EXISTENCE OF EQUILIBRIUM FOR WALRASIAN ENDOWMENT GAMES

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1. Introduction

In the framework of pure exchange economies it is well known that the competitive mechanism can be manipulated by individuals. Hurwicz (1972) had initiated this topic by proving that a consumer can misrepresent his declared preference and by this improve his state at the new resulting Walras (W) allocation. Postlewaite (1979) and Thomson (1979) proved analogous results for misrepresentation of endowments.

In view of these negative results, questions of the second-best type, concerning the degree of the manipulativity of the mechanism, may arise. Such questions aim to the set of Nash-equilibria (NE) of the associated manipulation game, and ask about its non-emptiness, its size and distance from the true W allocations, and about its behavior as the economy gets larger.

Results concerning the preference manipulation game are well known. Hurwicz (1979) proved that the set of NE of this game coincides with the lens bounded by the true offer curves when there are two consumers and two commodities. Thus, this set is big and contains allocations that are far from the W allocations. Thomson (1979) has shown that for such economies the set of NE allocations does not shrink when the economy is replicated. The results of Hurwicz were generalized by Otani and Sicilian (1982) for the cases of more commodities, or more consumers. (The general case, however, is not clear yet, and properties of the set of NE are not known).

Parallel results for endowment manipulation games are not available. In fact, even the existence of non-trivial NE for such games is not clear, and thus the main purpose of this paper will be to investigate this problem. Some results for these games were given by Thomson (1979) who characterized the NE allocations and supplied examples showing the arbitrariness of the locations of the NE allocations. Another interesting example was analyzed by Haller (1983) who has shown how (in that case) the unique non-trivial NE allocation converges to the true W allocation when the economy is replicated. Other results, like those of Roberts and Postlewaite (1976) on the diminishing incentives to manipulate when the economy gets large, can be applied to the endowment game also. These results, however, give us no indication on the set of NE allocation of this game.

Full information on the consumers' true endowments is not always available, and practically, getting such information might be very costly. Moreover, in a private economy it seems reasonable to endow agents with the ability of withholding part of their resources from the market. Since, in addition, manipulation with endowments is quite common (for example, farmers can sometimes benefit from holding, or even destroying part of their crops, and insured agents can improve by reporting on less wealth to the insurer) it seems that the need for more general results on the NE of endowment manipulation games is quite clear. In this paper we deal with one kind of such games, the no-destruction game, where consumers can declare on false initial endowments and then add the withheld part to the resulting W allocation. It is shown that if the economy is large enough (given fixed types of consumers) then NE do exist and are very close to regular W equilibria. Since these NE are also NE for the preference manipulation game, we gain another existence result that does not appear in the literature. The NE allocations of Theorem 1 converge to a regular W allocations, and a natural question that may rise now is whether any limit point of NE allocations is also a W allocation. Theorem 2 gives conditions for this, concerning the regularity of the limit W allocations.

The proofs of the theorems use tools that were developed for similar results in monopolistic competition, and excellent references are the papers of Mas-Colell (1982) and K. Roberts (1980).

Section 2 describes the model and gives the definitions. The existence theorem is proved in Section 3 while the limit one is proved in Section 4.

2. The Model

We deal with pure exchange economy that consists of m types of consumers. The economy E_k has k consumers with some distribution μ_k of the m types. The number of commodities is ℓ, the last one is the numeraire, and the price set is

$$S = \{p \in \mathbb{R}^{\ell} \mid p^i > 0, \quad p^{\ell} = 1\} \tag{1}$$

The consumption set of consumer i is $\mathbb{R}^{\ell}_{+}$, and he has a smooth utility function $u_i: \mathbb{R}^{\ell}_{+} \to \mathbb{R}$ with strictly positive gradient and quasi-concave. We also assume that each non-zero indifference surface is contained in $\mathbb{R}^{\ell}_{++}$ and has non-zero Gaussian curvature. Consumer i also has a bundle $\omega_i >> 0$ of initial endowments, and from the above it follows that consumer i has a c^1 demand function $f_i: S \times \mathbb{R}_{++} \to \mathbb{R}^{\ell}_{++}$ such that $f_i(p, p\omega_i)$ is the preferred bundle of consumer i given his budget set at the price-vector p.

