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CYCLICAL DETERMINANTS OF THE NATURAL LEVEL OF ECONOMIC ACTIVITY

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1. INTRODUCTION

Aggregate unemployment and output data in the United States in the 1970's and early 1980's suggest two basic sort of questions. First, why has the unemployment rate remained so high on average during this period? Second, why has the natural rate of unemployment apparently risen or, analogously, why has the level of potential or full-employment output consistent with physical availability of factors fallen?

Part of the answer is demographic, reflecting sharp increases in labor force participation of certain population groups. The process of absorbing into employment a large number of new workers with little work experience would be expected to raise actual and natural unemployment rates.

One may ask, however, whether the apparent change in natural (that is, full-employment or "potential") levels of economic activity reflect only demographic influences. The simultaneous occurrence of a continued period of low economic activity and of a fall in natural levels of activity (such as a higher natural unemployment rate) suggests that these phenomena may be related. The hypothesis to be explored in this paper is that a period of low economic activity may in fact lower the level of potential output itself.

The dependence, for example, of the natural rate of unemployment on fluctuations in actual unemployment has been suggested by Phelps (1973) and by Tobin (1980), who argued that it was:

> hard to resist or refute the suspicion that the operational NAIRU [non-accelerating-inflation rate of unemployment, i.e. the natural rate] gravitates toward the average rate of unemployment actually experienced. Among the mechanisms which produce that result are improvements in unemployment compensation and other benefits enacted in response to higher unemployment, loss of on-the-job training and employability by the unemployed, defections to the informal and illegal economy, and a slowdown in capital formation as business firms lower their estimates of needed capacity (p.61).

Empirically, Dickens (1982), in considering the productivity slowdown, has found evidence that recessions permanently affect the secular level of labor productivity, a phenomenon similar to that hypothesized here.

Our purpose here is to demonstrate rigorously the workings of one mechanism leading to a dependence of potential levels of output and employment on cyclical fluctuations in economic activity. The argument is based on the loss of training or skills (that is, human capital) which may occur because of a recession. Workers acquire much of their training on the job; skills may depreciate during periods out of the labor force or of unemployment. Therefore, the stock of human capital embodied in the labor force depends on employment histories. If, as seems reasonable, there is a dependence of potential output (positively) or the natural rate of unemployment (negatively) on the human capital stock, these variables will also depend on actual levels of economic activity.

The model presented is a highly stylized one, where all training is firm specific, where the representation of skill accumulation is very simple, and where there is no unemployment per se. One may either be employed or a non-participant (non-employment). This is dictated by our interest in tracing out rigorously how the above-mentioned dependence might operate, rather than in attempting to replicate the specific experience of the United States in the 1970's. Limiting ourselves to fully firm-specific human capital (rather than human capital with general components as well) means that the mechanism outlined might understate the costs of an economic downturn hypothesized here. A richer model of the labor market which explicitly included unemployment would allow us to model an increase in the natural rate due to an economic downturn. As it stands, the model can only be suggestive.

In Drazen (1985), a general multifactor model was presented in which transitory shocks could induce permanent changes in the steady-state composition of the labor force even if factor accumulation was the result of dynamic optimization. Since the optimal composition of heterogeneous factors other than labor was also affected, a continuum of labor force skill compositions was possible in steady state. Transitory downturns could have permanent effects. Here, tractability requires limiting ourselves to a single factor, meaning the optimal skill level of the labor force will be unique. Though transitory shocks will not have permanent effects, the effect of a shock on the skill composition of the labor force and on output will long "outlive" the shock itself. We now turn to a model of how this occurs.

2. THE BASIC FRAMEWORK

We consider a representative competitive firm which takes prices and wages as given. The firm uses one variable input, labor, to produce a single output. Before labor can produce at the firm, it must undergo a training process to acquire skills specific to the firm. We assume skilled labor must be internally generated, so that all training may be seen as on-the-job.

Since training is acquired on the job, the accumulation of skills is a form of learning-by-doing. In the original Arrow (1962) formulation, output per new machine depended positively on cumulative gross investment up to the time the new capital good was built. The analogue for on-the-job training is that the skill level (and, hence, productivity) of a new worker will depend on the length of time he has been on the job. Effective labor units per worker will rise during this training period until, at the end of training, he is fully skilled.

