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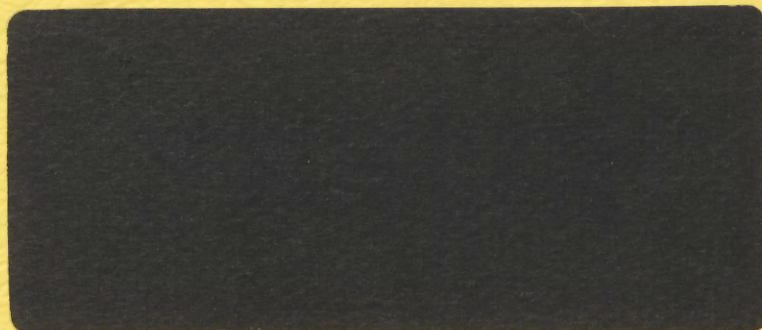
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TECHNOLOGICAL CHANGE, SECTORAL SHIFTS AND
THE DISTRIBUTION OF EARNINGS:
A HUMAN CAPITAL MODEL

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Discussion Paper No. 748

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ABSTRACT

The paper studies the long-run impact of technological change on the labour market in a two-sector model with heterogeneous workers. First it is assumed, in line with stylized facts, that inventions increase both productivity and skill requirements. Such skill-intensive inventions cause increases in inequality, shifts of labour out of the technologically dynamic sector, and relative price changes. In contrast, a skill-neutral invention causes neither changes in inequality nor sectoral shifts, while a skill-extensive invention reduces inequality and causes sectoral shifts toward the innovative sector. Only a skill-neutral invention leads unequivocally to a Pareto improvement.

KEYWORDS: Technological change, income distribution, inequality, sectoral shifts, human capital, skills.

JEL CLASSIFICATION NUMBER: 821 (Labour economics)

TECHNOLOGICAL CHANGE, SECTORAL SHIFTS AND THE DISTRIBUTION OF EARNINGS: A HUMAN CAPITAL MODEL

I INTRODUCTION

Several excellent analyses of the effects of technological change on the level of employment and wages have recently appeared (Neary (1981), Sinclair (1981), Katsoulacos (1986) and Karni and Zilcha (1988)). These studies for the most part assume that there is one sector, and that workers are equally skilled. They therefore implicitly assume that technical change proceeds at identical rates in all industries, and that the impact on specific workers does not depend on their individual characteristics.

Empirical studies throw the validity of both of these assumptions into doubt. First, the impact of recent technical changes on workers seem to have depended heavily on the occupation and skill level of the specific worker. Second, rates of innovation by industry have varied considerably.

Three recent surveys of French, British, West German and Canadian businesses concerning their adoption of new technologies related to micro-electronics suggest that the impact of innovation on workers depends very much on their skill levels. In particular, recent innovations related to micro-electronics have increased the demand for highly trained workers within the individual firm, while often reducing the demand for workers in less skilled occupations. (See Northcott et al. (1985), Economic Council of Canada (1987) and Ontario Task Force on Employment and Technology (1985).)

Indeed, a separate body of literature suggests that overall skill levels have slowly been rising. The time required for workers to learn an occupation has increased over the last few decades. A study of the skill level of Canadian workers by Myles (1988) confirms that the average skill level has increased over the period 1961-81, due both to changes in

industry's skill requirements and, of lesser importance, changes in the industrial mix. (Skill levels were measured by various criteria, including the training time needed.) Spenner (1983) surveys similar studies of American workers. All of the studies reviewed whose definitions of skills were based on the level of training required or the complexity of the task indicated either no change or a rise in skill levels in recent decades.

Note that these results suggest that recent technological changes have increased average skill levels. This evidence runs against the theory of technological "de-skilling" which has been advanced by, among others, Braverman (1974) and Marglin (1982).

Just as innovation affects various skill-classes of workers to different extents, it also affects industries to varying degrees. One indication of differing rates of innovation by industry is that rates of labour productivity growth have varied markedly between sectors. In Canada, for instance, labour productivity growth in the service sector (measured as changes in the constant dollar value of output per worker) has been anemic relative to that in the primary and industrial sectors. (Betts and McCurdy (1988)). Perhaps a more convincing demonstration that rates of innovation vary by industry comes from the three aforementioned surveys, which show strong variations in the adoption of new technologies between industries.

