RELATIVE PRICE VARIABILITY, INFLATION AND THE ALLOCATIVE EFFICIENCY OF THE PRICE SYSTEM

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A B S T R A C T

This paper develops a theory of the relationship between aggregate and relative price variability based on the inability of people, even in a rational world, to identify permanent changes in relative demands (whether caused by real or by monetary variability) and relative productivities as soon as they occur. The theory implies that the variance of the rate of inflation and the variance of relative price change are positively related. Although expectations are rational and markets always clear, production decisions respond sluggishly to changes in relative prices. Temporary monetary shocks to relative demands cause prolonged changes in the structure of production. Available information is used so as to maximize the efficiency of the price system. However, this efficiency decreases when any of the underlying variances increases. This decrease is more pronounced when the production lag is longer.

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INTRODUCTION

Recent literature suggests that the distributions of the general rate of inflation and that of relative prices are not independent. In particular there is empirical evidence which suggests that a) relative price change variability and the rate of inflation are positively related in a cross section of countries (Glejser (1965), Jaffe and Kleiman (1977)); b) relative price change variability and the variance of the rate of inflation are positively related over time in the U.S. (Vining and Elwertowski, (1976)); c) relative price change variability is positively related to the extent of unanticipated inflation (Parks (1978)); d) the level and the variance of the rate of inflation are positively related both cross sectionally and over time (Okun (1971), Gordon (1971), Logue and Willet (1976), Jaffe, Kleiman (1977), Foster (1978) and Blejer (1978)). At the theoretical level it has been suggested in Cukierman (1979b) that the Vining and Elwertowski empirical result can be explained within the context of a multimarket equilibrium model (of the type developed by Lucas (1973) and Barro (1976) in which such a relationship arises because individuals confuse between relative and aggregate movements in prices. An important welfare implication of all of the above is that one of the important costs of inflation may very well be related to the accompanying increase in relative price variability.[1] This is strongly emphasized in Friedman's (1977) Nobel lecture which suggests that: "The more volatile the rate of general inflation, the harder it becomes to extract the signal about relative prices from the absolute prices."

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[1] For a recent methodical listing of the costs of inflation see Fischer & Modigliani(1978).

(Friedman op.cit., p.467). Friedman further suggests that this reduction in the efficiency of market prices as coordinators of economic activity may reduce output and increase unemployment.[2]

This paper provides theoretical underpinnings for some of the above mentioned empirical regularities and develops other related results within a framework in which individuals detect permanent changes in relative excess demands and relative productivities only gradually as they persist through time. As a consequence they detect permanent changes in relative prices gradually as well. Any producer who plans his production in advance must make up his mind about how much of today's relative price is permanent and how much will have vanished away when he brings his product onto the market. Hence his current decisions depend on his perception concerning the permanence of the current situation. Since nature usually does not reveal in advance how much of a given shock is permanent the decisions of producers are subject to a certain amount of confusion between permanent and temporary changes. Unlike the aggregate-relative confusion, the permanent-transitory confusion is not dispelled by the publication of statistics about the general price level. It is therefore a longer lasting type of confusion which can explain persistence, since people learn whether a change is permanent mostly by observing whether it persists over time or not.[3] The framework used is anchored on a multimarkets, rational

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[2] For some preliminary evidence which supports this view see Evans (1978) and Blejer and Leiderman (1978). In a different but related direction, Barro (1976) shows that because of the confusion between aggregate and relative price variability, increased monetary variance tends to move output away from full information output.

[3] For an explanation of stagflation and the persistence of unemployment which is based on slow realization, by the public, of decreases in permanent productivity see Brunner, Cukierman and Meltzer (1980). Even though it seems to be a more lasting type of confusion, the permanent-transitory confusion was not applied much to the explanation of macroeconomic phenomena. A notable exception is Brunner and Meltzer (1978).

expectations, equilibrium model of the type developed by Lucas (1973) and Barro (1976) in which differential information across markets plays an important role. However, since the focus of the discussion here is on the implications of the permanent transitory confusion, individuals in all markets are endowed with the same current and past price and quantity information about all markets in the economy. But individuals have only imperfect knowledge about the permanence of those prices and quantities. Furthermore, supply and demand elasticities are allowed to differ across markets.[4] Monetary expansion has both temporary and permanent aggregate effects on prices. Although it is neutral in the long run, money has persistent temporary differential effects on demands across markets and on the composition of production. The major positive implications of the paper are:

a) There should be a positive association between the variance of inflation and the variance of relative price change across stochastic regimes.[5] This result is also consistent with the existence of a positive association between the variance of relative price change and the degree of unanticipated inflation across stochastic regimes.[6]

b) Both the variance of the rate of inflation and the variance of relative price change are increasing functions of the variances of real and nominal relative demand shocks, and of the variance of relative shocks to productivity.

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[4] By contrast most recent multimarkets equilibrium models which focus on the aggregate-relative confusion, assume that people are able to sort the permanent from the transitory shock one period after it occurs and that all markets have the same supply and demand elasticities. See Lucas (1973), Barro (1976), Cukierman (1979a) and Cukierman and Wachtel (1979). A notable exception is Hercowitz (forthcoming).

[5] This implication is potentially testable using a cross section of countries. The over time positive association found between these two variances by Vining and Elwertowski (1976) is suggestive in this context.

[6] That such a relationship exists over time for the U.S. is demonstrated by Parks (1978).

c) Although markets clear in each period the optimal (rational) forecast of the relevant future relative price is formed as some weighted average of the actual and the previously expected relative price. As a result production decisions respond only partially to actual changes in relative prices and prices respond sluggishly to changes in relative demands and relative productivities.

d) A tentative explanation for the observed positive relationship between the level and the variance of the rate of inflation in the general level of prices at low and intermediate rates of inflation is also offered.

In order to appraise quantitatively the efficiency of the price system as a coordinator of economic activity, measures of the divergence between actual and full information outputs are introduced. It is shown that this divergence is minimized when the differential effects of money on relative demands are minimized and when the variances of real relative demand shocks and relative productivity shocks are made as small as possible. The model implies that the effectiveness of current and past relative prices as guides for production decisions is smaller, ceteris paribus, the longer the production lag. Finally, for given exogenous variances of permanent or transitory source the adaptive predictions of permanent relative prices used by individuals also minimize the above measures of the costs of deviating from full information output.

The basic model including its deterministic and stochastic structure is presented in section I and the rationally expected permanent relative prices are characterized using a result due to Muth (1960). Most of the positive implications of the theory including the various relationships between the variance of inflation, the variance of relative price change and the extent of unanticipated inflation are derived and interpreted in section II. Section III shows that production decisions adapt to relative price change rather sluggishly and shows that the dispersion of actual output around full information output increases with any increase in either the variance of permanent or the variance of transitory demand or productivity shocks. Implications for monetary and other policies are spelled out as well.

I. THE MODEL

A. Supplies and Demands: The economy is composed of a large number of markets for different goods. All markets are competitive and clear instantaneously. Production takes one period so that suppliers of any given good have to decide today how much output they will put on the market in the next period. Each supplier makes his output decision on the basis of what he believes (presently), the relative price of his product will be in the next period.

More precisely supply of good v is given by

$$y_t^s(v) = \beta_v t + z_t(v) + \gamma_v(E_{t-1}p_t(v) - E_{t-1}Q_t) \qquad (1)$$

where $y_t^s(v)$ is the logarithm of output of good v supplied at time t, $E_{t-1}p_t(v)$ and $E_{t-1}Q_t$ are expected values (as of time t-1) of the logarithms of the price of good v in period t and of the general price level in period t respectively, and $z_t(v)$ is a random shock to production of good v in time t. $z_t(v)$ is the sum of a permanent component, $z_t^p(v)$, whose first difference is normally distributed with mean zero and constant variance σ_{zp}^2 and of a transitory component, $z_t^q(v)$, which is normally distributed with mean 0 and constant variance σ_{zq}^2 for all v's. Δz_t^p and z_t^q are serially and mutually uncorrelated. Formally

$$z_t(v) = z_t^p(v) + z_t^q(v), \Delta z_t^p(v) \sim N(0,\sigma_{zp}^2), \; z_t^q(v) \sim N(0,\sigma_{zq}^2) \quad \text{for all } v. \qquad (1a)$$

γ_v measures the positive supply elasticity of good v with respect to next period's expected relative price. The term $\beta_v t$ measures known with certainty permanent increases in the productivity of good v. Uncertain permanent differential changes in supply are expressed through the term z_t^p whose distribution is the same for all markets but whose realizations vary across markets. Demand for good v at time t is given by

$$y_t^d(v) = \psi_v(n_t(v) - p_t(v)) \tag{2}$$

where $y_t^d(v)$ is the logarithm of demand for good v at time t, $p_t(v)$ is the logarithm of the actual price of good v at time t, $-\psi_v$ is the (negative) elasticity of demand with respect to its own price and $n_t(v)$ is a shift parameter which is directly related to the logarithm of nominal income that people desire to spend on good v at time t.[7/] The shift parameter $n_t(v)$ reflects, for a given price of that good, both changes in the aggregate level of demand as well as changes which are specific to the demand for this good. More specifically

$$n_t(v) = x_t(v) + \delta t + mt + \varepsilon_t(v) \tag{3}$$

7/ This can be seen by noting that (2) is equivalent to

$$Y_t^d(v) = \left[\frac{N_t(v)}{P_t(v)}\right]^{\psi_v}$$ where the capital letters

here are the antilogs of the corresponding lower case letters in equation (2). For the particular case in which N_t does not depend on v this specification reduces to Parks' (1978) demand specification. See his equation (2).