This general case of a smooth learning curve turns out to be intractable here, for it would essentially require either treating trainees at each stage as a different input or keeping track of the employment history of each trainee. We therefore consider a special case in which a worker is totally unproductive until the end of the training period and then becomes fully skilled. There are then only two types of workers, namely skilled and trainees. Suppose it takes k periods to become trained. In steady state, where there are an equal number of workers at each stage of training, the gross addition to the stock of skilled workers would be αT where $\alpha = 1/k$ and T is the total number of trainees. Out of steady state, when T is changing, α will not equal $1/k$. For example, if, due to a downward shock, some workers previously in training were let go, α would exceed $1/k$ on the (reasonable) assumption that workers in an earlier stage of training are let go before those in a later stage. Conversely, if the number of trainees T is rising, α would be below $1/k$. We model this, inexactly, by letting α depend on T negatively. A more exact formulation would have α depending on the time derivative of T, but this formulation does not significantly enrich our results. α would also depend negatively on T if the training process exhibited decreasing returns.

We assume skilled workers retire at rate δ, taken as exogenous to the firm. Denoting the stock of skilled workers by L, the equation for the net change in available skilled workers at the representative firm is (where a dot over a variable represents a time derivative)

$$\dot{L} = \alpha(T) \cdot T - \delta L(t) \qquad (1)$$

where α is between 0 and 1 and $\frac{d\alpha}{dT}(=\alpha')$ is negative. For simplicity, we assume no direct costs of training. There will be indirect costs, as explained below.

Output produced by the firm can then be written $\mu f(L)$ where $f(.)$ is a production function with the standard properties (including the Inada conditions) and μ is a shift factor which may vary over time to represent fluctuations in aggregate economic activity or demand.[1] Output is taken as the numeraire.

The supply of potential workers to the firm is perfectly elastic at wage w, the marginal disutility of leisure. This supply price is assumed not to change over time. Since training is fully firm specific, the entire cost is borne by the firm, with trainees bearing none of the cost. Therefore, trainees receive a wage w even though they produce no output. Since firms bear all costs, they capture all the benefits, so that skilled workers are also paid w (which will in general be less than their marginal product). Firms also decide how many workers will be trained. (Of course, if training had both specific and general components, the allocation of costs between firm and trainee would be more complicated).

The firm's objective is to maximize the present discounted value of profits over an infinite horizon, subject to the initial value of the state variable L and the equation for its evolution, conditional on the expected path of $\mu(t)$. (We assume the firm has point expectations). To solve this problem we define a co-state variable $\pi(t)$ to be the value of an additional unit of skilled labor. We then maximize a current-value Hamiltonian defined by

$$H(t) = \mu(t)f(L(t)) - w(L(t) + T(t)) + \pi(t)[\alpha(T(t))\cdot T(t)-\delta L(t)] \quad (2)$$

subject to $L(0)$. We allow employment of skilled labor to jump down costlessly (skilled workers to be let go). The concavity of $H(t)$ in L ensures that jumps in L can occur only at the beginning of an optimal program. Here, this means only when there is an unforeseen change in $\mu(t)$.

First-order conditions for a solution are then

$$\Pi\alpha(1 + T\alpha'/\alpha) \leq w \qquad \text{with equality if } T > 0 \tag{3}$$

$$\Pi \geq 0 \qquad \text{with a jump in } L \text{ if } \Pi = 0. \tag{4}$$

Equation (3) indicates the relation between the current return to training and the cost. To understand it intuitively, note that if $\alpha' = 0$, (3) indicates that workers will be put into training until the per-period cost of training another worker, namely w, equals the value of a skilled worker, Π, multiplied by the number of workers who become skilled each period for each worker in training, α. That is, T is chosen to equate marginal cost to marginal benefit. When α' is negative, so that increasing T reduces the per-trainee productivity of the training process, the benefit must be adjusted to reflect this, the elasticity of α with respect to T determining the magnitude of this latter effect. Equation (4) says that workers will be let go if, and only if, their present discounted value to the firm is negative.

In addition, the state and co-state variables must obey transition laws. For the state variable, this is given by equation (1). For the co-state variable, we have

$$\dot{\Pi}(t) = (\rho+\delta)\Pi(t) - (\mu f' - w) \tag{5}$$

We may solve for Π to obtain

$$\Pi(t) = \int_t^\infty (\mu(\tau)f'(L(\tau)) - w)e^{-(\rho+\delta)(\tau-t)}d\tau \tag{6}$$

which is simply the present discounted value (modified for "depreciation") of the difference between the marginal product of skilled labor and the wage.