Given these stylized facts, it seems highly desirable to model technical change in a framework which allows both for heterogeneity among workers and for rates of innovation to differ between sectors. The theoretical literature on innovation and employment has yet to do this. On the other hand, the human capital literature has recognized the heterogeneous nature of labour markets. But it has yet to consider the consequences of innovation in much detail.

This paper therefore initiates a synthesis of these two strands of

literature. It studies the long term effects of technological change within a human capital model. Allowance is made for both worker heterogeneity and differing rates of productivity growth between sectors. It focuses on the effects of disembodied innovation on the following variables: i) the pattern of skill acquisition, ii) the distribution of employment between industrial sectors, and iii) the distribution of earnings. As such, it synthesizes the literature on technical change, which generally assumes that labour is homogeneous, with the human capital literature, which, while allowing for a heterogeneous work force, has not incorporated technical change.

II THE BASIC APPROACH IN CONTEXT OF THE LITERATURE

Workers are treated as self-employed. This is not an unreasonable assumption given that in the model there are constant returns to the single factor, labour. We thus abstract from the demand side of the labour market: it is the worker who chooses his level of training and the technology with which he works.

The model is not a short-run model of labour market adjustment to technical change. Rather, our concern is to show how innovation can alter workers' choice of industrial sector and of technology, given ability, *from one generation to the next.*

The basic premise is that workers differ in their ability to learn. The second major assumption is that innovation can alter the average training time required of workers of a given ability. It follows that a sorting process between technologies and workers will occur. Over time, new inventions, by increasing the number of available techniques, can significantly change the matching process and hence the distribution of

earnings. Based on the stylized facts listed above, the paper focuses on a type of invention which increases training requirements. But other types of invention, which either do not change training requirements, or decrease them, are also considered.

The third major assumption is that the distribution of innate abilities is static from one generation to the next, so that the only cause of changes in income distribution over time is technological change.

A fourth key feature of the model is that labour is supplied inelastically. There is no unemployment.

Fifth, the model distinguishes between "invention" and "innovation", and assumes that the rate of *invention* is exogenous. Despite this exogeneity, the actual rate of technological change, that is, the rate at which the economy innovates by adopting inventions, is assumed to be *endogenously determined* by workers who choose technologies and the accompanying training through a human capital decision. These assumptions bear some discussion in the context of the literature on technical change.

The Treatment of Technological Change in the Literature

Early neoclassical models of economic growth, such as Solow (1956) and Swan (1956), treated technical change as an exogenous shock. No distinction was made between an invention and a technical change -- i.e. an innovation.

A second branch of models, appearing mainly in the 1960's, endogenized technical change. For the most part this was done in one of two ways. The first method was to make productivity a non-stochastic increasing function of the labour resources devoted to research and development (R and D). Examples of this approach include Uzawa (1965), Phelps (1966) and Von Weizsäcker (1966). Romer (1986) makes an important contribution to this literature by studying the implications of increasing returns to R and D.

The second manner in which this literature endogenized technological change was to assume that productivity is positively related to cumulative gross investment. Arrow's (1962) well-known model of "learning by doing" used this technique. Kaldor and Mirrlees (1962) made a similar assumption, while distinguishing between vintages of technology.

There has emerged recently a third branch of models which extends the 1960's work on R and D by allowing for uncertainty (e.g. Aghion and Howitt (1989)). These models assume that the rate of invention is stochastic, but that the expected arrival rate is increasing with expenditure on R and D.

A fourth branch of modeling, on technology diffusion, endogenizes the rate of technical change by assuming that the rate of invention is exogenous, but that the rate of adoption of these new ideas is determined by the profit-maximizing decisions of firms. For instance, Soete and Turner (1984) model the rate of technical change as a function of the rents that can be obtained by adopting a new technology, and the firm's degree of uncertainty about the technology's effectiveness. Shleifer (1986) applies the insights of this literature to the issue of business cycles.

The present model is closest in spirit to this fourth branch of literature: it too assumes that the rate of invention is exogenous, and that the rate of adoption is determined endogenously by the optimizing decisions of economic agents. The novelty is that while the models cited in the above paragraph take the firm as the central agent, this paper focuses on the choices of workers. It views the rate of innovation as jointly determined by the distribution of workers' abilities and the exogenous rate of invention.

