COTTON AND WHEAT PRODUCTION IN THE EGYPTIAN AGRICULTURE;  
AN ECONOMETRIC ESTIMATION AND INTERPRETATION

By

Zvi Eckstein

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FOERDER INSTITUTE FOR ECONOMIC RESEARCH
Faculty of Social Sciences, Tel-Aviv University
Ramat Aviv, I s r a e l.
1. **INTRODUCTION**

In Egypt, as in all developing countries, agriculture is a key sector of the economy. This paper examines historical data on the production, yield, and prices of the two most important crops of Egyptian agriculture — cotton and wheat, in order to investigate the dynamic aspect of agricultural supply.

Following a brief description of the Egyptian economy and the main characteristics of its agricultural technology (Section 2), econometric time series techniques are used in order to analyze the aggregate data (Section 3). The estimation results indicate interesting oscillations in cotton and wheat as a response to an unexpected shift from the steady state. In Section 4, a simple dynamic optimization model that can interpret this result as a rational outcome is proposed: In earlier articles (Eckstein, 1981; 1984) this model has been fully described and estimated; in Section 5 of the present work it is tested by estimating the cotton dynamic production function using data from 1895-1976. The final section contains conclusions and some general remarks about future research.

2. **BACKGROUND**

Egypt is one of the poorest countries in the world, where high population growth is combined with a low rate of increase in real GDP. The per capita GDP is currently about 400 U.S. dollars, and the average growth rates (over the last 30 years) of population and real GDP are 2.4 and 4.6, respectively.
The agricultural sector currently accounts for 45 percent of total employment, 28 percent of GDP, and 60 percent of export earnings.\(^1\)

The most significant commodity in the agricultural sector is cotton. It was introduced to Egyptian peasants in the early 19th century, and Egypt gradually became a one-crop exporter, heavily sensitive to and integrated in the world market. A rapid increase in cotton production and exports from 1865 to 1914 represented a significant change in the traditional patterns of crop production; this was the primary reason for a period of growth and prosperity in Egypt at that time. During the intrawar period (1919-1939), the estimated growth of aggregate production and of agricultural production were lower than the growth in population. Immediately after the Second World War, cotton and aggregate production and export revenues increased as a result of the return to normal trade conditions, and later, the Korean Boom. Government policy after the 1952 revolution gradually turned the country toward a centralized planned economy. Among the most significant actions of the new regime with respect to the agricultural sector were: major land reforms in 1952 and 1964 which reduced the average acreage per farm; investment in the Aswan High Dam which was to improve the water supply and production of energy; the nationalization of trade in 1964, setting product prices, and area quotas in the late 1960s and the establishment of the collective farming structure in the villages. Egypt from 1967-1968 on can be considered a centralized planning economy.

Egyptian agriculture depends almost entirely on Nile water. Control of the irregularities in the Nile water supply, both seasonal fluctuations and large annual variations, has been regarded as the main concern of long-term development of the agricultural sector. Most of the public investment in
agriculture has been devoted to improvement of irrigation systems and the control of the flow of water in the Nile. The Aswan High dam is the most recent and most well-known project. Eighty percent of the annual water supply in the Nile passes through the Aswan High Dam during the period of mid-July to December. The control of the water supply during the year is important for the production of both cotton and wheat, since their long growing periods require constant irrigation in months when the natural water flow is low.

The ratio of crop area over cultivated area is probably the most significant indication for the movement toward continuous crop cultivation. As shown in Table 1, the greatest increase in this ratio occurred during the last quarter of the last century and the first of this century.2

| Table No. 1 About Here |

When land is under continuous cultivation, the issue of substitution and complementary effects in production of different crops become very important. For instance, cotton tends to deplete the nitrates from the soil, while clover builds the nitrate content of the land. In addition, clover is the major animal feed and can be produced in short periods. Various crops, and cotton, in particular, tend to build up the population of crop-specific insects and worms that may seriously affect output. Furthermore, the dates of sowing and harvesting impose several substitution and complementarity relationships between crops.
The Egyptian peasants are well aware of the above-mentioned characteristics of the technology and the advantages of crop rotation in order to maximize their profits. Rotation enables them to decrease the deterioration of land productivity due to cotton production and to cultivate the land continuously over the year. Hansen and Nashashibi (1974) described two-year and three-year traditional rotation systems, but mention that in practice farmers have deviated and changed the rotation over time.

This study emphasizes the role of the deterioration of land productivity in cotton production and the prices of cotton and wheat on the optimal allocation of land between cotton and wheat. In the 1920s cotton and wheat together shared about 60 percent of the cultivated area, and in the 1960s, about 50 percent. Recently their share in the cultivated area has been reduced to 40 percent. The production of cotton versus wheat has been a major issue for Egyptian agricultural policymakers for many years. Wheat is the main food product for the rapidly growing poor population. Self-sufficiency of food production has been regarded as an important objective. Over this century, the imports of wheat have generally increased gradually, but during the last decade (especially 1970-76), imports of wheat increased by a striking 300 percent.
Figures 1, 2, and 3 depict the aggregate time series observations on yield, land allocation, and relative prices of cotton and wheat in Egypt over the past 60 to 80 years. The observations on cotton area show frequent sharp fluctuations since 1912. At some points of time it is clear that wheat area increases as cotton area decreases. Although cotton yields have fluctuated, the average has stayed almost the same over 80 years. Wheat yields began to increase in 1969, but overall this trend is very low. The average acreage for both crops stayed almost the same over the last eight decades, as did the relative price of the crops and the ratio of the income per unit of area (Figure 3).