We may also add the transversality condition for $\Pi(t)$, namely

$$\lim_{t\to\infty} \Pi(t)e^{-\rho t} = 0 \tag{7}$$

(The condition that the marginal product of labor goes to zero as L goes to infinity assures L will be bounded.)

Sufficiency is ensured by the concavity of the production function. Equations (1) and (3) through (5) fully describe the optimal employment and training program over time of a representative, profit-maximizing firm.

Before considering the full-time paths in response to a temporary fall in μ, it may be illuminating to examine the optimal steady state. L and Π will be constant. Constant L implies that training will be just sufficient to replace skilled workers who retire. Setting (1) equal to zero, we find

$$\alpha(T)\cdot T = \delta L \tag{8}$$

From (3) we have

$$\Pi\alpha(1-\eta) = w \tag{9}$$

(where $\eta = -\frac{T}{\alpha}\frac{d\alpha}{dT}$), and from (5) we have, when $\dot{\Pi} = 0$,

$$\Pi = \frac{\mu f' - w}{\rho + \delta} \tag{10}$$

Combining (9) and (10), we obtain

$$\frac{w}{\alpha(T)(1-\eta(T))} = \frac{\mu f'(L) - w}{\rho + \delta} \tag{11}$$

Equations (8) and (11) could then be solved for steady-state T and L, given μ, w, ρ, and δ. Note that (11) yields the excess of marginal product over wage for skilled workers in steady-state.

3. DYNAMIC RESPONSE TO SHOCKS

We are now in a position to consider how past economic fluctuations can affect current potential output. We will show that a temporary period of low economic activity, even if perceived as temporary from its onset, will lower potential output well after the downturn is over. The virtue of the formal model of human capital accumulation that it allows us to derive explicitly the time path of behavior in response to a shock, here modelled as a fall in μ. For this we use standard phase diagrams.

To derive phase diagrams, we begin by noting that for given μ, the state of the firm at each point is fully determined by a single state variable L and the associated co-state variable Π. The motion of the system is described by equations (1) for $\dot{L}$ and (5) for $\dot{\Pi}$. Setting $\dot{L}$ equal to zero yields (8) where T is a function of Π and w from (9). To derive the locus of points such that $\dot{L} = 0$ (which we denote LL), we totally differentiate (8) to obtain

$$\frac{d\Pi}{dL}_{LL} = \frac{\delta}{\alpha(1-\eta)T_{\Pi}}. \tag{12}$$

T_{Π} denotes the partial derivative of T with respect to Π, defined implicitly by (9). This derivative will be positive, as long as αT is convex in T. (Note that αT represents the production function for human capital). Positive T_{Π} means that a higher return to training will raise the number of trainees. (12) will therefore be positive, meaning that LL slopes up in Π–L space. When $T = 0$, L must equal zero along LL. This is true at $\Pi = 0$ and for all Π no greater than $w/(\alpha(1-\eta))$ evaluated at $T = 0$. The LL curve may therefore be represented as in Figure 1. L will be falling at points above LL, rising at points below LL.

PUT FIGURE 1 ABOUT HERE

Similarly, setting the other dynamic equation, (5), equal to zero will yield a locus, denoted $\Pi\Pi$, along which Π is constant over time. This is equation (10) above. Differentiating it totally yields

$$\frac{d\Pi}{dL}_{\Pi\Pi} = \frac{\mu}{\rho+\delta}f''(L) \tag{13}$$

which is unambiguously negative, so that $\Pi\Pi$ slopes down in Π–L space. When $\Pi = 0$, $\mu f'(L)$ equals w, which yields a unique value of L. As L approaches zero, $f'(L)$ approaches infinity, so that Π approaches infinity as well. Hence $\Pi\Pi$ may be represented as in Figure 1. To the right of $\Pi\Pi$, Π will be rising; to the left, falling.

The equations of motion imply a saddle-point equilibrium. For any value of L, there is only one value of Π which implies convergence to equilibrium. The stable arm is denoted by the heavy line SS. All other paths diverge. Assuming the transversality condition to be satisfied (equation (7)) implies that the firm is on the convergent path when μ is fixed over an infinite horizon. The steady state is E_0.

