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Labor and wages

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INFLATION AND WAGE INDEXATION

by

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ABSTRACT

This paper studies a worker's and a firm's choice between a nominal and an indexed wage in a labor contract. It shows that the expected inflation does not influence the choice, but that the more uncertain the inflation, the more risk averse the worker, the more concave the production function, and the more imperfect the worker's access to the capital market, the more likely they are to choose an indexed wage; on the other hand, the higher the cost of indexation, the more likely they are to choose a nominal wage.

INTRODUCTION

Labor contracts often index the wage to the price level. The indexation may be full or partial, and it may be combined with a predetermined change in the wage base itself. The indexation serves as an important means of reducing the riskiness that an uncertain inflation causes in the real wage within the contract period, and it may therefore moderate the impact that an uncertain inflation would otherwise have on a worker's consumption and on the number of hours he would work. Accordingly, indexation of the wage may benefit a worker who is averse to risk; in addition, it may benefit a firm that produces under decreasing returns to labor.

Of course, the pre-eminence of indexation for the purpose of moderating the impact of an uncertain inflation presupposes the absence of a riskless asset, or at least that the worker's access to a riskless asset is limited. If, namely, the worker would have free access to a riskless asset, then he could moderate the impact of an uncertain inflation by exchanging his risky earnings with the safe asset, and he would not need the indexation of the wage for this purpose.

Full indexation of a constant base wage to the price level guarantees a constant real wage within the contract period. However, a worker's concern is with the riskiness of his consumption and employment, rather than with the variability of his real wage. A worker may borrow and save, so the time profile of his consumption need not follow the time profile of his real income. Therefore, even when inflation is the only uncertainty, there is no guarantee that a constant real wage reduces the riskiness of the worker's consumption and employment within the contract period.

A contract which combines full indexation with predetermined variations in the base wage can, however, guarantee a reduction in the riskiness that an uncertain inflation causes in a worker's consumption and employment. The full indexation provides a safe real wage at various points in time, while the predetermined variations in the base wage provides a safe way of transferring real income through time. Rather than having to borrow or lend nominal amounts in order to insure a preferred time profile of his consumption, the worker can, in effect, borrow or lend real amounts by contracting for suitable adjustments in the base wage. For instance, with full indexation and a base wage which decreases over time, the worker can be considered as first borrowing and later repaying a real amount to the firm.

A contract with predetermined variations in the degree of indexation — whether it has a constant or a varying base wage — can also guarantee a reduction in the riskiness that an uncertain inflation causes in a worker's consumption and employment. For instance, if the worker early in the contract period borrows in order to consume more than his real income at that time, then there is some degree of indexation of the wages later in the contract period which can insure that the worker's income later in the contract period will consist of a real component to be used for consumption, and a nominal component to be used for repaying the loan.

On the other hand, also a contract without indexation of the wage can be specified to reduce the riskiness that an uncertain inflation causes in a worker's consumption and employment. For instance, the worker could stabilize his consumption by working more hours, the higher the price level turns out to be. At the same time he would, of course, destabilize the number of hours he would work,[1/] but he may still prefer this to the greater variation in his consumption. It is also possible that the worker would prefer to work less hours, the higher the price level turns out to be; this would be the case if the increase in leisure would more than compensate him for the decrease in consumption. The general point is that without indexation, the working hours should not be fixed independently of the realized real wage. Working hours that are contingent on the realized real wage way help to reduce the overall impact of the uncertain inflation.

Furthermore, in a contract without indexation, predetermined variations in the nominal wage may also help reducing the riskiness of a worker's consumption and employment. For instance, the higher the nominal wage and the lower the savings from the earlier part of the contract period are at a particular time, the less variation in the working hours is needed to stabilize the worker's consumption.

An indexed contract, i.e., a contract with an indexed wage, is capable of eliminating the riskiness that an uncertain inflation causes in the worker's consumption and employment, if the predetermined changes in the base wage or in the degree of indexation may be as frequent as one may want. On the other hand, a nominal contract, i.e., a contract with a nominal wage, can reduce, but never eliminate the riskiness in the worker's consumption and employment. At the same time, the presumption is that indexation of the wage involves a cost. The cost is the administrative expense incurred in order to index the wage, and it may also include the greater difficulty in negotiating and administrating an indexed rather than a nominal contract. The benefit to the worker and the firm from eliminating the riskiness of the worker's consumption and employment must therefore be weighted against the cost of indexing the wage in order to decide whether an indexed or a nominal contract should be preferred.

A model to illustrate this choice between an indexed and a nominal contract will be presented below. The labor contract covers two intervals, and there is uncertainty only with respect to the price level in the second interval. The worker and the firm must therefore decide if they want to index the wage in the second interval. They choose a contract such that there is no other contract they would both prefer. It is assumed that the worker and the firm will fulfill their obligations in the labor contract.

The paper is related to Azariadis [1978], in which workers and firms by joint maximization determine the degree of indexation and the employment within a labor contract. It is also related to Danziger [1979], in which the number of times the wage should be indexed within a contract period is determined by joint maximization.

Various authors, e.g., Gray [1976, 1978], Cukierman and Razin [1976], Fischer [1977], and also Azariadis [1978] have studied the different effects that indexed and nominal contracts have on macroeconomic fluctuations. However, unless bargaining is centralized, workers and firms are not concerned with the feedback on macroeconomic variables that the combination of labor contracts might have. Accordingly, the choice between an indexed and a nominal contract is not based on the patterns of macroeconomic fluctuations that would follow if all contracts were of one type or another, but on microeconomic considerations of the type presented in this paper.

THE MODEL

A labor contract between a worker and a firm comprises two intervals. The price level in the first interval is unity, while the price level in the second interval, denoted by p, is uncertain. p is equally likely to become $(1+\mu)(1+\theta)$ and $(1+\mu)(1-\theta)$, where $\mu > -1$ and $0 < \theta < 1$. μ is the expected inflation, and the higher θ, the riskier is the inflation.

