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COMPARATIVE DYNAMICS OF MONETARY POLICY IN A FLOATING EXCHANGE RATE REGIME

by

Elhanan Helpman and Assaf Razin

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FOERDER INSTITUTE FOR ECONOMIC RESEARCH,
Faculty of Social Sciences,
Tel Aviv University, Ramat Aviv, Israel.

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1. INTRODUCTION

We study in this paper the dynamics of a simple two-country world economy. Each country produces traded and nontraded goods. Consumers maximize the discounted flow of utilities with the utility level depending on consumption of traded and nontraded goods. There exist in these economies well defined demand functions for money which are derived from the institutional structure in which money buys goods but goods do not buy goods. The use of national monies is modeled as in Stockman (1980) and Helpman (1981).

In contrast to conventional wisdom, we argue that expected changes in the money supply have no real effects. This statement is based on the assumption that economies evolve along a perfect foresight equilibrium path. Our result differs from that obtained by, say, Calvo and Rodriguez (1977), Liviatan (1980) and Dornbusch and Fischer (1980). In contrast with others, we fully specify the monetary mechanism and assign a major role to interest bearing assets. The adjustment of interest rates is an important ingredient in the neutrality result.

We investigate also the effects of unexpected changes in the money supply. These operate by means of a redistribution of real wealth which stems from capital gains and losses which arise due to the existence of nominal claims.[1/] They trigger a dynamic adjustment mechanism which is reflected in the behavior of all economic variables. The interesting features of these dynamic adjustments as well as the evolution of the economies along a perfect foresight equilibrium path are summarized in the last section. (Section 5). The interested reader can read, at this point, the summary of results which we provide in that section.

We describe our model in Section 2. The equilibrium time pattern of prices, interest rates and exchange rates is described in Section 3. We show that this equilibrium path is unique. Finally, the effects of monetary shocks are analyzed in Section 3. Our model lends itself also to other comparative dynamic exercises from which we abstain in this paper.

1/ See Boyer (1977) for similar effects on the analysis of devaluations.

2. THE MODEL

Assume that there are two countries, a home country (H) and a foreign country (F). Each country can be represented by a representative consumer. There is a traded good (T) whose supply in country i is $Q_{iT}(p_{iT}/p_{iN})$, i = H, F, and there is a nontraded good (N) whose supply in country i is $Q_{iN}(p_{iT}/p_{iN})$, i = H, F, where p_{iT} and p_{iN} are prices of traded and nontraded goods in country i. These supply functions are time invariant, which means that each country's transformation curve is constant over time.

Each country has its own currency, as well as an interest bearing asset which is denominated in its currency. Goods of a particular country can be bought only with that country's money. Similarly, the interest bearing asset of country i can be bought only with the help of country i's currency. The result is Clower-type liquidity constraints (see Clower (1967)).

Firms are assumed to produce the above described output levels. During a given period they sell their output and accumulate money. At the end of the period they distribute the accumulated money balances as factor and dividend payments. The supply of factors of production is completely inelastic. Home firms are owned by home residents while foreign firms are owned by foreign residents.

The decision problem of a representative consumer in the home country can be presented as follows:

(1) Choose $\{c_{HTt}, c_{FTt}, c_{HNt}, M_{Ht}, M_{Ft}, m_{Ht}, m_{Ft}, B_{Ht}, B_{Ft}\}_{t=0}^{\infty}$

to maximize $\sum_{t=0}^{\infty} \delta_H^t u_H(c_{HTt} + c_{FTt}, c_{HNt})$ subject to:

(1a) $M_{Ht} + e_t M_{Ft} \leq m_{Ht} + e_t m_{Ft} + p_{HTt-1} Q_{HTt} + p_{HNt-1} Q_{HNt} + X_{Ht}$

$$+ B_{Ht} + e_t B_{Ft} - (1+i_{Ht-1}) B_{Ht-1} - (1+i_{Ft-1}) e_t B_{Ft-1} \quad ,$$

(1b) $p_{HNt} c_{HNt} + p_{HTt} c_{HTt} + m_{Ht} = M_{Ht}$

(1c) $p_{FT} c_{FTt} + m_{Ft} = M_{Ft}$

(1d) $c_{HTt}, c_{FTt}, c_{HNt}, M_{Ht}, M_{Ft}, m_{Ht}, m_{Ft} \geq 0$

all $t = 0,1,2,\ldots$

(1e) $m_{H(-1)} = m_{F(-1)} = 0, \quad p_{HT0} Q_{HT0} + p_{NT0} Q_{HN0} = M_{H0}, \ (1 + i_{H(-1)}) B_{H(-1)} =$

$$= \overline{B}_{H0}, \ (1 + i_{F(-1)}) B_{F(-1)} = \overline{B}_{F0}$$

(1f) $\lim_{t\to\infty} \prod_{\tau=0}^{t-1} (1+i_{H\tau})^{-1} B_{Ht} \leq 0, \quad \lim_{t\to\infty} \prod_{\tau=0}^{t-1} (1 + i_{F\tau})^{-1} B_{Ft} \leq 0$

where:

c_{HTt} = home country consumption of home traded goods in period t

c_{FTt} = home country consumption of foreign traded goods in period t

c_{HNt} = home country consumption of nontraded goods in period t

M_{Ht} = home country beginning of period t holdings of home currency

M_{Ft} = home country beginning of period t holdings of foreign currency

m_{Ht} = home country end of period t holdings of home currency

m_{Ft} = home country end of period t holdings of foreign currency

B_{Ht} = home country beginning of period t short term borrowing in terms of home currency

B_{Ft} = home country beginning of period t short-term borrowing in terms of foreign currency

δ_H = home country discount factor, $0 < \delta_H < 1$

u_H = home country utility function

e_t = exchange rate -- price of F currency in terms of H currency -- in period t

p_{HTt} = home currency price of home traded goods in period t

Q_{HTt} = home output of traded goods in period t, $Q_{HTt} = Q_{HT}(p_{HTt}/p_{HNt})$

p_{HNt} = home currency price of home nontraded goods in period t

Q_{HNt} = home output of nontraded goods in period t, $Q_{HNt} = Q_{HN}(p_{HTt}/p_{HNt})$

p_{FTt} = foreign currency price of foreign traded goods

X_{Ht} = home money injection at the beginning of period t

i_{Ht} = period t interest rate on home currency denominated debt

i_{Ft} = period t interest rate on foreign currency denominated debt

The consumer maximizes the discounted flow of utilities, where the temporal utility level depends on the consumption of traded and nontraded goods. Foreign and home traded goods are perfect substitutes in consumption . Aggregate liquidity in period t is described by (1a). It is determined by monies carried over from the previous period, by dividend payments, by monetary transfers, and by borrowing net of debt repayments. This liquidity is split at the beginning of the period, via the foreign exchange market, into home and foreign currencies. Equations (1b) and (1c) describe the use of currencies during the period -- these are the liquidity constraints. Part of the money is spent on goods while the rest of it is carried forward to the next period. Condition (1d) is just a nonnegativity constraint while (1e) describes the initial conditions. In period $t = 0$ there are debt repayments $\overline{B}_{H0}$ and $\overline{B}_{F0}$ which have been inherited from the past and they are unalterable in period $t = 0$. Also, we assume that no monies were carried over by consumers from the past into period $t = 0$. This will turn out to be consistent with later developments. However, the quantity of money M_H was accumulated by firms in the previous period and this is paid out as dividends to the consumers. This is the initial quantity of home money in the system. Finally, (1f) describes terminal conditions of debt repayments (i.e., debt is not allowed to grow for ever at a rate that exceeds the rate of interest). This is the infinite horizon analogue of full debt repayment at the end of a finite horizon.

A problem similar to (1) exists also for the foreign country. We abstain from presenting it here. However, since we need foreign variables for future purposes, let us use the convention that variables with asterisks denote decision variables of the foreign country. For example, M^*_{Ht} denotes home currency holdings by foreign residents (the foreign representative consumer) at the beginning of period t.

An equilibrium of our world economy relative to given monetary transfer policies, consists of commodity prices, exchange rates and interest rates such that the maximizing variables of the individual decision problems satisfy:

(2a) $$c_{HNt} = Q_{HNt}$$

(2b) $$c^*_{FNt} = Q_{FNt}$$

(2c) $$c_{HTt} + c^*_{HTt} = Q_{HTt}$$

(2d) $$c_{FTt} + c^*_{FTt} = Q_{FTt}$$

(2e) $$M_{Ht} + M^*_{Ht} = M_H + \sum_{\tau=0}^{t} X_H$$

(2f) $$M_{Ft} + M^*_{Ft} = M_F + \sum_{\tau=0}^{t} X_H$$

(2g) $$B_{Ht} + B^*_{Ht} = 0$$

(2h) $$B_{Ft} + B^*_{Ft} = 0$$

all $t = 0,1,2,\ldots$. Conditions (2a) - (2b) describe equilibrium in markets for nontraded goods, (2c)-(2d) describe equilibrium in markets for traded goods, (2e)-(2f) describe equilibrium in money market, and (2g)-(2h) describe equilibrium in debt markets.