What happens when μ shifts? Consider an unanticipated fall in μ which is believed to be permanent. The $\Pi\Pi$ locus shifts to the left, as inspection of (10) indicates. This is the dashed curve in Figure 1. The LL locus will not shift if T is independent of μ. For example, if the supply price of trainees is simply w, so that the cost of training is independent of μ, this will be the case. The LL curve therefore will not shift. The new steady state is E_1 at a lower value of L. The new saddle path is denoted S'S.

If we enriched the model so that the supply price of trainees depended on μ (as would be the case if trainees had to be attracted from another sector in the economy), then LL would shift with changes in μ. This would introduce the possibility that the wages of trainees and skilled workers would differ. If the cost (i.e. the wage) of trainees to the firm fell with a fall in μ, the LL curve would shift left when μ fell. For a given $\Pi\Pi$ curve, this would imply an <u>increase</u> in L in response to a downward aggregate shock. The explanation is straightforward: the fall in μ lowers the opportunity cost of training. One can show that even in this case, the $\Pi\Pi$ curve will shift by more than the LL curve, as long as the wage of skilled workers is less sensitive to changes in μ than is the wage of trainees.[2] Skilled wages are, in fact, less sensitive to the cycle than unskilled wages, as evidenced by the narrowing wages differentials across skills in booms and the widening of wage differentials in downturns. In this case, as in the earlier one, where $\Pi\Pi$ shifts by more than LL, L must fall in the new steady state. The general nature of our results would therefore be the same.

The analysis of an unanticipated fall in μ, which is perceived as permanent after it occurs, is direct. Since the fall in μ is perceived as permanent, the firm's optimal reaction can be represented by a jump to the new saddle path S'S'. There are two cases to consider. If the fall in μ is sufficiently large, Π on S'S' may be negative at the existing L, violating condition (4). In this case, the firm will find it optimal to let some skilled workers go. More precisely, the firm will choose a level of L such that Π on S'S' is just equal to zero. Intuitively, the firm will let skilled workers go as long as their value is negative.

The other case (depicted in Figure 1) is where the fall in μ is not so large that (4) would be violated so that the firm finds it optimal to retain all of its already trained workers. In this case only Π jumps down, to the relevant point on S'S' (for example, from E_0 to A in the Figure). We then move along S'S' to E_1. The fall in Π means the number of trainees hired will fall, the firm adjusting to the shock by not replacing all skilled workers who retire. Starting from E_0, both total employment and the stock of skilled workers at the firm will be permanently lower.

The far more interesting question is what happens in response to a transitory shock, which is perceived as such. Specifically, suppose there is an unanticipated downward shock to μ at time t_0, but that as of t_0 it is anticipated that μ will return to its previous level at time t_0+n and remain there.

The crucial observation is that analyzing a change in the exogenous variables expected to be of fixed, finite duration can be decomposed into two problems, each of which is time autonomous. After t_0+n we have an infinite horizon problem with the original higher value of μ. The firm must therefore be on the path SS at t_0+n and after. Between t_0 and t_0+n, when μ is at the lower value, the motion of the system is described by the $\dot{\Pi}$ and $\dot{L}$ equations for the lower value of μ. Graphically, the lines of motion are those which would apply in the diagram referring to the lower μ (the arrows in Figure 2).

PUT FIGURE 2 ABOUT HERE

One further notes that the firm does <u>not</u> jump to S'S', for this would satisfy the transversality conditions only if μ were permanently lower. The transversality surface for the finite horizon problem (that is, until t_0+n) is clearly SS. An optimizing firm knows that as of t_0+n it must be back on SS. The firm therefore chooses the path consistent either with the given value of $L(t_0)$ (in the case where there is no jump down in L) or with the value of L which yields $\Pi=0$ for the lower μ (in the case where the firm finds it optimal to let go skilled workers) which will just get it to SS at t_0+n. Consider the case of no downward jump in L illustrated in Figure 2. If the firm was at E_0 before t_0 they would jump to a point like B, implying a path BC between t_0, and t_0+n. If they were out of steady state at t_0, at a point such as A, they would jump to G implying a path GH. Which path is chosen clearly depends on n, the length of time the downturn is expected to last. A larger n would imply paths such as DF or IJ.

The complete time paths for L and Π are given by the union of the solutions to the two problems. So, for example, the time path in response to a transitory shock of duration n if the firm starts at E_0 would be BCE_0.

