \quad \frac{dW_o^*}{dT_o} = \frac{(1-\gamma)}{\Delta} \frac{(1-\gamma\delta)}{R(1-R)(1-\gamma R)} [\theta(Y-G) + \omega(Y^*-G^*)] > 0$$

where, as shown in the Appendix, Δ , θ , and ω are positive.

Equation (31) shows that a budget deficit arising from a reduction in domestic taxes lowers R and raises the value of the world interest rate. The economic interpretation of this result is as follows. From equation (28) we note that, at the prevailing rate of interest, the fall in current taxes (which is accompanied by a corresponding rise in future taxes according to equation (30)), raises the value of domestic wealth by $(1-\gamma)(1-R)/R(1-\gamma R)$ and, thereby, raises

spending by the value of the marginal propensity to spend, $1-\gamma\delta$, times the change in wealth. The resulting excess demand for current goods raises their relative price in terms of future goods, and thus raises the world interest rate. As shown in the Appendix, $(1/\Delta)$ serves to translate excess demands for current goods into equilibrating changes in the rate of interest.

Equations (32)-(33) show that the domestic budget deficit raises the equilibrium-value of domestic wealth, W_0 , and lowers the corresponding value of foreign wealth, W_0^* . Thus, domestic budget deficits are transmitted negatively to the rest of the world. The international transmission mechanism is effected through the rate of interest. The rise in the world interest rate lowers the value of foreign wealth and mitigates the initial rise in the value of domestic wealth. These changes in wealth raise domestic spending, lower foreign spending and worsen the domestic current account of the balance of payments.

As may be seen from equation (31)-(33), when the probability of survival, γ , is unity budget deficits do not impact on interest rates and wealth; in this case the model yields the familiar Ricardian proposition.

V. The Impact of Future Budget Deficits

The previous analysis of the impact of budget deficits was conducted within a framework which consolidated the entire future into a single period. Consequently, in order to ensure solvency, a current budget deficit had to imply a corresponding rise in future taxes. The procedure of time-aggregation prevented an analysis of the impact of future budget deficits which are not accompanied by current changes in taxes. In order to examine the effects of future deficits we modify in this section the aggregation of the periods. Specifically, we divide the future into two parts -- the near future and the distant future. As before, the distant future

is represented by a consolidation of the entire period subsequent to that of the near future into a single period. In what follows a subscript 0 designates variables pertaining to the present period, a subscript 1 designates variables pertaining to the near future and, finally, variables which appear without a time subscript pertain to the distant future.

Consider an expected change in taxes occurring in the near future. Without changes in present period taxes government solvency requires that such a change must be accompanied by further offsetting changes in the distant future. Thus, solvency requires that

$$(30') \quad dT_1 = - \frac{R}{1-R} dT$$

where R is the constancy-equivalent interest factor in the distant future. In order to compute the effects of such future policies we note that the equilibrium system consists of the following requirements: (i) world output in the present period must be demanded (analogously to equation (29)) (ii) world output in the near future must be demanded (analogously to equation (21) for $t=1$),⁵ (iii) the equilibrium values of domestic and foreign wealth consist of the sums of the present values of disposable incomes in the present, the near future and the distant future adjusted for the initial debt (analogously to equation (28)).⁶

Using this system, assuming stationarity in the future and assuming that $\gamma=\gamma^*$ the impacts of the near-future deficit on domestic and foreign wealth are:

⁵The demand for output in the near future depends on the equilibrium values of human capital H_1 and H_1^* , which are obtained from equations analogous to (23)-(24) for $t=1$.

⁶The explicit inclusion of the requirement concerning foreign wealth replaces, by Walras Law, the condition analogous to equation (27) by which the sum of the present values of world net output in the distant future equals the corresponding sum of world demand.

$$(32') \quad \frac{dW_o}{dT_1} = - (1-\gamma) \frac{(Y_1 - G_1 + Y_1^* - G_1^*)}{\gamma(1-\gamma\delta)J} < 0$$

$$(33') \quad \frac{dW_o^*}{dT_1} = (1-\gamma) \frac{(Y_1 - G_1 + Y_1^* - G_1^*)}{\gamma(1-\gamma\delta^*)J} > 0$$

where

$$J = \frac{(Y-G)(Y_1 - G_1 + Y_1^* - G_1^*)(1-\gamma R)}{\gamma^2 R R_o} \left(\frac{1}{(1-\gamma\delta)(Y-G)} + \frac{1}{(1-\gamma\delta^*)(Y^* - G^*)} \right) > 0$$

As seen, analogously to the analysis of the effects of current deficits, future deficits (that are brought about through a reduction in taxes) also raise the equilibrium value of domestic wealth, W_o , and lower the corresponding value of foreign wealth W_o^* . Thus, domestic future budget deficits are also transmitted negatively to the rest of the world. As before, these changes in wealth do not occur when $\gamma=1$.