It is easy to show that in an equilibrium of our economy two parity conditions have to be satisfied:

(3a) $$p_{Ht} = e_t p_{Ft}$$

(3b) $$1+i_{Ht} = (1+i_{Ft})e_{t+1}/e_t$$

all $t = 0,1,\ldots$. The first parity condition describes the law of one price for traded goods, while the second describes the interest rate parity condition.

Let $c_{Tt} = c_{HTt} + c_{FTt}$ and $c^*_{Tt} = c^*_{HTt} + c^*_{FTt}$ be period t aggregate consumption of traded goods at home and abroad, respectively. Then, following the procedure employed in Helpman (1981) it can be shown that in an equilibrium with perfect foresight consumption levels are solutions to the following problems:

(4) Choose $\{c_{Tt}, c_{HNt}\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \delta_H^t u_H(c_{Tt}, c_{HNt})$$

subject to:

(4a) $$\sum_{t=0}^{\infty} d_t(p_{FTt}c_{Tt} + q_{HNt}c_{HNt}) \leq \sum_{t=0}^{\infty} d_t(p_{FTt}Q_{HTt} + q_{HNt}Q_{HNt}) - \overline{B}_{H0}/e_0 - \overline{B}_{F0}$$

(4b) $$c_{Tt}, c_{HNt} \geq 0$$

where $d_0 = 1$ and $d_t = \prod_{\tau=0}^{t-1}(1+i_{F\tau})^{-1}$, $t = 1,2,\ldots$, are discount factors, using <u>foreign</u> interest rates, and $q_{HNt} = p_{HNt}/e_t$ is the price of home nontraded goods in terms of foreign currency. For the foreign country the problem is:

(5) $$\text{Choose } \{c^*_{Tt}, c^*_{FNt}\}_{t=0}^{\infty} \text{ to maximize}$$

$$\sum_{t=0}^{\infty} \delta^t_F u_F(c^*_{Tt}, c^*_{FNt})$$

subject to:

(5a) $$\sum_{t=0}^{\infty} d_t(p_{FTt}c^*_{Tt} + p_{FNt}c^*_{FNt}) \leq \sum_{t=0}^{\infty} d_t(p_{FTt}Q_{FTt} + p_{FNt}Q_{FNt}) + \overline{B}_{H0}/e_0 + \overline{B}_{F0}$$

(5b) $$c^*_{Tt}, c^*_{FNt} \geq 0$$

Observe from (4a) and (5a) that each country's present value of spending is constrained to be smaller or equal to the present value of its output plus an indebtedness term. The indebtedness term stems from past borrowing and its net value across countries is zero; i.e., the debt of one country is the credit of the other.

It is clear from problems (4) and (5) that as of period $t = 0$ the perfect foresight equilibrium paths of <u>consumption</u> depend on monetary policies only to the extent that they affect the real value of the net indebtedness account $\overline{B}_{H0}/e_0 + \overline{B}_{F0}$. This is, in fact, the single channel through which monetary policy may have real effects in a perfect foresight world.

It can be shown that whenever the nominal interest rate on a particular currency denominated debt, say, currency j, is positive, no one will use in that period this currency for store of value purposes, i.e., in that period $m_{jt} = m^*_{jt} = 0$ (see Helpman (1981)). For simplicity, we will consider only monetary policies which assume positive interest rates in each country and in every period. In this case all money will be used for transaction purposes, which implies via (1b), (1c), similar condition for the foreign country, the parity condition (3a), the definition of q_{HTt}, and the equilibrium conditions (2a) - (2f):

(6a) $$p_{FTt}Q_{HTt} + q_{HNt}Q_{HNt} = \overline{M}_{Ht}/e_t$$

(6b) $$p_{FTt}Q_{FTt} + p_{FNt}Q_{FNt} = \overline{M}_{Ft}$$

for $t = 0,1,\ldots,$ where $\overline{M}_{jt} = M_j + \sum_{\tau=0}^{t} X_{j\tau}$ is the supply of currency j in period t. This completes the general description of our model. In the next section we specialize this model and provide a complete characterization of the equilibrium values of prices, interest rates and exchange rates.

3. THE TIME PATTERN OF PRICES, INTEREST RATES,AND EXCHANGE RATES

In order to demonstrate the response of prices, interest rates and nominal as well as real exchange rates to monetary shocks, we develop an explicit solution. First we assume that outputs of traded and nontraded goods are unalterable so that $Q_{jTt} = Q_{jT}$ and $Q_{jNt} = Q_{jN}$, $j = H, F,$ for all t. Second, we assume the following forms of utility functions:

(7a) $$u_H(c_T, c_{HN}) = \alpha_{HT}\log c_T + \alpha_{HN}\log c_{HN} + k_H$$

$$\alpha_{HT}, \alpha_{HN} > 0, \quad \alpha_{HT} + \alpha_{HN} = 1$$

$$k_H = -\alpha_{HT}\log\alpha_{HT} - \alpha_{HN}\log\alpha_{HN}$$

(7b) $$u_F(c_T^*, c_{FN}^*) = \alpha_{FT}\log c_T^* + \alpha_{FN}\log c_{FN}^* + k_F$$

$$\alpha_{FT}, \alpha_{FN} > 0, \quad \alpha_{FT} + \alpha_{FN} = 1$$

$$k_F = -\alpha_{FT}\log\alpha_{FT} - \alpha_{FN}\log\alpha_{FN}$$

In order to solve problems (4)-(5), we adopt a two-stage procedure. We concentrate only on problem (4) since the solution to (5), the foreign country, is similarly found. Our procedure consists of separating the temporal allocation of spending between the two goods from the intertemporal allocation of spending. This separation is possible with the utility functions described by (7), since it yields an indirect utility function which is additively separable in prices and spending.

Let:

$$Z_t = p_{FTt}c_{Tt} + q_{HNt}c_{HNt} \tag{8}$$

be the home country's spending in period t in terms of foreign currency, and let:

$$W_0 = \sum_{t=0}^{\infty} d_t(p_{FTt}Q_{HT} + q_{HNT}Q_{HN}) - \overline{B}_{H0}/e_0 - \overline{B}_{F0} \tag{9}$$

be the home country's net wealth in terms of foreign currency at the beginning of period zero. Observe that net wealth depends on both prices and interest rates that will prevail over the entire horizon. We assume at this stage that W_0 is finite, but we will show that this is indeed the case when we calculate the equilibrium values of prices and interest rates and the exchange rate e_0. However, W_0 is not a decision variable of the consumer at time $t = 0$ (although he influenced its value in period $t = -1$).

The indirect utility function of (7a) is:

$$v_H(p_{FT},q_{HN},Z) = -\alpha_{HT}\log p_{FT} - \alpha_{HN}\log q_{HN} + \log Z$$

Hence, the optimal allocation of spending as implied by (4) is the solution to:

$$\text{Choose } \{Z_t\}_{t=0}^{\infty} \text{ to maximize}$$

$$\sum_{t=0}^{\infty} \delta_H^t \log Z_t$$

subject to:

$$\sum_{t=0}^{\infty} d_t Z_t \leq W_0$$

$$Z_t \geq 0$$

The solution to this problem yields:

$$(10) \qquad Z_t = (1-\delta_H)(\frac{\delta_H^t}{d_t} W_0) \quad , \; t = 0,1,2,\ldots \;.$$

It is easy to show that $W_0 \delta_H^t / d_t$ is net wealth at the beginning of period t. 1/ Hence, our individual spends at each period a constant fraction $(1 - \delta_H)$ of his net wealth at the beginning of that period.

Now, using the demand functions which are derivable from (7a) in combination with (10), we obtain the following demands:

$$(11) \qquad c_{Tt} = \alpha_{HT}(1 - \delta_H) \frac{\delta_H^t}{d_t} W_0 / p_{FTt}$$

$$(12) \qquad c_{HNt} = \alpha_{HN}(1 - \delta_H) \frac{\delta_H^t}{d_t} W_0 / q_{HNt}$$

Applying a similar procedure to problem (5), we obtain the following demands by the foreign country:

1/ Changes in indebtedness can be directly calculated from changes in wealth (the current account). We will not present a solution for the debt variables.

(13) $$c^*_{Tt} = \alpha_{FT}(1 - \delta_F)\left(\frac{\delta^t_F}{d_t}W^*_0\right)/p_{FTt}$$

(14) $$c^*_{FNt} = \alpha_{FN}(1 - \delta_F)\left(\frac{\delta^t_F}{d_t}W^*_0\right)/p_{FNt}$$

where all $t = 0,1,...$ and

(15) $$W^*_0 = \sum_{t=0}^{\infty} d_t(p_{FTt}Q_{FT} + p_{FNt}Q_{FN}) + \overline{B}_{H0}/e_0 + \overline{B}_{F0}$$

Take now (6) and the equilibrium conditions in commodity markets combined with the demands (11)-(14), to obtain

(16a) $$p_{FTt}Q_{HT} + q_{HNt}Q_{HN} = \overline{M}_{Ht}/e_t$$

(16b) $$p_{FTt}Q_{FT} + p_{FNt}Q_{FN} = \overline{M}_{Ft}$$

(16c) $$q_{HNt}Q_{HN} = \alpha_{HN}(1 - \delta_H)W_0\delta^t_H/d_t$$

(16d) $$p_{FNt}Q_{FN} = \alpha_{FN}(1 - \delta_F)W^*_0\delta^t_F/d_t$$

(16e) $$p_{FTt}(Q_{HT} + Q_{FT}) = \alpha_{HT}(1 - \delta_H)W_0\delta^t_H/d_t + \alpha_{FT}(1-\delta_F)W^*_0\delta^t_F/d_t$$

Conditions (16c)-(16d) are equilibrium conditions in nontraded goods markets while (16e) is an equilibrium condition in the market for traded goods. Observe that, given the net wealth variables W_0 and W^*_0, (16) determines the equilibrium time path of prices, exchange rates and the foreign interest rate factors. (The domestic interest rate factors can be derived using the interest rate parity condition). Combining (16b) and (16d) with (16e), we obtain:

$$(17)\qquad d_t = \overline{M}_{FT}^{-1}[\theta_{FT}\alpha_{HT}(1-\delta_H)W_0\delta_H^t + (\alpha_{FN} + \theta_{FT}\alpha_{FT})(1-\delta_F)W_0^*\delta_F^t], \qquad t = 0,1,\ldots$$

where $\theta_{FT} = Q_{FT}/(Q_{HT} + Q_{FT})$ is the share of foreign traded goods in the world's supply of traded goods.