Thus the present model marks an advance over the first wave of models of technical change referred to above, since it endogenizes the rate of innovation. The way in which it does this is quite different from the methods

used in the second and third waves of models described above, but is quite close to that used in the fourth branch of work, the "diffusion" models. The main distinction between the diffusion models and the present work is that the latter explores interactions between the labour market and the rate of technical change.

The Assumption of Self-Employment

Our assumption of constant returns to scale simplifies the analysis considerably, since without loss of generality we can abstract from firms, instead focusing on self-employed agents choosing the optimal technology given their abilities.

One may object to the idea that workers "choose" technologies on the grounds that in the real world, firms choose technologies and impose them on workers. We would answer that the variation between technologies used by firms in the same industry, by different industries, and by different occupations is huge. Thus workers can choose the technology with which they will work, albeit in an indirect manner. They do this in three ways: by choosing the firm for which to work within an industry, by choosing the industry in which to work, and by choosing an occupation.¹

Outline of the Paper

In Section III, a one-sector model compares the outcomes between two generations of workers, the later generation choosing from a larger technology set than did their predecessors. In Section IV, a two-sector model is developed. The first sector is identical to that in the one-sector model presented in Section III. Sector 2, on the other hand, is technologically

¹For empirical evidence that different vintages of technology do co-exist at any one time in an industry, see the literature on the "S-curve" of technology adoption. A good review of this is Chapter 2 of Davies (1979).

stagnant. Given the above stylized facts, we can very loosely identify sector 1 as manufacturing and sector 2 as services.² The goals of this section are to trace the effects of different rates of innovation between sectors on earnings inequality, the sectoral allocation of labour, and relative goods prices. Section V considers types of innovation which either leave unchanged or reduce training needs, so as to show which of the predictions in Section IV result directly from the assumption that innovation increases training requirements. Policy conclusions are drawn. Section VI concludes.

The model's results will be related to the following two observations:

- a) The rate of employment growth in the service sector has exceeded that of the manufacturing and primary sectors in virtually all developed market economies since the 1960's. For instance, in Canada private sector employment in the service sector grew by 45.3% in the 1960's and 56.1% during the 1970's, compared to all-industry averages of 24.4% and 35.5% respectively. (Betts and McCurdy (1988)) For evidence from a broader range of developed countries, see for instance the labour force statistics in World Bank (1985, Table 21).
- b) The distributions of income in both the U.S. and Canada, but particularly the former, have become slightly more unequal over the last two decades. The income share of the lower groups has dropped, while that of the upper groups has risen. The share of the population whose income is within a given range

² We say that this identification is a loose one because, as Myles (1988) shows in the Canadian context, most industries in the service sector in fact have higher training requirements and average educational levels than the average component of the industrial sector. The element of the service sector which most closely fits our definition of "services" -- low productivity growth and low training needs -- is consumer services. Business and social services on the other hand exhibit higher levels of specific vocational preparation and general occupational development than any other industry.

of the average income has also declined somewhat, leading some to the interpretation that the middle class is declining. For the United States, Levy (1987) reports that between 1969 and 1984 the changes in the share of family income going to each quintile, from the bottom to the top of the income distribution, were -0.9%, -1.4%, -0.7%, +0.7%, and +2.3% respectively. Similarly, the distribution of men's earnings according to 9 income intervals in constant dollars between 1969 and 1984 shows that the percentage of men in the bottom three and the second highest income classes rose, while the percentage declined in all other income classes, suggesting increasing inequality of earnings among individual men. In Canada the percentage shares of total income of individuals whose major source of income is wages and salaries changed as follows between 1967 and 1987 (the numbers refer to share by quintiles, starting with the bottom quintile), 2.8 to 3.0, 10.9 to 10.4, 18.4 to 17.6, 25.4 to 25.7, and 42.5 to 43.2. (Statistics Canada, (1980, Table 58) and (1988, Table 66)) Leckie (1988, Table 4) reports that the proportion of Canadian individuals in the middle class, defined as those whose income is within 25% of the mean income, fell from 30.0% to 25.7% between 1971 and 1984. ³

III A ONE-SECTOR MODEL OF INNOVATION

Assumptions

Characteristics of Workers

A1 *Workers' utility depends on consumption, C , of the one good produced:*

$$(1) \quad U = U(C), \quad U_C > 0, \quad U_{CC} < 0.$$

A2 *Workers differ in innate ability. There is a well defined distribution of*

³ For a recent review of this literature, see Beach (1989).

abilities, a , among the population. The probability density function $m(a)$ has support $[\underline{a}, \bar{a}]$, where $\underline{a}, \bar{a} \in [0, \infty]$ and $\bar{a} > \underline{a}$. $M(a)$ denotes the corresponding distribution function.