In this paper we want to concentrate on the manipulation with the initial endowment and so we assume that the utilities of the consumers are well known. This, of course, is an extreme case because usually the lack of information covers both endowments and utilities. A strategy of consumer i is a declaration of some bundle y_i as his initial endowment and then, whatever will be his new net trade at the new resulting Walrasian prices, he will privately consume the amount $\omega_i - y_i$. Since the only test for the initial amounts of a consumer is given by his ability to perform the resulting Walrasian net trades, and since (by the above assumptions) the Walrasian allocations are strictly positive, we assume that a small upward misrepresentation is also possible. For simplicity, and following Thomson (1979), we assume that for each consumer i there exists some $\bar{\omega}_i >> \omega_i$ such that the i-th strategy set is

$$\Omega_i = \{y \in \mathbb{R}^{\ell}_{++} \mid y \leq \bar{\omega}_i\} \quad (2)$$

(0 is not an admissible strategy since we are interested in non-trivial Nash equilibria.)

When consumer i chooses to declare y_i, his demand excess function is $f_i(p,py_i) - y_i$, and we denote it by $Z_i(\cdot,y_i)$: $S \to \mathbb{R}^{\ell-1}$, (the demand for the 1-th commodity is given by Walras' Law), then, given a list of strategies $y = (y_1,\ldots,y_k) \in \prod_{i=1}^{k} \Omega_i$ at E_k, we say that p in S is a Walrasian price for y if

$$Z(p,y) = \frac{1}{k}\sum_{i=1}^{k} Z_i(p,y_i) = 0 \quad (3)$$

The pair (p,y) is called a Walrasian equilibrium with respect to y.

It is well known (Debreu (1970)) that generically every y has an odd number of isolated Walrasian prices. Since this number can be greater than one, we should use some selection mechanism (assuming that unilateral misrepresentation causes a change in prices that restores the Walrasian equilibrium).

Let y and y' be in $\Pi\Omega_i$ and p be a Walrasian price for y. The mapping $P: S \times \Pi\Omega_i \times \Pi\Omega_i \to S$ is defined as in Roberts (1980):

$$P(p,y,y') \in \arg\min \|p'-p\|, \text{ s.t. } Z(p',y') = 0 \quad (4)$$

It is clear that near a regular Walrasian equilibrium (i.e., $Z(p,y) = 0$ and Z_p is non-singular) the mapping P is really a function that describes the smooth selection of Walrasian equilibria that passes through (p,y). From the assumptions of the paper it is also clear that this function is C^1.

For simplicity we denote it by $P(y')$, when y' is the vector of the new strategies.

For each i the boundness of Ω_i implies that the set of admissible excess demand functions is also bounded. Thus, for k large enough the function P will be C^1 for any strategy of consumer i (the change in the aggregate average excess demand in E_k is $\frac{1}{k}(Z_i(\cdot,y_i') - Z_i(\cdot,y_i))$.

<u>Definition</u>: The pair $(p,y) \in S \times \Pi\Omega_i$ is <u>Nash Equilibrium</u> (NE) in the economy E_k if p is Walrasian price for y and if P is well defined and if for all i and for all $y_i' \in \Omega_i$

$$u_i(\omega_i + Z_i(p,y_i)) \geq u_i(\omega_i + Z_i(P(y'),y_i')) \tag{5}$$

where $P(y') = P(p,y,\{y_{-i},y_i'\})$ and $y_{-i} = (y_1,\ldots,y_{i-1},y_{i+1},\ldots,y_k)$ (see Figure 1 for the two agents case).

The definition is based on the assumption that the agents notice their effect on the prevailing prices and take it into account.

<u>Definition</u>: The pair (p,ω) is a <u>Competitive Equilibrium</u> (CE) if it is a Walrasian equilibrium w.r.t. ω.

The NE allocations (if they exist) might change with the replication of the economy. The CE allocation, however, and the Walrasian prices for ω remain fixed through the replication. Generally, there need not be any relation between NE and CE for the economy E_k (k finite). However, for the limit economy E ($E_k \xrightarrow[k\to\infty]{} E$ when the distribution of E is μ, and $\mu_k \xrightarrow[k\to\infty]{} \mu$) the relation does exist. With regularity every CE allocation is a NE allocation, and vice versa.