In order to have positive foreign interest rates in all periods, it is required that the stock of foreign money does not decline too fast. This is seen from (17) by observing that the expression in the square bracket declines over time $(0 < \delta_H, \delta_F < 1)$ while $\overline{M}_{Ft}$ is the stock of foreign money in circulation at the beginning of period t. Clearly, if foreign money increases over time, d_t declines over time implying positive interest rates in all periods. We assume that the money supply behaves so as to assure positive interest rates. Observe also that there is an initial condition $d_0 = 1$ which has to be satisfied. We will, however, take care of this condition at a later stage.

Finally, note that since $(1 + i_{Ft}) = d_t/d_{t+1}$, equation (17) implies $\lim_{t\to\infty} (1+i_{Ft}) = \lim_{t\to\infty}(\overline{M}_{Ft+1}/\overline{M}_{Ft})/\max(\delta_H,\delta_F)$, provided this limit exists. Suppose the rate of foreign monetary expansion goes to a finite limit μ_F and let $\delta_j = \frac{1}{1+\rho_j}$. Then $1 + i_{Ft}$ approaches $(1 + \mu_F)[1+\min(\rho_N,\rho_F)]$ which means that the long run nominal interest rate equals (approximately) the rate of monetary growth plus the minimal rate of time preference (which determines the long run real interest).

Now, combining (16e) with (17), we obtain the time path of traded goods' prices.

$$(18)\quad p_{FTt} = \frac{\overline{M}_{Ft}[\alpha_{HT}(1-\delta_H)W_0\delta_H^t + \alpha_{FT}(1-\delta_F)W_0^*\delta_F^t]}{(Q_{HT} + Q_{FT})[\theta_{FT}\alpha_{HT}(1-\delta_H)W_0\delta_H^t + (\alpha_{FN} + \theta_{FT}\alpha_{FT})(1-\delta_F)W_0^*\delta_F^t]}$$

In order to obtain some insight into the behavior of the time path of prices in relation to monetary policies, observe that (18) can be written in the form:

$$(19)\qquad p_{FTt} = \frac{\overline{M}_{Ft}}{Q_{HT}+Q_{Ft}} S[(W_0/W_0^*)(\delta_H/\delta_F)^t]$$

where the function $S(x)$ is defined as:

$$(20)\qquad S(x) \equiv \frac{1}{\theta_{FT}} \quad \frac{\alpha_{HT}(1-\delta_H)x + \alpha_{FT}(1-\delta_F)}{\alpha_{HT}(1-\delta_H)x + \alpha_{FT}(1-\delta_F) + (\alpha_{FN}/\theta_{FT})(1-\delta_F)}$$

The function $S(x)$ is increasing in x and bounded for all $0 \leq x \leq +\infty$. Using (19), we calculate:

$$(19a)\qquad \frac{p_{FT\ t+1}}{p_{FTt}} = \frac{\overline{M}_{Ft+1}}{M_{Ft}} \cdot \frac{S[(W_0/W_0^*)(\delta_H/\delta_F)^{t+1}]}{S[(W_0/W_0^*)(\delta_H/\delta_F)^t]}$$

The implication of (19a) is as follows: If $\delta_H > \delta_F$, then prices of traded goods in terms of foreign currency increase faster than the rate of monetary expansion in the foreign country, while if $\delta_H < \delta_F$ prices rise more slowly than the rate of monetary expansion. For $\delta_H = \delta_F$ prices rise exactly at the rate of money growth. However, independently of time preferences, the rate of price increase approaches in the limit (as $t \to \infty$) the rate of money growth.

From (16c), (16d) and (17), we calculate the time path of prices of the nontraded goods:

$$(21)\qquad q_{HNt} = \frac{\overline{M}_{Ft}}{Q_{HN}} \frac{\alpha_{HN}(1-\delta_H)W_0\delta_H^t}{\theta_{FT}\alpha_{HT}(1-\delta_H)W_0\delta_H^t + (\alpha_{FN}+\theta_{FT}\alpha_{FT})(1-\delta_F)W_0^*\delta_F^t}$$

$$(22)\qquad p_{FNt} = \frac{\overline{M}_{Ft}}{Q_{FN}} \frac{\alpha_{FN}(1-\delta_F)W_0^*\delta_F^t}{\theta_{FT}\alpha_{HT}(1-\delta_H)W_0\delta_H^t + (\alpha_{FN}+\theta_{FT}\alpha_{FT})(1-\delta_F)W_0^*\delta_F^t}$$

$$(23)\qquad e_t = \frac{\overline{M}_{Ht}}{\overline{M}_{Ft}}K[(W_0/W_0^*)(\delta_H/\delta_F)^t]$$

where

$$(24)\qquad K(x) \equiv \frac{\theta_{FT}\alpha_{HT}(1-\delta_H)x + (\alpha_{FN}+\theta_{FT}\alpha_{FT})(1-\delta_F)}{(\alpha_{NT}+\theta_{HT}\alpha_{HT})(1-\delta_H)x + \theta_{HT}\alpha_{FT}(1-\delta_F)}$$

Since $\theta_{FT}\alpha_{HT} < \alpha_{FN} + \theta_{FT}\alpha_{FT}$ and $\theta_{HT}\alpha_{FT} < \alpha_{NT} + \theta_{HT}\alpha_{HT}$, $K(x)$ is decreasing in x.1/

From (21), prices of home nontraded goods measured in foreign currency rise faster or slower than the rate of growth of foreign money supply, depending on whether $\delta_H > \delta_F$ or $\delta_H < \delta_F$. Alternatively, since the domestic currency price of home nontraded goods is $p_{HNt} = e_t q_{HNt}$, by combining (21) with (23)-(24), it is straightforward to see that the home currency price of home nontraded goods

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1/ Observe that $\theta_{FT}\alpha_{HT} - \alpha_{FN} - \theta_{FT}\alpha_{FT} = -1 + \theta_{FT}\alpha_{HT} + \theta_{HT}\alpha_{FT}$. The last two terms represent a weighted average of α_{HT} and α_{FT}, both smaller than one. Hence the weighted average is also smaller than one.

rises faster or slower than the rate of growth of domestic money supply depending also on whether $\delta_H > \delta_F$ or $\delta_H < \delta_F$. Similarly, from (22), prices of foreign nontraded goods (in terms of foreign currency) rise faster or slower than the rate of growth of the foreign money supply depending on whether $\delta_F > \delta_H$ or $\delta_F < \delta_H$.

From (23), and the fact that $K(\cdot)$ is a decreasing function, it is clear that the rate of depreciation of the local currency is larger or smaller than the difference in the rates of monetary growth at home and abroad depending on whether $\delta_H < \delta_F$ or $\delta_H > \delta_F$. However, independently of time preference, the rate of depreciation approaches the difference in the rates of monetary expansion as time goes to infinity (in the long run).

The domestic currency price of traded goods equals $e_t p_{FTt}$. Combining (18) with (23)-(24), we find that this price rises faster or slower than the rate of growth of the domestic money supply depending on whether $\delta_H < \delta_F$ or $\delta_H > \delta_F$.

Finally, consider the behavior of the real exchange rates; the home real exchange rate is p_{FTt}/q_{HNt} while the foreign real exchange rate is p_{FTt}/p_{FNt}. Combining (18) with (21) we find that the home real exchange rate increases or decreases over time as $\delta_H < \delta_F$ or $\delta_H > \delta_F$, while combining (18) with (22) we find that the foreign real exchange rate increases or decreases over time as $\delta_H > \delta_F$ or $\delta_H < \delta_F$.

In order to better understand the behavior of prices that was described above, consider the case $\delta_H > \delta_F$. In this case the foreign rate of time preference is larger than the home country rate of time preference.

