ON THE OPTIMAL POLICY OF PRICE ADJUSTMENTS WHEN DEMAND AND COST ARE UNCERTAIN

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Working Paper No. 12-81

March, 1 9 8 1

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ABSTRACT

This paper presents a model of a monopolistic firm's price adjustment. The firm's demand and cost are exposed to random shocks, and price adjustments are costly. It is shown that an increase in the riskiness of the shocks will have an ambiguous effect both on the expected size of the price adjustments and on the time interval between two consecutive price adjustments. It is also shown that an increase in the frequency of the shocks will have an ambiguous effect on the former and will decrease the latter, while an increase in the cost of price adjustments or in the interest rate will increase both the former and the latter.

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INTRODUCTION

When price adjustments are costly, a monopolistic firm may not want to adjust the price it charges for its output, even though the demand or other factors has changed since the firm first charged its present price. In order to maximize its discounted expected profit, the firm may prefer to adjust the price only when the cumulative change in the demand or other factors has reached a certain magnitude.

Barro (1972) has studied one aspect of a firm's price adjustment policy when price adjustments are costly. He assumes that each price adjustment involves a fixed cost, and that the demand function and the average (variable) cost function are linear. There is uncertainty, since the constant term in the demand curve is generated by a simple random walk which has a constant time interval between the steps and a constant size of the steps. Assuming that the firm determines ceiling and floor values for the changes in the demand that must occur before an adjustment in the price, Barro proves that the expected price change will follow the "Law of Supply and Demand."[1/]

Sheshinski and Weiss (1977) have studied another aspect of a firm's price adjustment policy when price adjustments are costly. In their model there is no uncertainty, but the general price level changes at a constant rate. They assume that each adjustment of the nominal price involves a fixed cost. They prove that the nominal price will be held constant during intervals of fixed duration, and that within each interval the real price will vary between a ceiling and a floor value that are the same for all intervals. Sheshinski and Weiss also analyze the effects of changes in the rate of inflation, the adjustment cost, and the interest rate on the length of the interval in which the nominal price will be held constant and on the values within which the real price will vary.

The present paper studies yet another aspect of a monopolistic firm's price adjustment policy when price adjustments are costly. It assumes that both the firm's demand and its cost are exposed to random shocks, and it focuses on two characteristics of the firm's policy: the expected size of the price adjustments and the expected time interval between two consecutive price adjustments. A major purpose of the paper is to show that contrary to much cursory theorizing, an increase in the riskiness of the shocks need not increase the expected size of the price adjustments and decrease the expected time interval between two consecutive price adjustments, but will affect both the former and the latter ambiguously.

Section 2 presents the model. Both the demand function and the average cost function are linear, and each price adjustment involves a fixed cost. The constant terms in the demand function and in the average cost function are generated by independent stochastic processes. The processes are generalizations of the simple random walk, and the time intervals between the shocks as well as the size of the shocks are variable. As in Barro and Sheshinski and Weiss, the model does not consider the interaction between pricing decisions of different firms, and it assumes that the firm has full knowledge of the profit it can obtain for alternative prices.[2/]

Section 3 derives the price adjustment policy which minimizes the firm's discounted expected loss due to the cost of a price adjustment. It proves that if the cost of a price adjustment is small, then every time the firm adjusts the price, it determines the new price such that there will be no forgone profit from the time of a price adjustment and until the first following shock. It also proves that the firm determines ceiling and floor values for the changes in the demand and the costs that must occur before an adjustment in the price. Furthermore, the section derives these ceiling and floor values expressed as changes in the price for which there will be no forgone profit.

Section 4 derives the expressions for the expected size of the price adjustments, and Section 5 derives the expression for the expected time interval between two consecutive price adjustments. Section 6 analyses the effects of increases in the riskiness of the demand and cost shocks; it proves that an increase in the riskiness of the shocks has ambiguous effects on the expected size of the price adjustments and on the expected time interval between two consecutive price adjustments. The following sections analyze the effects of increases in the other parameters of the model: Section 7 the effects of increases in the frequency of the shocks; Section 8 the effects of increases in the cost of price adjustment, and Section 9 the effects of increases in the interest rate. Finally Section 10 summarizes the results and also states the effects of changes in the riskiness of the shocks and in the values of the other parameters on the firm's discounted expected loss due to the cost of price adjustments.

2. THE MODEL

Consider a firm that at each time t ($t \geq 0$) produces a single non-storable good. The demand for the output at t depends on the price at t (p_t) and on a random variable. It is given by $D_t = \alpha_t - \beta p_t$, where β is a positive constant and α_t is the random variable at t. The value of α_o is known, while the value of α_t $(t > 0)$ is generated by a compound Poisson process.[3/] The changes in α_t ("shocks") occur at discrete intervals; if t_{i_D} is the time of the i_D'th demand shock $(i_D = 1,2,\ldots)$ and t_o denotes $t = 0$, then $\alpha_t = \alpha_{t_{i_D-1}}$ for $t_{i_D-1} \leq t < t_{i_D}$. The shocks occur with the frequency λ_D $(\lambda_D > 0)$. The inter-arrival times $t_{i_D} - t_{i_D-1}$ are mutually independent and have exponential distributions that depend on only λ_D. The demand shocks are mutually independent, and each shock (i.e., $\alpha_{t_{1_D}} - \alpha_{t_o}, \alpha_{t_{2_D}} - \alpha_{t_{1_D}}, \ldots$) is uniformly distributed with the probability density $(2h_D)^{-1}$, where $h_D > 0$, within the interval $[-h_D;h_D]$, and the probability density zero otherwise.

Let Y_t denote the output at t $(Y_t \geq 0)$. The average cost of producing Y_t depends on Y_t and on a random variable; it is given by $AC_t = \gamma_t + \delta Y_t$, where $\delta > -\beta^{-1}$ and γ_t is the random variable at t. The value of γ_o is known while the value of γ_t $(t > 0)$ is generated by a compound Poisson process. The process is analogous to the process generating α_t, but the two processes are independent. The cost shocks occur with the frequency λ_Y $(\lambda_Y > 0)$; the shocks are mutually independent, and each shock is uniformly distributed with the probability density $(2h_Y)^{-1}$, where $h_Y > 0$, within the interval $[-h_Y;h_Y]$, and the probability density zero otherwise.[4/]

A profit-maximizing firm would not produce if $D_t < 0 \Leftrightarrow \alpha_t\beta^{-1} < p_t$, since it would not be able to sell the output. Furthermore, the marginal cost of producing Y_t is $\gamma_t + 2\delta Y_t$, so the firm would not satisfy all demand if $p_t < \gamma_t + 2\delta D_t \Leftrightarrow p_t < (\gamma_t + 2\delta\alpha_t)(1 + 2\delta\beta)^{-1}$. Let the set I_t of p_t's be defined by $I_t \equiv \{p_t \geq 0 \mid (\gamma_t + 2\delta\alpha_t)(1 + 2\delta\beta)^{-1} \leq p_t \leq \alpha_t\beta^{-1}\}$. If $\gamma_t \leq \alpha_t\beta^{-1}$ and $\alpha_t > 0$, then I_t is not empty. It follows that for any $p_t \in I_t$ the firm will maximize its profit by choosing Y_t such that $p_t = \gamma_t + 2\delta Y_t$, and it will satisfy all demand, i.e., $D_t = Y_t$.