The worker and the firm can choose either a nominal or an indexed contract. A nominal contract determines a nominal wage and working hours for each of the intervals. A nominal contract need not fix the working hours for the second interval, but only specify how they will depend on p. The working hours in the second interval can therefore be used to reduce the impact of the riskiness of p on the worker's consumption in the second interval. An indexed contract determines a nominal (= real) wage and working hours for the first interval, and it determines a base wage, a degree of indexation, and working hours for the second interval. The working hours for the second interval can be fixed, since the worker and the firm can contract to avoid any riskiness of the worker's consumption in the second interval.

The capital market has no safe asset, and all borrowing and lending are in nominal amounts. The firm has perfect access to the capital market in which the expected real interest is r. The worker may have only imperfect access to the capital market. He borrows at the same or a higher expected real interest, and he lends at the same or a lower expected real interest than the firm. The expected real interest for the worker is r_W; for borrowing $r_W \geq r$ and for lending $r_W \leq r$. Because of inflation, the expected real value of \$ 1 in the second interval is

$\frac{1}{2}\left[\frac{1}{(1+\mu)(1+\theta)} + \frac{1}{(1+\mu)(1-\theta)}\right] = \frac{1}{(1+\mu)(1-\theta^2)}$. So taking the change in the expected real value of a nominal amount into account, \$ 1 borrowed or lent by the firm in the first interval must be repaid with \$ $(1+r)(1+\mu)(1-\theta^2)$ in the second interval, and \$ 1 borrowed or lent by the worker in the first interval must be repaid with \$ $(1+r_w)(1+\mu)(1-\theta^2)$ in the second interval.

The worker has a von-Neuman-Morgenstern utility function $U[c_1, h_1, c_2, h_2]$, where $c_i \geq 0$ and $0 \leq h_i \leq h_{max}$ are the consumption and the hours worked in the i'th interval, and h_{max} is the number of hours the worker has to his disposal. The utility increases in consumption and decreases in hours worked. The worker is averse to risk in the consumption and the hours worked in the second interval in the sense that for any α and β, not both zero, and any γ, $0 < \gamma < 1$,

$$\gamma U[c_1, h_1, c_2 + \frac{\alpha}{\gamma}, h_2 + \frac{\beta}{\gamma}] + (1-\gamma)U[c_1, h_1, c_2 - \frac{\alpha}{1-\gamma}, h_2 - \frac{\beta}{1-\gamma}] < U[c_1, h_1, c_2, h_2].$$

The worker uses all the income from the labor contract within the contract period. However, his consumption in an interval need not equal the real payment he receives from the firm in the same interval, since he may transfer income between the intervals.

The firm is risk neutral and is only concerned with its expected (discounted) real profit. The production function in the i'th interval is $f_i(h_i)$. The production in an interval increases with the hours worked in the interval, and there are non-increasing returns to scale. Let q denote the real value of a unit of the product. The real value of the production in the i'th interval becomes $qf_i(h_i)$.

To write out the worker's expected utility from a nominal contract, let $w_i \geq 0$ denote the nominal wage in the i'th interval. With the consumption c_1 in the first interval, the worker saves \$ $(w_1h_1-c_1)$, which becomes \$ $(w_1h_1-c_1)(1+r_w)(1+\mu)(1-\theta^2)$ in the second interval. The firm pays him $w_2h_2(w_2/p)$ for the $h_2(w_2/p)$ hours he works in the second interval. The hours worked are written as $h_2(w_2/p)$ to emphasize that they are contingent on the real wage in the second interval. The nominal amount available in the second interval is $w_2h_2(w_2/p) + (w_1h_1-c_1)(1+r_w)(1+\mu)(1-\theta^2)$, and the ensuing consumption in the second interval is therefore the risky $c_2 = (w_2h_2(w_2/p) + (w_1h_1-c_1)(1+r_w)(1+\mu)(1-\theta^2))\frac{1}{p}$. The expected utility from the nominal contract is

$$\frac{1}{2}\sum_p U[c_1, h_1, (w_2h_2(w_2/p) + (w_1h_1-c_1)(1+r_w)(1+\mu)(1-\theta^2))\frac{1}{p}, h_2(w_2/p)].$$

The corresponding expected real profit is

$$qf_1(h_1) - w_1h_1 + \frac{1}{2(1+r)}\sum_p \left\{qf_2(h_2(w_2/p)) - \frac{w_2h_2(w_2/p)}{p}\right\}.$$

A nominal contract in which the worker's expected utility cannot be increased without decreasing the firm's expected real profit is called a best nominal contract.

With an indexed contract, the worker's consumption in the second interval can be made safe. Any desired combination of the worker's consumption and employment in the two intervals can be obtained. For instance, besides determining the desired employment in the two intervals, the contract can determine a wage for the first interval and a base wage with full indexation for the second interval, such that the desired consumption in each interval is equal to the real payment the worker receives in the same interval. Alternatively, if the worker has perfect access to the capital market, besides determining the desired employment in the two intervals, the contract may determine a wage for the first interval and a base wage with a degree of indexation for the second interval, such that the worker by borrowing or saving to obtain the desired consumption in the first interval will also obtain the desired consumption in the second interval. Consequently, if the worker chooses a c_1 which makes c_2 safe, his expected utility from the indexed contract is simply

$$U[c_1, h_1, c_2, h_2].$$

Indexation of the second-interval wage to p costs the firm the real amount $a > 0$. The firm's corresponding maximal expected real profit, which requires that the worker does not frequent the capital market when $r_w \neq r$, is therefore

$$qf_1(h_1) - c_1 + \frac{1}{1+r}(qf_2(h_2) - a - c_2).$$

An indexed contract in which the worker's expected utility cannot be increased without decreasing the firm's expected real profit is called a best indexed contract. Clearly, in a best indexed contract the choice of c_1 will make c_2 safe.

The worker and the firm will choose a nominal or an indexed contract which is such that there is no other nominal or idexed contract which they both prefer. The chosen contract will either be a best nominal or a best indexed contract. In order to determine the exact contract that will be chosen, it is necessary to specify a rule for the relation between the worker's expected utility and the firm's expected real profit for alternative values of the various parameters. However, the purpose of the present model is only to determine the effects of changes in the parameters on the type of contract that will be chosen, so it is sufficient with a very general rule about the relation between the worker's expected utility and the firm's expected real profit.