To see it, let x be a CE allocation, i.e., $x_i = \omega_i + Z_i(p,\omega_i)$ where p is a regular Walrasian price for ω. Telling the truth is an admissible strategy, so that x can be realized. Now if agent i changes his strategy to y_i, the price p will not change, and he will get the bundle $\omega_i + Z_i(p,y_i)$, which cannot be preferred by him to x_i. Conversely, let x be a NE allocation. This means that there are vectors y (strategies) and p (Walrasian prices) such that $x_i = \omega_i + Z_i(p,y_i)$, and

$$\forall i \quad \forall y_i' \in \Omega_i \qquad u_i(x_i) \geq u_i(\omega_i + Z_i(P(y'),y_i')).$$

But here (assuming regularity) $P(y') = p$, and since $\omega_i \in \Omega_i$ we have

$$\forall i \qquad u_i(x_i) \geq u_i(\omega_i + Z_i(p,\omega_i)), \tag{6}$$

while $px_i = p\omega_i$. This implies that x is a CE allocation.

3. The Existence Theorem

The main aim of this paper is to prove existence of non-trivial NE for sufficiently large economies, and we do it by going backwards from the continuum economy E. We will show that generically any CE allocation of E is a limit of NE allocations of E_k $(k \to \infty)$.

Theorem 1. If (p,ω) is a regular CE of E then it is a limit of some $\{(p^k,y^k)\}_{k=k_0}^{\infty}$ where (p^k,y^k) is a NE for E_k.

Proof: Let (p,ω) be a regular CE and let y be a symmetric strategy vector that is close enough to ω (for each type) such that it falls in the neighborhood where the Walrasian price is given by the function $P(y)$. We can identify y with a vector in $\mathbb{R}^{\ell m}$.

(a) We will see here how the problem of each consumer can be linearized. At such y the excess demand of all the consumers besides i (in E_k) is given by

$$kZ(p,y) - Z_i(p,y_i) \quad (7)$$

and thus the first ℓ-1 coordinates of the bundles that i would be able to consume are given by $\hat{x} \in \mathbb{R}^{\ell-1}$ that satisfy

$$\hat{x} - \hat{\omega}_i = Z_i(p,y_i) - kZ(p,y) \quad (8)$$

where ^ denotes the projection onto $\mathbb{R}^{\ell-1}$. Thus

$$\hat{x} = \hat{\omega}_i + k(\frac{Z_i}{k} - Z) \quad (9)$$

and if y is close enough to ω then, because of regularity, $Z_p(P(y),y)$ is non-singular. This implies that for k large enough we have the non-singularity of

$$\hat{x}_p = k(\frac{1}{k}(Z_i)p - Z_p) = kA(k) \quad (10)$$

By the inverse function theorem, we get the existence of the function $\hat{p}(\hat{x})$ such that

$$\hat{p}_x = \frac{1}{k} A(k)^{-1} \quad \text{and} \quad \hat{p}_x \underset{k\to\infty}{\longrightarrow} 0 \quad (11)$$

Now, the constrained set of i is given by

$$C(k) = \{(\hat{x}, t) \in \mathbb{R}^{\ell} \mid t = p\omega_i - \hat{p}\hat{x}\} \quad (12)$$

(Later we will take care of the other constraints) and thus

$$t_x = -\hat{p} + \hat{p}_x(\hat{\omega}_i - \hat{x}) \underset{k\to\infty}{\longrightarrow} -\hat{p} \quad (13)$$

$$t_{xx} = -2\hat{p}_x + \hat{p}_{xx}(\hat{\omega}_i - \hat{x}) \underset{k\to\infty}{\longrightarrow} 0 \quad (14)$$

The limit in (14) is 0 since

$$\hat{p}_{xx} = \frac{1}{k}\frac{\partial}{\partial p}(A(k))^{-1} p_x = -\frac{1}{k} A(k)^{-1} A(k)_p \, A(k)^{-1} \frac{1}{k} A(k)^{-1} \to 0 \; Z_p^{-1} \, Z_{pp} (Z_p^{-1})^2 = 0$$

This means that the curvature of $C(k)$ goes to zero as $k \to \infty$.