Differences in ability do not affect the productivity of workers once trained for a given job. However, the length of time required to train for a given job is decreasing in ability. (See below.)⁴

A3 At the start of each generation, a cohort of workers is born, and after a fixed period of formal schooling, each worker enters the work force for one period. Without loss of generality, we normalize the population to 1.

Technology

A4 Agents choose a technology i from the technology set $[0, t]$, where t is a positive parameter denoting the state of technology.

A5 There are no new inventions made during the lifetime of a given generation. But the number of technologies available is non-decreasing over time, due to inventions which occur at the end of each generation.

The upshot of this assumption is that we can think of an increase in the technology parameter, t , as representing an unspecified passage of time. That is, the higher is the technology index t , the more recent is the period.

It is assumed that the production function obeys constant returns to scale. Therefore, we can consider the production of a single worker without loss of generality.

A6 The production function for one worker associated with the technology denoted by some $i \geq 0$ is:

⁴The assumption that workers, once trained, are equally productive with a given technology is not crucial for the model's results. If we assumed that the more able are more productive with a given technology after training, we would in fact strengthen the main results of the model, that the distribution of earnings is unequal, and that the more able gain more from new inventions.

$$(2) \quad Y = \delta i, \delta > 0.$$

This production function is gross of the opportunity cost related to training.

A7 The range of technologies available at any time, $i \in [0, t]$, is the same for all workers.

The Link Between Technology, Human Capital and On-the-Job Training

A8 In order to use a technology i the worker must acquire, as a minimum, the stock of human capital H given by

$$(3) \quad H = hi,$$

where h is a parameter obeying $h > 0$.

A9 The human capital production function of a worker with ability a is:

$$(4) \quad H = \frac{T(a+\beta)}{\omega}, \text{ where } \beta \text{ and } \omega \text{ are positive parameters, and where } T \text{ is the length of time spent on on-the-job training. } T \text{ obeys } T \in [0, 1].^5$$

Given that utility-maximizing workers will not devote any more time to training than required by their choice of technology i , equation (3) can be thought of as the worker's demand for human capital, conditional upon his or her choice of technology. Equation (4) determines the supply of human capital. We may thus equate (3) and (4). Defining $\alpha = h\omega$, and remembering the restriction that $T \leq 1$, we obtain the training function:

$$(5) \quad T(a, i) = \begin{cases} \frac{\alpha i}{a + \beta} & \text{if } \frac{\alpha i}{a + \beta} < 1, \quad \beta > 0, \alpha > 0 \\ 1, & \text{otherwise} \end{cases}$$

⁵ Note that in A3 we assume that the length of formal schooling is fixed. This explains why schooling does not enter the human capital production function explicitly. One may interpret the positive dependence of the human capital stock H on ability as reflecting in part the fact that more able people should gain more skills during the fixed period during which all workers attend school. It can be shown that when schooling becomes a choice variable, the main result of the one-sector model, concerning the impact of innovation on the distribution of earnings, continues to hold.

A10 The only cost of training is the opportunity cost -- the time spent. This is also the only cost related to acquisition of a technology.

Results

Since an invention $\delta t > 0$ introduces a new technique which requires that workers obtain more human capital than did earlier techniques, we will refer to $\delta t > 0$ as a *skill-intensive* invention.

The worker's problem involves choosing a technology $i \in [0, t]$ which will maximize his or her lifetime production: $\delta i(1 - T(a, i))$. We solve the problem in two steps: optimize assuming an interior solution, and then compare the resulting utility with the utility from the two corner solutions. ⁶

Step 1

The program to be solved is:

$$(6) \quad \text{Max}_i U \left[\delta i \left(1 - \frac{\alpha i}{a + \beta} \right) \right] \text{ s.t. } 0 \leq i \leq t.$$

F.O.C.:

$$(7) \quad U'(\cdot) \left[\delta \left(1 - \frac{\alpha i^*}{a + \beta} \right) - \left(\delta i^* \frac{\alpha}{a + \beta} \right) \right] = 0 \Leftrightarrow$$

$$a + \beta - 2\alpha i^* = 0$$

$$(8) \quad i^* = \frac{a + \beta}{2\alpha}, \quad t \geq i^* \geq 0 \quad ^7$$

⁶ Although we will henceforth focus on the choice of i and training T without explicit mention of the choice of the human capital stock H , the reader should bear in mind the links between the choices of human capital, on-the-job training and technology. The reader can learn about the comparative statics of human capital choices by examining those for technology, since $H = h_i$.