The impact of the future budget deficit on the current one-period interest factor, R_o , is:

$$(31') \quad \frac{dR_o}{dT_1} = \frac{(1-\gamma)(\delta^* - \delta)}{J}$$

Equation (31') shows that the effect of a future budget deficit on the current one-period rate of interest depends on the relation between the domestic and the foreign saving propensities, and, therefore, one may not argue that a future budget deficit per se raises short term interest rates. If $\delta < \delta^*$ the future budget deficit lowers R_o and raises the current one-period rate of interest and conversely if $\delta > \delta^*$. In interpreting this result we note that in the present period no government action takes place and, therefore, the change in the current rate of interest results only from changes in world savings which, in turn, stem

from changes in domestic and foreign wealth. As was shown in section IV the budget deficit occurring in period 1 raises domestic wealth and lowers foreign wealth in that period. It follows that at the prevailing current one-period interest rate the future deficit also raises W_0 and lowers W_0^* . These changes in wealth raise current-period world savings if $\delta > \delta^*$. Therefore, in order to restore world equilibrium with zero savings the current one-period interest rate must ^{rise} ~~rise~~. The opposite holds if $\delta < \delta^*$. Again, when $\gamma=1$ the future deficit does not alter the rate of interest.

Our analysis of the effects of budget deficits on the values of domestic and foreign wealth has demonstrated that the international transmission of the deficit must always be negative independent of whether the budget deficit occurs in the present or in the future. On the other hand we have shown that the impact of a future budget deficit on the current short-term rate of interest depends on whether the country which introduces the budgetary changes runs a surplus or a deficit in its international current account, i.e., if its saving propensity exceeds or falls short of the foreign saving propensity. Since it was shown that the directions of wealth changes do not depend on the direction of changes in the short-term rates of interest and since the link between foreign wealth and domestic deficit operates only through world capital markets, it follows that a future deficit must raise the overall "appropriate average" of short and long term rates of interest.

VI. The Impact of Fiscal Spending

The previous section analysed the effects of budget deficits on the equilibrium of the system. In that analysis changes in the deficit were brought about through changes in taxes while holding government spending intact. In the present section we analyse the effects of changes in the level of government

spending. In order to focus on the volume of fiscal spending rather than on the consequent changes in deficits, we suppose that all changes in government spending are accompanied by corresponding changes in taxes so as to yield balanced budgets. We also distinguish between transitory and permanent changes. Transitory changes alter only current spending by dG_0 whereas permanent changes also alter future spending by dG .

Using equations (27)-(29) the impact of a transitory balanced-budget change in domestic fiscal spending on the interest factor is:

$$(34) \quad \frac{dR}{dG_0} = - \frac{\gamma\delta(1-R)}{\Delta R} < 0 .$$

Equation (34) shows that a transitory rise in government spending lowers R and, therefore, raises the interest rate. This change in the interest rate is necessary in order to eliminate the excess demand for current goods consequent on the rise in government spending. This balanced-budget rise in government spending results in an excess demand for current goods because the rise in taxes lowers wealth by an amount equal to the rise in government spending and, since the private sector's marginal spending propensity is smaller than unity, private expenditure is reduced by only a fraction of the rise in government spending.

The rise in the rate of interest lowers both domestic and foreign wealth and, thereby, results in a reduction of domestic and foreign consumption. Thus, the current transitory balanced-budget rise in domestic fiscal spending is being paid for in part by the foreign country.

The same argument applies to the analysis of the effect of an expected future change in government spending. Formally, using equation (27)-(29) we obtain

$$(35) \quad \frac{dR}{dG} = \frac{\gamma(1-R)(1-\gamma\delta)}{\Delta(1-\gamma R)} > 0 .$$

As seen in equation (35) a future balanced-budget rise in government spending raises R and lowers the rate of interest. The fall in the rate of interest is necessary since, at the prevailing rate of interest, the expected rise in future taxes lowers wealth and reduces consumption demand and, thereby, results in an excess supply of current output. This excess supply is eliminated by the fall in the rate of interest which raises foreign wealth and which mitigates the initial fall in domestic wealth.