Since the curvature of every indifference surface of u_i is strictly positive, it now follows that there is a unique maximal point of u_i on $C(k)$. Call it x^*, then since $C(k) \underset{k\to\infty}{\longrightarrow} \{x \in \mathbb{R}^\ell \mid p(x-\omega_i) = 0\}$ we shall have that x^* is very close to $\omega_i + Z_i(p,\omega_i)$ (given that y is close enough to ω).

It is not difficult to see that $\omega_i + Z_i(p,\omega_i)$ is an interior point of the set

$$B = \{x \in \mathbb{R}^\ell \mid \exists y_i \in \Omega_i, \exists p \in S \text{ s.t. } x = \omega_i + Z_i(p,y_i)\} \qquad (16)$$

(Let x' be close to $\omega_i + Z_i(p,\omega_i)$, and let p' be such that $p'(x'-\omega_i) = 0$. $\omega_i + Z_i(p',\omega_i)$ is close to x', and we get that

$\omega_i + Z_i(p', \omega_i + Z_i(p',\omega_i) + \omega_i - x') = x'$, while $\omega_i + Z_i(p',\omega_i) + \omega_i - x'$ belongs to Ω_i.) This means that the only relevant constraint will be $x^* \in C(k)$.

The normal to $C(k)$ at x^* is

$$(\hat{p} + \frac{1}{k}(\frac{1}{k}(Z_i)_p - Z_p)^{-1}(\hat{\omega}_i - \hat{x}^*), 1), \text{ or } (\hat{p}+\alpha(k,y)(\hat{\omega}_i - \hat{x}^*), 1) \qquad (17)$$

while $\alpha(k,y) \underset{k\to\infty}{\longrightarrow} 0$. We also call it $p(k)$.

To summarize, x^* is also the solution of $\text{Max } u_i(x)$, s.t. $p(k)x \leq p(k)x^*$ (18)

or equivalently, there is $\lambda > o$ such that $\lambda \partial u_i(x^*) - p(k) = (\hat{P}(y) + \alpha(k,y)(\hat{\omega}_i - \hat{x}_i), 1)$

The situation is described in Figure 2.

(b) In this step we show how to get towards the desired NE. By the former step, if y is close enough to ω and k is large enough, then sufficient conditions for y to be a NE is the existence of x_i (for all i) such that

$$\text{(i)} \quad \lambda \partial u_i(x_i) = (\hat{P}(y) + \alpha(k)(\hat{\omega}_i - \hat{x}_i), 1)$$

$$\text{(ii)} \quad P(y)(x_i - \omega_i) = 0 \tag{19}$$

$$\text{(iii)} \quad \hat{x}_i - \hat{\omega}_i = Z_i(P(y), y_i)$$

Condition (iii) says that $\hat{x}_i - \hat{\omega}_i$ is the desired excess demand given the strategy y_i and the Walrasian price $P(y)$. Conditions (i) and (ii) say that x_i satisfies the necessary and sufficient condition for maximizing u_i on $C(k)$. (See Figure 3.) $\alpha(k) = 0$, $\bar{x}_i = \omega_i + Z_i(p,\omega_i)$, $p = P(\omega)$, and ω, satisfy (i) and (ii). The derivative of (i) and (ii) with respect to x and λ at the point $(y,\alpha,x,\lambda) = (\omega,0,\bar{x}_i,\bar{\lambda})$ (where $\bar{\lambda}$ is defined by (i) at ω,p and $\bar{x}_i$) is

$$\begin{vmatrix} \bar{\lambda}\partial^2 u_i(\bar{x}_i) & \partial u_i(\bar{x}_i) \\ p & 0 \end{vmatrix} = \bar{\lambda} \begin{vmatrix} \partial^2 u_i(\bar{x}_i) & \partial u_i(\bar{x}_i) \\ \partial u_i(\bar{x}_i) & 0 \end{vmatrix} \neq 0 \tag{20}$$

since u_i has the non-zero Gaussian curvature property.