⁷ The second-order condition is:

$$U''(\cdot) \left[\delta \left(1 - \frac{\alpha i^*}{a + \beta} \right) - \delta i^* \frac{\alpha}{a + \beta} \right]^2 + U'(\cdot) \left[- \frac{2\delta\alpha}{a + \beta} \right] < 0$$

Thus (8) gives the optimal interior solution. The second-order condition is quite robust to alternative specifications of $T(a, i)$ and the production function. If the production function for a worker, gross of production lost

Consider the first-order condition (7). The marginal return to an increase in technology i is $\delta(1 - T(a,i))$, which is the first term in brackets. The marginal cost, given by the second term in brackets, reflects the earnings forgone during training. More able workers choose a more recent technology (higher i), both due to higher marginal returns and, in the form of shorter training times, lower marginal costs.^{8 9}

Substituting (8) into the expression for income, the maximum earnings, assuming an interior solution, are given by

$$(9) \quad Y^* = \frac{\delta(a + \beta)}{4\alpha}$$

Step 2

Now consider the possibility of corner solutions. First, $i = 0$ will be chosen by no agent, since at $i = 0$,

$$\lim_{i \rightarrow 0^+} \partial U / \partial i = U'(\cdot) \delta > 0$$

Define τ as the optimal technology (at an interior solution) for the most able type of worker in the economy: $\tau = i^*|_a = \bar{a}$. Unless $t > \tau$, at least one type of agent will choose $i = t$, the other corner solution. This will occur for workers for whom:

due to training time, is given by $\gamma(i)$, then two sufficient but not necessary conditions for the second-order condition to obtain are $\gamma'' \leq 0$ and $T_{ii} \geq 0$.

⁸ The finding that more able workers choose a more recent technology is robust when the production function in the previous footnote, $\gamma(i)$, depends only on i . It can be shown that sufficient but not necessary conditions for the outcome are the two assumptions listed in the previous footnote, along with $T_{ai} \leq 0$. This new assumption states that the increase in training required by a marginally more productive technology is non-increasing in ability.

⁹ Note also that for all workers who choose an interior solution, $T(a, i^*) = \frac{1}{2}$. This follows from the assumption that the training function is linear in i . Because of this assumption, the objective function could be rewritten as a quadratic function in the training time T : $\gamma = \delta i(1-T) = qT(1-T)$ where q is a constant such that $qT = \delta i$. The person will maximize this objective function by choosing $T = \frac{1}{2}$. Becker (1985) obtains a similar result.

$$(10) \quad i^* = \frac{a + \beta}{2\alpha} \geq t$$

In other words, the corner solution $i = t$ will be chosen by workers for whom:

$$(11) \quad a \geq 2t\alpha - \beta$$

Thus, given the state of technology t , the worker of ability a chooses the optimal technology i^* following:

$$(12) \quad i^* = \begin{cases} t & \text{if } a > 2t\alpha - \beta, \\ \left(\frac{a + \beta}{2\alpha} \right) & \text{if } a \leq 2t\alpha - \beta \end{cases}$$

The Distribution of Earnings over Time

We can place workers into two categories: those with higher abilities, who choose $i = t$, and those with lower abilities, who choose an interior solution. We will subscript variables pertaining to these two classes of workers with H and L, respectively.

The Case of the More Able Workers

Define the "cross-over" ability $\tilde{a} = 2t\alpha - \beta$, so that from (12) all workers with ability above this cross-over level choose $i = t$. In a sense, these workers are "technology-rationed", in that the optimal technology for their ability level has not yet been invented. The proportion of workers who are technology-rationed, PR_H , will thus be $1 - M(\tilde{a})$.