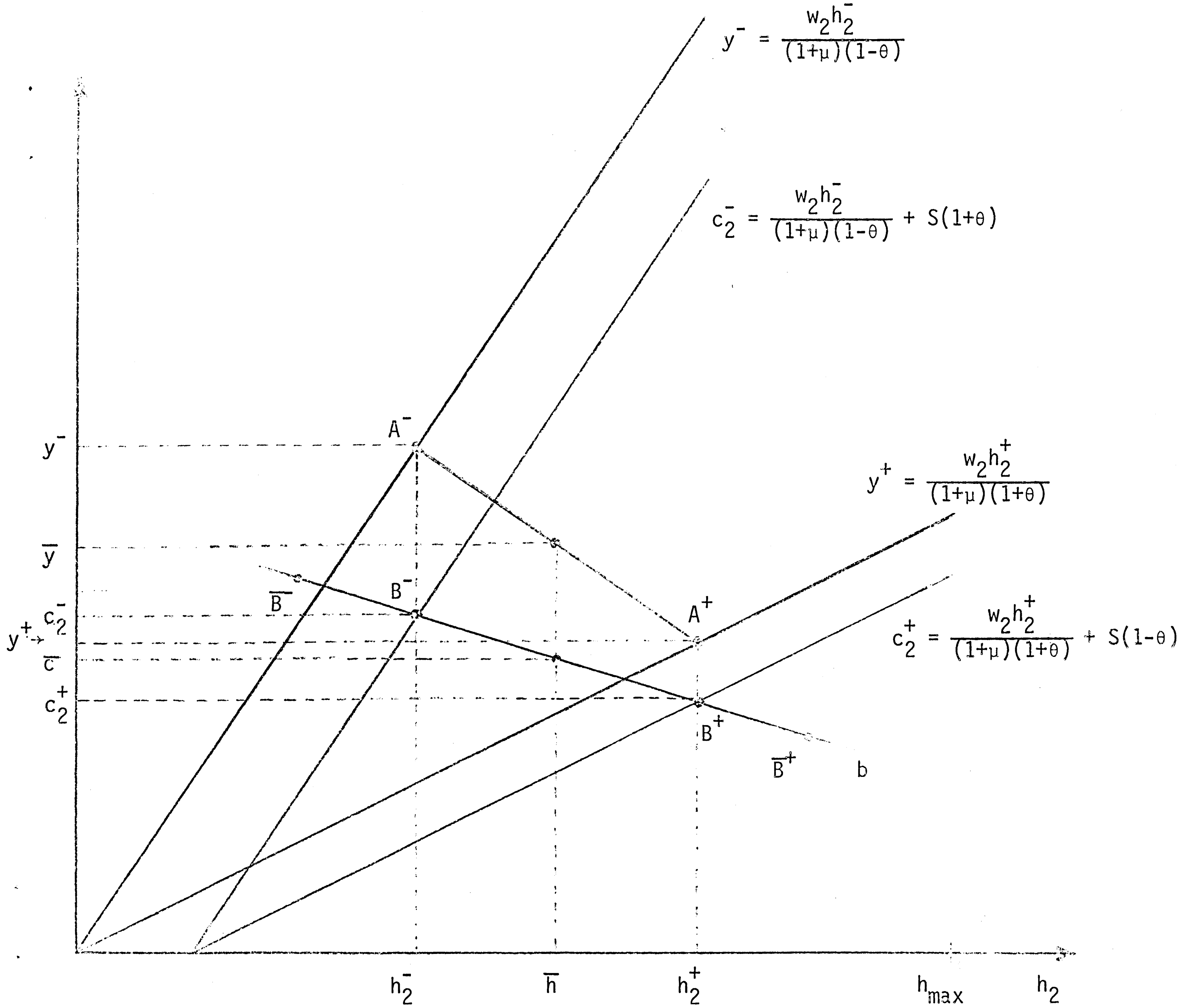
Consider two contracts which stem from different values of the parameters and are both best nominal or best indexed contracts. The rule is that the real amount the worker with the first contract should be given in the second interval in order to be indifferent between the two contracts does not have the opposite sign of the excess of the firm's expected real profit in the second contract over the first contract.[2/] In other words, a change in a parameter will either benefit both the worker and the firm, be harmfull to both the worker and the firm, or not affect at least one of them.

CHANGES IN THE PARAMETERS

This section determines how a change in a parameter would affect the relative attractiveness of the best nominal and the best indexed contracts, and consequently, how it would affect which type of contract that will be chosen.

The Expected Inflation. If the contract is nominal and μ is changed, then one can always change w_2 such that the distribution of $\frac{w_2}{p}$, and thus of $\frac{w_2 h_2(w_2/p)}{p}$, is unchanged. The distribution of $\frac{1+\mu}{p}$ is of course not changed by a change in μ. Since μ does not affect the real interest, it follows that μ does not affect the expected utility and the expected real profit that are obtained from the best nominal contracts. Likewise, μ does not affect the worker's utility and the firm's expected real profit that are obtained from the best indexed contracts. Consequently, the magnitude of the expected inflation does not affect the type of the chosen contract.

The Uncertainty of Inflation. Consider a nominal contract. If $p = (1+\mu)(1+\theta)$, let h_2^+ and y^+ denote the employment and the firm's real payment to the worker in the second interval. In Figure 1, h_2^+ and y^+ are found on the line segment $y^+ = \frac{w_2 h_2^+}{(1+\mu)(1+\theta)}$, $0 \leq h_2^+ \leq h_{max}$. The expected real value in the second interval of the worker's savings is $S \equiv (w_1 h_1 - c_1)(1+r_w)$, so when $p = (1+\mu)(1+\theta)$, the real value of the worker's savings is $\frac{S(1+\mu)(1-\theta^2)}{(1+\mu)(1+\theta)} = S(1-\theta)$. For the consumption to be non-negative it is therefore necessary that $\frac{w_2 h_2^+}{(1+\mu)(1+\theta)} + S(1-\theta) \geq 0 \iff h_2^+ \geq -\frac{S(1+\mu)(1-\theta^2)}{w_2}$.
If $S < 0$, then this constraint is not trivial. With c_2^+ denoting the consumption, h_2^+ and c_2^+ are therefore found on the line segment $c_2^+ = \frac{w_2 h_2^+}{(1+\mu)(1+\theta)} + S(1-\theta)$, $\max\{0, -\frac{S(1+\mu)(1-\theta^2)}{w_2}\} \leq h_2^+ \leq h_{max}$. In Figure 1 this line segment is drawn on the assumption that $S < 0$.