where

(3a) $x_t(v) = x_t^p(v) + x_t^q(v),\ \Delta x_t^p(v) \sim N(0,\sigma_{xp}^2),\ x_t^q(v) \sim N(0,\sigma_{xq}^2)$ for all v

(3b) $m_t = m_t^p + m_t^q,\ \Delta m_t^p \sim N(0,\sigma_{mp}^2),\ m_t^q \sim N(0,\sigma_{mq}^2)$

(3c) $\epsilon_t(v) \alpha N(0,\sigma_\epsilon^2)$ for all v

The term $\delta t + mt + \epsilon_t(v)$ summarize all the influences of money on the level of demand for good v. δ is the part of the permanent rate of growth in money supply which is known with certainty in advance. m_t is the stochastic part of money supply and is composed of a permanent component, m_t^p, and a transitory component, m_t^q, whose distributions are given in (3b). In addition to the term $\delta t + m_t$ through which money supply affects the total planned nominal spending on all goods in an identical manner, there is a term, $\epsilon_t(v)$, which reflects the fact that a given monetary impulse does not affect all markets with the same contemporaneous intensity. Some markets are affected sooner and others later. As a result the total monetary impact on the demand for a particular good may differ temporarily from its impact on the demand for other goods. This differential impact is captured by the transitory differential monetary noise, $\epsilon_t(v)$, whose distribution is given in (3c).

The size of the variance σ_ϵ^2 depends probably on the degree to which the demands facing the various sectors of the economy have developed similar institutional characteristics to incorporate monetary growth. At both high and low inflation σ_ϵ^2 will probably be relatively low. At high inflation all sectors would have introduced devices which incorporate the effects of monetary change quickly. At low inflation none would have introduced such devices. In <u>both</u> cases σ_ε^2 will therefore be relatively low since most sectors respond <u>uniformly</u> to monetary expansion. However, at intermediate rates of inflation some sectors are already adjusted to inflation while others are not. In this range monetary growth has more of a differential impact in the short run and therefore the variance σ_ε^2 should be relatively large.[8/]

However, this hypothesized relationship between the rate of inflation and σ_ε^2 is needed only in order to provide a tentative explanation for the observed positive relationship between the rate of inflation and the variance of relative price change. All the results below which are dignified by the title "proposition" do not depend on this hypothesis.

$x_t(v)$ is a real relative shock to the demand on market v. It contains both permanent $(x_t^p(v))$ and transitory $(x_t^q(v))$ effects on the demand for good v whose distributions are common to all v-s and given in (3a).

$\Delta z_t^p(v)$, $z_t^q(v)$, $\Delta x_t^p(v)$, $x_t^q(v)$, $\Delta m_t^p(v)$, $m_t^q(v)$ and $\varepsilon_t(v)$ are all serially and mutually uncorrelated. Although the distributions of the components of $z_t(v)$ and $x_t(v)$ are the same for all v-s, the realizations of those random variables across different markets usually differ.

8/ Different costs of developing mechanisms that incorporate monetary expansion swiftly have their stronger differential impact at intermediate rates of inflation. A stylized way to express this is to make σ_ε^2 an inverted U function of the average rate of inflation. Note also from (2) and footnote 7 that except for differential transitory monetary noise, $\varepsilon_t(v)$, money is neutral in the sense that a given increase in $\delta t + m_t$ which is matched by an increase of equal size in $p_t(v)$ for all v-s does not affect demand in any of the markets in the economy.

A basic inference problem which confronts people is to determine how much of a current change is period specific and how much will persist into the future. A possible, relatively simple, way to model this inference problem is the one used here. It is chosen because of its simplicity as well as because it provides a clean and easy distinction between permanent and transitory changes.

B. Equilibrium: Given the relative price of good v expected in period $t-1$ for period t, output decisions for period t are made. Those decisions together with the outcomes of the supply shock $z_t(v)$, the demand shocks $x_t(v)$ and the monetary shock $m_t + \varepsilon_t(v)$ determine the equilibrium price, $p_t(v)$, of good v at time t. Formally the equilibrium price may be obtained by equating (1) and (2) using (3) and solving for $p_t(v)$

$$p_t(v) = (\delta - \frac{\beta_v}{\psi_v})t + x_t(v) + m_t + \varepsilon_t(v) - \frac{z_t(v)}{\psi_v} - \frac{\gamma_v}{\psi_v}(E_{t-1}p_t(v) - E_{t-1}Q_t) \quad (4)$$

C. The General Price Level: The general price level is defined as a fixed weights index of the individual prices

$$Q_t \equiv \sum_v u(v)p_t(v) \quad , \quad u(v) \geq 0, \quad \sum_v u(v) = 1 \quad (5)$$

and the weight assigned to any one good is taken to be small in comparison to the sum of the weights since the number of goods in the economy is large.

D. The Structure of Information: Unlike recent multimarket models in which the information sets of individuals who operate in different markets are different[9/] any individual in the economy has access to current price information on all markets.

9/ See Lucas (1973), Barro (1976), Cukierman (1979a) and Cukierman and Wachtel (1979).

He therefore knows the current general price level and the current relative price of the good he is supplying. But his present production decision depends on what he currently believes about next period's relative price of the good he is selling. A rational forecast as of period t-1 of the relative price of the good for period t is that part of the actual relative price which is believed to be permanent given the information available in period $t-1$. That part of $p_{t-1}(v) - Q_{t-1}$ which is believed to be transitory is not relevant for forecasting $p_t(v) - Q_t$ since it will vanish as the next period unfolds. It follows from those considerations and (4) and (5) respectively that[10/]

$$E_{t-1}p_t(v) = (1-d_v)(\delta t + E_{t-1}x_t(v) + E_{t-1}m_t) - \frac{\beta_v t + E_{t-1}z_t(v)}{\psi_v + \gamma_v} + d_v E_{t-1}Q_t \quad (6)$$

$$E_{t-1}Q_t = \delta t + E_{t-1}m_t + \sum_v w(v) \cdot \psi_v E_{t-1}x_t(v) - \sum_v w(v)(\beta_v t + E_{t-1}z_t(v)) \quad (7)$$

where $d_v \equiv \frac{\gamma_v}{\psi_v + \gamma_v}$; $\bar{d} \equiv \sum_v u(v) d_v$ and $w(v) \equiv \frac{1}{1-\bar{d}} \frac{u(v)}{\psi_v + \gamma_v}$ and use has been made of the fact that

$$\sum_v w(v)\psi_v = \sum_v u(v) = 1 \quad (8)$$

$E_{t-1}z_t(v)$, $E_{t-1}x_t(v)$ and $E_{t-1}m_t$ are respectively the best forecasts, given the information available in period $t-1$, of the permanent level of productivity in market v, the permanent level of real relative demand in market v and the permanent level of the stochastic component of money supply. (6) and (7) express $E_{t-1}p_t(v)$ and $E_{t-1}Q_t$ in terms of these perceptions about permanent variables. We turn next to the characterization of the formation of these perceptions.

10/ (6) follows by using (3) and the fact that the best forecast as of period $t-1$ of $p_t(v)$ is $E_{t-1}p_t(v)$. (7) follows by substituting (6) into (5) and by using (8).

In period t-1 all people in the economy have observations about money supply up to and including period t-1 from which they can deduce m_{t-1}, m_{t-2} .... They also have observations on prices and quantities on all markets up to and including period t-1. Since the expectation, $E_{t-2}P_{t-1}(v) - E_{t-2}Q_{t-1}$ which was held in the past is known in period t-1 an observation on $y_{t-1}(v)$ amounts through (1) lagged one period to an observation on $z_{t-1}(v)$. Given m_{t-1} and $z_{t-1}(v)$ an observation on $p_{t-1}(v)$ amounts through (4) lagged one period to an observation on the sum $0_{t-1}(v) \equiv x_{t-1}(v) + \epsilon_{t-1}(v)$. Previous values of z(v) and 0(v) can be deduced from values of prices and quantities in preceding periods in a similar manner. The upshot is that observations on money prices and quantities up to and including period t-1 are equivalent to observations on

$$m_{t-1}, m_{t-2} \dots , \quad z_{t-1}(v), z_{t-2}(v) \dots , 0_{t-1}(v), 0_{t-2}(v) \dots \qquad (9)$$

from which people have to derive optimal forecasts of the permanent values of z_{t-1}, x_{t-1} and m_{t-1}. (The information in (9) is denoted, for future reference, by I_{t-1}.) The optimal forecasts of the permanent values of z_{t-1}, x_{t-1} and m_{t-1} are:

$$\text{(a) } E_{t-1}z_t(v) = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i z_{t-1-i}(v) \quad , \text{(b) } E_{t-1}x_t(v) = \theta \sum_{i=0}^{\infty} (1-\theta)^i 0_{t-1-i}(v) \qquad (10)$$

where

$$\text{(a) } \lambda = \sqrt{\frac{\sigma_{zp}^2}{\sigma_{zq}^2} + \frac{1}{4}\left(\frac{\sigma_{zp}^2}{\sigma_{zq}^2}\right)^2} - \frac{1}{2}\frac{\sigma_{zp}^2}{\sigma_{zq}^2}, \text{(b) } \theta = \sqrt{\frac{\sigma_{xp}^2}{\sigma_q^2} + \frac{1}{4}\left(\frac{\sigma_{xp}^2}{\sigma_q^2}\right)^2} - \frac{1}{2}\frac{\sigma_{xp}^2}{\sigma_q^2} \quad , \sigma_q^2 \equiv \sigma_{xq}^2 + \sigma_\epsilon^2 \qquad (11)$$