For present purposes, assume also that there are no changes in the supply of monies. This implies (via (10) and (17)) that home spending -- measured in terms of foreign currency -- increases over time, while foreign spending -- also measured in terms of foreign currency -- declines over time.[1/] These shifts in spending increase the demand for nontraded goods at home and reduce the demand for nontraded goods abroad, resulting in an appreciation of the real exchange rate at home and a depreciation of the real exchange rate abroad. Moreover, the increased demand for home nontraded goods increases the domestic currency price of nontraded goods, which requires more home money for transactions in the market for nontraded goods. This leaves less home money for transactions with home firms producing traded goods, thereby causing a reduction in the home currency price of traded goods. In the foreign country nominal prices move in the opposite direction. Since the foreign price of traded goods rises while the domestic price falls, the exchange rate appreciates at a rate that exceeds the rate of change (in absolute value) of either price of traded goods.

If there is growth in money supplies, the preceding discussion remains valid but needs a slight reinterpretation. Everything that we said about home prices should be interpreted as applying relative to the trend in home money growth while for foreign prices the relative stickyard is foreign money growth. Finally, the behavior of the exchange rate is relative to the differential trend in rates of money growth. The trend in real exchange rates is independent of monetary policies. Figures 1-4 describe the trend in prices and money supply for the case $\delta_H > \delta_F$.

1/ For $\delta_H < \delta_F$ this pattern is reversed and so are all other patterns discussed below.

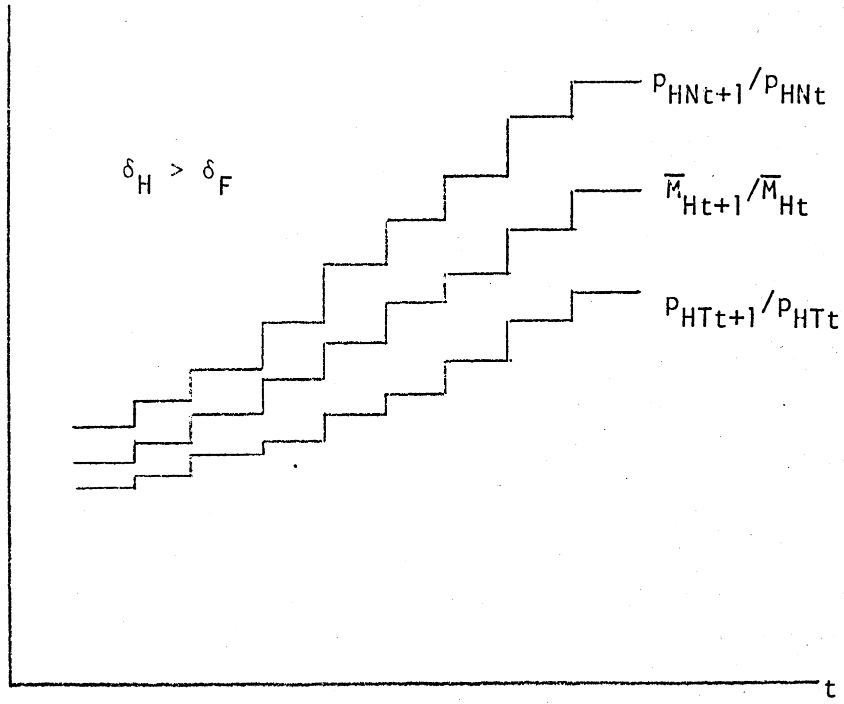

FIGURE 1

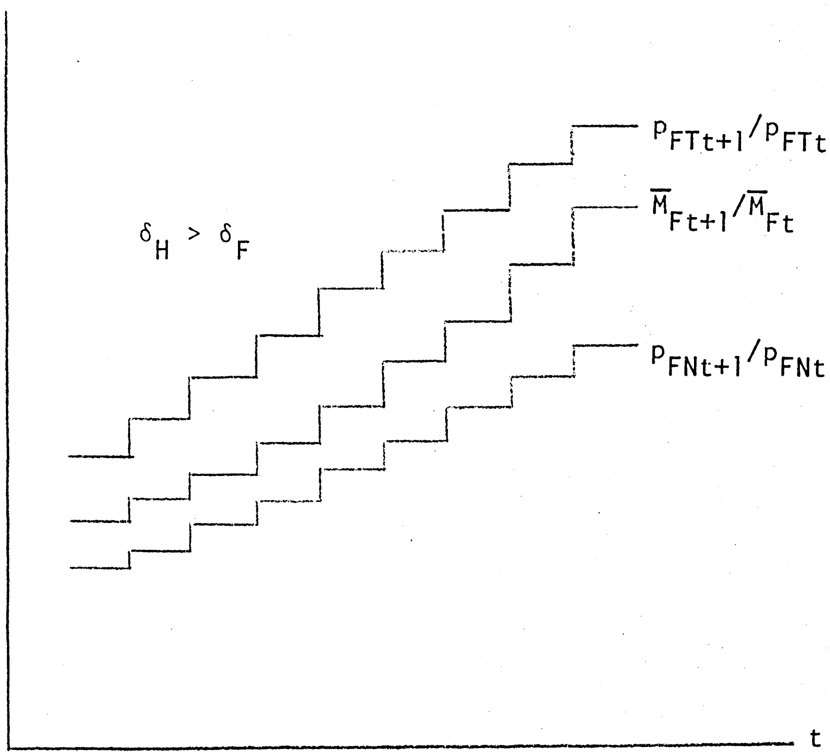

FIGURE 2

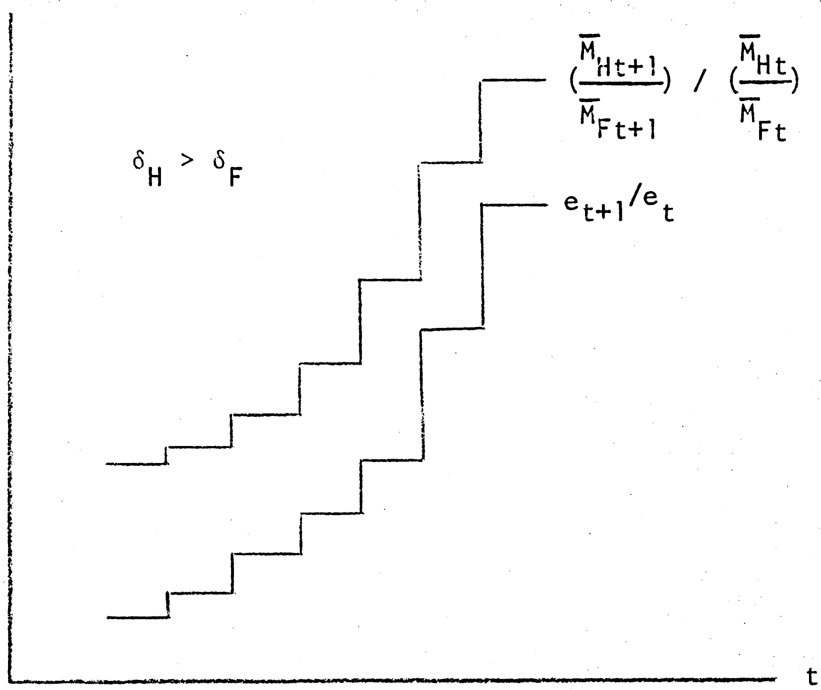

FIGURE 3

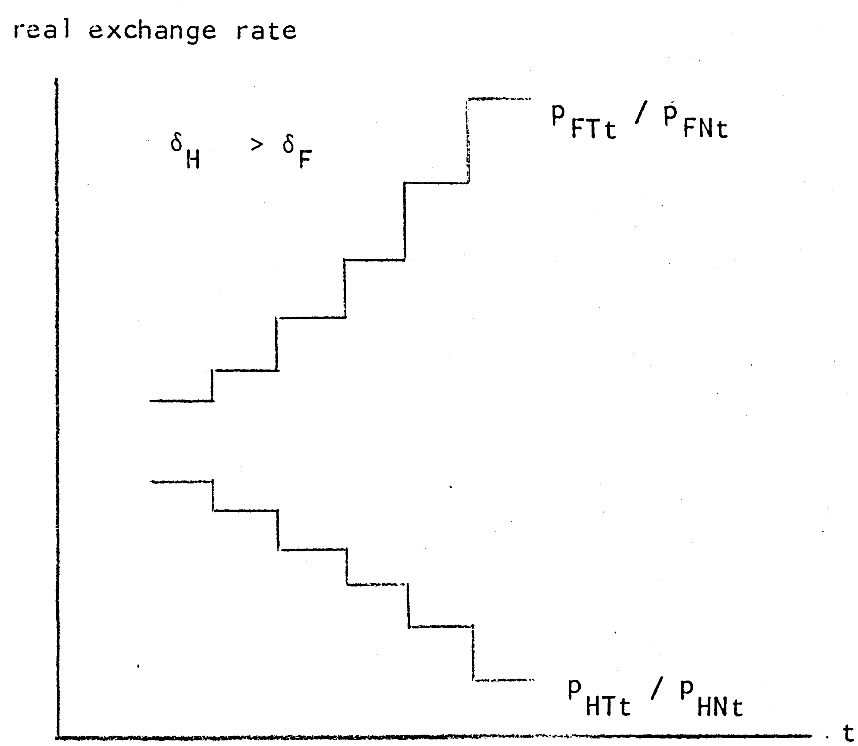

FIGURE 4

We close this section with a discussion of price indices. First, consider the GNP deflator. Since all money is used for transaction purposes, the rate of change in the GNP deflator equals in each country to the rate of growth of its money supply. Since the rate of change in the exchange rate is not equal to the differential in the rates of growth of the money supplies (unless $\delta_H = \delta_F$), PPP on GNP deflators does not hold, except in the limit (as t goes to infinity). However, the rate of change in the GNP deflator does not equal generally to the rate of change in the consumer price index (except in the limit as t goes to infinity). For the utility functions we have used there is an exact cost of living index, which is:

$$P_{Ft} = (p_{FTt})^{\alpha_{FT}}(p_{FNt})^{\alpha_{FN}}$$

for the foreign country. Call it the CPI. A similar index is defined for the home country. It is shown in Appendix I that the CPI increases faster than the rate of monetary expansion in the country that runs a deficit in the trade account (not the current account!) if it also has a higher rate of time preference and it rises slower than the rate of monetary expansion in the other country. If the deficit country has a lower rate of time preference its CPI increases slower than its rate of money growth with the opposite taking place in the surplus country.