The following will assume that $p_t \varepsilon I_t$ and $D_t = Y_t$.[5/] The profit at t becomes

$$[p_t - AC(D_t)]D_t = [p_t - \gamma_t - \delta(\alpha_t - \beta p_t)](\alpha_t - \beta p_t)$$

$$= - p_t^2\beta(1 + \delta\beta) + p_t[\gamma_t\beta + (1 + 2\delta\beta)\alpha_t] - \gamma_t\alpha_t - \delta\alpha_t^2$$

$$= - \beta(1 + \delta\beta)\left[p_t - \frac{(1+2\delta\beta)\alpha_t+\beta\gamma_t}{2\beta(1+\delta\beta)}\right]^2 - \gamma_t\alpha_t - \delta\alpha_t^2 + \frac{[(1+2\delta\beta)\alpha_t + \beta\gamma_t]^2}{4\beta(1 + \delta\beta)} .$$

Since $\beta(1+\delta\beta) > 0$, the maximum profit at t is obtained for $p_t = \dfrac{(1+2\delta\beta)\alpha_t + \beta\gamma_t}{2\beta(1 + \delta\beta)}$.

The profit forgone at t from the actual price being different from this value is $F_t \equiv k(p_t - a\alpha_t - b\gamma_t)^2$, where $k \equiv \beta(1 + \delta\beta)$, $a \equiv \dfrac{1+2\delta\beta}{2\beta(1 +\delta\beta)}$, and $b \equiv \dfrac{1}{2(1 +\delta\beta)}$ are constants. F_t depends on the firm's price p_t and on the random variables α_t and γ_t, but is independent of t and of everything that has happened before t.

The firm incurs a fixed adjustment cost $c > 0$ when it changes the price of its product [6/], and it may therefore not want to adjust the price even though $F_t \neq 0$. Let $\overline{p}_o$ denote the price charged from the last price adjustment before t_o. (It is therefore the price that will be changed until the first price adjustment for $t \geq 0$). Since F_t depends on p_t as well as on α_t and γ_t, the expected value at t_o of F_t depends both on the firm's policy of price adjustments and on the randomness of the shocks. Similarly, whether the firm at t adjusts the price or leaves it unchanged depends both on the firm's policy of price adjustments and on the randomness of the shocks. Let L_o denote the discounted expected loss at t_o that the firm incurs because it cannot adjust the price without incurring the cost c; L_o consists of the discounted expected value of all forgone profits plus the discounted expected outlays to price adjustments. If r denotes the interest rate $(r > 0)$, L_o becomes

$$L_o = \int_0^\infty (EF_t + \mu_t c)e^{-rt}dt \quad ,$$

where EF_t (the expectation at t_o of F_t) and μ_t (the probability at t_o of a price adjustment at t) depend on $\overline{p}_o$, the firm's policy of price adjustments and the randomness of the shocks. The firm will choose a policy of price adjustments which minimizes L_o. Such policy is called an optimal policy of price adjustments.

3. THE OPTIMAL POLICY

Define a new variable q_t as the difference between the actual and the profit-maximizing price at t, i.e., $q_t \equiv p_t - a\alpha_t - b\gamma_t$. The maximum profit at t is obtained for $q_t = 0$, and the profit forgone at t from not adjusting p_t is $F_t = kq_t^2$. Note that F_t is symmetrical around $q_t = 0$. If the last price adjustment before t took place at τ_o, the forgone profit can be written as $F_t = k\,(p_{\tau_o} - a\alpha_t - b\gamma_t)^2 = k[q_{\tau_o} - a(\alpha_t - \alpha_{\tau_o}) - b(\gamma_t - \gamma_{\tau_o})]^2$. Accordingly, the forgone profit at t depends on the profit-maximizing price at τ_o, and on the changes in α_t and γ_t from τ_o to t.

The waiting time from any t to the first following shock is, by virtue of the Poisson process, independent of t and everything that has happened before t. Similarly, the shock itself is independent of t and everything that has happened before t. In terms of q_t, the future for all t's with the same value of q_t therefore looks identical, and one suspects that the firm will adapt a double-sided (S,s) policy for q_t. This is called a (S,s_1,s_2) policy. In order to define this policy, let $\overline{q}_t$ denote the value of q_t if the price is held constant at t (i.e., $\bar{q}_t \equiv p_{\tau_o} - a\alpha_t - b\gamma_t$). According to a (S,s_1,s_2) policy the firm will choose three critical values: An initial value S for q_t such that when the firm adjusts the price, it determines the new price by $q_t = S \Leftrightarrow p_t = S + a\alpha_t + b\gamma_t$; a ceiling value s_1 for $\bar{q}_t$, and a floor value s_2 for $\bar{q}_t$ $(s_2 < s_1)$, such that the firm will adjust the price of $\bar{q}_t \notin [s_2;s_1]$, will not adjust the price if $\bar{q}_t \in (s_2;s_1)$, and will be indifferent whether or not to adjust the price of $\bar{q}_t = s_1$ or $\bar{q}_t = s_2$.

It has not been possible to derive the optimal policy without any restrictions on the parameters.[7/] If, however, the adjustment cost is "small", I will show that the optimal policy is indeed a (S,s_1,s_2) policy for q_t, and I will determine the values of S, s_1, and s_2 for this policy. The condition that the adjustment cost if "small" is stated formally in condition (c) which is assumed to hold in the remaining part of the paper:

Condition (c): $c \leq \dfrac{k}{4(r+\lambda)} \mathrm{Min}(a^2h_D^2;b^2h_Y^2)$,

where $\lambda \equiv \lambda_D + \lambda_Y$.

