ON THE CAPITALIZATION OF LAND IMPROVEMENT PROJECTS

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Working Paper No.41-83

December, 1983

Financial Assistance from the Foerder Institute for Economic Research is gratefully acknowledged. I am indebted to Ephraim Sadka for helpful comments. Any remaining errors are mine.

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1. INTRODUCTION

We know that the benefit of a land improvement project is fully capitalized if it directly affects only a small part of the population (see, for example, Polinsky and Shavell (1976), Pines and Weiss (1976), Helpman and Pines (1977), and Starrett (1981)). Therefore, in a system of many open cities (with free migration) maximization of land value minus the cost of the improvement, or the cost of the provision of the local public good, results in efficient resource allocation (see, for example, Sonstelie and Portney (1978) and Brueckner (1983)).

The full capitalization result is not valid, of course, if the size of the population directly affected by the improvement project is not negligible. In this case we do not a-priori know whether the value of the land, which is directly affected by the project, would increase, and, if it would, whether this increase exceeds or falls short of the social benefit.

In this paper I use a model of two standard monocentric cities to provide sufficient conditions for less than full capitalization of the social benefit. Under these conditions, profit maximizing results in a smaller project than is socially desirable.

2. THE MODEL

I consider here a closed system of two monocentric cities, 1 and 2, with N homogeneous households, which can move costlessly between them and which commonly and equally own $k(1 \geq k \geq 0)$ of the land in the two cities. Each household consumes a composite good with unitary price and housing with price $R^i(x)$, $(i = 1,2)$, which depends on the distance from the center of the respective city, x. $E^1(R^1(x),u)$ is the expenditure function of a household in city 1, and $E^2(R^2(x),u,\gamma)$ is the expenditure function of a household in city 2, where γ is a shift parameter indicating the level of the local public good and $\partial E^2/\partial\gamma < 0$. Households in city 1 spend $t^1(x)$ and households in city 2 spend $t^2(x,\alpha)$ on transport cost, where α is a shift parameter, indicating the level of service, such that $\partial t^i/\partial x > 0$, and $\partial t^2/\partial\alpha < 0$. Each household in the two cities is endowed with I units of the composite good. Thus the budget constraints in city 1 are:

(1-I) $$E^1(R^1(x),u) + t^1(x) = I + k(DLR_1 + DLR_2)/N,$$

and in city 2:

(I-II) $$E^2(R^2(x),u,\gamma)+t^2(x,\alpha) = I+k(DLR_1+DLR_2)/N,$$

where DLR_i = Profits derived from land transactions in city i, (i = 1,2).

Urban landlords rent land from agricultural landlords at a price R_A and supply it to urban households for $R(x)$. Then, with competition at the boundary of the city, we have:

(2) $$R^i(x) \geq R_A, \quad R^i(x_i) = R_A, \quad (i = 1,2),$$

where x_i is the minimal radius around the city center which contains all the developable land in city i, With the above relations:

(3-I) DLR_1 (Differential Land Rent in City 1) = $\int_0^{x_1} \theta^1(x)(R^1(x)-R_A)dx$,

(3-II) DLR_2 (Differential Land Rent in City 2) = $\int_0^{x_2} \theta^2(x,\beta)(R^2(x)-R_A)dx$,

where

$\theta^1(x)$ = the fraction of a circle of a radius x which is used in city 1 for housing times $2\pi x$,

$\theta^2(x,\beta)$= the fraction of a circle of a radius x, which is used for housing in city 2 times $2\pi x$,

β = a shift parameter, satisfying $\partial\theta^2/\partial\beta > 0$.

Equality of the demand for and the supply of housing in each location implies:

(4-I) $n^1(x)E^1_R - \theta^1(x) = 0$,

(4-II) $n^2(x)E^2_R - \theta^2(x,\beta) = 0$,

where n^i is the population density times θ^i and a subscript denotes partial derivative. In (4) we use the derivative property of the expenditure function which says that E^i_R = quantity demanded of housing.

Locating all the households in either city 1 or 2 implies:

(5) $\int_0^{x_1} n^1dx + \int_0^{x_2} n^2dx = N$

(1)-(5) solves for the equilibrium levels of $R^i(x)$, $n^i(x)$ x_i, DLR_i, and u $(i = 1,2)$, in terms of α, β and γ.[1]