4. RESPONSES TO MONETARY SHOCKS

We have described in the previous section the time paths of prices, exchange rates and interest rates. These time paths were based on the assumption that the economy evolves along a perfect foresight equilibrium path. Observe, however, that our equilibrium time path depends on the values of W_0 and W_0^* which represent present value wealth of each country, and these are endogenous variables which depend on the equilibrium time paths of prices, exchange rates and interest rates. This dependence is of no consequence for the qualitative description of the intertemporal equilibrium. But, it is of major importance for the description of responses of the economy to shocks. These issues are investigated in the present section. We consider unanticipated monetary shocks as well as anticipated monetary shocks.

In order to solve for W_0 and W_0^*, we also have to solve simultaneously for the exchange rate in period zero. We form, therefore, a three equation system which enables an explicit solution.

From (15), the definition of W_0^*, (16b) and (17), we obtain:

$$(25) \qquad \theta_{FT}\alpha_{HT}W_0 - \theta_{HT}\alpha_{FT}W_0^* + \bar{B}_{H0}e_0^{-1} = -\bar{B}_{F0}$$

Now, imposing the initial condition $d_0 = 1$, we get from (17):

$$(26) \qquad \theta_{FT}\alpha_{HT}(1-\delta_H)W_0 + (\alpha_{FN} + \theta_{FT}\alpha_{FT})(1-\delta_F)W_0^* = \bar{M}_{F0}$$

Finally, evaluating (23) at $t = 0$, and using (26), we obtain

$$(27) \qquad (\alpha_{HN} + \theta_{HT}\alpha_{HT})(1-\delta_H)W_0 + \theta_{HT}\alpha_{FT}(1-\delta_F)W_0^* - \bar{M}_{H0}e_0^{-1} = 0$$

Equations (25)-(27) provide a system of linear equations in W_o, W_o^* and e_0^{-1}. It is immediately clear from this system that the solution does not depend on the course ofmonetary policies beginning with period one. Hence, anticipated changes in monetary policy have no real effects, since we have shown in the previous section that real variables depend only on this triple (given the parameters of the system).

This observation is important because it had been argued in the literature that a change in the <u>rate</u> of monetary expansion (which means in effect a change in future quantities of money with no effect on the present stocks) as well as a future once and for all change in the quantity of money have immediate effects on the exchange rate as well as real effects on the equilibrium time path along which the economy evolves (see Calvo and Rodriguez (1977), Liviatan (1980) and Dornbusch and Fischer (1980), for example).[1/] In the present framework, in which the use of money in the economies has been explicitly modeled, there exist no effects of this nature. This finding is in line with the findings in Helpman (1981). It is, for example, easy to see that an anticipated increase in the money supply of the home country increases the exchange rate in period t, e_t, by the rate of increase in the stock of money, while an anticipated increase in the foreign money supply reduces the exchange rate in period t by the rate of increase in the stock of money. The exchange rate does not change in periods in which there are no changes in the stocks of monies.

In the present system unanticipated changes in the money supply have real effects which stem from the fact that they change the real value of international indebtedness. If the composition of debt is such that an unanticipated change in the money supply does not change the real indebtedness positions, then no real effects arise. For example, suppose $\overline{B}_{H0} = 0$, i.e., there are no international debts denominated in home currency, then it is easy to see from (25)-(27) that W_0 and W_0^* will not change as a result of an increase in $\overline{M}_{H0}$ and the exchange rate e_0 will increase in the same proportion as the increase

---

1/ We would like to point out that these studies assume full price flexibility. Clearly, with price rigidity real effects are expected to arise.

in $\overline{M}_{H0}$. This will have no real effects. On the other hand, if $\overline{B}_{F0} = 0$, i.e., there is no international debt in foreign currency, then an increase in $\overline{M}_{F0}$ causes an equiproportional increase in W_0 and W_0^* and an equiproportional fall in e_0. This has no real effects, since W_0 and W_0^* are denominated in terms of foreign currency.

Without surprises in monetary policy, equations (25)-(27) provide a solution of the initial conditions which are needed for a complete description of the equilibrium time paths of all economic variables that we described in the previous section. Note that these time paths as well as the initial conditions are uniquely determined. This implies that with perfect foresight there is a unique equilibrium, and we need not <u>assume</u> that the economy chooses a particular perfect foresight path (e.g., a stable arm path) as is done in some studies (see, for example, Calvo and Rodriguez (1977) and Liviatan (1980).)[1/]

Now suppose that at time $t = 0$ the monetary authority of the home country increases unexpectedly the domestic money supply for only one period. In order to compare the new equilibrium with the equilibrium that would have resulted had not the domestic money stock been increased, we need only to compute the change in W_0 and W_0^* and then see how this affects the course of all variables that were discussed in the previous section. From (25)-(27) we calculate:

$$\text{(28)} \qquad \frac{dW_0}{d\overline{M}_{H0}} = -\frac{1}{\Delta} e_0^{-1} \overline{B}_{H0}(\alpha_{FN} + \theta_{FT}\alpha_{FT})(1 - \delta_F)$$

1/ Our argument is valid only if there exists an equilibrium. But an equilibrium fails to exist only if the solution to (25)-(27) yields a negative value of W_0 or W_0^*. This may indeed happen if either $\overline{B}_{H0}$ or $\overline{B}_{F0}$ is too large in absolute value. In this case one of the countries is not able to repay its debts even if its consumption is zero in every period. See Appendix II.

(29) $$\frac{dW_0^*}{d\overline{M}_{HO}} = \frac{1}{\Delta} e^{-1} \overline{B}_{HO} \theta_{FT} \alpha_{HT} (1 - \delta_H)$$

(30) $$\frac{de_0^{-1}}{d\overline{M}_{HO}} \frac{\overline{M}_{HO}}{e_0^{-1}} = \frac{1}{\Delta} \overline{M}_{HO} \theta_{FT} \alpha_{HT} [(\alpha_{FN} + \theta_{FT}\alpha_{FT})(1 - \delta_F) + \theta_{HT}\alpha_{FT}(1 - \delta_H)]$$

where

(31) $$\Delta = - \overline{M}_{HO} \theta_{FT} \alpha_{HT} [(\alpha_{FN} + \theta_{FT}\alpha_{FT})(1 - \delta_F) + \theta_{HT}\alpha_{FT}(1 - \delta_H)]$$
$$- \overline{B}_{HO}(1-\delta_F)(1-\delta_H)[\alpha_{FN}(\alpha_{HN} + \theta_{HT}\alpha_{HT}) + \alpha_{HN}\theta_{FT}\alpha_{FT}]$$

It is explained in Appendix II that the existence of a solution with $W_0 > 0$ and $W_0^* > 0$ requires $\Delta < 0$. Hence, we confine our discussion to this case only.

It is clear from (30) that the exchange rate depreciates as a result of a domestic monetary expansion. However, combining (30) with (31) it can be seen that the rate of depreciation is smaller than the rate of monetary expansion when $\overline{B}_{HO} > 0$ and the rate of depreciation is larger than the rate of monetary expansion when $\overline{B}_{HO} < 0$. In the absence of indebtedness denominated in local currency the rate of depreciation is precisely equal to the rate of domestic monetary expansion. These results can be understood after the consideration of the effects of the monetary expansion on each country's wealth level.

From (28) and (29) it is clear that with $\overline{B}_{HO} = 0$ there are no wealth effects associated with an unexpected increase in domestic money. When $\overline{B}_{HO} > 0$ home wealth increases and foreign wealth declines, while when $\overline{B}_{HO} < 0$ home wealth declines and foreign wealth increases. Hence, the unexpected monetary expansion at home redistributes wealth in favor of the home country when it has a debt denominated in home currency and it redistributes wealth in favor of the foreign country when the home country has issued credit denominated in home currency. Without a wealth redistribution, the exchange rate depreciates precisely at the rate of monetary expansion. When the monetary injection redistributes wealth in favor of the home country, it causes an increase in aggregate spending at home and a fall in aggregate spending abroad (both in real terms and in terms of foreign currency). As a result, demand for non-traded goods increases at home and falls abroad, thereby inducing corresponding shifts in money uses in markets for nontraded goods. This means that relatively less home money and more foreign money are used for transactions in the traded goods' market. Hence, the exchange rate depreciates at a smaller rate than the rate of money growth. When wealth is redistributed in favor of the foreign country, the exchange rate depreciates at a rate which exceeds the rate of monetary expansion for reasons which can be explained in similar terms.