$y^- = \frac{w_2 h_2^-}{(1+\mu)(1-\theta)}$
$c_2^- = \frac{w_2 h_2^-}{(1+\mu)(1-\theta)} + S(1+\theta)$
$y^+ = \frac{w_2 h_2^+}{(1+\mu)(1+\theta)}$
$c_2^+ = \frac{w_2 h_2^+}{(1+\mu)(1+\theta)} + S(1-\theta)$
A^-
A^+
B^-
B^+
$\overline{B}^-$
$\overline{B}^+$
b
y^-
$\overline{y}$
c_2^-
y^+
$\overline{c}$
c_2^+
h_2^-
$\overline{h}$
h_2^+
h_{max}
h_2

Figure 1.

Likewise, if $p = (1+\mu)(1-\theta)$, let h_2^- and y^- denote the employment and the firm's real payment to the worker in the second interval. h_2^- and y^- are found on the line segment $y^- = \frac{w_2 h_2^+}{(1+\mu)(1-\theta)}$, $0 \leq h_2^- \leq h_{max}$. The real value of the worker's savings is $\frac{S(1+\mu)(1-\theta^2)}{(1+\mu)(1-\theta)} = S(1+\theta)$, and for the consumption to be non-negative, also h_2^- must satisfy $h_2^- \geq -\frac{S(1+\mu)(1-\theta^2)}{w_2}$. h_2^- and the consumption, denoted by c_2^-, are therefore found on the line segment $c_2^- = \frac{w_2 h_2^-}{(1+\mu)(1-\theta)} + S(1+\theta)$, $\max\{0, -\frac{S(1+\mu)(1-\theta^2)}{w_2}\} \leq h_2^- \leq h_{max}$.

With a given choice of w_2, h_2^+, and h_2^-, A^+ and A^- are the two equally likely combinations of employment and the firm's real payment to the worker in the second interval. The midpoint of A^+A^- is the expected employment, $\bar{h} \equiv \frac{h_2^+ + h_2^-}{2}$, and the firm's expected real payment to the worker, $\bar{y} \equiv \frac{y^+ + y^-}{2}$.

With also w_1, h_1, and c_1 chosen, S is determined, and B^+ and B^- are the two equally likely combinations of employment and consumption in the second interval. The midpoint of B^+B^- is the expected employment $\bar{h}$ and the expected consumption, $\bar{c} \equiv \frac{c_2^+ + c_2^-}{2} = \bar{y} + S$.

Assume now that θ increases, but that S, $\bar{h}$, and $\bar{y}$ are kept unchanged, and that w_2, h_2^+, and h_2^- are changed such that the new combinations of employment and consumption in the second interval are found on the line b through B^+ and B^-. The Appendix shows that this is practicable, that $\overline{B}^+$ and $\overline{B}^-$ are unique, and that the new combination of employment and consumption in the second interval are farther from $(\bar{h}, \bar{c})$, e.g., the combinations become $\overline{B}^+$ and $\overline{B}^-$ instead of B^+ and B^-.[3/]

The worker is risk averse and therefore prefers B^+ and B^- to $\overline{B}^+$ and $\overline{B}^-$.[4/]

The firm's expected real payment to the workers is the same for $\overline{B}^+$ and $\overline{B}^-$ as for B^+ and B^-, but the expected real value of the production is only the same if the production function in the second interval exhibits constant returns to scale; if f_2 exhibits decreasing returns to scale, then the expected real value of the production is lower in $\overline{B}^+$ and $\overline{B}^-$ than in B^+ and B^-. The firm's expected real profit in $\overline{B}^+$ and $\overline{B}^-$ does therefore not exceed its expected real profit in B^+ and B^-, and $\overline{B}^+$ and $\overline{B}^-$ are not better than B^+ and B^- for the firm.

The above argument is true for any initial B^+ and B^-. It follows that if θ increases, then at least the worker's expected utility or the firm's expected real profit must decrease in a best nominal contract.[5/] An increase in θ always makes the best nominal contracts less attractive.[6/]

Turning to the indexed contracts, a best indexed contract is specified such that the riskiness of inflation does not affect the worker's utility and the firm's expected real profit. Accordingly, the more uncertain the inflation is, the more likely it is that an indexed contract will be chosen.

The Cost of Indexation. The higher a is, the more expensive it is to insulate the worker from the riskiness of inflation in an indexed contract. So, the higher a is, the more likely it is that a nominal contract will be chosen.

In Figure 2 θ is measured on the vertical axis and a on the horizontal axis. The curve ℓ shows the combinations of θ and a for which the worker and the firm are indifferent between the two types of contract. For a combination of θ and a above ℓ, a nominal contract is preferred, and for a combination of θ and a below ℓ, an indexed contract is preferred. ℓ originates from $(0, 0)$ and is increasing, since the higher θ is, the higher must a be to make the worker and the firm indifferent between the two types of contract. For any riskiness of the inflation, there are small a's for which an indexed contract is preferred and large a's for which a nominal contract is preferred.

