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COLLECTIVE RATIONALITY, UNANIMITY AND LIBERAL ETHICS

by

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I. Introduction

The apparent inconsistency of the condition of unrestricted domain, the Pareto principle (the unanimity condition), the principle of liberalism and the existence of a social choice function, (SCF), that was pointed out by Sen [4], is indeed a disturbing feature of the social choice process in a free society. Recently, an attempt has been made to resolve this difficulty by reformulating the principle of liberalism.

In a recent contribution, Gibbard [2] presented a libertarian claim which is consistent with the conditions mentioned above and the existence of SCF. The essence of Gibbard's approach consists of a right-system which assigns alternative social states to individuals. The individual then possesses alienable right over the states assigned to him, which makes him decisive with regard to the social ranking of these states except under specified circumstances when this right is waived. Gibbard's libertarian claim, however, is inconsistent with all the conditions stated earlier and Arrow's condition of collective rationality. In addition, Gibbard's SCF does not satisfy the condition of cheat-proofness. Consequently, situations may arise where strategic behavior is called for. The proof that these observations regarding Gibbard's result are indeed valid is provided in the following section, where I also try to evaluate their ethical implications.

II. Gibbard's Libertarian Claim and the Conditions of Collective Rationality

To start with, consider the following definitions of Gibbard: 1/

Definition 1: (Right-System) A right-system, RS, is a set of ordered triples of the form $<x,y,b>$; if $<x,y,b> \varepsilon RS$ assigns the alternative social states $<x,y>$ to individual b.

Definition 2: Let Σ be a finite sequence, $y_1,\ldots,y_n$, of alternative social states. Then, other things relevant to the description of a social state being equal, for a given set of individual preference orderings, others beside b claim their right under RS to z over x through Σ, if and only if, $y_1 = z$, $y_n = x$, and for every i from 1 to n-1, at least one of the following holds:

$$(\forall c) \quad y_i P_c y_{i+1}$$

$$(\exists c) \quad [c \neq b \ \& < y_i, y_{i+1}, c > \varepsilon \text{ RS}, \ \& \ y_i P_c y_{i+1}]$$

This will be denoted $z >>_b x[\Sigma,RS]$.

Definition 3: Other things relevant for a description of a social state being equal, for a given set of individual preference orderings and a given set of available alternatives, S, b's right to x over y is waived under RS, if and only if for some z, $z \neq y$, yR_bz and for some sequence, Σ, of alternatives in S, $z >>_b x[\Sigma,RS]$. This will be written $xW_by[RS]$.

Definition 4: Other things relevant to the description of a social state being the same, for a given set of individual orderings, and a given set of available alternatives, S, a person b determines issue[2] <x,y>, under RS, if and only if, (i) <x,y,b> εRS

(ii) xP_by

(iii) $\sim xW_by$[RS]

This will be written xD_by[RS].

Definition 5: A SCF, f, realizes RS if and only if, for every set of individual preference orderings, a given set of available alternatives, S, and for every b, x&y, $xD_by[RS] \rightarrow xPy$, where xPy if and only if, $x\varepsilon S$ implies that y does not belong to the choice set.

Definition 6: f accords b an alienable right to x over y if and only if for some right system, RS, f realizes RS and <x,y,b> εRS. Using these definitions, Gibbard's libertarian condition, GL, may then be stated as follows:

Condition GL: "For every b, there is an [issue] j, such that for every pair of j-variants[3] x and y, f accords b an alienable right to x over y," The libertarian claim is that there exists SCF which satisfies the condition GL. The Pareto principle is:

Condition P: For any $x,y\ \varepsilon\Omega$, $[(\forall j)(xP_jy)] \rightarrow xPy$, where Ω is the set of all possible social states.