Hence, the figures depict a stationary oscillation process for total land allocations, land productivity, and the relative prices of cotton and wheat. In light of these observations, one can ask how economic theory, based on the presumption of optimizing agents can explain the main characteristics of these time series data. In particular, what are the possible sources of the frequent fluctuations in cotton land allocations?

3. AN ECONOMETRIC ANALYSIS OF THE DATA

In this section an econometric model for time series observations is presented that does not require many assumptions on the underlying economic and technological structure of the agricultural sector, but summarizes the data in a useful way for agriculture supply analysis. Specifically, annual
time series data on production, crop areas, and crop prices of the Egyptian agricultural sector are used with the same type of model to analyze the actual joint dynamic movement of crop prices, land allocations, production, and yields. In particular, the response of land allocations to shocks in land allocations is estimated.

The Egyptian agricultural production, as described above, involves many dynamic elements, where crop rotation is probably the most significant result. These dynamic elements suggest that land allocation and output observations should be correlated over time. Assuming that prices are correlated over time due to dynamic elements in world markets as well as in the Egyptian market, we would expect that land allocation and production would be correlated with past prices, since they contain information about future prices. The argument here is that if farmers try to behave optimally, then, in a dynamic environment, land allocation and production will be influenced by predictions of future variables such as prices. Further, we do not rule out the possibility that production or land allocation affects the market prices; that is, there may be a feedback on prices from the existence of a downward sloping demand curve for the output.

3.1 Methodology

Let \( X_t \) be an \( n \times 1 \) vector of outputs and/or land allocations, and let \( P_t \) be an \( m \times 1 \) vector of the relevant relative prices. If we subtract the deterministic parts, such as the constant and the trend, then we can define the vector \( Y_t = [\tilde{X}_t, \tilde{P}_t]' \), where \( \tilde{X}_t \) and \( \tilde{P}_t \) are the nondeterministic parts of \( X_t \) and \( P_t \), respectively.
Denote by \( \hat{Y}_t \) the best linear predictor of \( Y_t \) based on \( Y_s, s < t \). Assume that \( \hat{Y}_t \) can be well approximated by a linear combination of a finite number of past \( Y \)'s, with the weight on \( Y_{t-s} \) dependent on \( s \), not on \( t \).

The innovation in \( Y_t \) is \( U_t = Y_t - \hat{Y}_t \), and because \( U_t \) is itself a linear combination of current and past values of \( Y_t \) for all \( t \), \( U_t \) is serially uncorrelated. Assuming that \( U_t \) has a finite variance, we have the following linear vector regression equation:

\[
Y_t = \sum_{s=1}^{q} A_s Y_{t-s} + U_t
\]

(1) is called the vector autoregressive representation (VAR) of the vector \( Y \), where \( A_s \) is an \( (n+m) \times (n+m) \) matrix of parameters. The usual distribution theory applies to least-squares estimation of the \( A \)'s in this equation asymptotically, if it is assumed that \( \hat{Y} \) is an exact function of only \( q \) past \( Y \)'s.

The model regards all of the systems variables as being endogenous, and Zellner's seemingly unrelated regressions method is used in estimating the \( A \)'s. The lag length of the VAR is initially unspecified, and is determined by asymptotic \( x^2 \) statistic test on alternative lag lengths fitted to the model.

Once the \( A \)'s in (1) are estimated, \( Y_t \) can be expressed as a linear combination of current and past innovations \( (U \)'s), that is, as a distributed lag on \( U_t \). Then the moving average representation (MAR) for \( Y_t \) can be written
(2) \[ Y_t = \mathcal{E}_{s=0}^\infty B_s U_{t-s}, \]

where \( B_s \) is an \((n+m) \times (n+m)\) matrix of parameters.

Simulating (1) by setting \( U_{jt} = 1 \) and \( U_{t+s} = 0 \) for all \( s = 1, 2, 3, \ldots \), together with the initial conditions \( Y_{t-r} = 0 \) for \( r \leq 0 \), we get vectors of responses of all the variables in \( Y \) for as many periods in the future as we wish. It turns out that the \( j \)'s column of the corresponding \( B_s \) matrix is equal to the \( s \) period ahead simulated vector for \( Y \).

Therefore, one can regard the \( i, j \)'th component of \( B_s \), \( b_{ij}(s) \), as the "average" response, \( s \) period ahead, of the \( i \)'th variable, to an initial shock in the \( j \)'th variable. The sum of the \( b_{ij} \)'s, over all \( s \), gives rise to the contribution of the variable \( j \) to the variance of the variable \( i \) or, alternatively, the forecasting error variance due to shock in the \( j \)'th variable. Hence, we can decompose the forecasting error variance of each variable due to the shock in other variables.