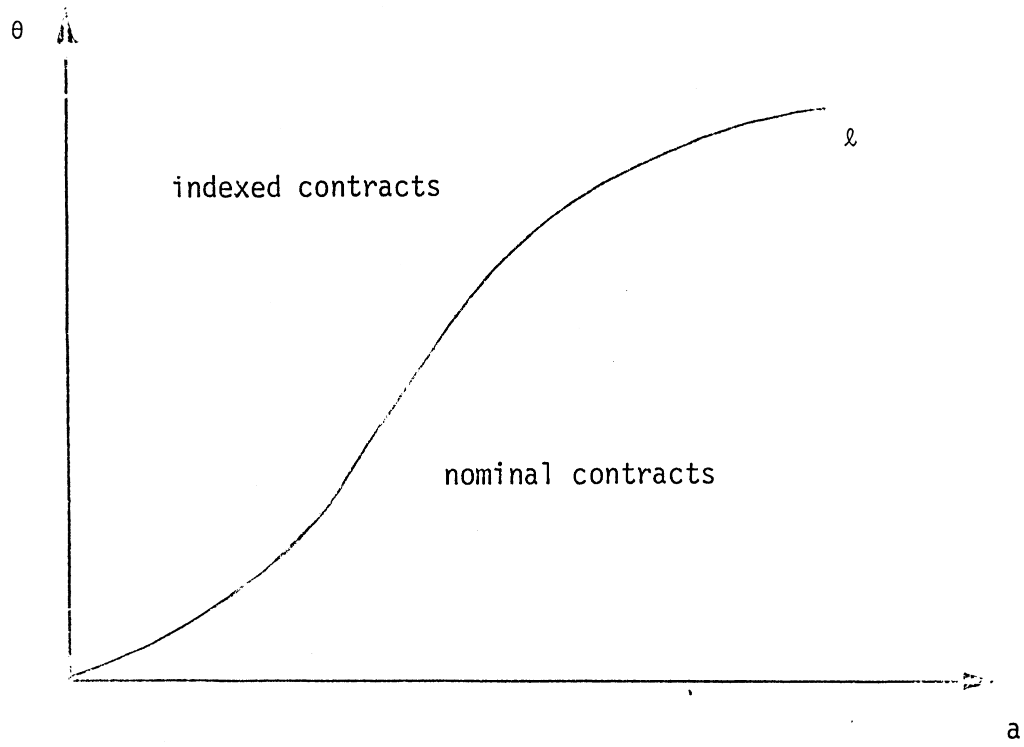


Figure 2.

Risk Aversion. The more risk averse the worker is,[7/] the higher is the risk premium he would pay to be insulated from the riskiness of the inflation. Consequently, the more risk averse the worker is, the lower is ℓ, and the more likely it is that an indexed contract is chosen.

Decreasing Returns. The more concave f_2 is, the larger is the loss of expected production caused by the variations in h_2 in a nominal contract. It follows that the more concave f_2 is, the lower is ℓ, and the more likely it is that an indexed contract is chosen.

Imperfection of the Capital Market. In a best nominal contract the impact of the uncertain inflation is reduced not only by h_2 being contingent on w_2/p, but also by the wage being different in the two intervals. For instance, assume that there were no imperfection in the worker's access to the capital market. For some nominal contract and choice of c_1, let B^+ and B^- again be the combinations of employment and consumption in the second interval when $p = (1+\mu)(1+\theta)$ and $p = (1+\mu)(1-\theta)$. If $h_2^+ > \bar{h}$, then the higher w_2 and the smaller w_1 and therefore S are, the smaller is the variation in h_2 needed to obtain a particular reduction in the variation of c_2. If $h_2^+ < \bar{h}$, then it is the exactly opposite.[8/] So a nominal contract with the same choice of c_1, and a higher w_2 and lower w_1 for $h_2^+ > \bar{h}$, and lower w_2 and higher w_1 for $h_2^+ < \bar{h}$, will have its B^+ and B^- closer to $(\bar{h}, \bar{c})$ on b. Since this remains true as long as $w_1 > 0$ for $h_2^+ > \bar{h}$ and $w_2 > 0$ for $h_2^+ < \bar{h}$, the implication is that unless the employment is fixed in the second interval, i.e., unless $h_2^+ = h_2^- = \bar{h}$, the best nominal contracts will have either $w_1 = 0$ (and $S < 0$ if $c_1 > 0$) or $w_2 = 0$ (and $S > 0$ if $c_2 < 0$).

However, if the worker's access to the capital market is imperfect, then he borrows to a higher and lends to a lower expected real interest than the firm. It is then less advantageous to delay the wage payment and have the worker borrow for consumption in the first interval, or to expedite the wage payment and have the worker save for consumption in the second interval. The more imperfect the worker's access to the capital market is, the less advantageous it is that the payment to the worker in the first interval is anything different from his consumption in that interval. The worker's access to the capital market may well be so imperfect that $S = 0$ even when $h_2^+ \neq \overline{h}$ in a best nominal contract.

In a best indexed contract the worker will never have to frequent the capital market, unless he can obtain as good conditions as the firm can obtain. The imperfection of the worker's access to the capital market is therefore of no consequence for the attractiveness of the best indexed contracts. It follows that unless $h_2^+ = \overline{h}$ in the best nominal contract, the more imperfect the worker's access to the capital market is, the lower is ℓ, and the more likely it is that the worker and the firm will choose an indexed contract.

CONCLUSION

This paper has studied a worker's and a firm's choice between a nominal and an indexed labor contract. It has shown that the expected inflation does not influence which type of contract will be chosen, but that the more uncertain the inflation, the more risk averse the worker, the more concave the production function, and the more imperfect the worker's access to the capital market, the more likely it is that the worker and the firm will choose an indexed contract; on the other hand, the higher the cost of indexation, the more likely it is that they will choose a nominal contract.

It was assumed that a predetermined change in the nominal wage is costless. If instead <u>any</u> adjustment of the wage — whether predetermined or not — is costly, then the worker and the firm would have to choose between a nominal contract in which the nominal wage is the same in the two intervals and an indexed contract. The best nominal contracts would then be less attractive than the best nominal contracts considered in this paper, and the choice of a nominal contract would therefore be less likely. Furthermore, the worker's expected utility from the best nominal contracts would no longer be independent of the expected inflation. This is because predetermined variations in the nominal wage can no longer be combined with the variations in the employment to reduce the overall riskiness of the worker's consumption and employment. Consequently, a change in the expceted inflation would change the attractiveness of the best nominal contracts, and it would therefore affect which type of contract that the worker and the firm would choose.[9]

In this paper, inflation has been the only source of uncertainty. A desirable extension of the analysis would be to include the real price of the firm's product as another source of uncertainty. If the real price of the firm's product were uncertain, then it would not be beneficial for an indexed contract to fix the employment in the second interval. The worker's consumption and employment would therefore be uncertain even with the best indexed contracts. However, with the additional source of uncertainty, a formal analysis of the choice between a nominal and an indexed contract would be no simple matter, and it is outside the scope of this paper.