These forecasts are best in the sense that each is the point estimate which, given I_{t-1}, minimizes the variance around the estimate.[11/] It is noteworthy that although expectations are fully rational the best forecasts take the form of distributed lags. Since permanent changes are observed together with transitory noise people learn about such changes by observing whether they persist through time or not. The coefficients λ and θ determine how quickly perceptions about permanent values change when new information about $z(v)$ and $O(v)$ is obtained. The higher those coefficients, the more weight is given to recent information. I shall therefore refer to them as coefficients of adaptation. It can be shown that each coefficient of adaptation is a monotonically increasing function of the ratio of the variance of the permanent component of a variable to the variance of the transitory component of that variable. Furthermore as this ratio varies from 0 to infinity the coefficient of adaptation varies from 0 to 1.[12/] Note that unlike $E_{t-1}z_t(v)$ and $E_{t-1}m_t$ which depend only on known values of z and m, $E_{t-1}x_t(v)$ depends on known values of $O(v)$. The reason is that $x(v)$ is never observed separately. The sum

$$O_{t-1}(v) = x_{t-1}(v) + \varepsilon_{t-1}(v) = x^p_{t-1}(v) + \underbrace{(x^q_{t-1}(v) + \varepsilon_{t-1}(v))}_{O^q_{t-1}(v)} \quad (12)$$

11/ This is a consequence of the fact that the first differences of the permanent components and the transitory components of each of the observations on the shocks, $z(v)$ and $O(v)$ are all normally distributed. (See (1a) and (3)). For a proof and further details see Muth (1960), pp.302-4 and Brunner, Cukierman and Meltzer (1980), pp. A similar forecast may be written for $E_{t-1}m_t$. However, since it is not needed for the subsequent analysis it is omitted.

12/ In general let the coefficient of adaptation be

$$a = \sqrt{r + \frac{r^2}{4}} - \frac{r}{2} \quad \text{where} \quad r = \frac{\sigma_p^2}{\sigma_q^2}$$ is the ratio of the variance of the permanent component to the variance of the transitory component. That a increases in r can be seen by noting that

$$\frac{\partial a}{\partial r} = \frac{1}{2}\left[\frac{1+\frac{r}{2}}{\sqrt{r + \frac{r^2}{4}}} - 1\right]$$ which is positive since $1 + \frac{r}{2} > \sqrt{r + \frac{r^2}{4}}$. That a varies between 0 and 1 follows by inspection of (11).

is observed instead. Since observations on $O(v)$ contain information about $x_t^p(v)$ these observations are used to generate $E_{t-1}x_t(v)$. However, since the transitory noise in (12) includes not only the real transitory relative demand shock, $x_t^q(v)$, but also the differential monetary noise, $\varepsilon_t(v)$, the variance of the transitory component of $O_{t-1}(v)$ is equal to $\sigma_q^2 \equiv \sigma_{xq}^2 + \sigma_\varepsilon^2$. It can be seen from (11b) that the existence of the differential monetary noise, $\varepsilon_t(v)$, makes learning about real relative permanent changes in demand slower by decreasing the coefficient of adaptation θ. The relative price expected for period t, as of $t-1$, can now be calculated by subtracting (7) from (6)

$$\begin{aligned} E_{t-1}p_t(s) - E_{t-1}Q_t &= (1-d_s)[E_{t-1}x_t(s) - \textstyle\sum_v w(v)\psi_v E_{t-1}x_t(v)] \\ &- \frac{1}{\psi_s + \gamma_s}[(\beta_s - \psi_s \textstyle\sum_v w(v)\beta_v)t + E_{t-1}z_t(s) - \psi_s \textstyle\sum_v w(v)\, E_{t-1}z_t(v)] \end{aligned} \tag{13}$$

This term can be expressed only in terms of observables by noting that $Ez_{t-1}(v)$ and $Ex_{t-1}(v)$ are given by (10a) and (10b) only in terms of observables. Note that the expected relative price does not depend on the expected permanent value of money supply. This is a reflection of the fact that money is neutral and people know it. (13) states that the expected relative price is an increasing function of the perceived permanent relative demand for good s and a decreasing function of the perceived permanent relative productivity of good s. It follows that when either relative demand for good s or the relative efficiency in the production of this good change permanently, perceptions about those changes adjust only gradually. Since those perceptions affect actual prices through (4) it follows that the equilibrium price on any market responds gradually to permanent changes in relative demand and relative productivity. In addition the perceived relative price is also affected by random deviations of the realization of differential monetary noise in market s from a weighted average of the differential monetary noise over markets. This can be seen more precisely by noting that the term $Ex_{t-1}(s) - \sum_v w(v)\psi_v Ex_{t-1}(v)$ on the right hand side of (13) may be rewritten using (10a) as

$$\theta \sum_{i=0}^{\infty} (1-\theta)^i \left[\left(x_{t-1-i}(s) - \bar{x}_{t-1-i} \right) + \left(\varepsilon_{t-1-i}(s) - \bar{\varepsilon}_{t-1-i} \right) \right] \tag{13a}$$

where $\bar{\varepsilon}_{t-1-i} \equiv \sum_v w(v)\Psi_v \varepsilon_{t-1-i}(v)$, $\bar{x}_{t-1-i} \equiv \sum_v w(v)\Psi_v x_{t-1-i}(v)$ for all i and t, $w(v)\ \Psi_v \geq 0$

and the sum of $w(v)\psi_v$ over markets is equal to 1 by (8). (13a) suggests that both current and past deviations of the differential monetary noise in market s from the economy wide average differential monetary noise[13/] affect the current perceptions about next period's relative price and therefore current production decisions. Positive deviations will cause, ceteris paribus, an overestimate of next period's relative price and therefore a higher level of output in industry s. Note also that past such deviations also have an effect on today's planned output. However the effect of any given past deviation on today's output decisions diminishes as the period in which it occurred recedes into the past. This suggests that differential monetary noise has persistent effects on the composition of output.

Although they have full current information on actual relative prices and use it as well as the known structure of the model to form rational forecasts, individuals are still subject to informational limitations for three reasons: first the realizations of period's t shocks are not known in period t-1; second, even with the information about actual relative prices for period t, individuals are not able to disentangle perfectly between the permanent and transitory component of the relative price; third, individuals cannot disentangle perfectly between changes in relative prices which are caused by differential monetary noise and between changes which are caused by real supply or demand factors.

13/ Note that although the expected values of both $\varepsilon_t(s)$ and $\bar{\varepsilon}_t$ are 0 their realizations in any given period differ in general from 0.

II. THE VARIABILITY OF RELATIVE PRICES AND OF THE RATE OF INFLATION

Recent literature suggests that there is a systematic relationship between the variance of relative price change and the variance of the rate of inflation. In particular Vining and Elwertowski (1976) present empirical evidence which supports the view that there is a positive relationship between those two variances. Cukierman (1979b) provides an explanation of this phenomena in terms of a multimarket model of the type developed by Lucas (1973) and Barro (1976). That explanation relies on a temporary inability of participants in localized markets to distinguish between general and relative price movements. Admittedly this confusion cannot last much longer than the time it takes to publish and take notice of general price indices. On the other hand the difficulty in distinguishing between permanent and transitory changes in relative prices cannot be expected to wither away so quickly. The statistics provide information only on actual changes in relative prices but there are no direct statistics on the permanent component of such a change. As noted in the discussion of (13) people learn about permanent changes in relative prices only gradually. It turns out that the permanent transitory confusion is able to generate an explanation for the relationship between general inflation variability and relative price variability, across stochastic regimes even when the supply and demand elasticities of different goods are allowed to vary across markets.[14]

A. The General rate of Inflation and its Variance; By definition the general rate of inflation is

$$\Pi_t = Q_t - Q_{t-1} = \sum_v u(v)\ (p_t(v) - p_{t-1}(v)) \tag{14}$$

Substituting (13) into (4), substituting the resulting expression into (14), using (12), (10a) and (10b) and rearranging we obtain:[15/]

14/ By contrast Lucas (1973), Barro (1976) and Cukierman (1979b) all assume implicitly that these elasticities are the same in all markets.