Collective Rationality: One of the conditions imposed on choice functions is that of collective rationality. The exact meaning of this term is itself a matter of judgement. The definition of collective rationality which is imposed by Arrow [1] implies that "the choice to be made from any set of alternatives can be determined by the choices made between pairs of alternatives" (Arrow [1] p.20). In other words, the choice set on any available set of alternatives is generated by a complete bineary relationship which itself is independent of the set of feasible alternatives.

It is convenient to present a formal statement of this and subsequent conditions within the framework of a mathematical model developed in Karni and Schmeidler [3]. Let A denote any finite set of no less than three alternatives. A conditional profile, which is the variable entering our social choice function is, by definition, a $(n+2)$ - List $(S/A, R_1,\ldots,R_n)$ where S is included in A, and $R_1,\ldots,R_n$ are the individual rankings of the elements of A. Let Π denote the set of all conditional profiles. A SCF, f assigns to every conditional profile an alternative in S.

Using the above notations and definitions we now define the condition of collective rationality as Independence of Non-Optimal Alternatives.

Condition INOA - A SCF, f, is said to be independent of Non-Optimal Alternatives if $f(S/A, R_1,\ldots,R_n) = f(S'/A,R_1,\ldots,R_n)$ on Π, whenever $f(S/A,R_1,\ldots,R_n) \in S'$ and $S' \subset S$.

In addition to fixed bineary relationship, Arrow [1] required that this relationship also satisfy full transitivity. The latter requirement is not necessary for obtaining social choice functions, acyclicity is the minimal additional requirement for that purpose.[4/]

Finally, Arrow's [1] condition of independence of irrelevant alternatives is similar in the present context to the following condition:

<u>Condition INFA</u>: A SCF, f, is said to be independent of Non-Feasible Alternatives, if $f(S/A, R_1,\ldots,R_n) = f(S/A, R_1',\ldots,R_n')$ on Π whenever $(\forall i)$ R_i and R_i' are identical on S.

<u>Proposition 1</u>: Let f be a SCF satisfying the conditions U, P, INOA and INFA. If f accords b an alienable right to some alternative x over some other alternative y, then f does not accord any other individual j, an alienable right to every alternative z over every other alternative w, such that w is j-variant of z.

<u>Proof:</u> Let $\langle x,y,b\rangle \varepsilon RS$, $x,y \varepsilon S$ and $xD_by[RS]$, for some set of individual preference ordering, a given S, and given all other things relevant to the description of a social state. Then, by definition, 6, $xP_by \rightarrow xPy$. Let the preference orderings of all other individuals be such that $(\forall c) yP_cx$. Then, by INFA and INOA, we must have:

(1) $[xP_by, \ \& \ (\forall c) yP_cx] \rightarrow xPy, \qquad c \neq b.$

Introduce a third alternative, w, and let R_b and R_c, $c \neq b$, be such that xP_by, yP_bw and $(\forall c)[yP_cw, wP_cx]$. It follows from (1) that xPy and by condition, P, yPw. Hence, by definition 5 y and w do not belong to the choice set. Suppose, with no loss of generality, that w and x are j-variants and $\langle w,x,j\rangle \varepsilon RS$. Suppose further that x is the least preferred alternative in the preference ordering of the j^{th} individual. Hence, by definition 3, $\sim wW_jx[RS]$, and the three conditions of definition 4 are satisfied. Thus, $wD_jx[RS]$ and by definition 5, wPx, which means that x does not belong to the choice set. Hence the choice set is empty, a contradiction, and f does not accord j an alienable right to w over x.

Q.E.D.