I first prove that every time the firm adjusts the price, it determines the new price by $S = 0$. Let L_t denote the value at t of the discounted expected future losses due to the cost of price adjustments. Among other things, L_t depends on α_t and γ_t, and on whether the price is adjusted at t. Furthermore, let $\bar{q}'_\tau$ and q''_τ, $|\bar{q}'_\tau| < |\bar{q}''_\tau|$, be two alternative values of $\bar{q}_\tau$.[8/] In order to prove that $S = 0$, it suffices to show that for any policy starting from $\bar{q}''_\tau$ and for which there is no adjustment at τ, there exists a policy τ starting from $\bar{q}'_\tau$ such that L_τ from the latter is less than L_τ from the former.

Assume first that the firm for $\bar{q}''_\tau$ adjusts the price after τ, but before the first shock in α_t or γ_t after τ, i.e., if the first shock after τ occurs at τ_1, the firm for $\bar{q}''_\tau$ adjusts the price at $t \varepsilon (\tau; \tau_1)$. Let the firm for $\bar{q}'_\tau$ adjust the price for the first time at the same time as the first adjustment for $\bar{q}''_\tau$, and from then on follow the same policy of price adjustments for $\bar{q}'_\tau$ as for $\bar{q}''_\tau$. Since $|\bar{q}'_\tau| < |\bar{q}''_\tau|$, the forgone profit at each t from τ and until τ_1 is less with $\bar{q}'_\tau$ than with $\bar{q}''_\tau$. Consequently, if the firm for $\bar{q}''_\tau$ adjusts the price after τ, but before τ_1, then for any policy for $\bar{q}''_\tau$, there exists a policy for $\bar{q}'_\tau$ such that L_τ for $\bar{q}'_\tau$ is less than L_τ for $\bar{q}''_\tau$.

Assume next that the firm for $\bar{q}''_\tau$ does not adjust the price before τ_1. Let the firm for $\bar{q}'_\tau$ also not adjust the price before τ_1. The forgone profit until τ_1 is then less with $\bar{q}'_\tau$ than with $\bar{q}''_\tau$. Let $\bar{q}'_{\tau_1}$ denote $\bar{q}_{\tau_1}$ if $\bar{q}_\tau = \bar{q}'_\tau$, and $\bar{q}''_{\tau_1}$ denote $\bar{q}_{\tau_1}$ if $\bar{q}_\tau = \bar{q}''_\tau$. The shock at τ_1 is either a demand shock or a cost shock. In case it is a demand shock, α_{τ_1} is uniformly distributed with the probability density $(2h_D)^{-1}$ in the interval $[\alpha_\tau - h_D; \alpha_\tau + h_D]$. Both $\bar{q}'_{\tau_1}$ and $\bar{q}''_{\tau_1}$ are therefore uniformly distributed with the probability density $(2ah_D)^{-1}$; $\bar{q}'_{\tau_1}$ within the interval $J'_D \equiv [\bar{q}'_\tau - ah_D; \bar{q}'_\tau + ah_D]$, and $\bar{q}''_{\tau_1}$ within the interval $J''_D \equiv [\bar{q}''_\tau - ah_D; \bar{q}''_\tau + ah_D]$. Similarly, in case the shock is a cost shock, both $\bar{q}'_{\tau_1}$ and $\bar{q}''_{\tau_1}$ become uniformly distributed with the probability density $(2bh_Y)^{-1}$; $\bar{q}'_{\tau_1}$ within the interval $J'_Y \equiv [\bar{q}'_\tau - bh_Y; \bar{q}'_\tau + bh_Y]$, and $\bar{q}''_{\tau_1}$ within the interval $J''_Y \equiv [\bar{q}''_\tau - bh_Y; \bar{q}''_\tau + bh_Y]$. Let $j = D$ if the shock is in α_t, and $j = Y$ if the shock is in γ_t. All $\bar{q}'_{\tau_1} \varepsilon J'_j$ and $\bar{q}''_{\tau_1} \varepsilon J''_j$ can be paired together by $\bar{q}'_{\tau_1} = \bar{q}''_{\tau_1}$ if $\bar{q}'_{\tau_1}$ and $\bar{q}''_{\tau_1}$ both lie within $J'_j \cap J''_j$ (which may be empty), and by $\bar{q}'_{\tau_1} + \bar{q}''_{\tau_1} = \bar{q}'_{\tau} + \bar{q}''_{\tau}$ otherwise.[9/] For pairs with $\bar{q}'_{\tau_1} = \bar{q}''_{\tau_1}$ it is immediate that for any policy with $\bar{q}''_{\tau_1}$, there exists a policy with $\bar{q}'_{\tau_1}$ for which L_{τ_1} with $\bar{q}'_{\tau_1}$ is less than L_{τ_1} with $\bar{q}''_{\tau_1}$. For the remaining pairs, $|\bar{q}'_{\tau}| < |\bar{q}''_{\tau}| \Rightarrow |\bar{q}'_{\tau_1}| < |\bar{q}''_{\tau_1}|$, so that a repetition of the above arguments for τ_1, and then for the time of the second shock after τ, etc., proves that for any policy with $\bar{q}''_{\tau_1}$ there exists a policy with $\bar{q}'_{\tau_1}$ for which L_{τ_1} is less than L_{τ_1} with $\bar{q}''_{\tau_1}$. Consequently, also if the firm for $\bar{q}''_{\tau}$ does not adjust the price before τ_1, then for any policy for $\bar{q}''_{\tau}$, there exists a policy for $\bar{q}'_{\tau}$ such that L_{τ} for $\bar{q}'_{\tau}$ is less than L_{τ} for $\bar{q}''_{\tau}$. It has therefore been proved that every time the firm adjusts its price, it determines the new price by $S = 0$.

I next prove that the firm will adjust the price at τ if $\bar{q}^2_{\tau} > (r + \lambda)ck^{-1}$, will not adjust the price if $\bar{q}^2_{\tau} < (r + \lambda)ck^{-1}$, and is indifferent whether or not to adjust the price if $\bar{q}^2_{\tau} = (r + \lambda)ck^{-1}$. The compound Poisson processes generating α_t and γ_t are independent, and the probability of the first shock after τ occuring after t $(t>\tau)$ is therefore $e^{-\lambda(t-\tau)}$. If the firm does not adjust the price before τ, the discounted expected profit lost in $[\tau;\tau_1)$ is

$$k\bar{q}^2_{\tau} \int_{\tau}^{\infty} e^{-(r+\lambda)(t-\tau)}\, dt = \frac{kq^2_{\tau}}{r+\lambda} \quad . \tag{1}$$

On the other hand, if the firm at $\tau' \in [\tau;\tau_1)$ adjusts the price so that $q_{\tau'} = S = 0$, the discounted expected profit lost in $[\tau;\tau_1)$ is

$$k\bar{q}^2_{\tau} \int_{\tau}^{\tau'} e^{-(r+\lambda)(t-\tau)} dt + ce^{-(r+\lambda)(\tau'-\tau)}$$

$$= \frac{k\bar{q}^2_{\tau}}{r+\lambda}[1 - e^{-(r+\lambda)(\tau'-\tau)}] + ce^{-(r+\lambda)(\tau'-\tau)} \quad . \tag{2}$$

Subtracting (2) from (1):

$$\left(\frac{k\bar{q}_\tau^2}{r+\lambda} - c\right)e^{-(r+\lambda)(\tau'-\tau)} ,$$

which is positive and decreases in τ' for $\bar{q}_\tau^2 > (r + \lambda)ck^{-1}$, negative and increases in τ' for $\bar{q}_\tau^2 < (r + \lambda)ck^{-1}$, and vanishes for $\bar{q}_\tau^2 = (r + \lambda)ck^{-1}$.