3.2. Results

Two systems of vector autoregressions (VAR) have been estimated using the data from the Egyptian agricultural sector. 4

**System 1:** Cotton lint price (COT-P) over wheat price (WT-P), cotton area (COT-AR), wheat area (WT-AR), cotton lint yield (COT-YLD), wheat yield (WT-YLD),
where COT—P and WT—P are prices of cotton and wheat during the previous year.

**System 2:** COT—P over corn price (CR—P), WT—P over CR—P, COT—AR, WT—AR, corn area (CR—AR).

The main reasons for using relative prices of crops are a) we want to eliminate the effects of general inflation and b) we do not have reasonable data on price indices. To see whether the choice of numeraire has a significant effect on the results, System 2 has been compared to a system where WT—P is the numeraire. The results are the same. Furthermore, System 1 uses WT—P and System 2 uses CR—P as numeraires. An alternative specification to System 1 has been estimated using cotton and wheat production levels instead of cotton and wheat yields. The results of this system are almost identical to those of System 1.

The asymptotic $\chi^2$ tests for the lag length reject specifications with less than five lags (See Eckstein, 1981). In order to test for non-Granger (1969) causality from areas and yields to the relative prices, F-tests are used for the separate equations. In System 2 the exclusion of lagged areas from COT—P over WT—P equation has $F(15,24) = 0.92$ with marginal significance level of 0.55, and the exclusion of lagged areas from WT—P over CR—P equation has $F(15,24) = 1.00$ with marginal significance level of 0.49. Hence, the hypothesis that lagged areas do not affect current prices is not rejected, and the exogeneity assumption on prices with respect to area decisions is supported.

In System 1, the test for the exclusions of lagged COT—AR, WT—AR, COT—YLD and WT—YLD have F values of 0.94, 1.17, 1.33, and 2.16 with significance levels of 0.47, 0.35, 0.28, and 0.09, respectively. Hence, the hypothesis of non-Granger (1969) causality from areas and yields on prices is not rejected.
Table 2 summarizes the results of 15 years ahead forecast error variance decomposed according to shocks in the variables of the different systems, as explained above.

Table No. 2 About Here

In System 2, prices account for 36-37 percent of the forecast error variance in land allocations. In System 1, prices account for 15-17 and 9-15 percent of the forecast error variance in land allocations and yields, respectively.

In both systems, the shock in any variable accounts for most of the variance error in the same variable. The shocks in prices are the second-most important factor in accounting for the variance error of land allocations.

These results support the claim that farmers in Egypt do respond to prices in making their decisions.

In System 2, land allocations account for less than 30 percent of variance error in prices, and COT-AR accounts for more than one half of it. However, in System 1, WT-AR is the prime factor in accounting for the variance error of prices. Land allocation and yield amount to 55 percent of forecast error in prices. Notice that the F-test for excluding lagged WT-AR from the price equation in System 1 has a low significance level and its contribution for forecast error variance in prices is relatively high.

The results of the estimated forecast error do not support the exogeneity of prices, and they indicate that the F-test support of the null hypothesis is due to a high variance of the estimated coefficients.

The MAR ($B_2$) coefficients in both systems converge to zero. Thus, the systems seem to be stationary. An interesting phenomenon that can be
observed from the MAR of both systems is the opposite responses of COT–AR and WT–AR to innovations in any variable (that is, when COT–AR increases, WT–AR decreases, and the frequent oscillation of both (see Figure 4).

Figures 4a and 4b show this result for innovations in COT–P over WT–P and in COT–AR for System 1. The positive (negative) one-step-ahead response of COT–AR (WT–AR) to an innovation in the relative price is as we can expect for almost any product. However the second step is a sharp decrease (increase) in COT–AR (WT–AR), and the third an increase, etc. Then the oscillations become less frequent. It turns out that this phenomenon exists in all of the estimated VAR's and in response to almost any variable.

Why do cotton and wheat land allocations respond to a positive shock in their relative price with frequent oscillations, that is an immediate positive (negative) response, and in the next year a negative (positive) response, and so forth? Most existing dynamic economic models predict that after a shock to the system, economic agents gradually move back to the "equilibrium" allocation. This is precisely the assumption in the traditional agricultural supply response model. Hence, the above phenomenon contradicts the presumption of the early supply response model as described by Nerlove (1958, p.61).

Under what conditions and why can we expect to get this cyclical phenomenon? Can this result be explained by rational optimizing behavior, or should we look to the Cobweb theorem for explanation? Is it the form of expectations or the technology that accounts for these observations?
Furthermore, should the government consider policies to stabilize crop prices? The model presented in the following section provides a simple, formal, and explicit rationale for the above observed phenomenon and some answers to the above questions.

4. A DYNAMIC MODEL

The model presented here fits well Schultz's (1975) description of farming sector within a traditional agricultural setting, that is, the farmers' choice of production "embodies a fine regard for marginal costs and returns." For simplicity, the model considers a representative farmer whose only factor of production is land.