This implies that, locally, $x_i(y,\alpha)$ exists such that (i) and (ii) are satisfied. Note that

$$\hat{x}_i(y,\alpha(k,y)) \underset{k\to\infty}{\longrightarrow} Z_i(P(y), \omega_i) + \hat{\omega}_i \tag{21}$$

(c) The last step is to show the existence of functions G_k that converge uniformly to a function G such that the only zero of G is the given ω and the zeros of G_k are strategies of NE. By a version of the implicit function theorem we shall then get the existence of the desired sequence of NE.

Let V' be a small neighborhood of ω for which the above results hold, and let $V = \{ x \in V' | p(x_i - \omega_i) = 0 \text{ for all } i\}$ (i.e. $V = V' \cap$ {the budget hyperplane of p through ω}).

Define $G_k: V \to \mathbb{R}^{m(\ell-1)}$ by $((G_k)_i: V \to \mathbb{R}^{\ell-1})$

$$(G_k(y))_i = \hat{x}_i(y, \alpha(k,y)) - \hat{\omega}_i - Z_i(P(y), y_i) \tag{22}$$

The zeros of G_k satisfy (iii) and thus they give the NE strategies.

The limit function $G: V \to \mathbb{R}^{m(\ell-1)}$ is

$$(G(y))_i = Z_i(P(y), \omega_i) - Z_i(P(y), y_i) \tag{23}$$

It is clear that $G(\omega) = 0$. If there was another y in V such that $G(y) = 0$ then $P(y)$ should be a Walrasian price for ω. From the regularity assumption we get $P(y) = p$ and $Z_i(p, \omega_i) = Z_i(p, y_i)$. Since we are in V it implies that $\omega = y$. Thus G has a unique zero in V.

We now prove that $G_y(\omega)$ is non-singular (or informally, regularity of Z implies regularity of G). G can be written as

$$(G(y))_i = \hat{f}_i(P(y), P(y)\omega_i) - \hat{\omega}_i - \hat{f}_i(P(y), P(y)y_i) + \hat{y}_i \tag{24}$$

and (when the upper index is for the commodity, and w_i is i-th income)

$$\frac{\partial G_i^h}{\partial y_i^h}(\omega) = \sum_j \frac{\partial f_i^h}{\partial p^j} \frac{\partial p^j}{\partial y_i^h} + \frac{\partial f_i^h}{\partial w_i} \sum_j \frac{\partial p^j}{\partial y_i^h} \omega_i^j - \sum_j \frac{\partial f_i^h}{\partial p_j} \frac{\partial p^j}{\partial y_i^h} - \frac{\partial f_i^h}{\partial w_i} \left(\sum_j \frac{\partial p^j}{\partial y_i^h} \omega_i^j + p^h\right) +$$

$$+ 1 = 1 - \frac{\partial f_i^h}{\partial w_i} p^h \tag{25}$$

$$\frac{\partial G_i^h}{\partial y_i^t}(\omega) = - \frac{\partial f_i^h}{\partial w_i} p^t, \text{ when } t \neq h, \text{ and}$$

$$\frac{\partial G_i^h}{\partial y_j^t}(\omega) = 0, \quad \text{when } j \neq i.$$

This means that $G_y(\omega)$ can be written as

$$G_y(\omega) = \begin{pmatrix} M_1 & & 0 \\ & \ddots & \\ 0 & & M_m \end{pmatrix} \tag{26}$$

where M_i is of order $(\ell-1) \times (\ell-1)$ and is defined by

$$M_i = \begin{pmatrix} 1 - \frac{\partial f_i^1}{\partial w_i} p^1 & \cdots & - \frac{\partial f_i^1}{\partial w_i} p^{\ell-1} \\ \vdots & \ddots & \vdots \\ - \frac{\partial f_i^{\ell-1}}{\partial w_i} p^1 & \cdots & 1 - \frac{\partial f_i^{\ell-1}}{\partial w_i} p^{\ell-1} \end{pmatrix} \tag{27}$$

It is easy to see that

$$|M_i| = 1 - \sum_{j=1}^{\ell-1} \frac{\partial f_i^j}{\partial w_i} p^j = \frac{\partial f_i^\ell}{\partial w_i} \tag{28}$$

by differentiating the budget constraint $pf_i = w_i$ with respect to w_i. Each consumer i has a commodity j such that $\frac{\partial f_i^j}{\partial w_i} \neq 0$ (it need not be the same for all of them). Call it ℓ for the i-th consumer (i.e. use another parametrization for the budget hyperplane) and we get that $|M_i| \neq 0$. This implies that $G_y(\omega)$ is non-singular.