15/ B is some combination of parameters of no particular interest.

$$
\begin{aligned}
\pi_t = {} & B + \Delta m_t^p + m_t^q - m_{t-1}^q \\
& + \sum_v u(v)(\Delta x_t^p(v) + 0_t^q(v)) - \sum_v u(v)\left(1 + \theta \frac{d_v - \bar{d}}{1-\bar{d}}\right) 0_{t-1}^q(v) \\
& - \sum_v \frac{u(v)}{\Psi_v} (\Delta z_t^p(v) + z_t^q(v)) + \sum_v \frac{u(v)}{\Psi_v} \left(1 + \lambda \frac{d_v - \bar{d}}{1-\bar{d}}\right) z_{t-1}^q(v) \\
& + \sum_v u(v)\left(\frac{\bar{d}-d_v}{1-\bar{d}}\right)\left(\theta \sum_{i=0}^{\infty} (1-\theta)^i \Delta x_{t-1-i}^p(v) - \theta^2 (0_{t-2}^q(v) + (1-\theta) 0_{t-3}^q(v) + \ldots)\right) \\
& - \sum \frac{u(v)}{\Psi_v} \left(\frac{\bar{d}-d_v}{1-\bar{d}}\right)\left(\lambda \sum_{i=0}^{\infty} (1-\lambda)^i \Delta z_{t-1-i}^p(v) - \lambda^2 (z_{t-2}^q(v) + (1-\lambda) z_{t-3}^q(v) + \ldots)\right) ,
\end{aligned} \tag{14a}
$$

(14a) expresses the rate of inflation as a function of the rate of growth of money supply, the weighted sum, over goods, of the first differences of specific demand factors and the weighted sum of the first differences of specific shocks to productivity. The general rate of inflation also depends on the weighted sum over markets of the first differences of perceptions about the permanent levels of specific demand factors and the first differences of perceptions about the permanent levels of specific shocks to productivity. The expressions for these perceptions, as embodied in (10) give rise to the dependence of (14a) on past permanent and transitory shocks to productivity and aggregate demand. The unconditional expected value of Π_t is B since by (1a) and (3a)-(3c) the unconditional expected value of each of the other terms on the right hand side of (14a) is 0. Noting that each of the stochastic terms on the right hand side of (14a) is uncorrelated with each of the other stochastic terms and using (1a) and (3a)-(3c) the conditional variance of Π_t can be calculated to be

$$V(\pi) = \sigma_{mp}^2 + 2\,\sigma_{mq}^2 + \sigma_{xp}^2 \sum_v (u(v))^2 \left(1 + \frac{\theta}{2-\theta}\left(\frac{\bar{d} - d_v}{1-\bar{d}}\right)^2\right)$$

$$+ \sigma_q^2 \sum_v (u(v))^2 \left(1 + \frac{\theta^3}{2-\theta}\left(\frac{\bar{d}-d_v}{1-\bar{d}}\right)^2 + \left(1 + \theta\,\frac{d_v-\bar{d}}{1-\bar{d}}\right)^2\right) \qquad (15)$$

$$+ \sigma_{zp}^2 \sum_v \left(\frac{u(v)}{\Psi_v}\right)^2 \left(1 + \frac{\lambda}{2-\lambda}\left(\frac{\bar{d}-d_v}{1-\bar{d}}\right)^2\right) + \sigma_{zq}^2 \sum_v \left(\frac{u(v)}{\Psi_v}\right)^2 \left(1 + \frac{\lambda^3}{2-\lambda}\left(\frac{\bar{d}-d_v}{1-\bar{d}}\right)^2 + \left(1 + \lambda\,\frac{d_v-\bar{d}}{1-\bar{d}}\right)^2\right)$$

from which it follows that the variance of the rate of inflation depends on σ_{mp}^2, σ_{mq}^2, σ_{xp}^2, $\sigma_q^2 = \sigma_{xq}^2 + \sigma_\epsilon^2$, σ_{zp}^2 and σ_{zq}^2. It is immediately obvious from (15) that the variance of the general rate of inflation is an increasing function of both the variance, σ_{mp}^2, of the permanent component of money supply as well as of the variance, σ_{mq}^2, of the transitory component of money supply. The effects of the other variances is not so immediately clear because changes in σ_{xp}^2 and σ_q^2 affect $V(\pi)$ directly but also by changing the coefficient of adaptation θ. Similarly changes in σ_{zp}^2 and σ_{zq}^2 change $V(\pi)$ directly but also by changing the coefficient of adaptation λ. The following proposition gives rather weak sufficient conditions for $V(\pi)$ to increase in all the underlying variances.

Proposition 1 [16/]: a. The variance of the rate of inflation increases in both σ_{mp}^2 and σ_{mq}^2. b. Provided the non stochastic distributions of $u(v)$ and d_v are independent or have a positive correlation the variance of the rate of inflation is increasing in σ_{xp}^2, σ_{zp}^2, σ_q^2 and σ_{zq}^2. In particular it is increasing in σ_ϵ^2.

16/ In this and subsequent propositions it is implicitly assumed that there is some degree of permanent-transitory confusion. That is $0 < \theta < 1$ and $0 < \lambda < 1$.

Proof: See part A of appendix.

$u(v)$ is the weight assigned to market v in the computation of the general price index. $d_v = \gamma_v/(\Psi_v+\gamma_v)$ is the ratio of the supply elasticity of good v to the sum of the (absolute value of the) demand elasticity and the supply elasticity of that good. I will refer to d_v as the relative supply elasticity of good v. The condition of proposition 1 requires that there be no systematic relationship between the weights $u(v)$ and the relative supply elasticities, d_v, across goods (or that there be a positive association between them). This seems like a reasonably weak condition. In any case it is only sufficient but not necessary. Even if it does not hold there is a good chance that the results of Proposition 1 hold.

B. Relative Price Change and its variance

By definition the change in the relative price of good s is

$$RPC_t(s) = p_t(s) - Q_t - (p_{t-1}(s) - Q_{t-1}) \quad . \tag{16}$$

Substituting (13) into (4), substituting the resulting expression into (16), using (10a) and (10b) and rearranging, the relative price change of good s can be expressed only in terms of actual, current past shocks as,

$$\begin{aligned} RPC_t(s) &= C \\ &+ (\Delta x_t^p(s) + O_t^q(s))(1-u(s)) - \sum_{v\neq s} u(v)[\Delta x_t^p(v) + O_t^q(v)] \\ &- \frac{1}{\Psi_s}(1-u(s))(\Delta z_t^p(s) + z_t^q(s)) + \sum_{v\neq} \frac{u(v)}{\Psi_v}[\Delta z_t^p(v) + z_t^q(v)] \\ &- A(s)[\theta \sum_{i=0}^{\infty}(1-\theta)^i \Delta x_{t-1-i}^p(s) - \theta^2(O_{t-2}^q(s) + (1-\theta)O_{t-3}^q(s) + \dots)] \\ &+ \sum_{v\neq s}\frac{u(v)}{1-\bar{d}} B(s,v)\,(\theta \sum_{i=0}^{\infty}(1-\theta)^i \Delta x_{t-1-i}^p(v) - \theta^2(O_{t-2}^q(v) + (1-\theta)O_{t-3}^q(v) + \dots)) \end{aligned} \tag{17}$$

$$+ \frac{A(s)}{\Psi_s}(\lambda \sum_{i=0}^{\infty}(1-\lambda)^i \Delta z^p_{t-1-i}(s) - \lambda^2(z^q_{t-2}(s) + (1-\lambda)\, z^q_{t-3}(s) + \ldots))$$

$$- \sum_{v\neq s}\frac{u(v)}{\Psi_v(1-\bar{d})} B(s,v)(\lambda \sum_{i=0}^{\infty}(1-\lambda)^i \Delta z^p_{t-1-i}(v) - \lambda^2(z^q_{t-2}(v) + (1-\lambda)z^q_{t-3}(v) + \ldots))$$

$$- [1-u(s) + \theta A(s)]0^q_{t-1}(s) + \frac{1}{\Psi_s}[1-u(s) + \lambda A(s)]z^q_{t-1}(s)$$

$$+ \sum_{v\neq s} u(v)[1+\theta\frac{B(s,v)}{1-\bar{d}}]\, 0^q_{t-1}(v) - \sum_{v\neq s}\frac{u(v)}{\Psi v}[1+\lambda\frac{B(s,v)}{1-\bar{d}}]z^q_{t-1}(v)$$

where $B(s,v) \equiv 1-\bar{d} - (1-d_v)(1-d_s)$, $A(s) \equiv d_s - \frac{u(s)}{1-\bar{d}}B(s,s)$ and C is a known combination of parameters of no particular interest.[17/]

The unconditional expected value of $RPC_t(s)$ is equal to C since the unconditional expected value of all the other terms on the right hand side of (17) is zero.[18/] Noting that each of the stochastic terms in (17) is uncorrelated with each of the other stochastic terms and using (1a) and (3a)-(3c) the unconditional variance of $RPC_t(s)$ can be calculated as:

$$V(RPC(s)) = \sigma^2_{xp}[(1-u(s)^2 + \sum_{v\neq s} u(v))^2 + \frac{\theta}{2-\theta}((A(s)^2 + \sum_{v\neq 2}(\frac{u(v)B(s,v)}{1-\bar{d}})^2)]$$

$$+ \sigma^2_{zp}[(\frac{1-u(s)}{\Psi_s})^2 + \sum_v(\frac{u(v)}{\Psi_v})^2 + \frac{\lambda}{2-\lambda}((\frac{A(s)}{\Psi_s})^2 + \sum_{v\neq s}\frac{u(v)B(s,v)}{\Psi_v(1-\bar{d})})]$$

$$+ \sigma^2_q[(1-u(s))^2 + \sum_{v\neq s}(u(v))^2 + \frac{\theta^3}{2-\theta}((A(s))^2 + \sum_{v\neq s}(\frac{u(v)B(s,v)}{1-\bar{d}})^2)$$

$$+ (1-u(s) + \theta A(s))^2 + \sum_{v\neq s}(u(v))^2(1+\theta\frac{B(s,v)}{1-\bar{d}})^2] \qquad (18)$$

$$+ \sigma^2_{zq}[(\frac{1-u(s)}{\Psi_s})^2 + \sum_{v\neq s}(\frac{u(v)}{\Psi_v})^2 + \frac{\lambda^3}{2-\lambda}((\frac{A(s)}{\Psi_s})^2 + \sum_{v\neq s}(\frac{u(v)B(s,v)}{\Psi_v(1-\bar{d})})^2)$$

$$+ (\frac{1-u(s) + \lambda A(s)}{\Psi_s})^2 + \sum_{v\neq s}(\frac{u(v)}{\Psi_v})^2(1+\lambda\frac{B(s,v)}{1-\bar{d}})^2] \quad .$$

17/ $C = \frac{1}{(1-d)(\Psi_s + \gamma_s)}[\Psi_s \sum_v u(v)\frac{\beta_v}{\Psi_v + \gamma_v} - (1-\bar{d})\beta_s]$

18/ For $\Psi_v = \Psi$, $\gamma_v = \gamma$ and $\beta_v = \beta$ for all v-s c is equal to zero.