The results in equations (34)-(35) provide the ingredients necessary for the analysis of the effects of a permanent balanced-budget rise in government spending. In that case $dG_0 = dG \equiv d\bar{G}$, and the impact of this fiscal expansion on the rate of interest can be obtained by adding the expressions in equations (34) and (35). It follows, therefore, that

$$(36) \quad \frac{dR}{d\bar{G}} = \frac{\gamma(1-R)}{\Delta R(1-\gamma R)}(R-\delta)$$

Thus, the effect of a permanent rise in government spending on the rate of interest depends on whether the interest factor, R , exceeds or falls short of the discount factor, δ . The interpretation of this result can be given in terms of the effect of the fiscal spending on the excess demand for current goods or, equivalently, in terms of its effect on world savings. As a result of the permanent rise in fiscal spending, at the prevailing interest rate, both the supply of output net of government absorption as well as the demand for consumption are reduced by the same proportion. Therefore, the excess supply of current goods or, alternatively, the level of savings also change by the same proportion. If the initial level of domestic savings was negative, then the rise in fiscal spending raises the value of savings (by making it less negative) and, thereby, induces a fall in the relative price of present goods, that is, a fall in the

rate of interest. On the other hand, if the initial value of savings was positive the permanent rise in government spending lowers domestic savings and results in a higher rate of interest. These results are analogous to those obtained for the effect of a future budget deficit on the current one-period rate of interest. The link between this explanation and the expression in equation (36) is completed by noting that if $R > \delta$ the initial value of the home country's savings is negative and the economy runs a current account deficit, whereas the opposite holds if $R < \delta$.

The change in the rate of interest alters the values of domestic and foreign wealth and consumption. If the rate of interest rises, then foreign wealth falls and if the rate of interest falls foreign wealth rises. As is obvious the permanent rise in domestic fiscal spending lowers domestic wealth; the fall in wealth occurring at the initial rate of interest is being reinforced or mitigated depending on whether the equilibrium rate of interest rises or falls.

Finally, it is relevant to note that in contrast with the analysis of budget deficits in sections IV and V, here, no qualitative result depends on whether the probability of survival, γ , is smaller from or equal to unity. In fact, the qualitative results of the effect of fiscal spending are identical to those in Frenkel and Razin (1984) where the analysis was conducted under the assumption that $\gamma=1$. In the present section, however, we restricted ourselves only to balanced-budget changes in government spending. This restriction was introduced in order to isolate the impact of spending from the impact of deficits since, when $\gamma < 1$ the Ricardian proposition concerning the irrelevance of budget deficits ceases to apply.

VII. Concluding Remarks

In this paper we analysed the effects of budget deficits and government spending, on world interest rates, and wealth. We examined in detail the nature of the international transmission mechanism. The model that was used embodied the assumptions that world capital markets are fully integrated and that individuals behave rationally on the basis of self-fulfilling expectations. Economic behavior of individuals and governments was assumed to be governed by temporal and intertemporal budget constraints. In order to capture the effects of budget deficits on world equilibrium, we assumed that the probability of survival is less than unity and, therefore, individuals behave as if their horizons was finite. The formulation assured that the model was not subject to the Ricardian proposition according to which budget deficits do not matter. Our analysis demonstrated that the dependence of world rates of interest, wealth, and consumption, on fiscal policies is highly sensitive to the detailed specification of the policies in terms of the level and the time path of government spending and taxes.

The departure from the Ricardian proposition in this paper stems from the difference between the effective interest-factor that individuals use in discounting future taxes and the corresponding market interest-factor that governs governments' behavior. In the present analysis the specific reason responsible for the difference between the two interest factors was that individuals were assumed to be mortal while governments were implicitly assumed to be immortal. In general, of course, the probability that governments and their commitments survive is also less than unity. Under such circumstances the impact of budget deficits are governed by the relation between the survival probabilities of individuals and governments. It is relevant to note that while the formulation in this paper has focused on the survival probability as the reason for the departure from the

Ricardian proposition, the key argument need not hinge on this specific formulation. In general, independent of the reasons for the differences between effective and market interest factors, such differences imply that, within the present framework, budget deficits exert real effects on the world economy.

Rather than summarizing the results it may be instructive to outline some possible extensions. Among such extensions would be (i) a more refined degree of commodity aggregation that would result in possible changes in relative prices and would introduce an additional channel for the international transmission of fiscal policies; (ii) an incorporation of investment which responds to interest-rate changes and which influences the future path of output;⁷ and (iii) a modification of the utility function so as to generate a saving function possessing life-cycle characteristics. Specifically, the possibility that savings respond negatively to the rate of interest would influence the impact of budget deficits on interest rates.