Gibbard's social choice function satisfies condition U, P, GL and INFA. Gibbard's exposition tends to hide the fact that his SCF is consistent with INFA. To see this, note that the relation P in definition 5 is defined relative to the feasible set S, so that xPy means no more than $\sim(x\varepsilon S \,\&\, y\varepsilon C)$, where C denotes the choice set. Note next that according to definition 3, the sequence Σ through which others besides b claim rights must consist entirely of feasible alternatives. Thus, whether a right is waived depends on individual preferences among feasible alternatives alone, and hence whether $xD_by[RS]$ holds, (by definition 4) depends on individual preferences among feasible alternatives alone. From all this it should be clear that the SCF of Gibbard satisfies INFA.[5/]

It follows, therefore, that Gibbard's SCF is inconsistent with the condition INOA. More specifically, this function violates the above definition of collective rationality by not being generated by fixed bineary relationship. This result has the implication that the choice set is not independent of the set of feasible alternatives. To the extent that the choice set represents <u>just</u> social states, then justice becomes a relative concept (i.e., relative to the feasible set of alternatives). In addition, Gibbard's SCF does not satisfy the condition of cheat-proofness which is defined as follows:

Condition CP: A SCF, f, is said to be manipulable at the conditional profile $(S/A,R_1,\ldots,R_n)$ by individual j, if there is a ranking R'_j of A such that $f(S/A,R_1,\ldots,R_{j-1}, R'_j, R_{j+1},\ldots,R_n)\ P_j\ f(S/A,R_1,\ldots,R_n)$ where P_j denotes strict preference relation induced by R_j. f is said to be cheat-proof if it is nowhere manipulable.

Proposition 2: Gibbard's SCF does not satisfy the condition of cheat-proof.
Proof: Consider the following example[6/], let x,y and z represent three social states, and assume that $<x,y,b>\varepsilon RS$, $<z,x,a>\varepsilon RS$ and $<z,y,a>\varepsilon RS$. Suppose further that person's b sincere preference ordering of the three alternatives in the example given above is $xP_by\ P_bz$, and person's a sincere preferences are $yP_ax\ P_az$. (To the extent that there are other individuals we assume that their preferences agree with that of person a.) Clearly in this case xPz and yPz by condition P, and xPy since, $xD_by[RS]$ and definition 5. Hence, the choice set consists of x. Suppose now that individual a misrepresents its preferences as follows $yP'_az\ P'_ax$. In this case, yPz by condition P, $zD_ax[RS]$, and by definition 5, zPx. Hence, by definition 2, b waives his right to x over y, and the choice set consists of y. But yP_ax, hence the SCF is manipulable.

Q.E.D.

The admission of strategic manipulation of individual preference ordering raises a serious difficulty with Gibbard's SCF. To see this consider individual b in the above example. Facing individual a's announced preference ordering, he may decide to waive his right for x over y, in which case the outcome is y. Or he may decide to change his own preference ordering interchanging the rankings of z and y so that $xP'_bz\ P'_by$. In which case he is not required to waive his right for x over y. But, the outcome in this case is z, unless individual a decides to restore the original preference ordering, in which case the outcome will again be x.

As this example clearly shows, the possibility of strategic behavior under Gibbard's SCF give rise to different outcomes which depend on the personality characteristics of the players, the amount of information they possess and the rules according to which they negotiate the outcome. The extent to which the outcome of such a social choice process may be regarded as just depends upon the ethical evaluation of choice sets which are based upon threats, counter-threats, and cheating.

FOOTNOTES

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1. Throughout this paper I shall use Gibbard's notations and definitions. This is in order to shorten the exposition. I shall assume therefore, that the reader is acquainted with Gibbard's paper.

2. An issue is defined as a choice among alternative's possible features which determine a social state.

3. Definition: x is a j-variant of y, if and only if, $(\forall_i)[i \neq j \rightarrow x_i = y_i]$ where x_i and y_i are the ith entry of the vectors x and y respectively.

4. See Sen [4], p.16, Lemma 1*1.

5. This observation bears heavily upon private correspondence with Gibbard, who indicated to me that he was prodded to make his libertarian condition consistent with INFA by Thomas Schwartz.

6. In this example, taken from Gibbard, x, y and z may be interpreted as follows:

   x: Both Edwin and Angelina remain single,

   y: Edwin weds Angelina,

   z: The judge weds Angelina and Edwin remains single.

   person b is Edwin, and person a is Angelina.

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