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OPTIMAL HEDGING IN THE FUTURES MARKET UNDER PRICE UNCERTAINTY

by

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1. INTRODUCTION

Discussions in the literature of optimal hedging strategies have been largely inconclusive. When one ventures beyond the traditional view that the hedger should have a hedged position equal in quantity (and opposite in direction) to his spot position, most authors stress that the hedging decision is essentially one which depends on preferences, expectations, and the behavior of futures versus spot prices. An optimal hedging strategy exists but cannot be specified.

In this paper we show that definitive statements may be made about the optimal hedging ratio for all risk-averse individuals provided prices (both spot and futures) fulfill certain conditions. The conditions we require are two:

1. The futures price today is an unbiased predictor of the futures price tomorrow.
2. Randomness in the difference between the futures price and the spot price is not correlated to the spot price.

Although these conditions cannot be said to hold universally, they appear to hold empirically for a large number of cases. The remainder of this introduction will survey a number of these cases.

Unbiasedness: Although the Keynes-Hicks "normal backwardation" theory holds that futures prices must rise (if hedgers are short) or fall (if hedgers are long) over time, it has proven exceedingly difficult to show that this phenonemon actually holds. In fact, Tomek and Grey (1970) have shown that for corn and soybeans, the futures price at planting time is an unbiased predictor of the harvest spot price. Kofi (1973) has extended this result; it would appear from his results that if $\tilde{P}_h$ is the harvest spot price of wheat, and $\tilde{F}_t$ is the futures price of wheat in month t (we use ~ to denote random variables), then for $t \leq h$, the null hypothesis

$$H_o: \ F_t = E_t(P_h) \ ,$$

where E_t the expectation at time t, cannot be rejected. Similar results hold for a variety of goods with continuous inventories. A particular corollary of this hypothesis (and one needed for our results) is that the futures price at any moment in time, F_t, is itself an unbiased predictor of future futures prices, F_{t+a}, $a \geq 0$.

Basis uncorrelated with spot prices: The second condition needed for our results relates to the relation between the futures price and the contemporaneous spot price, F_t and P_t. We require that $\tilde{P}_t = \alpha_t + \beta_t \tilde{F}_t + \tilde{b}_t$, where $\tilde{b}_t$ is uncorrelated with $\tilde{F}_t$ (either the futures price or the spot price.)

The condition required is consistent with a cost of storage theory. Working (1948), for example, has postulated that F_t/P_t is a constant, and this appears to be borne out by statistical research (Kendall and Scobie (1980)). Further support for the condition is lent by Ward and Dasse's (1977) research into the basis on citrus futures. Of six variables used to explain the basis in frozen concentrated orange juice futures, only one relates to the spot price of the commodity, and this variable was found to have no explanatory power.

2. THE OPTIMAL HEDGE RATIO

Consider a risk-averse producer who will have a fixed inventory Q of a commodity in period 1. The commodity will be sold at the prevailing market price $\tilde{P}_1$, so that (in the absence of hedging) the producer's income will be $Q\tilde{P}_1$. The producer need not hold the inventory unhedged, however, as there exists a futures market for the good. The current futures market price F_0 relates to delivery at some later time 2; we denote by $\tilde{F}_1$ the futures price at time 1 (unknown at $t = 0$) for time 2 delivery. By selling quantity X short on the futures market, the producer will have income in time 1 of $Q\tilde{P}_1 + X(F_0 - \tilde{F}_1)$. We shall assume that there is no futures contract for time 1 delivery. If such a contract exists, then there is a forward market for the good, and if this forward market is unbiased, our results show that even in the absence of separability (see below) the optimal hedge consists of $X = Q$.

We assume that the producer is a risk-averse decision maker who maximizes the expected utility of time 1 income; in particular, denoting the utility function by U, we assume that $U' > 0$, $U'' < 0$. Thus the producer's problem may be written:

$$\max_X \ EU(Q\tilde{P}_1 + X(F_o - \tilde{F}_1)). \tag{1}$$

We now make two assumptions which are essential for our results:

(A.1) Unbiasedness: $F_o = E\tilde{F}_1 = E\tilde{P}_2$, where E denotes the expectation operator at time o.

(A.2) Separability: We assume that $\tilde{P}_t = \alpha_t + \beta_t\tilde{F}_t + \tilde{b}_t$

where the joint probability density function of $\tilde{F}_t$ and $\tilde{b}_t$ is separable.

A special case of separability is when $\tilde{F}_t$ and $\tilde{b}_t$ are independent; in this case the joint probability density function is the product of the two densities. Separability is a weaker condition than independence.

We may now prove the following proposition:

Proposition: Under assumptions (A.1) and (A.2) the optimal hedge is $X^*=\beta_1 Q$.