Consider the definitions of the following variables:

- \( X_{it} \) is the production of crop at time \( t \),
- \( P_{it} \) is the price of crop \( i \) at time \( t \),
- \( A_{it} \) is the land allocated to crop \( i \) at time \( t \),
- \( \overline{A}_t \) is the total cultivated land that is available at time \( t \),
- \( 0 < \beta < 1 \) is the objective discount factor,
- \( a_{it} \) is the shock to production of crop \( i \) at time \( t \),
- \( f_1, f_2, d_0, d_1 \) are positive parameters of the production functions,
- \( E \) is the mathematical expectation operator, where \( E_t(X) = E(X | I_t) \),
- \( I_t \) is the information set at time \( t \), and
- \( L \) is the lag operator, which is defined by the property \( L^k x_t = x_{t-k} \).
The farmer is assumed to maximize his discounted expected profit in terms of the price of crop 1 (cotton). Hence, the farmer's objective is to maximize

\[ E_0 \sum_{t=0}^{\infty} \beta^t (X_{1t} + \frac{p_{2t}}{p_{1t}} X_{2t}). \]

The maximization is subject to three technological constraints:

- Land constraint
  \[ A_{1t} + A_{2t} = \bar{A}. \]
  
- The production function of crop 1
  \[ X_{1t} = ((f_1 + a_{1t}) - \frac{d_0}{2} A_{1t} + d_1(\bar{A} - A_{1t-1}))A_{1t}. \]
  
- The production function of crop 2 (wheat)
  \[ X_{2t} = (f_2 + a_{2t})A_{2t}. \]

The production function of crop 1 (5) includes a linear shock to productivity, \( a_{1t} \), which is uncontrollable and random, and a dynamic term, \( d_1(\bar{A} - A_{1t-1}) \), which is meant to approximate the deterioration of productivity due to successive cultivation of cotton on the land. The last term implies that, on the average, land productivity at time \( t \) increases proportionately to the quantity of current soil which has not been used for crop 1 in the previous period.

The deterioration of land yield is due to exhaustion of the soil and accumulation of crop-specific insects and worms, arising from growing the same crop in successive periods. This element is captured by the term \(-d_1 A_{1t-1} \) in (5), but if more land is available for cultivation (\( \bar{A} \) is
increasing), farmers are more flexible and can avoid deterioration of the yield. Notice that this term introduces a dynamic element into the production function. In the following it is shown that a positive $d_1$ gives rise to a land allocation process that can be regarded as crop rotation. This is a well-known practice in agriculture where land deterioration in some crops is severe.

On the other hand, a negative $d_1$ implies that it is better to use the same land for the same crop and, therefore, implies an adjustment cost argument for dynamic behavior (Eckstein, 1984).

Substituting (4)-(6) into (3), the farmer's problem becomes: Maximize

$$J = E_{\tau=0}^{\infty} \sum_t \left( (f_1 + a_{1t})A_{1t} - \frac{d_0}{2} A_{1t}^2 + d_1 (\bar{A} - A_{1t-1}) A_{1t} - R_t A_{1t} + R_t \bar{A} \right)$$

by choice of $A_{10}, A_{11}, A_{12}, \ldots$, where $R_t = \frac{1}{P_{1t}} (P_{2t} (f_2 + a_{2t}))$ is the "real shadow price" for crop 1 land allocations, and $I_t$ is the farmer's information set at time $t$ which is assumed to include all past realizations of the variables in the model.

The optimization is subject to a given level of $A_{1-1}$ and the given stochastic processes of $a_{1t}$ and $R_t$. In Eckstein (1981, 1984) the dynamic demand for $A_{1t}$ and the land allocation decision rules are analytically solved for a general form of the exogenous stochastic processes. Here the role of the model in interpreting the results of the previous section is demonstrated by using a numerical example. Particular
emphasis is placed on the role of the dynamic aspect of the production function, \( d_1 \), on the cyclical movements of land allocations.

Consider, for example, the following values of the parameters:

\[ d_0 = .25, \ \beta = .9, \ f_1 = 20 \ \text{and} \ \bar{A} = 80 \]

for all \( t = 1, 2, \ldots \)

and let the stochastic processes of \( R_t \) and \( a_{lt} \) be:

\[
\begin{align*}
R_t &= 5 + .5R_{t-1} + U_t^R \\
a_{lt} &= .4a_{lt-1} + U_t^a
\end{align*}
\]

For the above specification we consider two cases.

**Case 1:** \( d_1 \geq .1 \), then \( \lambda_1 = -.48 \).

Hence, the decision rule is:

\[ A_{lt} = -.48A_{lt-1} + 79.0 - 1.97R_{t-1} + 1.63a_{lt-1}. \]

**Case 2:** \( d_1 \leq -.1 \), then \( \lambda_1 = .48 \).

Hence, the decision rule is:

\[ A_{lt} = .48A_{lt-1} + 49.0 - 3.08R_{t-1} + 2.33a_{lt-1}, \]

where \( \lambda_1 \) is the smaller root in absolute value that solves

\[
\frac{1}{\lambda_1} = - \frac{d_0}{d_1} - \beta \lambda_1.
\]
The above decision rules demonstrate the implications of the sign of \( d_1 \). The responses to prices and to shocks in productivity are much higher in Case 2 than in Case 1 and the first-order serial correlation in \( A_1 \) is negative (positive) for \( d_1 \) positive (negative).