In order to analyze the effect of the unexpected increase in the stock of domestic money on the time path of prices, exchange rates and interest rates, assume for concreteness $\overline{B}_{H0} > 0$, i.e., there is a wealth redistribution in favor of the home country. From (17) and (28)-(29) we see that the discount factors on foreign currency d_t increase for all $t = 1,2,\ldots$ if $\delta_H > \delta_F$ and they decrease if $\delta_H < \delta_F$. Moreover, since $d_t/d_{t+1} = 1 + i_{Ft}$, a direct calculation reveals that interest rates on foreign currency denominated debt decline in <u>all</u> periods if $\delta_H > \delta_F$ and they increase in <u>all</u> periods if $\delta_H < \delta_F$.

Now consider the response of prices in the foreign country. It is clear from (19) and (22) that the monetary expansion in the home country shifts upward the entire profile of traded goods' prices and downwards the profile of nontraded goods' prices in the foreign country. Hence the real exchange rate is higher in every period. Since prices of traded goods rise and prices of nontraded goods fall, the CPI may either rise or fall. The result differs according to whether there is a deficit or a surplus in the trade account (see Appendix I). However, since there are no changes in the foreign money supply, the profile of its GNP deflator does not change.

In the home country price movements are somewhat more complicated. Combining (19) and (22) with (23), it can be seen that starting with period one -- when stocks of domestic money remain unchanged -- domestic currency prices of nontraded goods are higher while domestic currency prices of traded goods are lower. The entire profile of real exchange rates, including period zero, is lower. In period zero, when the unanticipated increase in the money supply takes place, the price of nontraded goods rises at a rate that exceeds the rate of monetary expansion while the price of traded goods rises, but at a rate which is smaller than the rate of monetary expansion (remember that both p_{FT0} and e_0 increase). The CPI rises, therefore, in period zero, but it may rise proportionally more or less in the following periods, depending on whether there is a deficit or a surplus in the trade account (see Appendix I). The GNP deflator rises in period zero at the rate of money growth and does not change in subsequent periods. In Figures 5-10 we present the course of prices and real exchange rates with and without the unexpected increase in home money. Solid lines describe variables with the unexpected monetary injection, while broken lines describe variables which would have obtained without the unexpected monetary injection.