Condition (c) implies that both for $q_\tau = 0$ (i.e., the price is adjusted at τ), and for $q_\tau = \bar{q}_\tau$ (i.e., the price is not adjusted at τ), the probability density of any $\bar{q}_{\tau_1} \varepsilon [q_\tau - [(r + \lambda)ck^{-1}]^{1/2} ; q_\tau + [(r + \lambda)ck^{-1}]^{1/2}]$ is $(2ah_D)^{-1}$ if the shock at τ_1 is in the demand, and $(2bh_Y)^{-1}$ if the shock at τ_1 is in the cost. In other words, L_{τ_1} is independent of whether the price is adjusted or kept unchanged at τ. It follows that the firm will adjust the price at τ if $\bar{q}_\tau^2 > (r + \lambda)ck^{-1}$, will not adjust the price if $\bar{q}_\tau^2 <(r + \lambda)ck^{-1}$, and will be indifferent whether or not to adjust the price if $\bar{q}_\tau^2 = (r + \lambda)ck^{-1}$. Setting $s_1^2 = s_2^2 = (r+\lambda)ck^{-1} \Leftrightarrow s_1 = -s_2 = [(r+\lambda)ck^{-1}]^{1/2}$,this can be expressed equivalently as follows: At τ the firm will adjust the price if $\bar{q}_\tau \notin [s_2; s_1]$, will not adjust the price if $\bar{q}_\tau \ \varepsilon (s_2;s_1)$, and will be indifferent whether or not to adjust the price if $\bar{q}_\tau = s_1$ or $\bar{q}_\tau = s_2$. It therefore completes the proof that under condition (c) the optimal policy of price adjustments is a (S,s_1,s_2) policy for q_t.10/ If $s \equiv [(r+\lambda)ck^{-1}]^{1/2}$, the optimal policy of price adjustments in terms of q_t is

$$(S,s_1,s_2) = (0,s, - s). \tag{3}$$

The symmetry in the optimal policy for q_t stems from the symmetric nature of the forgone profit and the shocks, as well as from the cost of a price adjustment being independent of its size and direction. The expected profit forgone from τ until the first shock after τ is $aq_\tau^2(r + \lambda)^{-1}$ when discounted to τ. The firm therefore determines S such that there will be no forgone profit from the time of a price adjustment and until the first following shock, and it determines s_1 and s_2 such that at these values of q_τ, the discounted expected profit forgone from τ and until the first following shock is equal to the adjustment cost.

In terms of p_t, the optimal policy requires that every time the firm adjusts the price the new price is determined by $p_t = a\alpha_t + b\gamma_t$. Both a and b are positive and decrease in β and δ. Consequently, the higher the highest possible demand $(D_t = \alpha_t$ for $p_t = 0)$, and the higher the smallest possible cost $(AC_t = \gamma_t$ for $Y_t = 0)$, the higher is the price after an adjustment; furthermore, the steeper the demand curve and the faster the increase in the costs, the higher is the price after an adjustment.

In addition the optimal policy requires that the firm adjusts the price every time the profit-maximizing price has changed by more than s, keeps the price unchanged if the profit-maximizing price has changed by less than s, and is indifferent whether to adjust the price or keep it unchanged when the profit-maximizing price has changed by precisely s.[11/] The effects of the parameters on s are straightforward: The more frequent the shocks are, the more often will the firm reach a given ceiling and floor value for $\overline{q}_t$. Since price adjustments are costly, the firm reacts to an increase in λ_D or λ_Y by increasing s. The higher the adjustment cost, the less attractive it is to adjust the price for any given $\overline{q}_t$, so an increase in c implies an increase in s. The higher the interest rate, the less important is the forgone earnings in the future relative to an immediate outlay for a price adjustment; an increase in r therefore also implies an increase in s.

On the other hand, the steeper the demand curve and the faster the increase in the costs, the more profit is forgone by not adjusting the price. Accordingly, if β or δ increase, the firm will reduce s. Note that h_D and h_Y do not affect the optimal policy. The reason is that under condition (c) the distribution of the values of $\overline{q}_{\tau_1}$ for which there will be no adjustment at τ_1 is independent of h_D and h_Y.

4. THE EXPECTED SIZE OF THE PRICE ADJUSTMENTS

Let $T_o \in [0;\infty)$ be the time of a price adjustment, and let T_i denote the time of the i'th shock $(i = 1,2,\ldots)$ after T_o, where both demand and supply shocks are counted. Because of the repetitive nature of the optimal policy, the expected size of a price adjustment in $[0,\infty)$ is simply the expected size of the first price adjustment after T_o. If the shock at T_1 is a demand shock, then q_{T_1} will be uniformly distributed with the probability density $(2ah_D)^{-1}$ in the interval $[-ah_D; ah_D]$; the probability of an adjustment will be $(ah_D - s)(ah_D)^{-1} = 1 - s(ah_D)^{-1}$, and the expected size of the adjustment will be $\frac{1}{2}(ah_D + s)$. Correspondingly, if the shock at T_1 is a cost shock, then q_{T_1} will be uniformly distributed with the probability density $(2bh_Y)^{-1}$ in the interval $[-bh_Y; bh_Y]$; the probability of an adjustment will be $1 - s(bh_Y)^{-1}$, and the expected size of the adjustment will be $\frac{1}{2}(bh_Y + s)$. Since the probability is $\lambda_D\lambda^{-1}$ that the shock at T_1 is a demand shock, the probability that the price will not be adjusted at T_1 is

$$1 - \lambda_D\lambda^{-1}[1 - s(ah_D)^{-1}] + 1 - \lambda_Y\lambda^{-1}[1 - s(bh_Y)^{-1}]$$