Although this expression looks rather formidable it turns out that, under relatively weak conditions, it is unambiguously increasing in all the underlying variances. The details appear in the following proposition.

Proposition 2: a. If the following conditions are satisfied

$$\text{(i)} \quad d_s > u(s) \ , \ \text{(ii)} \quad \frac{1-\bar{d}}{1-d_s} > \frac{1}{2} > u(s) \ , \ \text{(iii)} \ \sum_{v \neq s} (u(v))^2 > \frac{1-d_s}{1-\bar{d}} \sum_{v \neq s} (u(v))^2 (1-d_v)$$

the variance of relative price change is an increasing function of σ^2_{xp}, σ^2_{zp}, σ^2_{xq}, σ^2_{zq} and σ^2_{ε} .

b. The variance of relative price change does not depend on the aggregate monetary variances σ^2_{mp} and σ^2_{mq} .

Proof: a. See Appendix B. b. Immediate by inspection of (18).

Condition (i) of the Proposition states that the relative size of the supply elasticity of good s (in comparison to the sum of the absolute values of the demand and supply elasticities of this good) has to be larger than the weight given to good s in the general price index. Since this weight is small and most supply and demand elasticities do not take extreme values (of 0 or infinity) this condition is quite likely to be fulfilled. The part $1/2 > u(s)$ of condition (ii) is just a restatement of the idea that u(s) is small. The first inequality in (ii) restricts the relative demand elasticity of good s, $1-d_s$, to be no larger than two times the average relative demand elasticity in the economy.

For any s whose relative demand elasticity is equal to, or smaller than, the average relative demand elasticity in the economy condition (iii) is always fulfilled since $1-d_v < 1$ for all v-s. It will be fulfilled for markets with relative demand elasticities which are larger than the average relative demand elasticity as well provided $1-d_s$ exceeds $1-\bar{d}$ by an amount which is not too large and whose exact form can be derived from condition (iii). In any case all three conditions are only sufficient conditions. Inspection of table 1 in Appendix B and of (18) suggests that even when one or more of these conditions are violated it is still likely that the result of Proposition 2a holds. It is noteworthy that the aggregate monetary variances σ^2_{mp} and σ^2_{mq} do not affect the variance of relative price change. This is a consequence of the underlying monetary neutrality of the model and the fact that people are subject only to the permanent transitory confusion and not to the aggregate relative confusion.[19/] However, the differential (across markets) transitory monetary noise, σ^2_ε, does affect the variance of relative price change directly through σ^2_q and indirectly by affecting the coefficient of adaptation θ. Proposition 2a suggests that the total effect of an increase in σ^2_ε is to increase the variance of relative price change.

It is interesting that the variance of relative price change of different goods differs even though the stochastic structure of all markets is identical. This is a consequence of the differences in elasticities across markets.[20/]

19/ By contrast in models which deal exclusively with the aggregate relative confusion, the variance of relative price change does depend on the aggregate monetary variance. See Barro (1976) and Cukierman (1979a). This suggests that a framework which incorporates both types of confusion simultaneously may be of interest.

20/ By contrast the variance of relative price change in Barro (1976), Cukierman (1979a) and Cukierman (1979b) is uniform across markets. Obviously if we substitute uniform (across markets) demand and supply elasticities and $u(v) =$ constant into (18) the variance of relative price change will also become uniform here as well.

C. The relationship between the variance of the rate of inflation and the variance of relative price change:

A direct consequence of propositions 1b and 2a is

Proposition 3: Under the conditions of Propositions 1 and 2 changes in either or some combination of the following exogenous variances:

(i) The variance, σ^2_{xp}, of permanent relative real demand shocks.

(ii) The variance, σ^2_{xq}, of transitory relative real demand shocks.

(iii) The variance, σ^2_{zp}, of permanent relative shocks to productivity.

(iv) The variance, σ^2_{zq}, of transitory relative shocks to productivity.

(v) The variance, σ^2_{ε}, of the differential monetary noise,

cause a positive relationship between the variance of relative price change and the variance of the rate of inflation.

The proposition implies that there should be a positive relationship between the variance of relative price change and the variance of the rate of inflation in a cross section of countries, each with a different, but constant over time set of exogenous variances. This is in principle an empirically testable proposition which is indirectly supported by the work of Glejser (1965) and that of Jaffe and Kleiman (1977). It is found in both studies that there is, cross sectionally, a positive relationship between the variance of relative price change in a country and its average inflation. This finding together with the widely documented positive relationship between the variance and the level of inflation yield empirical support to Proposition 3. The positive, over time, relationship found between the variance of relative price change and the variance of inflation by Vining and Elwertowski (1976) is also suggestive in this context. However, it is not quite a test of Proposition 3 since it is done over time whereas the proposition applies to different regimes for which data across countries is a better approximation.

The result of Proposition 3 also implies that there should be a positive association across regimes between relative price change variance and the extent of unanticipated inflation; when the variance of inflation is larger it becomes more difficult to forecast it and unanticipated inflation is therefore larger. By Proposition 3 the variance of inflation and the variance of relative price change are positively related. Hence unanticipated inflation and relative price variability should be positively related across regimes.[21/]

This section is concluded by interpreting the model somewhat broadly in order to provide a tentative explanation for a widely documented empirical regularity. Recent empirical literature suggests that the level and the variance of the rate of inflation are positively related both cross sectionally and over time (Okun (1971), Gordon (1971), Logue and Willet (1976), Jaffee and Kleiman (1977), Foster (1978) and Blejer (1978). In terms of the model presented here this could arise if either or both of the following hold;

a. An increase in the rate of monetary expansion is accompanied by an increase in the variance of money. Such a relationship may arise if governments tend to use stop go policies more and with a higher frequency when the rate of inflation is higher. In terms of the model a positive relationship between the variances σ^2_{mp}, σ^2_{mq} on one hand and the rate of monetary inflation immediately implies through (15) a positive relationship between the level and the variance of the rate of inflation. This conjecture is easily tested empirically.

b. If the average rate of monetary expansion, δ , and the variance of the differential monetary noise, σ^2_{ε}, are positively related. At least for low and medium rates of inflation (like those experienced by the U.S.) this seems to be

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21/ Parks (1978) finds such a positive relationship over time for the U.S. However, there is no work, to my knowledge, which makes this comparison cross sectionally.

22/ As suggested by Flemming (1976), this could be caused by the tendency of governments to announce and attempt, at least partially, to implement more unrealistic stabilization programs at higher rates of inflation.

a reasonable conjecture because at medium rates of inflation it pays for some sectors to make institutional adjustments to the inflationary environment but it does not pay other sectors to do that. However, past a certain threshold high enough inflation all sectors will introduce the institutional adjustments. In that range we should probably expect a negative or no relationship between the rate of inflation and the variance, σ_ε^2. But for the low and medium inflation countries a positive relationship between δ and σ_ε^2 could provide another reason for the observed (mostly in such countries) positive relationship between the variance in the general rate of inflation and the rate of general inflation.

III. RELATIVE PRICES AS SIGNALS FOR PRODUCTION DECISIONS

A fundamental function of the price system and of relative prices in particular, as emphasized by Hayek (1945) is to transmit efficiently the information that agents in the economy need in order to decide what to produce and how much to produce. Since in most cases production decisions have to be made before the relevant relative prices are fully known it becomes important for producers to separate between the temporary and the permanent component of the relative price of the good that they currently observe on the market. Only the permanent component is relevant for their decisions since it is the relative price at which they are most likely to sell the good on whose production they are currently deciding. The question of efficiency of relative prices as production signals then becomes: to what extent are today's predictions of tomorrow's prices a good guide to production decisions?

This section is devoted to the investigation of the above issue. More specifically the two following questions are handled: 1. How much of current relative price information do people use in making production decisions and what are the factors which determine the extent to which they use it? 2. How well does the price system perform its signalling function and what are the factors which determine its efficiency?