Now, as in Mas-Colell (1982) we define $M = \{\infty, 1, 2, \ldots\}$, $d(n,m) = \left|\frac{1}{n} - \frac{1}{m}\right|$, $G(y,k) = G_k(y)$, and use the following implicit function theorem for completing the proof.

Implicit function theorem (Schwartz (1967), Ch. 38): Let $V \subset \mathbb{R}^s$ be an open set and M a metric space. Let $G: V \times M \to \mathbb{R}^s$ be continuous. Suppose that $G_y(y,k)$ exists and depends continuously on (y,k) for all (y,k) in $V \times M$.

Suppose that $G(\bar{y},\bar{k}) = 0$ and $G_y(\bar{y},\bar{k})$ is non-singular. Then there are neighborhoods $\bar{y} \in V' \subset V$, $\bar{k} \in M' \subset M$ and a continuous function $y: M' \to V'$ such that $G(y(k),k) = 0$ for all $k \in M'$. (In fact, if $G(y,k) = 0$ and $(y,k) \in V' \times M'$ then $y = y(k)$).

Corollary: If (p,ω) is a regular CE of E then it is the limit of a sequence of NE for the Walrasian preference game

Remark: When $k \to \infty$ there are CE of E_k that converge to (p,ω). This implies that the NE allocations of Theorem 1 become more efficient as k gets large.

Remark: Usually the strategies that give the NE allocations of the theorem are not unique. Any $\bar{y}_i$ for which $Z_i(p^k,\bar{y}_i) = Z_i(p^k,y_i^k)$ will give a NE strategy for consumer i.

4. Limit of NE

We show here that under some conditions the limit of NE allocations is a CE allocation.

Theorem 2 Let $\{(p^k, y^k)\}_{k=k_0}^{\infty}$ be a sequence of NE for E_k such that $(p^k, y^k) \xrightarrow[k\to\infty]{} (p, y)$, and let x^k be the corresponding NE allocations. Then

(a) p is Walrasian price for y at E.

(b) $\{x^k\}$ has a converging subsequence.

(c) If p is regular for y and $x^k \to x$ then x is a CE allocation for E.

Proof: (a) Follows from the continuity of Z and since every p^k is a Walrasian price for y^k (at E_k).

(b) For all k and i, $y_i^k + Z_i(p^k, y_i^k)$ is bounded by $\Sigma\omega_i$, and thus $x_i^k = \omega_i + Z_i(p^k, y_i^k)$ is bounded too. This implies that $\{x^k\}$ belongs to some compact set, and has a converging subsequence. Without loss of generality, $x^k \to x$.

(c) Suppose that there is i (or a set with positive measure) such that $x_i \neq \omega_i + Z_i(p,\omega_i)$. Thus there is v such that $p(v - \omega_i) = 0$ and $u_i(v) = u_i(x_i) + \varepsilon$ $(\varepsilon > 0)$. If v is chosen close enough to x then there is strategy $\bar{y}_i$ for i such that $v = \omega_i + Z_i(p,\bar{y}_i)$ (i.e. i can obtain this v).

Look now at the sequence $\{\bar{y}^k\} = \{y_{-i}^k, \bar{y}_i\}$. It is clear that $Z(\cdot, \bar{y}^k) \xrightarrow[k\to\infty]{} Z(\cdot,y)$ and from the regularity we get $P(\bar{y}^k) \to p$. But then we have

$$\omega_i + Z_i(P(\bar{y}^k), \bar{y}_i) \xrightarrow[k\to\infty]{} v \tag{29}$$

such that for k large enough i can improve by declaring $\bar{y}_i$:

$$u_i(\omega_i + Z_i(P(\bar{y}^k), \bar{y}_i) > u_i(x_i^k) = u_i(\omega_i + Z_i(p^k, y_i^k), \tag{30}$$

and x^k is not a NE allocation.