I have chosen the constants in the example so as to have a similar mean for the land allocations of the two cases. If \( R_t \) and \( a_{lt} \) are constants then the sign of the root \( \lambda_1 \) does not change the variance of the decision rule, but does change the frequencies in which the land allocations cross their mean.

Assuming that no shocks to productivity are observed, the above decision rules can be written as regression equations by operating on both sides of the decision rules by \((1 - .4L)\).

The regression equation for Case 1 is

\[
A_{1t} = -.08A_{1t-1} + 1.92A_{1t-2} + 47 - 1.97x_{t-1} + .79R_{t-2} + 1.63u_{t-1}.
\]

The regression equation for Case 2 is

\[
A_{1t} = .88A_{1t-1} - 1.92A_{1t-2} + 30 - 3.08R_{t-1} + 1.23R_{t-2} + 2.33u_{t-1}.
\]

Assuming that \( u_{t}^a \) and \( u_{t}^R \) are distributed normally, with mean zero and variance of one, I stimulate the model for 100 observations, given the
same initial value of $A_1$'s and $R_S$. The results are illustrated in Figure 5. The line with symbols represents Case 2, and the other, Case 1.

The mean in Case 1 is 40.2 and the variance is 4.8, while the mean in Case 2 is 39.6 and the variance is 29.0. This fact is explained by the difference in the responses to changes in prices and shocks to productivity. When the shocks to productivity are ignored, this result is retained. However, in Case 1 the land allocations cross the mean more frequently than in Case 2; this is explained by the difference in the sign of $\lambda_1$.

In order to demonstrate the use of this example in interpreting the Egyptian data, the MAR of the implied VAR of the two cases is computed. The VAR's are the joint processes of (10) and (8) for Case 1 and (11) and (8) for Case 2. The responses of the area to shocks in the area equation are given in Figure 6 (the responses to innovation in the price reveal the same pattern).

For Case 1, where we have deterioration ($d_1 > 0$), the area shows frequent oscillations and for Case 2, where we have adjustment cost ($d_1 < 0$), the first response is higher and there is a smooth convergence to the mean. Further, the one-step-ahead response to the once-but-not-for-all shock in prices is higher in Case 2 and is consistent with the response to the once-and-for-all shift in prices.
The reverse oscillations of wheat in the example are almost trivial since total cultivated land ($A$) is a constant. However, if $A$ is a stochastic process and $d_1 > 0$, the phenomenon can be captured from the data.

The above dynamic model implies that the dynamic technological constraints on Egyptian cotton production may account for the cyclical patterns of the cotton and wheat land allocations. In the next section a particular statistical test of this hypothesis is suggested.

5. ESTIMATION AND TEST OF THE COTTON PRODUCTION FUNCTION

Attempts to estimate the land allocation decision rule jointly with the exogenous processes were reported in Eckstein (1981, 1984). The results give some support to the model but some of the assumptions were rejected by the statistical tests. Here, I report an econometric test of the dynamic cotton production function. As explained above, the hypothesis of this work is that the dynamic aspects of the cotton technology account for most of the dynamics observed in land allocations and production.

Notice that in the present model, even if current land allocation stays constant over time, the average product according to equation (5) is not constant. In particular, it decreases and oscillates in response to the shocks. However, this phenomenon does not necessarily indicate a technological change according to our definition. On the other hand, it is interesting to test the hypothesis that the cotton production technology, as it is expressed by equation (5), has not been changed during the sample period, 1896-1976.

The production function (5) can be written as:
(12) \[ Y_{lt} = f_1 + d\bar{A}_t - \frac{d_0}{2} A_{lt} - d_1 A_{lt-1} + a_{lt} \]

where it is assumed that \( \bar{A} \) can change over time, and that \( a_{lt} \) has a first-order Markov process. Thus,

(13) \[ a_{lt} = a + \rho a_{lt-1} + U^a_t \]

and

\[ E(U^a_t U^a_{t-j}) = 0 \quad \text{for all} \quad j = 0 \]

\[ E(U^a_t) = \nu_a > 0 \]

\[ E(U^a_t^2) = 0. \]

If \( a = 0 \), (12) seems to have all the properties of a regression equation with serially correlated errors and no lag-dependent variables. However, the right-hand-side variables in (12) are random and the minimum requirement for (12) to be a regression equation is that \( E[a_{lt} A_t, A_{lt}, A_{lt-1}] = 0 \). This condition is not satisfied for (12) since \( a_{lt} \) is correlated with \( a_{lt-1} \) and so is \( A_{lt} \) from the optimal decision rule (e.g., equation (10)). Therefore, the GLS procedure is not correct, since the initial estimates of \( a_{lt} \) are not consistent.