3. FULL OWNERSHIP, k = 1

Differentiating both sides of (1), with respect to α, β and γ multiplying the resulting expression by the respective n^i, integrating the result and using (3) and (4) yield for Marginal Social Benefit (MSB):

$$(6) \qquad MSB \equiv N\bar{E}_u du = -\left[\int_0^{x_2} n^2 t_\alpha^2 dx\right] d\alpha + \left[\int_0^{x_2} \theta_\beta^2(x,\beta)(R^2 - R_A)dx\right] d\beta$$

$$- \left[\int_0^{x_2} n^2 E_\gamma^2 dx\right] d\gamma > 0$$

where income equivalence of the change in utility is:

$$\bar{E}_u = \left[\int_0^{x_1} n^1 E_u^1 dx + \int_0^{x_2} n^2 E_u^2 dx\right]/N$$

The expressions on the right-hand side of (6) are the direct incremental social benefit of a transportation improvement, a land augmenting (e.g. land reclamation) and a provision of the public good, respectively. The indirect effects resulting from the reallocation do not appear in (6). This reflects the fact that there is no market failure and each activity is correctly priced. Observe also that the changes in land values do not appear in (6), since DLR_1 and DLR_2 are simultaneously both cost and income, and, therefore, they are washed out in the calculation of the social benefit.

To derive the relation between the increase in land value in city 2 and MSB, differentiate (1-I), with respect to α, β and γ, multiply the result by

n^1, integrate, and use (3), (4), (5), and (6) to obtain, after rearranging terms:

(7) $$dDLR_2 = \frac{\bar{E}_u^1}{\bar{E}_u} MSB + \frac{N_2}{N_1} dDLR_1$$

where $\bar{E}_u^1$ = average income equivalence of the marginal utility in city 1.

$N_i = \int_0^{x_0} n^i dx$, (i = 1,2).

(7) is essentially the formula derived in Pines and Weiss (1976) for a non spatial case. As explained there, there are two sources for the bias in using $dDLR_2$ for estimating MSB. The first is the difference between the average income equivalence of the incremental benefit of the population in city 2 and the total population; the second is the change in land value in city 1. Each of these two sources of the bias can contribute to both over, and undercapitalization, depending on the specific structure of the exogenous functional relations and parameters.[2] The only statement that can a priori be made without further specification is that both the sources of the bias vanish when N_2 tends to zero. This is the full capitalization case, i.e. $dDLR_2$ = MSB, mentioned in the introduction about many small open cities with free mobility.

Now, I turn to a more specific case:

<u>Proposition</u>: If:

(a) $\dot{E}_u^1 = \dot{E}_u^2 = 0$, $E_u^1 = E_u^2$, $E_{u\gamma}^2 = 0$,[3/] (b) $\dot{\varepsilon}^1$, $\dot{\varepsilon}^2 > 0$[4/], and

(c) $\int_0^{x_2} d\dot{\varepsilon}^2 R^2 dx - d\varepsilon^2(x_2) R_A \geq 0$[5/], where $\varepsilon^1 = \theta^1/\dot{t}^1$, $\varepsilon^2 = \theta^2/t_x^2$ and a dot denotes a differentiation with respect to distance, x, then: DLR_2 MSB.

Proof: Proceed in steps:

(i) R^1 can either increase everywhere or decrease everywhere, or remain unchanged everywhere: Differentiating (1-I) with respect to α, β and γ yields:

$$dR^1(x) = [\frac{1}{N}(dDLR_1 + dDLR_2) - E_u^1 du]/E_R$$

Hence, $$\text{Sign } dR^1(x) = \text{sign}[\frac{1}{N}(dDLR_1 + dDLR_2) - E_u^1 du]$$

In view of assumption (a) the expression in the brackets is constant which proves the assertion.

(ii) If $DLR_1 \geq 0$, then there exists $x^{**} \in [0,x_2]$ such that $dR^2(x^{**}) < 0$: Following Wheaton (1974) and Pines and Sadka (1983) we can derive:

$$(8) \qquad \int_0^{x_1} \dot{\epsilon}^1 dR^1 dx + \int_0^{x_2} \dot{\epsilon}^2 dR^2 dx = d\epsilon^2(x_2)R_A - \int_0^{x_2} R^2 d\dot{\epsilon}^2 dx < 0.$$

The inequality follows from (2) and assumption (c) of the proposition. Then, it follows from (i), assumption (b), and $DLR_1 \geq 0$ that the second term on the left-hand side of (8) must be negative. In view of assumption (b), this is possible only if dR^2 falls somewhere. This proves (ii).

(iii) $dDLR_1 < 0$: Suppose not. Then (i) and (ii) imply that there exist x^* and x^{**} such that $dR^1(x^*) \geq 0$ and $dR^2(x^{**}) < 0$. Differentiating (1-I) and (1-II) with respect to α, β and γ using assumption (a), it follows that:

$$dR^2(x^{**}) = [E_R^1 dR^1\Big|_{x=x^*} - (t_\alpha^2 \, d\alpha + E_\gamma^2 d\gamma)\Big|_{x=x^{**}}]/E_R^2 > 0,$$

(since $dR^1(x^*) \geq 0$, and $t^2_\alpha, E^2_\gamma < 0$). Contradiction.