The productivity of these workers, once trained, is identical: δt . This can be thought of as the wage. But the distribution of lifetime income will be unequal since the more able of these technology-rationed workers can finish the required training more quickly. The lifetime income (or consumption) of these workers with ability a is given by:

$$(13) \quad C_H = \delta t \left(1 - \frac{\alpha t}{a + \beta} \right)$$

There are five things to note about this earnings or consumption function:

1) The $C_H:a$ locus is upwardly sloping and strictly concave for all $t > 0$:

$$\frac{\partial C_H}{\partial a} = \delta t^2 \frac{\alpha}{(a+\beta)^2} > 0 \quad \forall t > 0; \quad \text{and} \quad \frac{\partial^2 C_H}{\partial a^2} = -\frac{2\delta t^2 \alpha}{(a+\beta)^3} < 0 \quad \forall t > 0$$

Thus in this region, income increases with ability, but at a decreasing rate.

2) The slope of the $C_H:a$ locus steepens between one generation and the next, simply because the state-of-the-art technology becomes more productive as the technology index t increases:

$$\frac{\partial^2 C_H}{\partial t \partial a} = \frac{2\delta t \alpha}{(a+\beta)^2} > 0 \quad \forall t > 0$$

3) Over time the $C_H:a$ locus becomes more concave because the state-of-the-art technology t advances:

$$\frac{\partial}{\partial t} \frac{\partial^2 C_H}{\partial a^2} = \frac{-4\delta t \alpha}{(a+\beta)^3} < 0 \quad \forall t > 0$$

4) Over the generations, the share of workers who are "technology-rationed" declines as t rises. This is so because the cross-over ability \tilde{a} rises with t : $\partial \tilde{a} / \partial t = 2\alpha$ until PR_H reaches zero.

5) Workers of a given ability who are technology-rationed are made better off from one generation to the next as the technology index t increases:

$$\frac{\partial C_H}{\partial t} = \delta \left(1 - \frac{2t\alpha}{a+\beta} \right) > 0,$$

The inequality follows since for technology-rationed workers $a > 2t\alpha - \beta$.

The Case of the Less Able Workers

The proportion of workers who choose an interior solution is given by $PR_L = M(\tilde{a}) = M(2t\alpha - \beta)$, by the definition of \tilde{a} . Since these workers choose an interior solution, equations (8) and (9) give their choice of i^* and their corresponding income. Note the following:

1) The proportion of workers who choose an interior solution increases with time, as new technologies become available.

2) Unlike the income of technology-rationed workers, the income of these less

able types of workers remains constant between generations.

3) The slope of the $C_L:a$ locus is constant for all abilities and values of t :

$$\frac{\partial C_L}{\partial a} = \delta/4\alpha; \quad \frac{\partial^2 C_L}{\partial a^2} = 0; \quad \frac{\partial^2 C_L}{\partial t \partial a} = 0$$

Finally, it can be shown that at the cross-over ability \tilde{a} , at which workers begin to become technology-rationed, the $C:a$ locus is continuous.

The path of income distribution over time is shown in Figure 1 over leaf. (If we were to assume that ability were distributed uniformly, the graph of $C:a$ would also be a graph of $C:(\text{percentile of population})$.)

Workers of lower ability find a technology with the optimal training and productivity characteristics at an early point in history. Once this technology has emerged, it is always chosen by workers with that ability level at later times. More able workers are technology-rationed; they choose the latest available technology. But due to the assumption that, from the origin, the training time is convex in ability for a given i , the increase in income with rising ability is concave beyond the cross-over ability \tilde{a} .