Observe that \( a_{lt} = a/1 - \rho + U^a_t / (1 - \rho L) \); substituting that in (12) and operating by \((1-\rho L)\), we get the following equation:

(14) \[ Y_{lt} = \rho Y_{lt-1} + (\bar{d} - \rho \bar{d}L)\bar{A}_t - [\frac{d_0}{2} - (\rho - \frac{d_0}{2} - d_1)L - d_1 \rho L^2] \]

\[ \cdot A_{lt} + (1 - \rho)f_1 + a + U^a_t. \]

\( U^a_t \) has been assumed to be serially uncorrelated and orthogonal to the stochastic processes of \( \bar{A}_t \) and \( R_t \). \( U^a_t \) is not observed at time \( t \);
hence, it is orthogonal to current and past land allocation, current and past total cultivated land, and past $Y_{1t}$'s, since all these variables can be expressed as linear combinations of past $U_t$'s. Therefore, the following orthogonality condition holds for (14).

\[(15) \quad E(U_t^\delta Y_{1t-1}, Y_{1t-2}, \ldots, A_{1t}, \bar{A}_{t-1}\ldots, A_t, \bar{A}_{t-1}) = 0.\]

Condition (15), together with some mild assumptions on $Y_{1t}, A_{1t},$ and $\bar{A}_t$ give rise to consistent estimates of the production parameters by applying ordinary least squares (OLS) to (14).

The constant parameters in (14), $(f_1$ and $a)$, cannot be identified separately, and are of no major interest in this study. The other parameters, $d_0, d_1, \bar{d},$ and $\rho$ can be estimated efficiently by using a two-step method.

**Step 1**: Apply OLS to (14) to get initial consistent estimates of the parameters.

**Step 2**: Take a Taylor expansion of the non-linear part in (14) around the estimates of Step 1, and apply OLS to the transformed equation. 

Let $\rho^*, d_0^*, d_1^*, \bar{d}^*, (\rho d_0)^*, (\rho d_1)^*, (\rho \bar{d})^*$ be the OLS estimates for Step 1. Then the Taylor expansion of (14) is:

\[
Y_{1t} + \frac{(d_0 \rho)^*}{2} A_{1t-1} + (d_1 \rho)^* A_{1t-2} - (\bar{d} \rho)^* \bar{A}_t - (1-\rho)f_1
\]

\[
+ a + \rho[Y_{1t-1} + \frac{d_0^*}{2} A_{1t-1} + d_1^* A_{1t-2} - \bar{d}^* \bar{A}_{t-1}]
\]

\[
- \frac{d_0^* [A_{1t} - \rho^* A_{1t-1}]}{2} - d_1^* [A_{1t-1} - \rho^* A_{1t-2}]
\]

\[
+ \bar{d} (\bar{A}_t - \rho^* \bar{A}_{t-1}) + U_t^a
\]

\[(16)\]
From Step 1 there are many consistent estimates and we are free to choose one that is consistent with the model's assumptions.

Table 3 summarizes the results for the sample period 1895-1976 as well as for two sample sub-sample periods: pre-and post-World War II.

We use the second-step estimates as the asymptotically efficient estimates for testing the null hypothesis of no structural change in the production function between the two periods. Using the resulting SSR from the second step (see Table 2), we see that the F(9,66) statistic for this test is equal to 1.37, which implies a marginal significance level greater than .01. Thus, the hypothesis that the cotton production function in Egypt stayed the same from 1895 until 1976 is not rejected.

The estimates of the production parameters using the entire sample are consistent with the model. \( \rho = .6 \) implies that the stochastic process for the productivity shocks is stationary, \( d_0, d_1, \bar{d} \) are positive and \( d_0/d_1 > 2 \) holds. This last result supports a restriction on the model of Section 3 for the existence of a land allocation decision rule. The confidence that \( d_1 > 0 \) is low; however, the contemporaneous decreasing effect on the yield (\( d_0 \)) turns out to be significant.

The assumptions of land fertility deterioration (\( d > 0 \)), as well as the existence of one production function for the sample period are supported. If one accepts the approach of dynamic rational optimization as a way of describing economic outcomes, then the model of this paper provides one explanation that is based on a dynamic production function that fits the data very well.
However, it should be emphasized that a much stronger test of the model is an outcome of estimating the implied decision rule from the model of Section 4. The attempts made in Eckstein (1981, 1984) gave only a mild support to this model and much more work is necessary before the approach can be fully evaluated.

5. **CONCLUDING REMARKS**

A major issue in Egyptian agricultural policies has been that of cotton vs. wheat production. The rapid increase in domestic demand for food has made Egypt a net importer of wheat since the beginning of the twentieth century. Hansen (1964) considered this issue, recognizing the interdependencies between the two crops. In a framework of a static and deterministic model, he described the optimal allocation of land between cotton and grain as a problem of optimum tariffs, assuming a downward sloping demand for Egyptian cotton. In this study it is shown that past data indicate a strong substitution between cotton and wheat in the allocation of land. The inherent dynamics of the production imply that responses to policies may turn out to be different than the predictions of static models.

Results from the econometric estimation here seem to indicate that future research may be fruitful for quantitative policy analysis. First, the existence of frequent oscillations in land allocations of cotton and wheat indicate the existence of negative serial correlations in land allocations. It is shown that the depletion of land fertility may account for the negative serial correlation, and this is consistent with the practice of crop rotation.
In order to understand the performance of land productivity, it is desirable to know whether the hypothesis presented here is correct or if the frequent fluctuations are mostly due to effects that are not controlled by farmers' decisions, such as, feedback policy rules on prices and taxes, or weather and water supply. Estimation of this type of model using regional data should help to improve our understanding of the dynamics of cotton and wheat supply and allocations of land. Thus we may be able to better evaluate policies, as well as identifying the regions where the technological constraint has had a larger effect on the growth of cotton land productivity.