Throughout the analysis in this paper we assumed that the foreign government followed a balanced-budget policy. This assumption was adopted in order to ensure that changes in world rates of interest which result from domestic fiscal policies do not impact on the solvency of the foreign government. If the foreign government does not follow a balanced-budget policy, then interest rate changes would necessitate secondary changes in foreign spending or taxes in order to restore solvency. An extension of the analysis would relax the foreign balanced-budget assumption and would allow for the necessary adjustment of foreign fiscal management. A more major extension would recognize that the interdependencies among the various open economies provide incentives for strategic behavior by

⁷For an analysis emphasizing the role of capital formation see Buiter (1984).

individual countries attempting to exploit monopoly power and to extract maximum seigniorage from debt issue. Such behavior could then be incorporated into a more elaborate game-theoretic framework. Further, the interdependencies and the strategic behavior could result in inefficient outcomes from a global perspective that may call for harmonization of fiscal policies. In such an extended framework, government spending and the timing of taxes would become endogenous variables that are determined in the context of world equilibrium.

APPENDIX

A. The Solution of the System with $\gamma=1$

In this Appendix we examine the properties of the model for the special case in which the probability of survival is unity. In that case, with $\gamma=1$, the market interest factor equals the effective interest factor and, therefore, from equations (22) and (25) in the text, H_0 depends on the path of output net of government spending and is independent of the time pattern of taxes. Furthermore, in that case the equilibrium condition (21) of the text, depends on the initial values of human wealth and on the paths of outputs net of government spending, and is not influenced directly by the values of human wealth in all other periods. Therefore, in that special case, equations (23)-(24) of the text may be left out of the system. The resulting equilibrium conditions are:

$$(A-1) \quad (1-\delta) W_0 + (1-\delta^*) W_0^* = \bar{y}_0$$

$$(A-2) \quad (1-\delta)\delta^t W_0 + (1-\delta^*)\delta^{*t} W_0^* = \alpha_t \bar{y}_t$$

$$(A-3) \quad W_0 = \sum_{t=0}^{\infty} \alpha_t y_t - B_0$$

where

$$W_0 = H_0 + B_{g0} - B_0, W_0^* = H_0^* + B_{g0}^* + B_0$$

$$y_t = Y_t - G_t \quad \text{and} \quad \bar{y}_t = \bar{Y}_t - G_t - G_t^*$$

Equations (A-1)-(A-3) correspond to equations (20)-(22) of the text.

The solution of this system is

$$(A-4) \quad W_0 = \frac{(1-\delta^*) [\bar{y}_0 \sum_{t=0}^{\infty} \delta^{*t} \lambda_t - B_0]}{(1-\delta)\Delta}$$

$$(A-5) \quad W_0^* = \frac{\bar{y}_0 \sum_{t=0}^{\infty} \delta^t \lambda_t^* + B_0}{\Delta}$$

$$(A-6) \quad \alpha_t = \frac{(1-\delta^*) \{ \bar{y}_0 [\delta^{*t} \sum_{\tau=0}^{\infty} \delta^{\tau} \lambda_{\tau}^* + \delta^t \sum_{\tau=0}^{\infty} \delta^{*t} \lambda_{\tau}] + (\delta^{*t} - \delta^t) B_0 \}}{\bar{y}_t \Delta}$$

where $\Delta = [1 - (1-\delta^*) \sum_{t=0}^{\infty} (\delta^{*t} - \delta^t) \lambda_t^*] > 0$, and where λ_t^* denotes the share of foreign product net of government spending, i.e., $\lambda_t^* \equiv y_t^*/\bar{y}_t$ and, λ_t denotes the corresponding share of the home country, i.e., $\lambda_t \equiv (y_t/\bar{y}_t) = 1 - \lambda_t^*$.

As is evident by inspection of equations (A-4)-(A-6), the system conforms with the Ricardian proposition. Thus, given the path of government spending (which governs the paths of y and y^*) the time pattern of taxes and government debt issue is irrelevant for the determination of world equilibrium. These solutions (which are discussed in detail in Frenkel and Razin (1984)), provide answers to questions about the effects of current or future changes in government spending on the paths of world rates of interest and consumption and, thereby, on the paths of the current account and the balance of indebtedness.