APPENDIX

In a nominal contract the firm's average real payment to the worker in the second interval is

$$\bar{y} = \left[\frac{1}{2}\,\frac{w_2 h_2^+}{(1+\mu)(1+\theta)} + \frac{w_2 h_2^-}{(1+\mu)(1-\theta)}\right] = \frac{w_2(h_2^+(1-\theta) + h_2^-(1+\theta))}{2(1+\mu)(1-\theta^2)}.$$

Assuming that $h_2^+ + h_2^- > 0$, this implies that

$$w_2 = \frac{2\bar{y}(1+\mu)(1-\theta^2)}{h_2^+(1-\theta) + h_2^-(1+\theta)}.$$

Using that $h_2^+ + h_2^- = 2\bar{h} \iff h_2^- = 2\bar{h} - h_2^+$,

$$w_2 = \frac{2\bar{y}(1+\mu)(1-\theta^2)}{h_2^+(1-\theta) + (2\bar{h}-h_2^+)(1+\theta)} = \frac{\bar{y}(1+\mu)(1-\theta^2)}{\bar{h}(1+\theta) - h_2^+\theta}.$$

The nominal value in the second interval of the worker's savings is $(w_1h_1-c_1)(1+r_w)(1+\mu)(1-\theta^2)$, so c_2^+ becomes

$$c_2^+ = \frac{\dfrac{\bar{y}(1+\mu)(1-\theta^2)h_2^+}{\bar{h}(1+\theta) - h_2^+\theta} + (w_1h_1-c_1)(1+r_w)(1+\mu)(1-\theta^2)}{(1+\mu)(1+\theta)} = \frac{\bar{y}(1-\theta)h_2^+}{\bar{h}(1+\theta) - h_2^+\theta} + S(1-\theta).$$

Of course, if $h_2^+ = h_2^- = 0$, then $c_2^+ = S(1-\theta)$.

B^+ and B^- are the (h_2^+, c_2^+) and (h_2^-, c_2^-) which lie on b and yield the expected employment $\bar{h}$ and the expected consumption $\bar{c}$. On b, if w_2 would be higher, then the expected consumption would higher than $\bar{c}$; B^+ and B^- are therefore unique.

Assume now that $\bar{h}$, $\bar{y}$, and S are given and that neither of B^+ and B^- lie on the axes, i.e., that $h_2^+ > 0$, $h_2^- > 0$, $c_2^+ > 0$, and $c_2^- > 0$. The following will then show that the bigger θ is, the farther away are B^+ and B^- from $(\bar{h}, \bar{c})$.

Consider first the case where $h_2^+ = h_2^-$, i.e., when there is no variation in employment; b is vertical. c_2^+ then reduces to $(\overline{y}+S)(1-\theta)$, which decreases in θ. So in this case it is immediate that the higher θ is, the farther away are B^+ and B^- from $(\overline{h}, \overline{c})$.

Consider next the more complicated case where $h_2^+ \neq h_2^-$, i.e., where there is variation in employment. Let g denote the slope of b, and e the intersection of b with the vertical axis. Using the above expression for c_2^+,

$$\frac{\overline{y}(1-\theta)h_2^+}{\overline{h}(1+\theta) - h_2^+\theta} + S(1-\theta) = e + gh_2^+,$$ and since $(\overline{h}, \overline{c})$ is also on b, $\overline{y} + S = e + gh$.

Subtracting,

$$\frac{\overline{y}(1-\theta)h_2^+}{\overline{h}(1+\theta) - h_2^+\theta} - \overline{y} - \theta S = g(h_2^+-\overline{h})$$

$$\text{(A1)} \iff \frac{\overline{y}(h_2^+ - \overline{h}(1+\theta))}{(\overline{h}(1+\theta) - h_2^+\theta)(h_2^+-h)} - \frac{\theta S}{h_2^+ - \overline{h}} = g.$$

The direction of the effect of a change in θ on h_2^+ on b is found to be differentiating this with respect to θ:

$$\frac{\partial g}{\partial \theta}d\theta + \frac{\partial g}{\partial h_2^+}dh_2^+ = 0$$

$$\iff \left[\frac{\overline{y}(-\overline{h}(\overline{h}(1+\theta) - h_2^+\theta) - (h_2^+ - \overline{h}(1+\theta))(\overline{h}-h_2^+))}{(\overline{h}(1+\theta) - h_2^+\theta)^2(h_2^+-\overline{h})} - \frac{S}{h_2^+ - \overline{h}}\right]d\theta$$

$$+ \left[\frac{\overline{y}((\overline{h}(1+\theta) - h_2^+\theta)(h_2^+ - \overline{h}) - (h_2^+ - \overline{h}(1+\theta))(-2h_2^+\theta + \overline{h}(1+2\theta)))}{(\overline{h}(1+\theta) - h_2^+\theta)^2(h_2^+-\overline{h})^2} + \frac{\theta S}{(h_2^+-\overline{h})^2}\right]dh_2^+ = 0$$

$$\iff \left[\frac{\overline{y}h_2^+(h_2^+-2\overline{h})}{(\overline{h}(1+\theta) - h_2^+\theta)^2} - S\right]d\theta$$

$$+ \theta \left[\frac{\bar{y}(h_2^{+2} + 2(1+\theta)\bar{h}^2 - 2(1+\theta)h_2^+\bar{h})}{(\bar{h}(1+\theta) - h_2^+\theta)^2(h_2^+-\bar{h})} + \frac{S}{h_2^+ - \bar{h}} \right] dh_2^+ = 0.$$