(iii) and (7) imply $DLR_2 < MSB$. Q.E.D.

Observe that Proposition 1 does not exclude the possibility that DLR_2 itself is negative.[6] Thus, either $dDLR_2$ is negative or, at least, underestimates MSB.

4. ZERO OWNERSHIP (ABSENTEE LANDLORDS) k = 0

This is the case discussed by Polinsky and Shavell (1976). Repeating the calculation from Section 3 for k = 0, we obtain:

$$(6') \qquad MSB \equiv MHB + MLB = -[\int_0^{x_2} n^2 t^2_\alpha dx]d\alpha + [\int_0^{x_2} \theta^2_\beta (R^2 - R_A)dx]d\beta - [\int_0^{x_2} n^2 E_\gamma dx]d\gamma > 0,$$

where MHB (Marginal Household's Benefit) $= N\bar{E}_u du$,

MLB (Marginal Landlords Benefit) $= dDLR_1 + dDLR_2$,

and

$$(7)' \qquad dDLR_2 = MSB + \frac{N_2}{N_1}\frac{\bar{E}_u}{\bar{E}^1_u} dDLR_1 = MSB - \frac{N_2}{N} = MLB.$$

Comparing (6) and (6)' we realize that the formula does not change. Most probably, however, the value in (6)' does differ from that of (6), since the allocations differ. Regarding (7)' we can prove:

Proposition 2: If assumptions (b) and (c) of Proposition 1 are satisfied, then

$$DLR_2 < MSB.$$

<u>Proof</u>: It follows from assumptions (b) and (c) of Proposition 1 and from (8) that there must exist either some $x^* \varepsilon [0,x_1]$ such that $dR^1(x^*) < 0$, or, if not, $x^{**} \varepsilon [0,x_2]$ such that $dR^2(x^{**}) < 0$. In the first case, differentiate (1-I) at x^* with respect to α, β and γ to get:

$$duE_u^1 \Big|_{x=x^*} = -E_R^1 dR^1 \Big|_{x=x^*} > 0.$$

But then for any $x \varepsilon [0,x_1]$ we have

$$\text{(9)} \qquad DR^1 = - E_u^1 du/E_R^1 < 0 \quad \text{for all} \quad x \varepsilon [0,x_1) \text{ since } du,\ E_R^1 > 0.$$

Hence R^1 declines everywhere and so does DLR_1.

In the second case we obtain from differentiating (1-II) and rearranging terms:

$$du\ E_u^2\Big|_{x=x^{**}} = - (E_R^2 dR^2 + t_\alpha^2 d\alpha + E_\gamma^2 d\gamma)\Big|_{x=x^{**}} > 0,$$

since $dR^2(x^{**})$, t_α^2, $E_\gamma^2 < 0$. Households' utility does not vary with location. Therefore we must have in equality (9) satisfied. Hence, in the case of absentee landlords, R^1 declines everywhere and therefore $dDLR_1 < 0$. This and (7)' prove the proposition. Q.E.D.

5. <u>SUMMARY AND CONCLUDING COMMENT</u>

Constant utility of income or absentee landlords imply less-than-full capitalization of the benefit of land improvement projects. These results are derived here for a model of two standard monocentric cities. They are, of course, valid for more than two cities as long as the population of th city in which the improvement is carried out or the local public good is supplied, is not negligibly small, relative to the total population of all the cities. On the basis of these findings, one can conjecture that profit maximizing leads to underinvestment, which is equivalent to the underproduction of a monopolist.

FOOTNOTES

1. By Walras' Law, the market for the composite good is also cleared.

2. The author can provide examples for both $\bar{E}_u^1/\bar{E}_u > 1$ and $\bar{E}_u^1/\bar{E}_u < 1$ and for $dDLR_1 > 0$ and $dDLR_1 < 0$.

3. This is the case discussed by Strotz (1968) and Starrett (1981). The utility functions which yield this relation are $u^1 = f^1(H^1) + Z^1$, $u^2 = f^2(H^2, \gamma) + Z^2$, where H^i is housing and Z^i the composite good $(i = 1,2)$.

4. This relation is satisfied when $\dot{\theta} > 0$ and $\ddot{t}^1, t_{xx}^2 \leq 0$, which are standardly assumed.

5. This assumption is satisfied, for example, whenever $t^2(x,\alpha) = b(x)/\alpha$ and $\theta^2(x,\beta) = \beta c(x)$.

6. An example can be provided by the author upon request.

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