$$= s\lambda^{-1}[\lambda_D(ah_D)^{-1} + \lambda_Y(bh_Y)^{-1}] .$$

If the price is not adjusted from T_o and until T_{n+1} $(n \geq 1)$, and the shock at T_{n+1} is a demand shock, then $q_{T_{n+1}}$ will be uniformly distributed with the probability density $(2ah_D)^{-1}$ in the interval $[q_{T_n} - ah_D; q_{T_n} + ah_D]$, where $q_{T_n} \in [s_2; s_1]$. The probability of an adjustment will be $[(q_T + ah_D - s) + (-q_{T_n} + ah_D - s)](2ah_D)^{-1} = 1 - s(ah_D)^{-1}$, which is the same as the probability of an adjustment with a demand shock at T_1. So for <u>given</u> q_{T_n}, the expected (absolute) size of the price adjustment will be

$$\frac{\frac{1}{2}(q_{T_n} + ah_D + s)(q_{T_n} + ah_D - s) + \frac{1}{2}(-q_{T_n} + ah_D + s)(-q_{T_n} + ah_D - s)}{(q_{T_n} + ah_D - s) + (-q_{T_n} + ah_D - s)}$$

$$= \frac{q_{T_n}^2 + (ah_D)^2 - s^2}{2(ah_D - s)} \quad .$$

Since q_{T_n} itself is uniformly distributed [with probability density $(2s)^{-1}$] within the interval $[-s;s]$, the expected size of the price adjustment will be

$$\frac{1}{2s}\int_{-s}^{s} \frac{q_{T_n}^2 + (ah_D)^2 - s^2}{2(ah_D - s)} dq_{T_n} = \frac{s^2/3 + (ah_D)^2 - s^2}{2(ah_D - s)} \quad .$$

Correspondingly, if the price is not adjusted from T_o to T_{n+1}, and the shock at T_{n+1} is a cost shock, then the probability of an adjustment will be $1 - s(bh_Y)^{-1}$ (the same as the probability of an adjustment with a cost shock at T_1), and the expected size of the price adjustment will be $[s^2/3 + (bh_Y)^2 - s^2][2(bh_Y - s)]^{-1}$.

The expected size of a price adjustment (P) therefore becomes:

$$P = \frac{1}{2}(ah_D + s)\frac{\lambda_D}{\lambda}(1 - \frac{s}{ah_D}) + \frac{1}{2}(bh_Y + s)\frac{\lambda_Y}{Y}(1 - \frac{s}{bh_Y}) + \left[\frac{s^2/3 + (ah_D)^2 - s^2}{2(ah_D - s)} \cdot \frac{\lambda_D}{\lambda}(1 - \frac{s}{ah_D}) + \frac{s^2/3 + (bh_Y)^2 - s^2}{2(bh_Y - s)} \cdot \frac{\lambda_Y}{\lambda}(1 - \frac{s}{bh_Y})\right] \sum_{n=1}^{\infty} [\frac{s}{\lambda}(\frac{\lambda_D}{ah_D} + \frac{\lambda_Y}{ah_Y})]^n ,$$

which reduces to

$$P = \frac{\frac{\lambda_D}{2}(ah_D - \frac{s^2}{ah_D}) + \frac{\lambda_Y}{2}(bh_Y - \frac{s^2}{bh_Y}) + \frac{s^3}{6\lambda}(\frac{\lambda_D}{ah_D} + \frac{\lambda_Y}{bh_Y})^2}{\lambda - s(\frac{\lambda_D}{ah_D} + \frac{\lambda_Y}{bh_Y})} . \qquad (4)$$

5. THE EXPECTED TIME INTERVAL BETWEEN TWO CONSECUTIVE PRICE ADJUSTMENTS

The probability that i shocks $(i = 0,1,\ldots)$ will occur in $(T_o; t]$ is $e^{-\lambda(t - T_o)}\lambda^i(t - T_o)^i/i!$, and the probability that the firm will not adjust the price before the i'th shock after T_o is $\{s\lambda^{-1}[\lambda_D(ah_D)^{-1} + \lambda_Y(bh_Y)^{-1}]\}^i$. Since $\int_{T_o}^{\infty} e^{-\lambda(t-T_o)}\lambda^i(t - T_o)^i/i!dt = \lambda^{-1}$,12/ the expected time interval between two consecutive price adjustments (Δ) becomes

$$\Delta = \lambda^{-1} \sum_{i=o}^{\infty} [\frac{s}{\lambda}(\frac{\lambda_D}{ah_D} + \frac{\lambda_Y}{bh_Y})]^i = [\lambda - s(\frac{\lambda_D}{ah_D} + \frac{\lambda_Y}{bh_Y})]^{-1} . \qquad (5)$$

6. INCREASES IN THE RISKINESS OF THE SHOCKS

An increase in the riskiness of a shock may be taken to mean an increase in h_D for a demand shock and an increase in h_Y for a cost shock. Recall that h_D and h_Y do not affect the optimal policy for q_t. An increase in h_D therefore increases the expected size of the adjustment when a demand shock is the cause of the adjustment, but does not affect the expected size of the adjustment when a cost shock is the cause of the adjustment. It also increases the probability that a price adjustment will be caused by a demand shock. (It is analogous for an increase in h_Y). If the expected size of the adjustment is higher for a demand shock than for a cost shock, an increase in h_D will clearly increase P, but if the expected size of the adjustment is lower for a demand shock than for a cost shock, it is possible that an increase in h_D will decrease P because of the increase in the probability that an adjustment will be caused by a demand shock. Hence the effect of an increase in h_D on P (and analogously, the effect of an increase in h_Y on P) is ambiguous. Differentiating (4) with respect to h_D, one obtains that dP/dh_D has the same sign as

$$\lambda_D[(ah_D)^2 + s^2 - \frac{s^3}{3ah_D} - 2sah_D] + \lambda_Y[(ah_D)^2 + s^2 - \frac{s^3 + 3s(ah_D)^2}{3bh_Y} - sbh_Y]$$

$$+ (\frac{\lambda_D}{ah_D} + \frac{\lambda_Y}{bh_Y})^2 \frac{s^4}{6\lambda}. \qquad (6)$$

To show that (6) may be positive, consider the special case where $\lambda_D = \lambda_Y$, $h_D = h_Y$, $\beta = 1$, $\delta = 0$; it follows that $a = b$ and that $dP/d\lambda_D$ has the same sign as

$$(ah_D)^2 + s^2 - \frac{s^3}{3ah_D} - 2sah_D + \frac{s^4}{3(ah_D)^2} = ah_D(ah_D - 2s) + \frac{s^2}{ah_D}(ah_D - \frac{s}{3}) +$$

$$+ \frac{s^4}{3(ah_D)^2} \quad ,$$

which is positive from condition (c).