To determine the sign of the coefficient to $d\theta$, first use that

$$c_2^+ > 0 \iff \frac{\bar{y}h_2^+}{\bar{h}(1+\theta) - h_2^+\theta} > -S:$$

$$\frac{\bar{y}h_2^+(h_2^+-2\bar{h})}{(\bar{h}(1+\theta) - h_2^+\theta)^2} - S < \frac{\bar{y}h_2^+(h_2^+-2\bar{h})}{(\bar{h}(1+\theta)-h_2^+\theta)^2} + \frac{\bar{y}h_2^+}{\bar{h}(1+\theta) - h_2^+\theta}$$

with the same sign as

$$h_2^+[h_2^+ - 2\bar{h} + \bar{h}(1+\theta) - h_2^+\theta] = h_2^+(h_2^+-\bar{h})(1-\theta).$$

Accordingly, if $0 \leq h_2^+ < \bar{h}$, then the coefficient to $d\theta$ is negative. Next, use that $c_2^- > 0 \iff \frac{\bar{y}(2\bar{h}-h_2^+)}{\bar{h}(1+\theta) - h_2^+\theta} > -S:$

$$\frac{\bar{y}h_2^+(h_2^+-2\bar{h})}{(\bar{h}(1+\theta) - h_2^+\theta)^2} - S < \frac{\bar{y}h_2^+(h_2^+-2\bar{h})}{(\bar{h}(1+\theta) - h_2^+\theta)^2} + \frac{\bar{y}(2\bar{h}-h_2^+)}{\bar{h}(1+\theta) - h_2^+\theta}$$

with the same sign as

$$h_2^+(h_2^+-2\bar{h}) + (\bar{h}(1+\theta) - h_2^+\theta)(2\bar{h}-h_2^+) = (1+\theta)(h_2^{+2}+2\bar{h}^2-3h_2^+\bar{h}).$$

Differentiating with respect to h_2^+ shows that this reaches its minimum value for $2h_2^+ - 3\bar{h} = 0 \iff h_2^+ = \frac{3}{2}\bar{h}$. Since the value of $h_2^{+2} + 2\bar{h}^2 - 3h_2^+\bar{h}$ is zero for both $h_2^+ = \bar{h}$ and $h_2^+ = 2\bar{h}$, it follows that of $\bar{h} < h_2^+ \leq 2\bar{h}$, then the coefficient to $d\theta$ is negative. It has therefore been proved that the coefficient to $d\theta$ is negative for $h_2^+ \neq \bar{h}$.

To determine the sign of the coefficient to dh_2^+, one again for $h_2^+ < \bar{h}$ uses that $\dfrac{\bar{y}h_2^+}{\bar{h}(1+\theta) - h_2^+\theta} > -S$:

$$\theta\left[\frac{\bar{y}(h_2^{+2} + 2(1+\theta)\bar{h}^2 - 2(1+\theta)h_2^+\bar{h})}{(\bar{h}(1+\theta) - h_2^+\theta)^2(h_2^+-\bar{h})} + \frac{S}{h_2^+ - \bar{h}}\right] < \theta\left[\frac{\bar{y}(h_2^{+2} + 2(1+\theta)\bar{h}^2 - 2(1+\theta)h_2^+\bar{h})}{(\bar{h}(1+\theta) - h_2^+\theta)^2(h_2^+-\bar{h})}\right.$$

$$\left. - \frac{\bar{y}h_2^+}{(\bar{h}(1+\theta) - h_2^+\theta)(h_2^+-\bar{h})}\right],$$

with the same sign as

$$-[h_2^{+2} + 2(1+\theta)\bar{h}^2 - 2(1+\theta)h_2^+\bar{h} - h_2^+(\bar{h}(1+\theta) - h_2^+\theta)] = -[h_2^{+2} + 2\bar{h}^2 - 3h_2^+\bar{h}](1+\theta).$$

Since this reaches its maximum value for $h_2^+ = \frac{3}{2}\bar{h}$ and $h_2^{+2} + 2\bar{h}^2 - 3h_2^+\bar{h} = 0$ for $h_2^+ = \bar{h}$, the coefficient to dh_2^+ is negative for $0 \leq h_2^+ < \bar{h}$. If $h_2^+ > \bar{h}$ one again uses that $\dfrac{\bar{y}(2\bar{h}-h_2^+)}{\bar{h}(1+\theta) - h_2^+\theta} > -S$:

$$\theta\left[\frac{\bar{y}(h_2^{+2} + 2(1+\theta)\bar{h}^2 - 2(1+\theta)h_2^+\bar{h})}{(\bar{h}(1+\theta) - h_2^+\theta)^2(h_2^+-\bar{h})} + \frac{S}{h_2^+ - \bar{h}}\right] > \theta\left[\frac{\bar{y}(h_2^{+2} + 2(1+\theta)\bar{h}^2 - 2(1+\theta)h_2^+\bar{h}}{(\bar{h}(1+\theta) - h_2^+\theta)^2(h_2^+-\bar{h})}\right.$$

$$\left. - \frac{\bar{y}(2\bar{h}-h_2^+)}{(\bar{h}(1+\theta) - h_2^+\theta)(h_2^+-\bar{h})}\right],$$

with the same sign as

$$h_2^{+2} + 2(1+\theta)\bar{h}^2 - 2(1+\theta)h_2^+\bar{h} - (2\bar{h}-h_2^+)(\bar{h}(1+\theta) - h_2^+\theta) = (1-\theta)h_2^+(h_2^+-\bar{h}).$$

The coefficient to dh_2^+ is therefore positive from $\bar{h} < h_2^+ \leq 2\bar{h}$, and it has been established that $\dfrac{dh_2^+}{d\theta} \lesseqgtr 0$ as $h_2^+ \lesseqgtr \bar{h}$. It follows that for a higher θ, h_2^+ on b will be farther away from $\overline{h}$. So, also in the case where $h_2^+ \neq h_2^-$, the higher θ, the farther away will B^+ and B^- be from $(\overline{h}, \overline{c})$.