Several basic points become clear when we consider the segment CE_0 and compare C and E_0. Both points represent the position of the representative firm (and hence, in a sense, of the economy) for the same level of μ, but for different histories. Let us suppose that this value of μ corresponds to a high level of economic activity, let's say that consistent with economic variables being at their "potential" or "natural" levels. E_0 may be thought of as the potential economic activity state of the system after a period of high economic activity, while C represents the potential activity state following a period of low activity.

Comparing C and E_0 one notices that L is lower at the first point, so that total output willl, of course, be lower. If the fall in μ induced the firm to let go workers at t_0, this effect would be reinforced. Hence, an economic downturn will depress the level of output well after the downturn is over because of its effects on the skill level of the labor force.[3] Put another way, when output falls below its high activity level, this may lower the natural level of output itself. This general mechanism could be used to determine the short-and long-run effects on output and employment of any time path of shocks (that is, of μ). (It is the ability to find employment and output paths for any path of μ which may justify the highly technical treatment.) We have considered only one simple example to show how the methodology could be applied and to demonstrate theoretically the basic hypothesis of the paper.

Since the only labor force states considered are employment and non-participation, the basic model cannot be applied directly to unemployment. However, the sensitivity of the stock of skilled labor to past economic fluctuations would appear to have important implications about unemployment rates. If there is an inverse relation between the skill level of the labor force and the natural rate of unemployment (as various pieces of evidence seem to support), the basic model would support a dependence of the natural rate of unemployment on past fluctuations. Or, in terms of unemployment, the natural rate of unemployment would depend on fluctuations in actual unemployment, an increase in the actual rate tending to raise the natural rate. This would be in line with the Tobin supposition quoted at the beginning of the paper. It would also be consistent with the empirical evidence on actual and natural rates in the 1970's cited at the beginning of the paper. Of course, to demonstrate this formally, one would need to incorporate unemployment into the model. This would require a significant extension of the framework, perhaps by including an unskilled sector in which unskilled workers have higher unemployment propensities because of a higher probability of layoff in a downturn or longer search time (in the case where training had a non-firm-specific component).

4. CONCLUSIONS

The proposition that cyclical fluctuations may affect the level of potential output or the natural rate of unemployment strikes many as surprising. When one considers the mechanisms by which this may work, it should seem more plausible. Given the importance of on-the-job training in human capital accumulation and the obvious necessity of being employed to receive such training, past employment should theoretically be important in determining current skill levels. This, in turn, would affect potential output and, conceivably, the natural rate of unemployment. Though 'natural' and actual levels of economic variables cannot diverge permanently, periods of high economic activity may raise output and lower employment over the longer term. Therefore, even if one accepts the natural rate hypothesis, short-run countercyclical policy may be useful in terms of longer run effects.

Of course, any policy conclusions are tentative. What has been called the new classical macroeconomics presents a strong argument against the indiscriminate use of countercyclical, macroeconomic policy. The micro-based resulted presented here indicate that the case for or against policy needs careful rethinking. A goal of this paper was not to prejudge the conclusion, but simply to encourage such a reexamination.

FOOTNOTES

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1. One possible interpretation is that when aggregate demand is low, marketing a given quantity of output is costly, so that μ represents the ratio of revenue received from production, net of marketing costs, to actual output produced.

2. Let us denote the wage received by skilled workers as w^s, that by trainees as w^t. The horizonal shift in the $\Pi\Pi$ curve, found by differentiating (10) by μ for given L is

$$\frac{d\Pi}{d\mu} = \frac{1}{\rho+\delta}\left(f' - \frac{dw^s}{d\mu}\right)$$

or, as an elasticity

$$\left.\frac{d\Pi}{d\mu} \cdot \frac{\mu}{\Pi}\right|_{\Pi\Pi} = \frac{\mu f' - w^s\left(\frac{\mu}{w^s}\frac{dw^s}{d\mu}\right)}{\mu f' - w^s} .$$

To find the horizontal shift in the LL curve, we note from (8) that if L is to be constant along LL, T must be constant. From (9), where w is replaced by w^t, this requires that w^t and Π change proportionally in response to changes in μ, so that

$$\left.\frac{d\Pi}{d\mu}\frac{\mu}{\Pi}\right|_{LL} = \frac{dw^t}{d\mu} \cdot \frac{\mu}{w^t} .$$

Comparing these two expressions, we see that a sufficient condition for $\left.\frac{d\Pi}{d\mu}\frac{\mu}{\Pi}\right|_{\Pi\Pi} > \left.\frac{d\Pi}{d\mu}\frac{\mu}{\Pi}\right|_{LL}$ (that is, that the horizontal shift in $\Pi\Pi$ is greater than in LL) is that $\frac{dw^t}{d\mu} \cdot \frac{\mu}{w^t} > \frac{dw^s}{d\mu} \cdot \frac{\mu}{w^s}$.