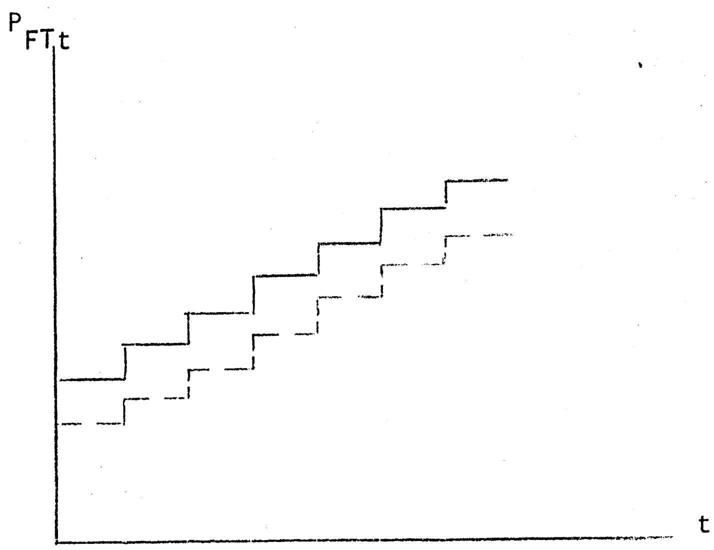

FIGURE 5

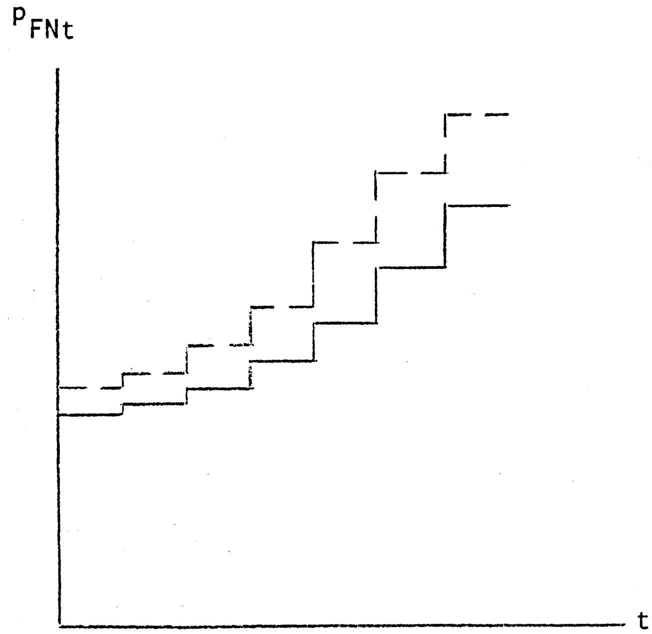

FIGURE 6

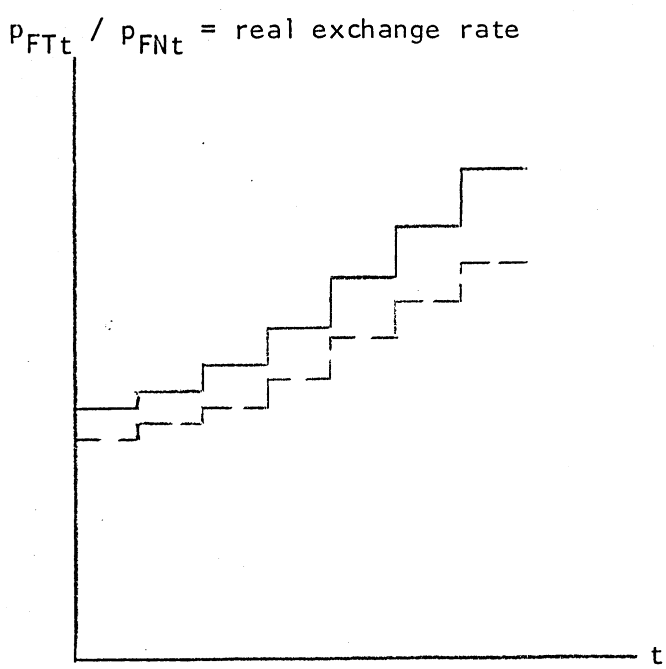

FIGURE 7

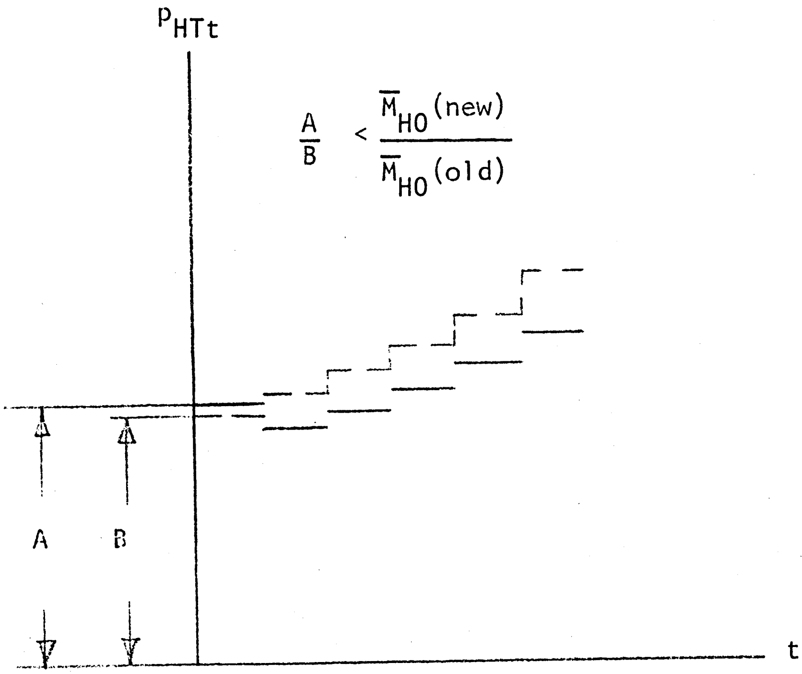

FIGURE 8

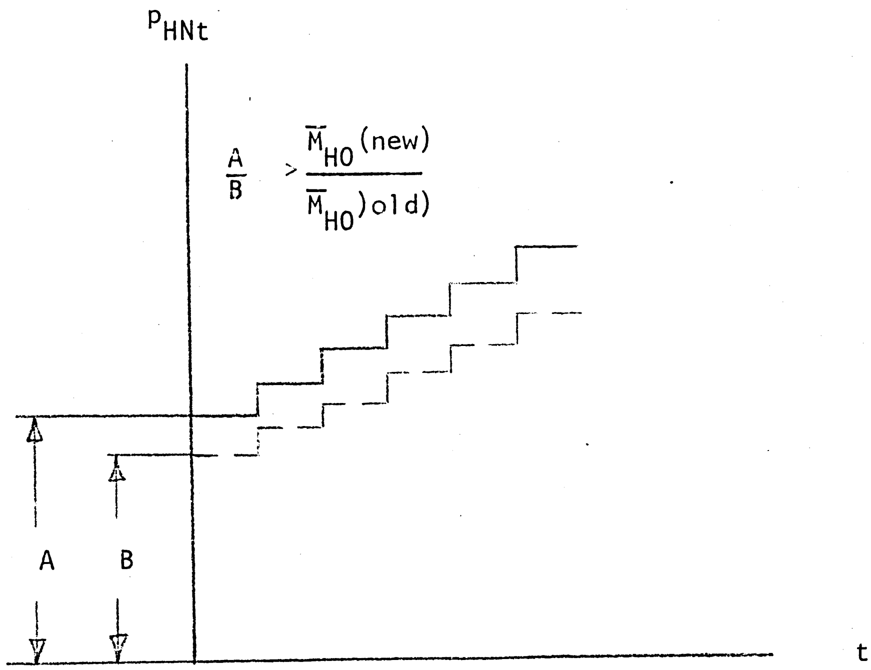

FIGURE 9

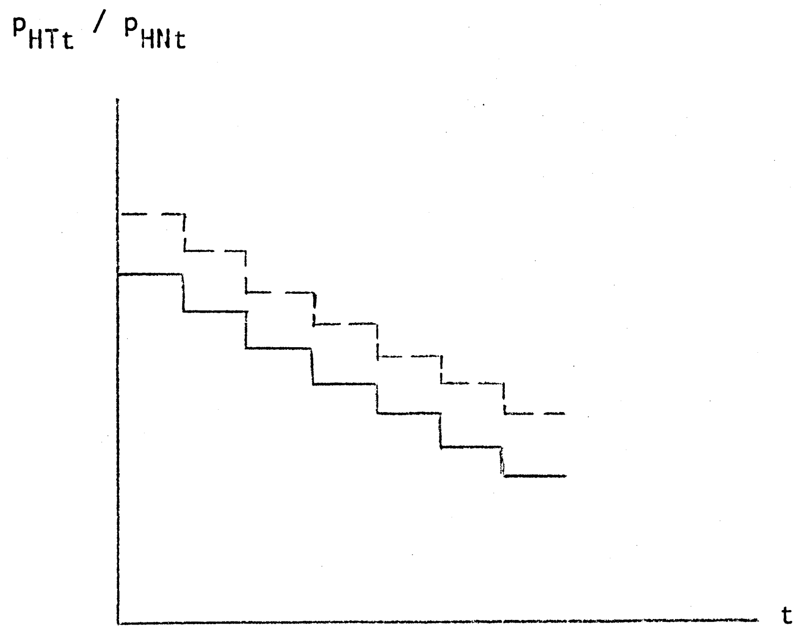

FIGURE 10

In the case considered so far, i.e., $\overline{B}_{H0} > 0$, it is clear from (23) and (30) that following the unexpected monetary expansion in period zero the entire profile of the exchange rate shifts upward. However, in period zero the exchange rate rises at a higher rate than the rate of monetary injection (a type of overshooting). If, however, $\overline{B}_{H0} < 0$, then starting from period one, the entire profile of exchange rates is lower than without the monetary injection, while in period zero, it is higher but by a rate which falls short of the rate of the monetary injection. These two cases are described in the following figures.

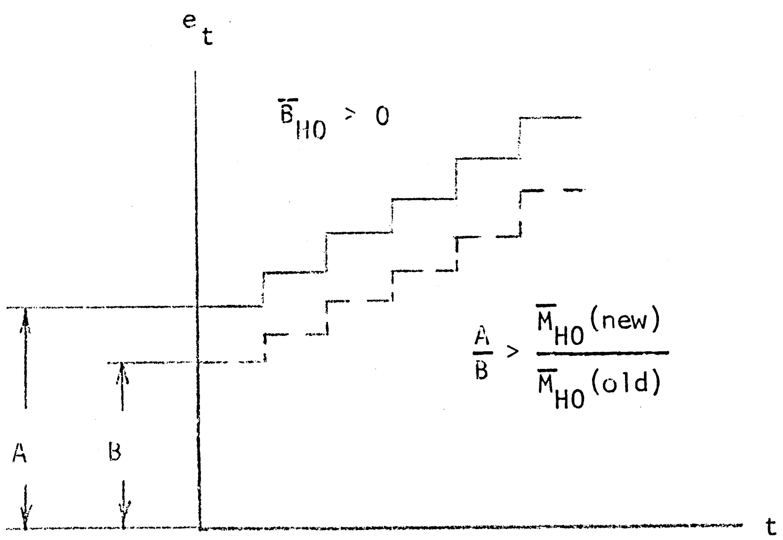

FIGURE 11

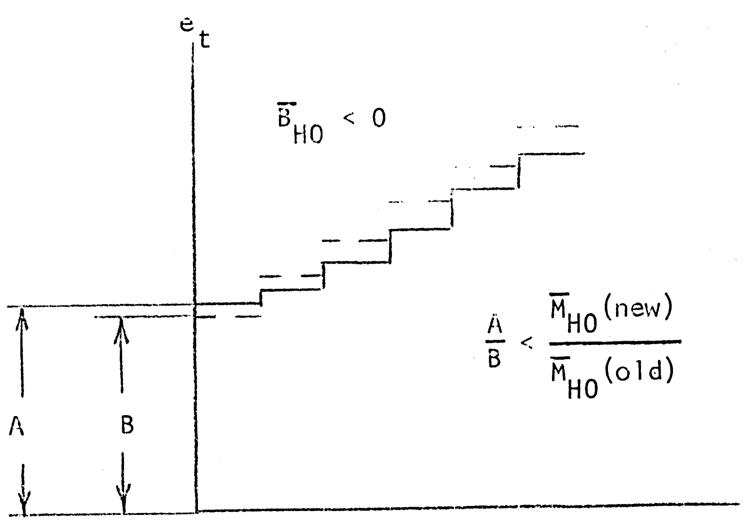

FIGURE 12

5. SUMMARY

We have presented, in this paper, a complete two-country model with a floating exchange rate. Due to the explicit specification of the monetary mechanism, we were able to show that expected changes in the money supply have no real effects while unexpected changes do have real effects. The real effects that are induced by unexpected changes in the money supply stem from unexpected redistribution of wealth that takes place as a result of the change in money stocks. The redistribution of wealth results from the existence of nominal obligations whose real value changes with prices. No real redistribution takes place when changes in monetary policy are preannounced.

If an unexpected increase in the stock of home money takes place at, say, period zero, and this is a temporary change so that in all subsequent periods money stocks do not change, real wealth is redistributed from foreigners to home residents if the home country has debts denominated in home currency. If foreigners have debts denominated in home currency, wealth is redistributed in favor of the foreign country.

For concreteness, suppose that wealth is redistributed in favor of the home country. Then, in the foreign country the price of traded goods will rise and the price of nontraded goods will fall in <u>all</u> periods, thereby increasing the real exchange rate in all periods. The foreign GNP deflator will not change and its CPI will rise or fall depending on whether we deal with periods of import surpluses or export surpluses.

In the home country we have to distinguish between the effects in period zero and effects in all subsequent periods. In all periods following period zero the responses are opposite to those that take place in the foreign country. For example, the price of traded goods falls while the price of nontraded goods rises, resulting in an appreciation of the real exchange rate. However, in period zero the price of traded goods as well as the price of nontraded goods rises, but the rate of increase in the price of traded goods falls short of the rate of monetary expansion while the rate of increase in the price of nontraded goods exceeds the rate of monetary expansion. Thus, in period zero both the CPI and the GNP deflator increase.

It is clear from the above described responses of prices of traded goods that the home currency appreciates in all periods that follow the period of the monetary surprise. However, in period zero the domestic currency depreciates and its rate of depreciation exceeds the rate of increase in the stock of home money. Hence, the exchange rate moves on impact in a direction which is opposite to its following trend.

The preceding discussion was based on the assumption that the home country has a debt denominated in home currency. If indebtedness in terms of home currency is zero, there is no redistribution of real wealth and no real effects as a result of an unexpected increase in home money. There are, however, nominal responses which take place only in period zero and which amount to an equiproportional increase in the price of traded goods, nontraded goods and the exchange rate, with the rates of increase being equal to the rate of increase in the stock of money.

Our model generates also interesting dynamics along the perfect foresight path. Suppose that the home country has a lower rate of time preference. Then over time at home the price of traded goods rises slower than the rate of home money growth while the price of nontraded goods rises faster than the rate of home money growth. In the foreign country there are opposite trends. As a result, the rate of depreciation of the home currency (appreciation if negative) is smaller than the differential rate of money growth at home and abroad. Hence, the rate of currency depreciation does not reflect the differential rates of monetary expansion. Moreover, since the GNP deflator rises in each country at the rate of money growth, PPP based on GNP deflators is invalid. PPP also does not hold if we use CPI's in our calculations, since in periods in which the home country has consecutive import surpluses its CPI rise more slowly than its rate of money growth with the opposite taking place in the foreign country, while in periods in which the home country has consecutive export surpluses its CPI rises faster than its rate of money growth with the opposite taking place in the foreign country.