A. <u>The utilization of current relative prices</u>: Substituting (4) into (5), and using the resulting expression together with (4) again, the actual relative price of good s can be written:

$$p_t(s) - Q_t = O_t(s) - \sum_v u(v) O_t(v) - \frac{z_t(s)}{\psi_s} + \sum_v \frac{u(v)}{\psi_v} z_t(v) - \left[\frac{\gamma_s}{\psi_s}(E_{t-1}p_t(s) - E_{t-1}Q_t) - \sum_v u(v)\frac{\gamma_v}{\psi_v}(E_{t-1}p_t(v)\right] - E_{t-1}Q_t) - K_{st} \tag{19}$$

where K_{st} is some combination of parameters of no interest for the discussion.

By plugging into the right hand side of (19) perceived permanent values instead of actual values it is possible to compute $E_{t-1}p_t(s) - E_{t-1}Q_t$, that part of the observed relative price which people currently believe to be permanent. Using the fact that, by (10), permanent perceptions are formed adaptively and rearranging terms it is possible to write the believed in permanent relative price as

$$E_tP_{t+1}(s) - E_tQ_{t+1} = \theta(p_t(s) - Q_t) + (1 - \theta)(E_{t-1}p_t(s) - E_{t-1}Q_t) + (\theta - \lambda)\left[\frac{z_t(s)}{\psi_s} - \sum_v u(v)\frac{z_t(v)}{\psi_v} - \left(\frac{E_{t-1}z_t(s)}{\psi_s} - \sum_v u(v)\frac{E_{t-1}z_t(v)}{\psi_v}\right)\right] \tag{20}$$

Abstracting for a moment from the last term in (20) by assuming $\theta = \lambda$ we see that the believed in permanent relative price of period t is simply a weighted average of the actual relative price in period t and of what was believed to be the permanent relative price in the previous period. It is apparent from (20) that it is not optimal to base the forecast of the permanent component of the relative price[23/] on the actual relative price only. Intuitively, since relative price movements are caused by both transitory and permanent shocks, the optimal forecast of the permanent component of a relative price gives some weight to current relative prices but also to preconceptions about permanent relative prices. Moreover the higher the variance of permanent relative demand and productivity shocks

<u>23/</u> which determines the level of output for period t + 1.

in comparison to the variance of temporary relative demand and productivity shocks the higher is the common value of θ and λ (see footnote 12) and the more weight is given to current relative prices. For higher values of θ production decisions become more sensitive to changes in actual relative prices. When the variances of permanent shocks are relatively large there is a good chance that any given movement in actual relative prices reflects mostly permanent changes. It therefore makes good sense to give a large weight to the actual movements in relative prices. On the other hand when this variance is small in comparison to the variance of transitory shocks, actual movements in relative prices reflect mostly temporary effects and it pays to stick to preconceptions.

It is also apparent from (20) and the supply functions in (1) that production decisions overreact to purely transitory changes and underreact to purely permanent changes in relative demands and relative productivities. This is a direct consequence of the inability of individuals to distinguish perfectly between permanent and transitory changes.

The picture gets a bit more complicated when θ and λ differ from each other. In addition to the simple adaptation of beliefs just described, the difference between the two coefficients of adaptations is applied to the difference between actual relative productivity in good s and last period permanent perceptions about relative productivity in this market.

It is interesting that the beliefs about the permanent relative price in (20) have an adaptive structure. Unlike most of the literature in which adaptive expectations have appeared those beliefs are fully rational since they are derived by using the structure of the economy in conjunction with all the currently available information.[24]

B. The efficiency of relative prices as signals for production: Ideally producers would have liked to gear their production to the actual relative price of their good in the period in which they sell it. This, however, is not feasible for two reasons; first they do not know what shocks are going to materialize in the future. Second, even at the present period they have only partial knowledge of the degree of permanence of the current relative price. Hence actual output in each industry will deviate, in general, from the output that producers would have been producing if they had more information. Following Barro (1976), I will refer to this level of output as full information output.[25/] However, in the present context two alternative concepts of full information output can be distinguished. The first is the level of output that would be produced if producers knew in period t the true decomposition of the current relative price into its permanent and transitory components. I will refer to this level of output as full current information output since it arises when individuals have full current information about the permanence of current shocks but no information beyond the structure of the model about shocks that are going to materialize only in the future. The second concept to which I shall refer as full future information output is the level of output that would have been produced if the actual relative price of period $t + 1$ was already known in period t. This concept endows individuals not only with perfect knowledge about the

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24/ The precise adaptive structure in (20) is a consequence of the particular stochastic assumptions made about the distributions of the permanent and transitory components of the various shocks. However, the result that when people confuse between transitory and permanent changes, perceptions about permanent magnitudes adjust slowly using all past information transcends the particular stochastic structure used here. For example, if for any stationary stochastic process of the "ARIMA" type the perception of the permanent component is defined as the predicted value of the series as the forecast horizon goes to infinity, then this perception is usually a distributed lag of all past realizations of the process. For such a definition and other details see Nelson and Plosser (1979).

25/ However, in Barro's model the informational confusion is between aggregate and relative price movements and it is assumed that all markets have identical demand and supply functions.

composition of shocks that have occurred but also with perfect foresight about next period shocks. If we take the view that full information can refer only to the components of shocks that have materialized already the first concept is the relevant one.26/ Full current information output, $y^*_t(s)$ can be calculated from (1) as:

$$y^*_t(s) = \beta_s t + z^p_{t-1}(s) + \gamma_s(p^p_{t-1}(s) - Q^p_{t-1}) \tag{21}$$

where $p^p_{t-1}(s) - Q^p_{t-1}$ is the best forecast of the relative price for period t as of period $t-1$ provided the true values of the permanent levels of relative demand and productivity shocks of period $t-1$ are known in that period. Subtracting (21) from (1) we obtain the difference between actual and full current information output as

$$D_c(s) = y_t(s) - y^*_t(s) = \Delta z^p_t(s)+z^q_t(s)+\gamma_s[E_{t-1}P_t(s)-E_{t-1}Q_t-(P^p_{t-1}(s)-Q^p_{t-1})] \tag{22}$$

which suggests that actual output is larger (smaller) than full current information output if the believed in permanent relative price of the previous period is larger (smaller) than the actual permanent relative price of that period. By going through a calculation analogous to that which led to equation (13), with true permanent values replacing perceptions about permanent values, it is possible to express $p^p_{t-1}(s) - Q^p_{t-1}$ as a function of the true permanent values of the shocks in period $t-1$. Substituting this expression as well as (13) into (22) and rearranging

$$\begin{aligned} D_c(s) &= \Delta z^p_t(s) + z^q_t(s) \\ &+ \frac{\gamma_s\psi_s}{\psi_s+\gamma_s}\Bigg[\left(1-u(s)\frac{1-d_s}{1-\bar{d}}\right)\left(E_{t-1}x_t(s) - x^p_{t-1}(s)\right) - \frac{(1-u(s))(E_{t-1}z_t(s) - z^p_{t-1}(s))}{\psi_s + \gamma_s} \\ &- \sum_{v\neq s} u(v)\frac{1-d_v}{1-\bar{d}}\left(E_{t-1}x_t(v) - x^p_{t-1}(v)\right) + \sum_{v\neq s}\frac{u(v)}{\psi_v+\gamma_v}\left(E_{t-1}z_t(v) - z^p_{t-1}(v)\right)\Bigg] \end{aligned} \tag{23}$$

26/ Both Barro (1976) and Cukierman (1979a) use full current information output as a benchmark. However, since a comparison of actual to full future information output is of independent interest some remarks about its implications are made at the end of this section.

By substituting (10a) and (10b) into (23) it can be checked that the unconditional expected value of (23) is zero. Rearranging terms after this substitution so that the terms in (23) are all mutually uncorrelated it is possible to compute the variance of $D_c(s)$ as the sum of the variances of those terms. The result appears in equation (24).27/

$$V(D_c(s)) = \sigma_{zp}^2 + \sigma_{zq}^2 + (\psi_s d_s)^2 \Bigg[\Big[\Big(1-u(s)\,\frac{1-d_s}{1-\bar{d}}\Big)^2 + \sum_{v \neq s} \Big(u(v)\,\frac{1-d_v}{1-\bar{d}}\Big)^2 \Big] \underbrace{\frac{\theta^2\sigma_q^2 + (1-\theta)^2\,\sigma_{xp}^2}{\theta(2-\theta)}}_{G(\theta)} + \Big(\Big(\frac{1-u(s)}{\psi_s+\gamma_s}\Big)^2 + \sum_{v \neq s}\Big(\frac{u(v)}{\psi_v+\gamma_v}\Big)^2\Big) \underbrace{\frac{\lambda^2\sigma_{zq}^2 + (1-\lambda)^2\sigma_{zp}^2}{\lambda(2-\lambda)}}_{G(\lambda)} \Bigg] \qquad (24)$$

This variance is the variance of actual around full current information output and as we saw it is positive because people confuse between permanent and transitory movements in relative demands and relative productivities. If producers had known the actual permanent value of relative demands and relative productivities in each period the variance in (24) would have been zero. However since such perfect information is unavailable, actual output usually deviates from $y_t^*(s)$. The variance in (24) is a possible quantitative measure of the extent of this deviation. I will take it as a measure of the signalling efficiency of relative prices for production decisions. The larger it is the less efficient is the price system in generating optimal production decisions and the more costly is the permanent-transitory confusion for the economy. Friedman (1977) suggests that increased volatility of inflation makes market prices less efficient coordinators of economic activity. 28/ The model presented here makes it possible to formalize and quantify those costs by using measures like that in (24). It is therefore interesting to

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27/ In the derivation use has been made of the formula for the sum of an infinite geometric progression.