To show that (6) may be negative, consider the special case where $\lambda_D = 1$, $\lambda_Y = 2$, $h_D = 2$, $h_Y = 5$, $\beta = 1$, $\delta = 0$, $r = 1$, and $c = 1/4$. It follows that $a = b = 1/2$ and $s = 1$, and a small numerical computation will show that (6) becomes negative. Hence it has been shown that an increase in h_D (and by anology in h_Y) has an ambiguous effect on P.

An increase in h_D or h_Y will increase the probability that the price will be adjusted when a shock occurs, and it must therefore decrease Δ. Differentiating (5) with respect to h_D and h_Y,

$$\frac{d\Delta}{dh_D} = - \frac{s\lambda_D}{{}^2ah_D^2} < 0, \quad \text{and}$$

$$\frac{d\Delta}{dh_Y} = - \frac{s\lambda_Y}{\Delta^2 bh_Y^2} < 0 \quad .$$

Defining increases in the riskiness of the shocks as increases in h_D and h_Y is of course very special. A more general definition of increasing riskiness is the Rothschild and Stiglitz (1970) definition. Using their definition, increases in h_D and h_Y would be only one of many ways of increasing the riskiness, and one suspects that the effect of increases in the riskiness of the demand and cost shocks on Δ will not be as clear as when considering only increases in h_D and h_Y. In particular this is so because without the uniform distributions of the shocks and condition (c), if a shock occurs at τ, the probability density of any $\bar{q}_\tau \in [s_2;s_1]$ would in general not be independent of the price determined at the last adjustment before τ.

Due to the specification of the model, it is not possible to consider increasing riskiness in general. Within the model it is, however, possible to consider somewhat more general increases in riskiness than the increases resulting from increases in h_D and h_Y, and this is sufficient to establish that the effects of increases in the riskiness of the demand and the cost shocks on Δ are not as unambiguous as when only increases in h_D and h_Y are considered.

The model assumes that the demand shocks are generated by a compound Poisson process with intensity λ_D and the shocks uniformly distributed with mean zero and probability density $(2h_D)^{-1}$. Let λ_D and h_D be written as $\lambda_D = \theta^{-1}\rho$ and $h_D = \theta\ell$, where $\rho > 0$, $\ell > 0$, and $\theta \geq 1$. One may now interpret the process in the following way: The demand shocks are generated by a compound Poisson process with intensity ρ, while θ^{-1} is the probability that a shock is uniformly distributed with the probability density $(2\theta\ell)^{-1}$ within the interval $[-\theta\ell;\theta\ell]$, and $1-\theta^{-1}$ is the probability that a shock is degenerate and takes the value zero.

The advantage of this reformulation is that it is no longer true that if a shock occurs at τ, the probability density of any $\bar{q}_\tau \in [s_2;s_1]$ is independent of the price determined at the last adjustment before τ. When a demand shock occurs, the probability <u>density</u> of any $\bar{q}_\tau \in [s_2;s_1]$ except $q_{\tau'}$ is $(2\theta a\ell)^{-1}$, while the probability <u>atom</u> of $\bar{q}_\tau = q_{\tau'}$ is $1 - \theta^{-1}$, where τ' is the time of the last change in q_t before τ. One can now increase the riskiness of the demand shocks not only by increasing h_D, but also by increasing θ.[13/] If one increases the riskiness by increasing θ, it will decrease the frequency of the non-degenerate shocks, which will tend to increase Δ. It will, of course, also imply a smaller s and a bigger h_D which will tend to affect Δ in the opposite direction. The overall effect will, however, be to decrease Δ. Substituting $\lambda_D = \theta^{-1}\rho$ and $h_D = \theta\ell$ into (5) and differentiating with respect to θ,

$$\frac{d\Delta}{d\theta} = - \left[- \frac{\rho}{\theta^2} - \frac{s}{2(\theta^{-1}\rho+\lambda_Y+r)} \left(\frac{\rho}{\theta^2 a\ell} + \frac{\lambda_Y}{6h_Y}\right) + \frac{2s\rho}{\theta^3 a\ell} \right]$$

$$> \frac{\rho}{\theta^2} - \frac{2s\rho}{\theta^3 a\ell} = \frac{\lambda_D}{\theta} - \frac{2s\lambda_D}{\theta\, ah_D} = \frac{\lambda_D}{\theta}\left(1 - \frac{2s}{ah_D}\right) \geq 0$$

by condition (c); hence $d\Delta/d\theta > 0$. Increasing the riskiness of the demand shocks in this way implies an increase in Δ, and it has therefore been shown that an increase in the riskiness of the demand shocks (and by analogy in the riskiness of the cost shocks) has an ambiguous effect on Δ.

The reason for this ambiguity is easily understood: If an increase in the riskiness of the shocks results from an increase in h_D, it <u>increases</u> the probability that the price will be adjusted when a demand shock occurs. On the other hand, if an increase in the riskiness of the shocks results from an increase in θ, it <u>decreases</u> the probability that the price will be adjusted when a demand shock occurs.

The ambiguous effects of an increase in the riskiness of one type of shock on P and Δ do not depend on the existence of the other type of shock, e.g., the ambiguous effects of an increase in the riskiness of the demand shocks do not depend on $\lambda_Y > 0$. This is immediate for the effect on Δ, since $d\Delta/dh_D < 0$ and $d\Delta/d\theta > 0$ even if $\lambda_Y = 0$. It is less clear for the effect on P, since an increase in h_D will always increase P if $\lambda_Y = 0$. However, it is simple to show that an increase in the riskiness of the demand shocks will affect P ambiguously even if $\lambda_Y = 0$.[14/]

7. INCREASES IN THE FREQUENCY OF THE SHOCKS

The more frequent the shocks are [15/], the higher is s. An increase in the frequency of one type of shock therefore affects P in two different ways: It increases the expected size of the adjustment when either type of shock is the cause of the adjustment. In addition it changes the probability that the price will be adjusted for one type of shock rather than for the other; an increase in λ_D or λ_Y may therefore either increase <u>or</u> decrease the probability of an adjustment taking place for the type of shock for which the expected size of the adjustment is highest. In the first case P clearly increases; in the second case P may increase, but it is also possible that the increase in λ_D or λ_Y will decrease P. The effect of an increase in λ_D or λ_Y on P is therefore ambiguous. To prove this for λ_D, one needs to show that the sign of $dP/d\lambda_D$ is ambiguous. Since P is continuous in c, it is sufficient to show that the sign of $\lim_{c\to 0} dP/d\lambda_D$ is ambiguous. Using that $\lim_{c\to 0} s = 0$, one obtains from (4) that

$$\lim_{c\to 0} \frac{dP}{d\lambda_D} = d(\lim_{c\to 0} P)/d\lambda_D = d\left(\frac{\lambda_D a h_D + \lambda_Y b h_Y}{2\lambda}\right)/d\lambda_D$$

$$= \frac{\lambda_Y(ah_D - bh_Y)}{2\lambda^2} \gtrless \text{ as } \quad ah_D \gtrless bh_Y \quad .$$