3. These results do require αT to depend on T, which, for example would not be true if $\alpha = 1/T$. In this latter case, equation (1) is independent of Π, and L would be invariant to shocks if we started in steady state. Technically, LL would be horizontal.

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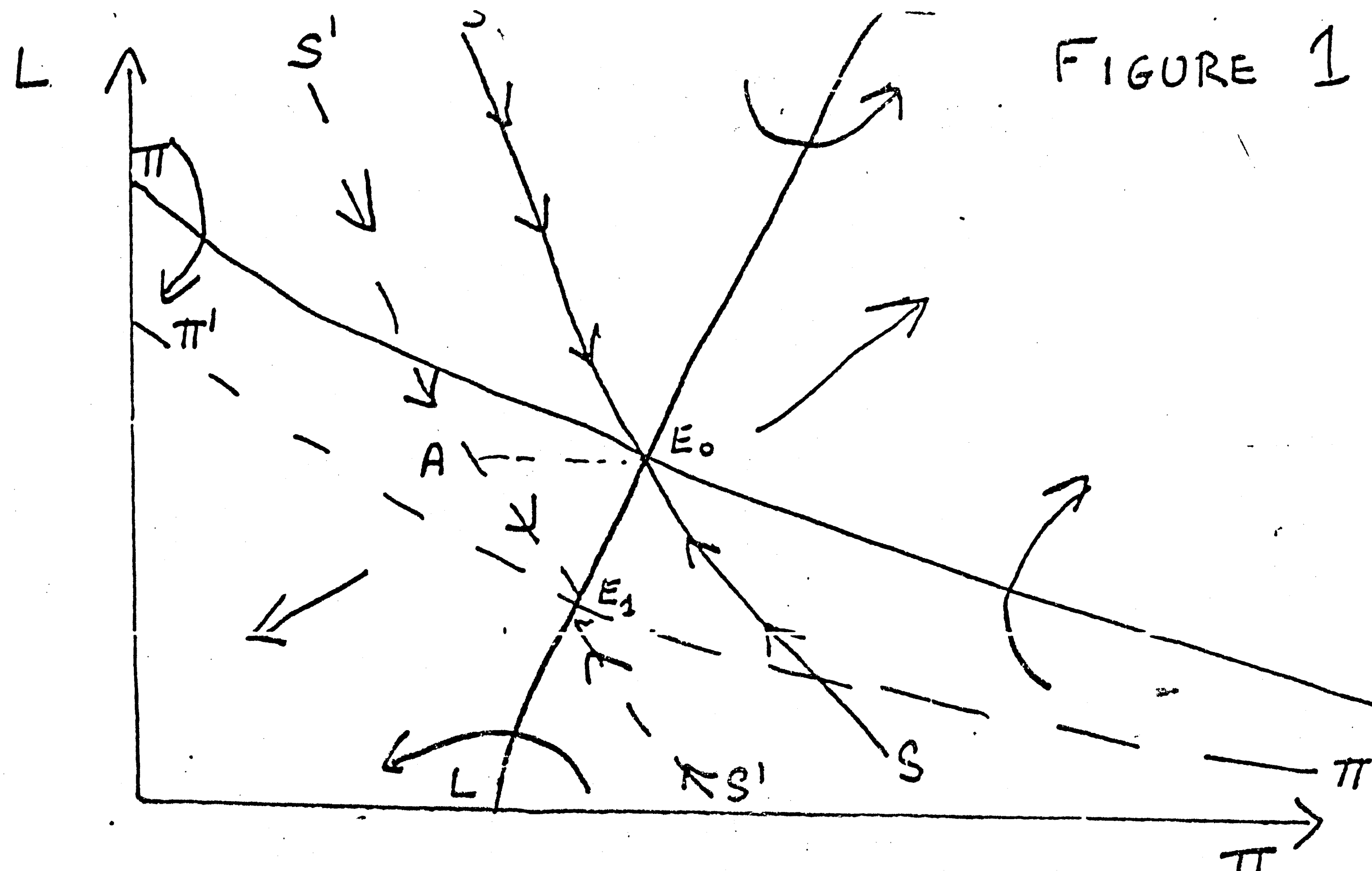