Remark: If every vector of strategies y has only one Walrasian price, then Theorem 2 holds even if p is not regular

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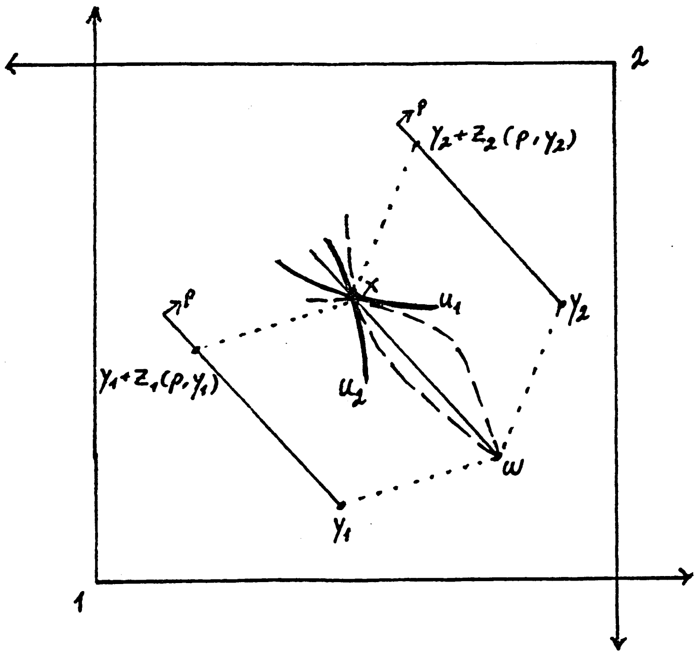
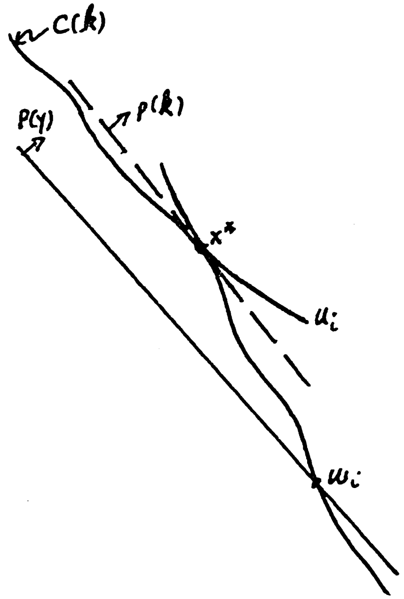
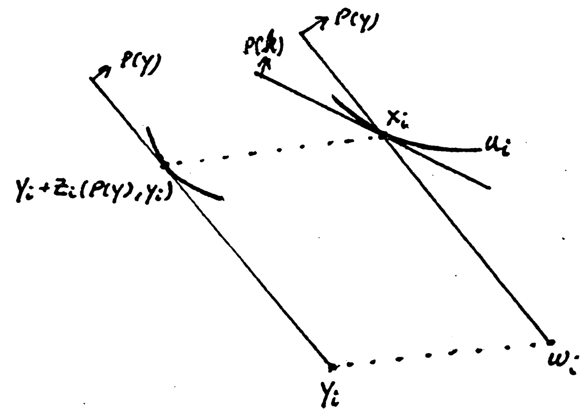


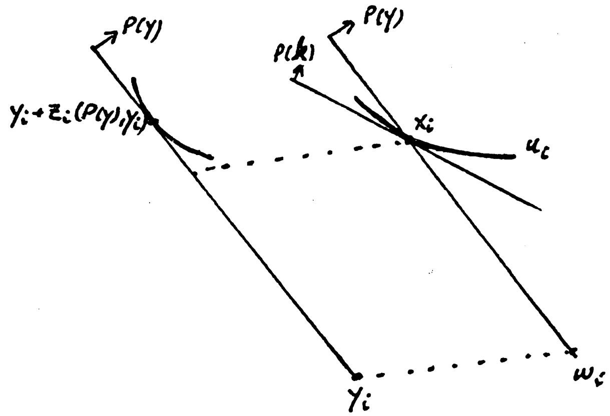
<u>Figure 1</u>: x is a NE allocation and p=P(y). The broken curves are the translated offer curves.



<u>Figure 2</u>: C(k) is the relevant part of the offer surface of the rest.



<u>Figure 3a</u>: x_i satisfies (i) - (iii).



<u>Figure 3b</u>: x_i satisfies (i) and (ii).