The same proof can be applied for an increase in λ_Y, so it has been proved that an increase in λ_D or λ_Y has an ambiguous effect on P.[16/]

An increase in the frequency of the shocks will decrease Δ if s is held constant, but since s increases with λ_D and λ_Y, it will also have an indirect effect in the opposite direction. However, the overall effect of an increase in λ_D or λ_Y is always to decrease Δ : Differentiating (5) with respect to λ_D,

$$\frac{d\Delta}{d\lambda_D} = -\frac{1}{\Delta^2}\left[1 - \frac{s}{2(\lambda+r)}\left(\frac{\lambda_D}{ah_D} + \frac{\lambda_Y}{bh_Y}\right) - \frac{s}{ah_D}\right]$$

$$< -\frac{1}{\Delta^2}\left[1 - \frac{s}{2}\left(\frac{1}{ah_D} + \frac{1}{bh_Y}\right) - \frac{s}{ah_D}\right]$$

$$= -\frac{1}{4\Delta^2}\left[3\left(1 - \frac{2s}{ah_D}\right) + \left(1 - \frac{2s}{bh_Y}\right)\right] < 0,$$

since each of the parentheses in the last expression is positive from condition (c). It can be proved in the same way that $d\Delta/d\lambda_Y < 0$.

8. INCREASES IN THE ADJUSTMENT COST

An increase in the adjustment cost makes it less attractive to adjust the price, and it therefore increases s. As a result, there is an increase in the expected size of the adjustment both when a demand shock and when a cost shock is the cause of the adjustment; in addition there is an increase in the probability that the price will be adjusted for the type of shock for which the expected size of the adjustment is highest. Consequently, an increase in c implies an increase in P and Δ . To establish this, differentiate first (4) with respect to s; one obtains that dP/ds has the same sign as

$$\lambda_D^2(1 - \frac{s}{ah_D})^2 + \lambda_Y^2(1 - \frac{s}{bh_Y})^2 + \lambda_D\lambda_Y(\frac{2s^2}{ah_D bh_Y} + \frac{bh_Y - 2s}{ah_D} + \frac{ah_D - 2s}{bh_Y}) +$$

$$+ (\frac{\lambda_D}{ah_D} + \frac{\lambda_Y}{bh_Y})^2 \left[\lambda_D(1 - \frac{2s}{3ah_D}) + \lambda_Y(1 - \frac{2s}{3bh_Y})\right],$$

which is always positive when condition (c) holds. It follows that

$$\frac{dP}{dc} = \frac{dP}{ds}\frac{ds}{dc} = \frac{dP}{ds}\frac{s}{2c} > 0.$$

Furthermore,

$$\frac{d\Delta}{dc} = \frac{1}{\Delta^2}\ \frac{s}{2c}(\frac{\lambda_D}{ah_D} + \frac{\lambda_Y}{bh_Y}) > 0.$$

9. INCREASES IN THE INTEREST RATE

The higher the interest rate is, the lower is the weight of the future in the firm's discounted expected loss. The cost of a price adjustment is incurred at the time of the adjustment, while the benefits are obtained only afterwards. An increase in the interest rate therefore increases the profitability of keeping the price unchanged from any t and until the first following shock, rather than adjusting the price at t. As a consequence, the firm postpones price adjustments and s increases. As for an increase in c, this increases the expected size of the adjustment when either type of shock is the cause of the adjustment, and it also increases the probability that the price will be adjusted for the type of shock for which the expected size of the adjustment is highest. Both P and Δ therefore increase with r,

$$\frac{dP}{dr} = \frac{dP}{ds}\frac{ds}{dr} = \frac{dP}{ds}\frac{s}{2(r+\lambda)} > 0, \quad \text{and} \quad \frac{d\Delta}{dr} = \frac{1}{\Delta^2}\frac{s}{2(r+\lambda)}(\frac{\lambda_D}{ah_D} + \frac{\lambda_Y}{bh_Y}) > 0.$$

10. CONCLUSION

This paper presents a model of a monopolistic firm's price adjustment. The firm's demand and cost are exposed to random shocks, and each price adjustment involves a fixed cost. The firm minimizes the discounted expected loss due to the cost of price adjustments. The paper proves that if the cost of a price adjustment is small, then the firm will adapt a (S, s_1, s_2) policy of price adjustments. The major interest of the paper is to examine the effect of an increase in the riskiness of the shocks on the expected size of the price adjustments (P) and on the expected time interval between two consecutive price adjustments (Δ). It is proved that contrary to what might have been expected, an increase in the riskiness of the shocks will have an ambiguous effect on both P and Δ. The reason is that it is not the overall riskiness of the shocks which is important for the determination of P and Δ: only the riskiness of the shocks within the areas where the firm will adjust the price is important for the the determination of P, and only the probability that a shock will fall within the areas where the firm will adjust the price is important for the determination of Δ.

It is also proved that an increase in the frequency of the shocks will have an ambiguous effect on P and will decrease Δ, and that an increase in the cost of a price adjustment or in the interest rate will increase P and Δ. Even though these results are derived from a special model, the logic behind the results seems to indicate that they will be true also in a more general framework.

It may be of interest to determine the effects of changes in the riskiness of the shocks and in the values of the other parameters on the firm's discounted expected loss due to the cost of price adjustments (L_o). It is straightforward to show that an increase in the riskiness of the shocks will have an ambiguous effect on L_o. The reason is that as for P and Δ, it is not the overall riskiness of the shocks which is important for L_o; rather it is the riskiness of the shocks within the area where the firm will not adjust its price together with the probability that a shock will fall within the area where the firm will not adjust its prices that are important for the determination of L_o. It is also straightforward to show that an increase in the frequency of the shocks will increase L_o if the firm starts by adjusting the price, that an increase in the cost of a price adjustment will increase L_o, and that an increase in the interest rate will decrease L_o.

FOOTNOTES

1. The "Law of Supply and Demand" states that the price will rise when there is excess demand and fall when there is excess supply. Barro's result relates to the expected price change, and he defines supply as the output level at the time of the last price adjustment.