It was assumed in the above that neither B^+ nor B^- was at an axis. However, abstracting from the non-negativity of employment and consumption, the derivations show that if B^+ and B^- are different points and each could be any point except (0, 0), then an increase in θ would still cause B^+ and B^- to be farther away from $(\overline{h}, \overline{c})$. So, if B^+ and B^- would be different points, at least one of them at an axis, but not at (0, 0), then the non-negativity of employment and consumption would imply that an increase in θ would bring B^+ and B^- out of existence. If B^+ and B^- were identical, then it must be for $h_2^+ = h_2^-$ and $c_2^+ = c_2^- = 0$. In this case a change in θ would not change B^+ and B^-. If B^+ of B^- would be at (0, 0), then $h_2^+ = c_2^+ = 0$ or $h_2^- = c_2^- = 0$; also in this case a change in θ would not change B^+ and B^-.

NOTES

1. For instance, the real income would be constant, if the working hours were proportional to the price level.

2. Specification of the rule in this way — rather than directly on the differences in the expected utilities and expected discounted real profits — permits an examination of a change in the worker's risk aversion.

3. The exceptions are noted below.

4. Consider two lotteries. The first lottery has the mean $\overline{B}^+$ and the outcome B^+ with probability $\frac{|\overline{B}^+B^+|}{|B^+B^-|}$ and B^- with probability $\frac{|\overline{B}^+B^-|}{|B^+B^-|}$; the second lottery has the mean $\overline{B}^-$ and the outcome B^+ with probability $\frac{|\overline{B}^-B^+|}{|B^+B^-|}$ and B^- with probability $\frac{|\overline{B}^-B^-|}{|B^+B^-|}$. If each lottery has the probability $\frac{1}{2}$ of being played, then B^+ and B^- are equally likely outcomes. For instance, the probability of B^+ is $\frac{|\overline{B}^+B^+| + |\overline{B}^-B^+|}{2|B^+B^-|} = \frac{|\overline{B}^+B^+| + |B^-\overline{B}^+|}{2|B^+B^-|} = \frac{1}{2}$. The risk-averse worker prefers $\overline{B}^+$ to any lottery with mean $\overline{B}^+$ and prefers $\overline{B}^-$ to any lottery with mean $\overline{B}^-$. As a consequence, he prefers $\overline{B}^+$ and $\overline{B}^-$ to B^+ and B^-.

5. There are two exceptions to the statement that a higher θ implies a B^+ and B^- farther from $(\overline{h}, \overline{c})$. The first exception is if B^+ or B^- were different points, at least one of which were at an axis, but not at (0, 0). An increase in θ would then bring B^+ and B^- out of existence. This exception does not modify the conclusion in text. The second exception is if B^+ and B^- were identical at the horizontal axis, or B^+ or B^- were (0, 0). An increase in θ would then not move B^+ and B^-; the attractiveness of the best nominal

contracts would not be affected. However, this exception to the conclusion in text is not very interesting, since it only comes about when $c_2 = 0$ in at least one state of nature.

6. Increasing w_2 without changing h_2^+ or h_2^- increases the worker's expected utility and decreases the firm's expected real profit. w_2 can therefore always be changed to satisfy the rule for the relation between the worker's expected utility and the firm's expected real profit. This also applies when the changes in the other parameters are considered.

7. The utility function U' exhibits more risk aversion than the utility function U if $U' = \phi(U)$, $\phi' > 0$ and $\phi'' < 0$. This implies that the two-dimensional risk premiums $(\tilde{c}_2, \tilde{h}_2)$ and $(\tilde{c}_2', \tilde{h}_2')$ are such that $\tilde{c}_2' < \tilde{c}_2'$, whenever $\tilde{h}_2 = \tilde{h}_2$. See Kihlstrom and Mirman [1974] and Paroush [1975].

8. The formal proof is similar to the proof that the higher θ is, the farther away are B^+ and B^- on b from $(\overline{h}, \overline{c})$. c_1, $\overline{c}$, and $\overline{h}$ are unchanged, so w_1 and w_2 must change such that $d\overline{y} + dS = 0$. In the case where $h_2^+ = h_2^-$, $c_2^+ = (\overline{y}+S)(1-\theta)$. c_2^+ would not be affected, so if the employment is fixed, a change in w_1 and w_2 would not change B^+ and B^-. In the case where $h_2^+ \neq h_2^-$, the direction of the effect of a change in w_1 and w_2 on h_2^+ on b is found by differentiating (A1) with respect to S:

$$\frac{\partial g}{\partial S}dS + \frac{\partial g}{\partial \overline{y}}d\overline{y} + \frac{\partial g}{\partial h_2^+}dh_2^+ = 0$$

$$\Leftrightarrow -\left[\frac{h_2^+ - \overline{h}(1+\theta)}{(\overline{h}(1+\theta) - h_2^+\theta)(h_2^+ - \overline{h})} + \frac{\theta}{h_2^+ - \overline{h}}\right]dS + \frac{\partial g}{\partial h_2^+}dh_2^+ = 0$$

$$\Longleftrightarrow \frac{(\overline{h}-h_2^+)(1-\theta^2)}{\overline{h}(1+\theta) - h_2^+\theta} dS + \frac{\partial g}{\partial h_2^+}(h_2^+-\overline{h})dh_2^+ = 0.$$

The coefficient to dS is positive for $h_2^+ < \overline{h}$ and negative for $h_2^+ > \overline{h}$; the Appendix showed that the opposite is true for the coefficient to dh_2^+. Thus, $\frac{dh_2^+}{dS} > 0$, and since $\frac{dS}{dw_1} > 0$, it follows that B^+ and B^- will be close to $(\overline{h}, \overline{c})$, for $h_2^+ > \overline{h}$, the lower w_1, and for $h_2^+ < \overline{h}$, the higher w_1.

9. A more uncertain inflation increases the expected value of $\frac{1}{p}$. A higher θ would therefore have two effects: The higher riskiness of the inflation, which would decrease the attractiveness of the best nominal contracts, and the lower expected value of $\frac{1}{p}$, which might change the attractiveness of the best nominal contracts in either direction.

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