28/ See Friedman Op.Cit., p.467.

inquire how is the variance of actual around full current information output affected by various factors. It is apparent that this variance is affected by either of σ_{xp}^2 or σ_q^2 only through its dependence on the term $G(\theta)$ on the right hand side of (24). Differentiating G partially w.r.t. σ_{xp}^2 and w.r.t. σ_q^2

$$\text{a.} \quad \frac{\partial G}{\partial \sigma_{xp}^2} = \frac{(1-\theta)^2}{(2-\theta)^2\theta^3} \qquad\qquad \text{b.} \quad \frac{\partial G}{\partial \sigma_q^2} = \frac{\theta}{2-\theta} \tag{25}$$

Both partial derivatives are obviously positive. Hence an increase in either the variance of transitory relative demand shocks or the variance of permanent relative excess demand shocks, whether caused by real or monetary factors, causes an increase in the variance of the deviation of actual from full current information output. Similarly since $G(\lambda)$ is the same function of λ as $G(\theta)$ is of θ it follows from (25) that any increase in either the variance of the permanent component of relative shocks to productivity, σ_{zp}^2, or the variance of the transitory component of relative shocks to productivity, σ_{zq}^2 cause an increase in the variance of the deviation of actual from full current information output.

Since σ_q^2 is at least partially induced by the differential monetary noise, σ_ϵ^2, (25b) implies that it should be kept as low as possible.[29/]

More generally to the extent that government can decrease the real relative variances of demand and productivity shocks (σ_{xp}^2, σ_{xq}^2, σ_{zp}^2, σ_{zq}^2) by promoting stable policies in other areas this is desirable as well. Any policies which increase those variances make it more difficult for producers

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29/ By contrast Barro (1976) and Cukierman (1979a) obtain, using a similar criterion of efficiency, that the variance of aggregate monetary shocks should be kept at zero. In the present framework this variance summarized here by σ_{mp}^2 and σ_{mq}^2 does not affect the allocative efficiency of the price system since the aggregative-relative confusion has been assumed away.

to adapt their production decisions to those irreducible variations in relative demands and productivity which are caused by nature.

It seems intuitively plausible that a lower σ_q^2 (or σ_{zq}^2) by decreasing the amount of "noise" in the economy will make the dispersion of actual around full current information output smaller. It is less clear why a decrease in σ_{xp}^2 (or σ_{zp}^2) also has the same effect since such a decrease by decreasing the coefficient of adaptation, θ, (or λ) makes the detection of permanent changes more difficult. The answer to this seeming paradox is that a decrease in θ (or λ) also reduces the amount of uncecessary transitory noise that the expectations formation process in (10b) picks. This effect taken in isolation should work to decrease $V(D_c)$.[30/] It turns out that those two conflicting effects of a change in θ on $V(D_c)$ exactly cancel each other leaving only the direct positive effect of σ_{xp}^2 on $V(D_c)$. The reason for this cancelling out is brought out more clearly in the following proposition which I believe is also of independent interest.

<u>Proposition 4</u>: For given values of σ_{xp}^2, σ_q^2, σ_{zp}^2 and σ_{zq}^2 the values of the coefficients of adaptation, θ and λ which minimize the variance of actual around full current information output are given by equations (11b) and (11a) respectively.

Proof: $V(D_c(s))$ is minimized for the value of θ which minimizes the expression $G(\theta)$ on the right hand side of (24). The first and second partial derivatives of $G(\theta)$ w.r.t. θ are

$$\frac{\partial G}{\partial \theta} = \frac{2(\theta^2\sigma_q^2 - (1-\theta)\,\sigma_{xp}^2)}{(2-\theta)^2\,\theta^2} \tag{26}$$

$$\frac{\partial^2 G}{\partial \theta^2} = \frac{2[((2-\theta)\theta + 4(1-\theta)^2)\,\sigma_{xp}^2 + \theta^3\sigma_q^2]}{((2-\theta)\theta)^3}$$

30/ For notational convenience the index s of $V(D_c(s))$ is deleted.

respectively. Since the second partial is positive for any θ it follows that $G(\theta)$ will be minimized by the value of θ for which the first partial is 0. Solving for this value of θ from (26) we obtain that it is equal to the expression for θ in (11b). The proof for λ is analogous. Q.E.D.

For the individual producer σ^2_{xp}, σ^2_q, σ^2_{zp} and σ^2_{zq} constitute exogenous datum but he is free to use the data he observes in any way he chooses. Proposition 4 suggests that he uses the available information so as to make the variance of the deviation between actual output and the output he would be producing if he had full current information as small as possible. Intuitively the individual increases θ up to the point at which the marginal decrease in $V(D_c)$ as a result of quicker detection of permanent changes is equal to the marginal increase in $V(D_c)$ caused by the fact that the higher θ also pick up more of the transitory noise. Since, at the margin, there is no net effect of θ on $G(\theta)$ the total effect of a change in σ^2_{xp} or σ^2_q on $V(D_c)$ reduces to their direct effects which are always positive. A similar argument holds for λ.

C. A remark on the deviation of actual around full future information output; All the results that were just demonstrated for the deviation of actual around full current information output hold also for the deviation of actual from full future information output.[31/] More precisely the variance of the deviation of actual output from full future information output, $V(D_f)$, is an increasing function of all the underlying variances σ^2_{xp}, σ^2_{xq}, σ^2_{zp}, σ^2_{zq} and in particular of σ^2_{ε}. Furthermore the coefficients of adaptation θ and λ from equation (11) also minimize $V(D_f)$.

An advantage of $V(D_f)$ over $V(D_c)$ is that it is sensitive to the length of the production lag. In particular the longer the production lag the higher, ceteris

31/ Recall that future information output is the level of output that would have been produced, had the producer known at the time he made production decisions what will be the actual relative price at which he will sell.

paribus, is $V(D_f)$. This implies that firms which, because of the nature of their business have to plan production further in advance will be more adversely affected by uncertainty. Furthermore a given increase in σ^2_{xp} or σ^2_{zp} will increase $V(D_f)$ more in firms with longer production lags. The production lag, whether short or long is assumed given here. However, the result that firms with longer production lags are more adversely affected by uncertainty holds even when the production lag is responsive to policy provided some difference in production lags across firms persists. For brevity the proof of the above statements is omitted.[32/]

V. CONCLUDING REMARKS

This paper has presented a theory of the relationship between the aggregate price level and relative prices which is based on the inability of individuals to identify permanent changes immediately as they occur.[33/] Since the optimal forecast of the permanent relative price turns out to be a combination of distributed lags on past realizations of relative demand and productivity shocks production decisions respond only partially to changes in relative prices. As a result the supply elasticity w.r.t. the permanent relative price is larger than the supply elasticity w.r.t. the actual relative price. The larger are the variances of permanent relatively to the variances of transitory shocks, the more do production decisions rely on current relative prices. It should be stressed that although expectation formation is fully rational it is optimal for individuals to use information on all the past thus introducing sluggishness into their formation and into production and pricing decisions.[34/]

32/ The proof is available from the author upon request.

33/ Most of the positive implications of this theory appear in the introduction and are not duplicated here.

34/ A similar point is made in the context of learning about the changing parameters of a single equation by B.Friedman (1979) and in the context of a macro model by Brunner, Cukierman and Meltzer (1980). See also Shiller (1978) Section 5.

This paper also considers questions relating to the efficiency of relative prices as coordinators of economic activity and introduces two alternative quantitative measures of this efficiency. In particular it is shown that any increase in variance, whether caused by transitory or permanent, monetary or real factors, increases the dispersion of actual around full information output.

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APPENDIX

A. Proof of proposition 1b.

1. Effect of a change in σ_q^2 : Differentiating $V(\pi)$ from (15) partially with respect to σ_q^2 , taking (11b) into consideration, and rearranging

$$\text{(A1)}\quad \frac{\partial V(\pi)}{\partial \sigma_q^2} = 2\left[J_\theta + \theta(1-\theta)\right]\theta \frac{\sum_v (u(v)a_v)^2}{(2-\theta)(1-\theta)} + 2\sum_v (u(v))^2(1+\theta a_v)$$

$$\text{where}\quad J_\theta \equiv \frac{\theta^2}{2(1-\theta)}\left(1-\sqrt{\frac{1+\frac{\theta^2}{1-\theta}\left(1+\frac{\theta^2}{4(1-\theta)}\right)}{\frac{\theta^2}{1-\theta}\left(1+\frac{\theta^2}{4(1-\theta)}\right)}}\right),\quad a_v \equiv \frac{d_v-\bar{d}}{1-\bar{d}}$$

and use has been made of the relationship

$$\frac{\sigma_{xp}^2}{\sigma_q^2} = \frac{\theta^2}{1-\theta}$$ which can be derived from (11b).

Since $0 < \theta < 1$ the first expression on the right hand side of (A1) is positive iff

$$\text{(A2)}\quad J_\theta + \theta(1-\theta) > 0$$

which in turn is equivalent to the condition $4(1-\theta(1-\theta)) > 1+\theta+\theta^3$. It can be checked that this condition is always satisfied for $0 \leq \theta \leq 1$. Hence the first term on the right hand side of (A1) is positive. The second term on the right hand side of (A1) can be rewritten

$$\sum_v (u(v))^2 + \frac{\theta}{1-\bar{d}}\sum_v (u(v))^2(d_v-\bar{d})$$ which is positive if

$$\text{(A3)}\quad \sum_v u(v))d_v\left[u(v) - \sum_s (u(s))^2\right] > 0$$

If d_v was constant (A3) would be equal to zero. When it is changing from market to market independently of u(v) or so as to produce a positive correlation between u(v) and d_v the first term on the R.H.S. of (A3) becomes bigger than the second making (A3) positive. It follows $\frac{\partial V(\pi)}{\partial \sigma_q^2} > 0$.