Before proving the proposition, we note that our result holds for all risk averse individuals, irrespective of their preferences and the particular distributions of $\tilde{P}_1$ and $\tilde{F}_1$. This contrasts with results in the literature (e.g., Johnson (1960), Ederington (1979), Rolfo (1980)) in which either the optimal hedge depends on a specific utility function or in which the tradeoffs between return and risk from hedging are presented, but with no specific solution.

Let us first prove the following Lemma.

Let $\tilde{X}(\omega)$ be a random variable, defined on Ω , which satisfies:

$$\tilde{X} > 0 \quad \text{with probability } 1 \ , \quad E\tilde{X} < \infty \quad \text{and} \quad \sigma^2_{\tilde{X}} > 0. \quad \text{Then}$$

<u>Lemma</u>: Let A and B be constants and

$$E\tilde{X}U'(A\tilde{X} + B) = (E\tilde{X})EU'(A\tilde{X} + B) \qquad (2)$$

Then we must have $A = 0$.

<u>Proof of the Lemma</u>: Denote by $\mu = E\tilde{X}$. Define the following two sets on the sample space:

$$K_1 = \{\omega \mid \tilde{X}(\omega) > \mu\}$$
$$K_2 = \{\omega \mid \tilde{X}(\omega) \leqq \mu\}$$

It is clear that the following inequality holds if $A > 0$ (since U' is a decreasing function):

For <u>all</u> $\omega_1 \in K_1$, $U'(A\tilde{X}(\omega_1) + B) < U'(A\tilde{X}(\omega_2) + B)$ for <u>all</u> $\omega_2 \in K_2$

But since

$$\int_{K_1} (\tilde{X}(\omega_1) - \mu) = -\int_{K_2} (\tilde{X}(\omega_2) - \mu) \qquad (3)$$

we obtain that if $A > 0$,

$$\int_{K_1} (\tilde{X}(\omega_1) - \mu)U'(A\tilde{X}(\omega_1) + B) < -\int_{K_2} (\tilde{X}(\omega_2) - \mu)U'(A\tilde{X}(\omega_2) + B)$$

which is a contraction since $K_1 \cup K_2 = \Omega$, and by (2) we have $E(\tilde{X} - \mu)U'(A\tilde{X}(\omega) + B) = 0$. Now assume that $A < 0$. This implies that:

For <u>all</u> $\omega_1 \in K_1$ $U'(A\tilde{X}(\omega_1) + B) > U'(A\tilde{X}(\omega_2) + B)$ for <u>all</u> $\omega_2 \in K_2$

which, together with (3) yields

$$\int_{K_1} (\tilde{X}(\omega_1) - \mu)U'(A\tilde{X}(\omega_1) + B) > -\int_{K_2} (\tilde{X}(\omega_2) - \mu)U'(A\tilde{X}(\omega_2) + B)$$

Again in contradiction to (2). Therefore the only case where (2) holds is when $A = 0$.

Proof of the Proposition. Let us rewrite (1) as follows:

$$\underset{X}{\text{Max}}\; E_{\tilde{F}_1,\tilde{b}_1} \left\{ U[(\beta Q - X)\tilde{F}_1 + XF_o + \alpha Q + Q\tilde{b}_1] \right\} , \tag{4}$$

where the subscripts denote expectation with respect to $\tilde{F}_1$ and $\tilde{b}_1$.

The first order condition is

$$E_{\tilde{F}_1,\tilde{b}_1} \{(F_o - \tilde{F}_1)U'[(\beta_1 Q - X^*)\tilde{F}_1 + X^*F_o + \alpha_1 Q + Q\tilde{b}_1]\} = 0 \tag{5}$$

By (A.2) (5) can be written as:

$$E_{\tilde{b}_1} \left\{ E_{\tilde{F}_1} (F_o - \tilde{F}_1)U'[(\beta_1 Q - X^*)\tilde{F}_1 + X^*F_o + \alpha_1 Q + Q\tilde{b}_1] \right\} = 0 \tag{6}$$

6) certainly holds if for almost all ω,

$$E_{\tilde{F}_1} \{(F_o - \tilde{F}_1)U'[(\beta_1 Q - X^*)\tilde{F}_1 + X^*F_o + \alpha_1 Q + Q\tilde{b}_1(\omega)]\} = 0 \tag{7}$$

However (7) holds at $\beta_1 Q - X^* = 0$, which is the unique solution to (7). Since (1) has a unique optimum it must be at $X^* = \beta_1 Q$.

Q.E.D.

3. CONCLUSION

We have shown that in a wide class of cases the optimal hedging ratio in futures markets is independent of preferences. The optimal hedging ratio in futures markets may well be different from that in forward markets (they will be the same only if $\beta_1 = 1$).

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