Second, estimation of the production function separately provided a test of the hypothesis that the production function has stayed stable over 80 years. The results do not reject the hypothesis. Moreover, specification of cotton land productivity as a function of past and present land allocations did not result in rejection of the hypothesis that the form of this function stayed unchanged inbetween pre- and post-World War II periods.

Over the entire period, the use of fertilizer input per unit of crop area has changed from 3 kilograms in 1907 to 188.7 kilograms in 1966. Hence, it may be that I underestimated the actual deterioration effect on yield, as the use of fertilizer was ignored. The enormous increase in the use of fertilizer prevented further deterioration but did not eliminate it. Therefore, further research should be addressed to evaluation of the effect of deterioration and of the use of fertilizers and pesticides on productivity.

Incorporating the effects of cotton prices and input prices in the model may be very helpful in evaluating the effects of policies on the productivity and allocations of land for cotton and wheat. It may be that interventions in the use of fertilizers and pesticides, directly by quotas and indirectly by
setting prices, account for most of the existing deterioration in the land productivity. If so, it has constrained the development and growth of the entire agricultural sector.

One of the most interesting questions is why the average cotton yield in Egypt remained almost constant from 1895 to 1960. This question is still open, and it deserves more study. In the past it has been believed that water projects, such as the Aswan High Dam, would remove most of the constraints on development of the agricultural sector. This work emphasizes another constraint: the inherent deterioration in productivity due to past cultivation. The results suggest that further investigation of this aspect may prove useful to the policymakers, in Egypt, as well as in other developing countries.
APPENDIX — THE DATA

Sources


Variables

Cotton Area (COT-AR), thousand feddans, (1) for 1895-1960 and (3) for 1961-1976.
Cotton Lint Yield (COT-LY-YLD), cantar per hundred feddans, (1) for 1895-1960 and (3) for 1961-1976.
Total Cultivated Land (TOT-AR), thousand feddans, (1) for 1895-1960 and (3) for 1961-1969. For 1970-1976 the data are not available and are computed by taking the crop area from (3) and multiplying the average ratio of cultivated area to crop area during 1961-1969.
Cotton Seeds Yield (COT-SD-YLD), ardebs per 100 feddans, (1) for 1913-1960 and (2) for 1961-1969.
Cotton Lint Price (COT-LT-P), milliemes per cantar (1) for 1913-1960 and (2) for 1961-1969.
Wheat Price (WT-P), milliemes per ardeb, (2) for 1913-1969.
Wheat Yield (WT-YLD), ardebs per hundred feddans, (1) for 1913-1960 and (3) for 1961-1969.

Cotton Seeds Price (COT-SD-P), milliemes per ardeb, (1) for 1913-1960 and (2) for 1961-1969.

Wheat Area (WT-AR), thousand feddans, (1) for 1895-1960 and (3) for 1961-1976.

Units:

1 cantar = 44.928 kg.
1 feddan = 1,038 acres.
1 cotton seed ardeb = 123 kg.
1 wheat ardeb = 150 kg.
1000 milliemes = 1 Egyptian pound.
Financial assistance from the Foerder Institute for Economic Research is gratefully acknowledged.

1. The interested reader is referred to Owen (1969), Hansen and Marzouk (1965), Hansen and Nashashibi (1975) and Marbo (1979) for detailed discussions of the Egyptian economy and the agricultural sector.

2. It is interesting to note that as a result of the American Civil War, the price of cotton increased by approximately 90 percent from 1860 to 1865, and during that period the Egyptian exports of cotton increased by 300 percent.

3. Sims (1980) discusses this type of econometric modeling and applies it to a macroeconomic question. The reader is referred to Sims' paper for additional information on the econometric methodology.

4. The data sources are described in the Appendix. For the VAR the series included the years from 1913 to 1969.

5. A complete discussion of the approach presented here versus the conventional Nerlovian (1958) approach is contained in Eckstein (1981 and 1983).

6. The method for the solution is according to Hansen and Sargent (1980).

7. This result is due to the rational expectations solution we proposed for the model in the previous section (see Eckstein, 1984).
8. For consistency, the following three conditions should be met:

(i) \( E(U_t^2, X_t, X_{t-1}, \ldots,) = \sigma_a^2 \)

(ii) \( \{E[X_T'X_T]^{-1}\}_{T=\infty} \to 0 \) in quadratic mean; and

(iii) \( \{E[X_T'X_T][X_T'X_T]^{-1}\}_{T=\infty} \to I \) in probability as \( T \to \infty \),

where \( X_T \) is the stacked vector on \( T \) observations on \( X_t \) and

\[
X_T' = [Y_{t-1}, A_{1t}, A_{1t-1}, \hat{A}_t, \hat{A}_{t+1}].
\]

9. This two-step procedure is according to Hatanaka (1974).

10. Recently, Sagi (1982) estimated dynamic agricultural production functions with Egyptian post-World War II data. His results support the hypothesis of the negative effect of past areas on current production (\( d_1 > 0 \)).
REFERENCES