2. In Iwai (1974) the firm is uncertain about the profit it can obtain for alternative prices.

3. See Feller (1943) and Prabhu (1965).

4. The sum of two independent compound Poisson processes is itself a compound Poisson process. One may therefore consider the demand and cost shocks as being generated by a compound Poisson process where the shocks occur with the frequency $\lambda \equiv \lambda_D + \lambda_Y$, and a particular shock is a demand shock with probability $\lambda_D\lambda^{-1}$ and a cost shock with probability $\lambda_Y\lambda^{-1}$. Besides the mutual independence of the inter-arrival times and of the shocks, the properties of a Poisson process used in this paper is that the probability of precisely i $(i = 0,1,2,\ldots)$ shocks occurring in $(\tau;t]$ is $e^{-\lambda(t-\tau)}\lambda^i(t-\tau)^i/i!$, while the probability of the i'th shock $(i = 1,2,\ldots)$ after τ occurring at t is $e^{-\lambda(t-\tau)}\lambda^i(t-\tau)^{i-1}/(i-1)!$.

5. The derivations therefore apply only as long as $p_t \varepsilon I_t$. The non-stationary nature of the stochastic processes implies that I_t will become empty for some t with probability one. However, it is possible to choose the parameter values such that for any $t > 0$ and $\varepsilon > 0$, the probability that I_t becomes empty before t is less than ε.

   The stochastic processes may be modified to be stationary. For instance, it would not affect the optimal policy of price adjustments and the comparative-static results derived below if it were assumed that α_t is generated by a stationary compound Poisson process where the shocks occur with the frequency λ_D, and the i_D'th shock is uniformly distributed with the probability density $(2h_D + |\alpha_{t_{i_D}} - \bar{\alpha}|)^{-1}$ within the interval $[-h_D - \mathrm{Max}\,(0,\alpha_{t_{i_D}} - \bar{\alpha});\ h_D + \mathrm{Max}\,(0,\bar{\alpha} - \alpha_{t_{i_D}})]$. ($\bar{\alpha}$ is the stationary mean of α_t).

6. See Barro (1972) for a discussion of the appropriateness of modeling the price adjustment cost as fixed.

7. For an inventory problem in which all shocks are undirectional, Scarf (1959) proves that the optimal policy is of the (S,s) type. Sheshinski and Weiss (1977) prove that when there is a constant rate of inflation, but no uncertainty, then the optimal policy of nominal price adjustments is of the (S,s) type. In his study of monopolistic price adjustment Barro (1972) assumes that the firm will follow a (S,s_1,s_2) policy, but does not prove that this is optimal. I am not aware of any general proof of the optimality of a (S,s_1,s_2) policy when the shocks are not undirectional.

8. The alternative values of $\bar{q}_\tau$ may result from different policies of price adjustments before τ.

9. Values of $\bar{q}^{1}_{\tau_1}$ and $\bar{q}^{''}_{\tau_1}$ obtained for the same shock are thus in general not paired together.

10. The assumption that $p_t \in I_t$ translates into $q_t \in I'_t$, where $I'_t \equiv \{q_t \geq -a\alpha_t + b\gamma_t | (\gamma_t - \alpha_t\beta^{-1})[2(1+\delta\beta)(1+2\delta\beta)]^{-1} \leq q_t \leq (\alpha_t\beta^{-1} - \gamma_t)[2(1+\delta\beta)]^{-1}\}$. The derivation of the optimal policy therefore requires that S, s_1, $s_2 \in I'_t$; this is satisfied if $c \leq (\alpha_t\beta^{-1} - \gamma_t)^2\beta[4(1+\delta\beta)(1+2\delta\beta)^2(\lambda + r)]^{-1}$.

11. The latter possibility shows that although the (S,s_1,s_2) policy in (3) is unique, the optimal policy itself is not unique.

12. As can be seen by repeated partial integration.

13. After the reformulation the cumulative distribution $G(x)$ of a demand shock x is $G(x) = \frac{1}{2\theta}(\frac{x}{\theta\ell} + 1)$ for $-\theta\ell \leq x < 0$, $G(0) = 1 - \frac{1}{2\theta}$, and $G(x) = 1 + \frac{1}{2\theta}(\frac{x}{\theta\ell} - 1)$ for $0 < x \leq \theta\ell$. Consequently, $\int_{-\theta\ell}^{x} G(u)du = \frac{x^2}{4\theta^2\ell} + \frac{x}{2\theta} + \frac{\ell}{4}$ for $-\theta\ell \leq x \leq 0$, and $\int_{-\theta\ell}^{x} G(u)du = \frac{\ell}{4} + \frac{x^2}{4\theta^2\ell} + x - \frac{x}{2\theta}$ for $0 < x \leq \theta\ell$. It follows that $d[\int_{-\theta\ell}^{x} G(u)du]/d\theta \geq 0$ for all θ, where the inequality is strict except for $x \varepsilon \{-\theta\ell; 0; \theta\ell\}$. A higher θ therefore implies riskier demand shocks according to the Rothschild and Stiglitz definition.

14. For instance, assume that each demand shock has the probability ϕ of being uniformly distributed with probability density $(2h_D')^{-1}$ within the interval $[-h_D'; h_D']$, and the probability $1 - \phi$ of being uniformly distributed with probability density $(2h_D'')^{-1}$ within the interval $[-h_D'', h_D'']$, where $h_D' < h_D''$ and $c \leq ka^2 h_D'^2 [4(r + \lambda)]^{-1}$. The expected adjustment is lower for the shock distributed within $[-h_D'; h_D']$, than for the shock distributed within $[-h_D''; h_D'']$. An increase in h_D' increases the riskiness of the demand shock, and it is possible that it will decrease P, since it increases the probability that an adjustment will be caused by the shock for which the expected adjustment is lowest. In fact the expression for P will be as (5) with a substituted for b, ϕ for λ_D, $1 - \phi$ for λ_Y, and 1 for λ.

15. Changing the frequency of the shocks may be considered another way of changing the riskiness of the shocks.

16. If λ_D and λ_Y increase in the same proportion, then P is certain to increase. Since $\lambda_D \lambda^{-1}$ is unchanged, the only effect on P comes from the increase in s, and it is shown below that $dP/ds > 0$.

The proof that the sign of $dP/d\lambda_D$ is ambiguous depends critically on the presence of two types of shocks. If $\lambda_Y = 0$, then $dP/d\lambda_D$ would always be positive. I conjecture that if $\lambda_Y = 0$, then $dP/d\lambda_D$ would be positive for <u>any</u> distribution of the demand shocks, and not only when they are uniformly distributed.

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