B. Derivation of the Impact of Budget Deficits

In this Appendix we derive the formal solutions to the impact of budget deficits on R , W_0 and W_0^* . Differentiating equations (29), (27) after multiplying it by $(1-R)/R$, and (28) yields:

$$\begin{pmatrix} 1-\gamma\delta & 1-\gamma^*\delta^* & 0 \\ \frac{(1-R)\gamma\delta}{R} & \frac{(1-R)\gamma^*\delta^*}{R} & a \\ 1 & 0 & \frac{-\gamma(Y-G)}{(1-\gamma R)^2} \end{pmatrix} \begin{pmatrix} dW_0 \\ dW_0^* \\ dR \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{1-R}{R} \\ -1 \end{pmatrix} \frac{1-\gamma}{1-R} dT_0$$

where

$$a = -\frac{1}{R(1-R)} [\nu(Y-G) + \omega(Y^*-G^*)]$$

$$\nu = \frac{(1-\gamma R)^2 - (1-\gamma)(1-\gamma R^2)}{(1-\gamma R)^2} > 0$$

$$\omega = \frac{(1-\gamma^* R)^2 - (1-\gamma^*)(1-\gamma^* R^2)}{(1-\gamma^* R)^2} > 0$$

The determinant of the matrix is denoted by $-\Delta$. The value of Δ is:

$$\Delta = \frac{(1-\gamma^*\delta^*)}{R(1-R)} [\psi(Y-G) + \omega(Y^*-G^*)] > 0$$

where

$$\psi = \gamma \frac{(1-\gamma R^2) - (1-R)^2 (\gamma\delta - \gamma^*\delta^*)}{(1-\gamma R)^2} > 0$$

The solutions of the system are given in equations (31)-(33) in the text where

$$\theta = \frac{(1-\gamma R)^2 - \gamma(1-R)^2 - (1-\gamma)(1-\gamma R^2)}{(1-\gamma R)^2} > 0$$

REFERENCES

- Bailey, Martin J., National Income and the Price Level, New York: McGraw-Hill, 1962.
- Barro, Robert J., "Are Government Bonds Net Wealth?" Journal of Political Economy 82, No. 6 (November/December 1974): 1095-1117.
- _____, "On the Determination of the Public Debt," Journal of Political Economy 87, No. 5, Part 1 (October 1979): 940-71.
- Buiter, Willem H., "Fiscal Policy in Open Interdependent Economies", Unpublished manuscript, London School of Economics, May 1984.
- Blanchard, Olivier J., "Debt, Deficits and Finite Horizons," Unpublished manuscript, MIT, April 1984.
- Frenkel, Jacob A. and Assaf Razin. "Fiscal Policies, Debt and International Economic Interdependence," National Bureau of Economic Research, Working Paper Series No. 1266, January 1984.
- Diamond, Peter A., "National Debt in a Neoclassical Growth Model," American Economic Review 60, No. 5 (December 1965): 1126-50.
- Fleming, J. Marcus. "Domestic Financial Policies Under Fixed and Floating Exchange Rates," International Monetary Fund Staff Papers 9, (November 1962): 369-79.
- Hall, Robert E., "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," Journal of Political Economy 86, No. 6 (December 1978): 971-87.
- Kydland, Finn E. and Edward C. Prescott. "A Competitive Theory of Fluctuations and the Feasibility and Desirability of Stabilization Policy" in Fischer Stanley (ed.) Rational Expectations and Economic Policy, Chicago: University of Chicago Press, 1980.
- Lucas, Jr., Robert E. and Nancy L. Stokey, "Optimal Fiscal and Monetary Policy in an Economy Without Capital," Journal of Monetary Economics 12, No. 1 (July 1983): 55-93.
- Mundell, Robert A., International Economics, New York: MacMillan, 1968.
- Razin, Assaf and Lars E.O. Svensson. "The Current Account and the Optimal Government Debt," Journal of International Money and Finance 2, No. 3 (August 1983): 215-24.
- Samuelson, Paul A., "An Exact Consumption Loan Model of Interest With or Without the Social Contrivance of Money," Journal of Political Economy 66, No. 6 (December 1958): 467-82.
- Stiglitz, Joseph E., "On the Relevance or Irrelevance of Public Financial Policy: Indexation, Price Rigidities, and Optimal Monetary Policy," National Bureau of Economic Research, Working Paper Series, No. 1106, April 1983.
- Yaari, Menahem E., "Uncertain Lifetime, Life Insurance and the Theory of the Consumer," Review of Economic Studies 32, No. 2 (April 1965): 137-50.