__________ and Girgis A. Marzouk (1965), Development and Economic Policy in the UAR (Egypt) (Amsterdam: North Holland).


TABLE 1
RATIO OF CROP AREA TO CULTIVATED AREA 1877-1965

<table>
<thead>
<tr>
<th>Year</th>
<th>Ratio of Crop Area to Cultivated area</th>
</tr>
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<tbody>
<tr>
<td>1877</td>
<td>1.0</td>
</tr>
<tr>
<td>1900</td>
<td>1.4</td>
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<td>1913</td>
<td>1.5</td>
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<td>1940</td>
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<td>1955</td>
<td>1.7</td>
</tr>
<tr>
<td>1965</td>
<td>1.7</td>
</tr>
<tr>
<td>1972</td>
<td>1.6</td>
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</table>
TABLE 2
PERCENTAGE OF FORECAST ERROR VARIANCE 15 YEARS AHEAD
PRODUCED BY EACH INNOVATION*

Triangularized Innovation in:
System 1

<table>
<thead>
<tr>
<th></th>
<th>System 1</th>
<th></th>
<th>System 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>COT—P</td>
<td>COT—AR</td>
<td>WT—AR</td>
<td>COT—YLD</td>
</tr>
<tr>
<td>WT—P</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COT—P</td>
<td>45</td>
<td>9</td>
<td>21</td>
<td>17</td>
</tr>
<tr>
<td>WT—P</td>
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<td>8</td>
</tr>
<tr>
<td>COT—AR</td>
<td>17</td>
<td>52</td>
<td>9</td>
<td>14</td>
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<tr>
<td>WT—AR</td>
<td>15</td>
<td>14</td>
<td>48</td>
<td>9</td>
</tr>
<tr>
<td>COT—YLD</td>
<td>9</td>
<td>10</td>
<td>18</td>
<td>57</td>
</tr>
<tr>
<td>WT—YLD</td>
<td>15</td>
<td>18</td>
<td>14</td>
<td>8</td>
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</tbody>
</table>

* Detailed tables can be found in Eckstein (1981).
## TABLE 3
ESTIMATED PARAMETERS OF THE PRODUCTION FUNCTION*

<table>
<thead>
<tr>
<th>Period</th>
<th>Estimate 1</th>
<th>Estimate 2</th>
<th>Estimate 3</th>
<th>Estimate 4</th>
<th>Estimate 5</th>
<th>Estimate 6</th>
<th>Estimate 7</th>
<th>Estimate 8</th>
<th>Estimate 9</th>
<th>Estimate 10</th>
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<th>Estimate 12</th>
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<tr>
<td>1895-1976</td>
<td>.5179</td>
<td>.0881</td>
<td>.0181</td>
<td>.0372</td>
<td>241,781.80</td>
<td>.72</td>
<td>2.31</td>
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<td>T = 80</td>
<td>.6127</td>
<td>.0719</td>
<td>.0212</td>
<td>.1310</td>
<td>262,085.19</td>
<td>.66</td>
<td>2.33</td>
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<tr>
<td></td>
<td>(.088)</td>
<td>(.029)</td>
<td>(.029)</td>
<td>(.032)</td>
<td>(59.1)</td>
<td>(59.1)</td>
<td>(59.1)</td>
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<td>1895-1941</td>
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<td>.0370</td>
<td>.0393</td>
<td>.0265</td>
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<td>.46</td>
<td>2.60</td>
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<td>T1 = 45</td>
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<td>.0331</td>
<td>.0377</td>
<td>.0249</td>
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<td>.43</td>
<td>2.54</td>
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<tr>
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<td>(.122)</td>
<td>(.038)</td>
<td>(.037)</td>
<td>(.070)</td>
<td>(53.7)</td>
<td>(53.7)</td>
<td>(53.7)</td>
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<tr>
<td>1944-1976</td>
<td>.2992</td>
<td>.1540</td>
<td>.0361</td>
<td>.0303</td>
<td>99,652.8</td>
<td>.72</td>
<td>2.03</td>
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<tr>
<td>T2 = 31</td>
<td>.4605</td>
<td>.1650</td>
<td>.0111</td>
<td>.1227</td>
<td>110,060.22</td>
<td>.64</td>
<td>2.27</td>
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<tr>
<td></td>
<td>(.177)</td>
<td>(.077)</td>
<td>(.065)</td>
<td>(.063)</td>
<td>(65.1)</td>
<td>(65.1)</td>
<td>(65.1)</td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

* All three regressions include constants and the data have not been detrended.

** The consistent estimates in Step 1 for \( p_{d0}, p_{d1} \) and \( p_{d} \) are taken as \( p^*d_0, p^*d_1, p^*d \).
Figure 1:

Sources: See Appendix
Figure 2:

Sources: See Appendix
Figure 3:

Sources: See Appendix
Figure 4(a): Response of Area in System 1

RESPONSES OF COTTON AND WHEAT AREAS

- cotton area
- wheat area

INNOVATION IN COT-P OVER WT-P

K STEP AHEAD

0 4.00 8.00 12.00 16.00 20.00 24.00
Figure 4(b): Response of Area in System 1
Figure 5: Simulated Land Allocations

- Case 1
- Case 2
Figure 6:
Response of Area in the Example