FIGURE 1

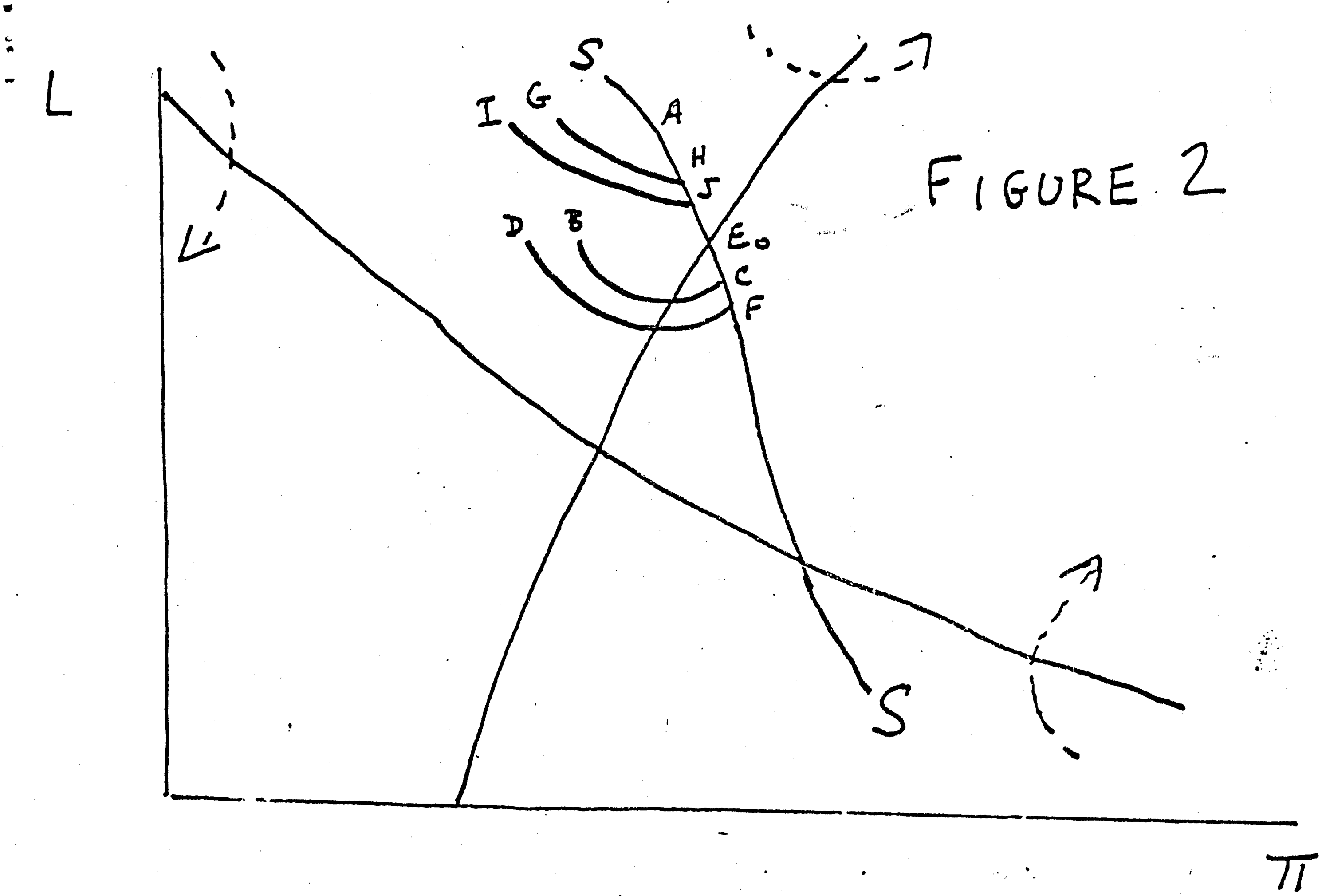

FIGURE 2