Finally, we would like to point out that in our model there exists a <u>unique</u> perfect foresight equilibrium path of prices, consumption, exchange rates and interest rates. We do not have to rely on arbitrary choices from a set of many perfect foresight equilibria.

APPENDIX I

We discuss in this appendix the behavior of the true cost of living index, which is defined for the foreign country as:

$$(1.1) \qquad P_{Ft} = (p_{FNt})^{\alpha_{FN}}(p_{FTt})^{\alpha_{FT}}$$

A similar CPI can be defined for the home country. Using (18) and (22), we can write P_{Ft} in the following form:

$$(1.2) \qquad P_{Ft} = \frac{[\alpha_{FN}(1-\delta_F)(Q_{FN})^{-1}]^{\alpha_{FN}}}{\theta_{FT}(Q_{HT} + Q_{FT})^{\alpha_{FT}}} \quad \overline{M}_{Ft}\ L\ [(W_0/W_0^*)(\delta_H/\delta_F)^t]$$

where

$$(1.3) \qquad L(x) \quad \frac{[\alpha_{HT}(1 - \delta_H)x + \alpha_{FT}(1 - \delta_F)]^{\alpha_{FT}}}{[\alpha_{HT}(1 - \delta_H)x + \alpha_{FT}(1 - \delta_F) + \alpha_{FN}(1 - \delta_F)/\theta_{FT}]}$$

Logarithmic differentiation of (1.3) yields (where $\varepsilon_L(\cdot)$ is the elasticity of $L(\cdot)$):

$$\text{sign } \varepsilon_L(x) = \text{sign}[\alpha_{FT}(1 - \delta_F)\theta_{HT}/\theta_{FT} - \alpha_{HT}(1 - \delta_H)x]$$

$$= \text{sign } [\frac{Q_{HT}}{Q_{FT}} - \frac{\alpha_{HT}(1 - \delta_H)}{\alpha_{FT}(1 - \delta_F)} x]$$

Using $x = (W_0/W_0^*)(\delta_H/\delta_F)^t$, (11) and (13), we obtain:

$$\text{(1.4)} \qquad \text{sign } \varepsilon_L[W_0/W_0^*)(\delta_H/\delta_F)^t] = \text{sign } \left(\frac{Q_{HT}}{Q_{FT}} - \frac{c_{Tt}}{c_{Tt}^*}\right)$$

The right hand side of (1.4) is positive if the foreign country runs a deficit in its trade account and it is negative if the foreign country runs a surplus in its trade account. This means that if in periods t and $t+1$ the foreign country runs a deficit in its trade account, then

$$\frac{P_{Ft+1}}{\overline{M}_{Ft+1}} > \frac{P_{Ft}}{\overline{M}_{Ft}} \quad \text{if} \quad \delta_H > \delta_F$$

$$\frac{P_{Ft+1}}{\overline{M}_{Ft+1}} < \frac{P_{Ft}}{\overline{M}_{Ft}} \quad \text{if} \quad \delta_H < \delta_F$$

and opposite for a surplus. In other words:

The foreign CPI rises faster than its rate of monetary expansion between two consecutive periods if the foreign country has an import surplus in both periods and a higher rate of time preference or if it has an export surplus in both periods and a lower rate of time preference. Its CPI rises slower than its rate of monetary expansion between two consecutive periods if it has an import surplus in both periods and a lower rate of time preference or if it has an export surplus in both periods and a higher rate of time preference.

A similar result can be derived for the home country. Clearly, the CPI of one country rises faster than its rate of monetary expansion while the other country's CPI rises slower than its rate of monetary expansion.

It is also clear from this discussion that if W_0/W_0^* rises as a result of an unexpected monetary disturbance in the home country, then the foreign CPI rises in periods of foreign deficits and falls in periods of foreign surpluses. If W_0/W_0^* falls, the foreign CPI falls in periods of foreign deficits and rises in periods of foreign surpluses. Similar conclusions can be derived for the home country for periods which follow the period in which the disturbance takes place. In the period in which the disturbance takes place, we have to add to the CPI the proportional effect of the unexpected monetary disturbance.

It is also easy to see from (1.2) and (1.3) that a once and for all unexpected temporary change in the home money supply that redistributes wealth (i.e., changes W_0/W_0^*) has lasting effects on the rate of inflation in the foreign country in terms of the CPI. The rate of inflation as measured by the GNP deflator does not change. Similar conclusions apply to the home country, except that the rate of inflation as measured by the GNP deflator changes only in the vicinity of the time period in which the shock occurs.

APPENDIX II

The purpose of this appendix is to show that $\Delta < 0$ where Δ is defined in (31) is required for the existence of a solution in which net wealth of every country is positive. Clearly, negative wealth is inconsistent with an equilibrium since consumption is proportional to wealth and consumption has to be nonnegative:

From (25)-(27) we can solve for W_0, W_0^*, and e_0^{-1}. The solution for the wealth variables is:

$$(II.1)\quad W_0 = \Delta^{-1}[\overline{B}_{HO}\overline{M}_{FO}\theta_{HT}\alpha_{FT}(1-\delta_F) + \overline{B}_{FO}\overline{M}_{HO}(\alpha_{FN} + \theta_{FT}\alpha_{FT})(1-\delta_F) - \overline{M}_{HO}\overline{M}_{FO}\theta_{HT}\alpha_{FT}]$$

$$(II.2)\quad W_0^* = \Delta^{-1}[-\overline{B}_{HO}\overline{M}_{FO}(\alpha_{HN} + \theta_{HT}\alpha_{HT})(1-\delta_H) - \overline{B}_{FO}\overline{M}_{HO}\theta_{FT}\alpha_{HT}(1-\delta_H) - \overline{M}_{HO}\overline{M}_{FO}\theta_{FT}\alpha_{HT}]$$

where (compare to (31)):

$$(II.3)\quad \Delta = -\overline{M}_{HO}\theta_{FT}\alpha_{HT}[(\alpha_{FN} + \theta_{FT}\alpha_{FT})(1-\delta_F) + \theta_{HT}\alpha_{FT}(1-\delta_H)] - \overline{B}_{HO}(1-\delta_F)(1-\delta_H)[\alpha_{FN}(\alpha_{HN} + \theta_{HT}\alpha_{HT}) + \alpha_{HN}\theta_{FT}\alpha_{FT}]$$

Now, suppose $\Delta > 0$. Then (II.3) implies:

$$(II.4)\quad \overline{B}_{HO} < -\frac{\overline{M}_{HO}\theta_{FT}\alpha_{HT}[(\alpha_{FN} + \theta_{FT}\alpha_{FT})(1-\delta_F) + \theta_{HT}\alpha_{FT}(1-\delta_H)]}{(1-\delta_F)(1-\delta_H)[\alpha_{FN}(\alpha_{HN} + \theta_{HT}\alpha_{HT}) + \alpha_{HN}\theta_{FT}\alpha_{FT}]}$$

However, with Δ 0 equations (II.1) and (II.2) imply that W_0 and W_0^* are positive if and only if the following two conditions are satisfied:

$$(II.5) \qquad \overline{B}_{F0} > \frac{\overline{M}_{F0}\theta_{HT}\alpha_{FT}[\overline{M}_{H0} - \overline{B}_{H0}(1-\delta_F)]}{\overline{M}_{H0}(\alpha_{FN} + \theta_{FT}\alpha_{FT})(1-\delta_F)}$$

$$(II.6) \qquad \overline{B}_{F0} < \frac{[\overline{M}_{H0}\theta_{FT}\alpha_{HT} + \overline{B}_{H0}(\alpha_{HN} + \theta_{HT}\alpha_{HT})(1-\delta_H)]\overline{M}_{F0}}{\overline{M}_{H0}\theta_{FT}\alpha_{HT}(1-\delta_H)}$$

From (II.4) and (II.5), we obtain $\overline{B}_{F0} > 0$,

which implies that the right hand side of (II.6) has to be positive. But this is so if and only if the following holds:

$$(II.7) \qquad \overline{B}_{H0} > -\frac{\overline{M}_{H0}\theta_{FT}\alpha_{HT}}{(\alpha_{HN} + \theta_{HT}\alpha_{HT})(1-\delta_H)}$$

Combining (II.4) with (II.7), we obtain:

$$(II.8) \qquad \frac{\overline{M}_{H0}\theta_{FT}\alpha_{HT}[(\alpha_{FN} + \theta_{FT}\alpha_{FT})(1-\delta_F) + \theta_{HT}\alpha_{FT}(1-\delta_H)]}{(1-\delta_F)(1-\delta_H)[\alpha_{FN}(\alpha_{HN} + \theta_{HT}\alpha_{HT}) + \alpha_{HN}\theta_{FT}\alpha_{FT}]} < \frac{\overline{M}_{H0}\theta_{FT}\alpha_{HT}}{(\alpha_{HN} + \theta_{HT}\alpha_{HT})(1-\delta_H)}$$

which implies the contradiction:

$$\theta_{HT}\alpha_{FT}[\theta_{FT}\alpha_{HT}(1-\delta_F) + (\alpha_{HN} + \theta_{HT}\alpha_{HT})(1-\delta_H)] < 0$$

Hence, $\Delta > 0$ leads to a contradiction and, therefore $\Delta < 0$.

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