2. Effect of a change in σ_{zq}^2 : Differentiating, similarly, V(π) partially with respect to σ_{zq}^2 using (11a) and rearranging

$$\text{(A4)} \quad \frac{\partial V(\pi)}{\partial \sigma_{zq}^2} = 2\left[J_\lambda + \lambda(1-\lambda)\right]\lambda \frac{\sum_v\left(\frac{u(v)}{\psi_v} a_v\right)^2}{(2-\lambda)(1-\lambda)} + 2\sum_v \left(\frac{u(v)}{\psi_v}\right)^2\left(1+\lambda\frac{d_v-\bar{d}}{1-\bar{d}}\right)$$

where J_λ is the same function of λ as J_θ is of θ. It follows from (A2) that $J_\lambda + \lambda(1-\lambda) > 0$. The first term on the R.H.S. of (A4) is therefore positive. (A3) implies that without the division by ψ_v the second term on the right hand side would have been positive too. Since there is, by definition, a negative relationship between ψ_v and d_v the division by ψ_v gives relatively higher weights to positive values of $d_v-\bar{d}$ making this term even more positive.

3. Effect of a change in σ_{xp}^2: Inspecting V(π) and noting that θ is an increasing function of σ_{xp}^2 it can be seen that all the terms on the right hand side of (15) unambiguously increase in σ_{xp}^2 except for $\theta \sum_v (u(v))^2(d_v-\bar{d})$ which was shown to be positive in (A3). Hence an increase in σ_{xp}^2 increases V(π).

4. Effect of a change in σ_{zp}^2 : Using an argument similar to that in 3 and noting again that ψ_v and d_v are negatively related it can be shown that V(π) increases in σ_{zp}^2 .

B. Proof of Proposition 2a

1. Effects of changes in σ_q^2 and σ_{zq}^2.

The partial derivatives of V(RPC(s)) with respect to σ_q^2 and σ_{zq}^2 appear in table 1. They are derived by differentiating (19) with respect to σ_q^2 and σ_{zq}^2, using the relationships $\sigma_{zp}^2/\sigma_{zq}^2 = \lambda^2/(1-\lambda)$ and $\sigma_{xp}^2/\sigma_q^2 = \theta^2/(1-\theta)$ which can be derived from (11a) and (11b) and by rearranging. It is convenient to prove two lemmas first

Lemma 1; $K_\theta \geq -1$, $K_\lambda \geq -1$ (See definitions in table 1)

Proof: Rearranging the elements in K_θ it can be shown that the condition $K_\theta > -1$ is equivalent to the condition

$$(1-\theta)^3 \left[\theta^7 + \theta^5(2-\theta) + 2\theta^4 + 3\theta^3 + \theta^2 + 4(1-\theta)^2\right] \geq 0$$

It is easy to see that for $0 \leq \theta \leq 1$ this expression is non negative. The proof of $K_\lambda > -1$ is analogous. Q.E.D.

TABLE 1

(A5) $$\frac{\partial V(RPC(s))}{\partial \sigma_q^2} = (1-u(s))^2 + K_\theta\Big(\underbrace{d_s - u(s)\Big(1-\frac{(1-d_s)^2}{1-\bar{d}}\Big)}_{A(s)}\Big)^2 + \sum_{v\neq s}(u(v))^2\Big[1+\Big(1-\frac{(1-d_v)(1-d_s)}{1-\bar{d}}\Big)^2 K_\theta\Big] + (1-u(s)+\theta A(s))(1-u(s)+A(s)(\theta+2J_\theta))+2(\theta+J_\theta)\sum_{v\neq s}(u(v))^2\Big[1-\frac{(1-d_v)(1-d_s)}{1-\bar{d}}\Big] + \sum_{v\neq s}(u(v))^2\Big[1+\theta(\theta+2J_\theta)\Big(1-\frac{(1-d_v)(1-d_s)}{1-\bar{d}}\Big)^2\Big]$$

(A6) $$\frac{\partial V(RPC(s))}{\partial \sigma_{zq}^2} = \Big(\frac{1-u(s)}{\psi_s}\Big)^2 + K_\lambda\Big(\frac{A(s)}{\psi_s}\Big)^2 + \sum_{v\neq s}\Big(\frac{u(v)}{\psi_v}\Big)^2\Big[1+\Big(1-\frac{(1-d_v)(1-d_s)}{1-\bar{d}}\Big)^2 K_\lambda\Big] + \Big(\frac{1}{\psi_s}\Big)^2(1-u(s)+\lambda A(s))\;(1-u(s)+A(s)(\lambda+2J_\lambda)+2(\lambda+J_\lambda)\sum_{v\neq s}\Big(\frac{u(v)}{\psi_v}\Big)^2\Big[1-\frac{(1-d_v)(1-d_s)}{1-\bar{d}}\Big] + \sum_{v\neq s}\Big(\frac{u(v)}{\psi_v}\Big)^2\Big[1+\lambda(\lambda+2J_\lambda)\Big(1-\frac{(1-d_v)(1-d_s)}{1-\bar{d}}\Big)^2$$

where $$J_\theta \equiv \frac{\theta^2}{2(1-\theta)}\left(1-\sqrt{\frac{1+\frac{\theta^2}{1-\theta}\left(1+\frac{\theta^2}{4(1-\theta)}\right)}{\frac{\theta^2}{1-\theta}\left(1+\frac{\theta^2}{4(1-\theta)}\right)}}\right), \quad K_\theta \equiv \theta^2\left[\frac{\theta}{2-\theta}+\frac{2}{1-\theta}J_\theta\right]$$

J_λ and K_λ are respectively the same functions of λ as J_θ and K_θ are of θ.

Lemma 2: $\theta + 2J_\theta > 0$, $\lambda + 2J_\lambda > 0$

It can be shown by rearranging the condition $\theta + 2J_\theta > 0$ that this condition is equivalent to the condition

$$(1+\theta)\left(1 + \frac{\theta^2}{4(1-\theta)}\right) > 0$$

For $0 \leq \theta < 1$ this expression is always positive. The proof of $\lambda + 2J_\lambda > 0$ is analogous. Q.E.D.

The sum of the first two terms on the right hand side of (A5) is positive. This is proved by showing that even when K_θ assumes its lower bound in Lemma 1 this sum is still positive. For this case the condition that the sum of the first two terms is positive is

$$(1-u(s))^2 > \left(d_s - u(s)\left(1 - \frac{(1-d_s)^2}{1-\bar{d}}\right)\right)^2$$

Since $d_s > u(s)$ this condition is equivalent to the same condition without the second powers which in turn is implied by condition (ii) of the proposition.

The next term on the right hand side of (A5) is positive even in the extreme case $K_\theta = -1$ provided

$$\sum_{v \neq s} (u(v))^2 (1-d_v)\left(2 - \frac{1-d_s}{1-\bar{d}}(1-d_v)\right) > 0$$

which is implied in turn by condition (ii) of the proposition.

The fourth term on the right hand side of (A5) is also positive since, by Lemma 2 $\theta + 2J_\theta > 0$ provided $A(s) > 0$. But condition (i) of the proposition implies that $A(s) > 0$.

The fifth term is positive since $\theta + J_\theta > \theta + 2J_\theta > 0$ by Lemma 2 and the remaining term in the product is positive by condition (iii of the proposition.

The positivity of the last term on the right hand side of (A5) is a direct consequence of Lemma 2. It follows that an increase in σ_q^2 increases V(RPC(s)).

The proof that (A6) is positive is analogous except for one before the last term on the right hand side of (A6). Without the division by ψ_v this term would have been positive by condition (iii) of the proposition. Since ψ_v and $(1-d_v)$ are positively correlated the division by ψ_v results in smaller weights to smaller terms (of the type $1-(1-d_s)(1-d_v)/(1-\bar{d})$) and in higher weights to larger such terms. Hence the entire term must be larger than that which appears in condition (iii) and positive afortiori. It follows that an increase in σ_{zq}^2 increases V(RPC(s)).

2. Effects of changes in σ_{xp}^2 and σ_{zp}^2

By inspecting (18) and noting that θ increases in σ_{xp}^2 * it can be seen that V(RPC(s)) unambiguously increases in σ_{xp}^2 through all its terms except possibly through the following two terms.

$$\text{(A7)} \quad (1-u(s) + \theta\, A(s))^2$$

$$\text{(A8)} \quad \sum_{v \neq s} (u(v))^2 \left(1 + \theta\left(1 - \frac{(1-d_v)(1-d_s)}{1-\bar{d}}\right)\right)^2$$

But condition (i) of the proposition implies that $A(s) > 0$ from which it follows that (A7) increases in θ. Condition (iii) of the proposition implies that (A8) is increasing in θ as well. It follows that V(RPC(s)) increases in σ_{xp}^2. The proof for σ_{zp}^2 is analogous.

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* See footnote 12.