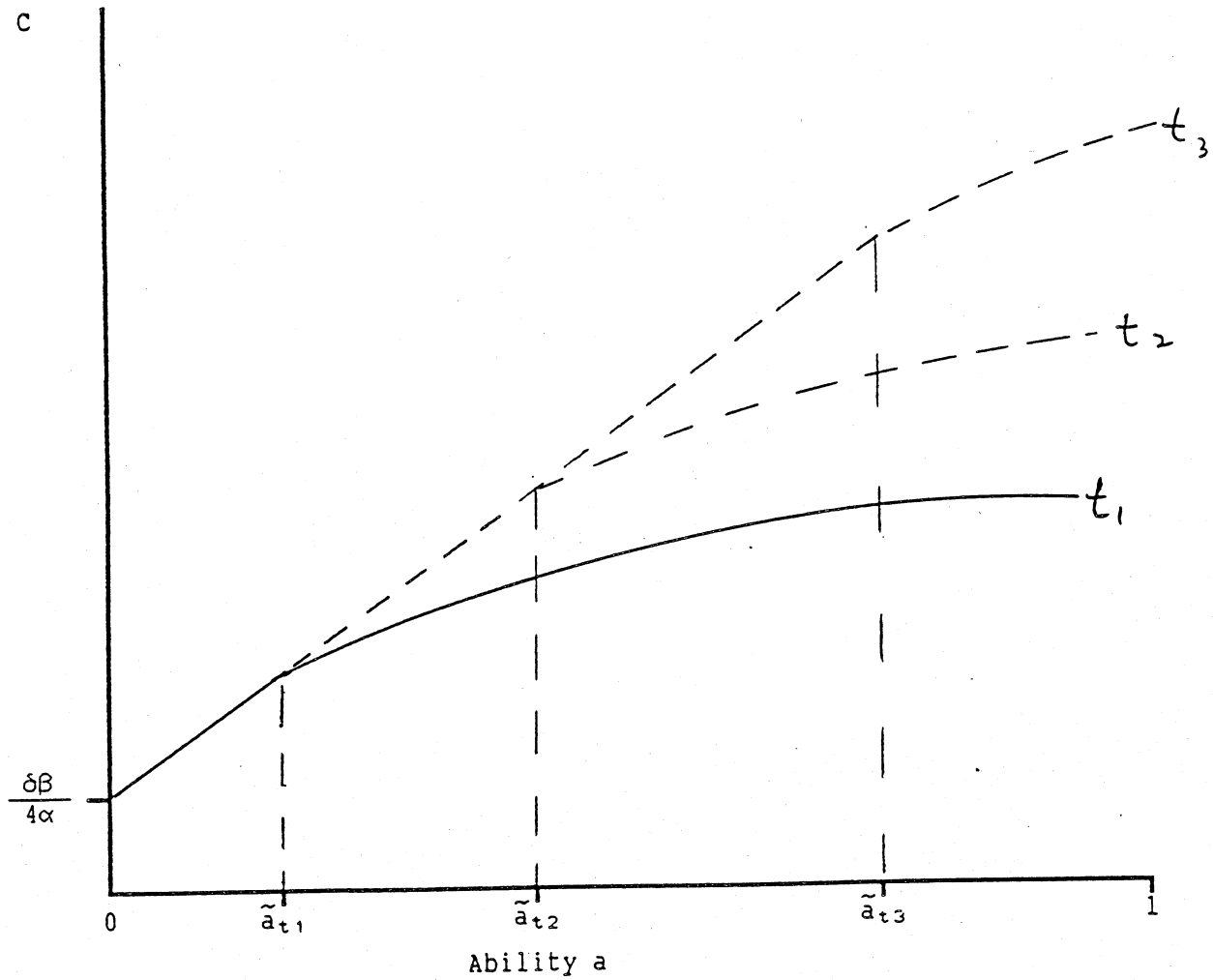
Between generations, new inventions make all the more able workers better off, while the utility of less able workers with $a < \tilde{a}$ remains unchanged. The proportion of workers who are technology-rationed declines between generations until the technology parameter t reaches the value which is the optimal technological choice for the most able workers. In this final steady state, consumption is distributed linearly with ability.

Thus, we conclude that technological change of the type studied here leads to an increasingly uneven distribution of earnings from one generation to the next. The less able maintain their real income, but their share of income falls, as the real income of more able workers increases over time.

It can be shown that if changes in income distribution are assessed by

Figure 1

The Distribution of Output (and Hence Consumption) (C) by Ability (a) for Three Different States of Technology in the One-Sector Model.
 Note: $t_3 > t_2 > t_1$, i.e. higher curves reflect more advanced technologies.
 It is assumed that $\underline{a} = 0$ and $\bar{a} = 1$ in the figure.



measuring the shares of income among n segments of the population, the shares of those "n-tiles" for which all members are at an interior solution decline by an equal percentage between two periods. For income groups whose members are technology-rationed, the decline in share of national income tends further toward zero for each successive group as one moves up the income distribution. For the top group(s), the share of total income will rise. This follows because for those who are technology-rationed, $\frac{\partial}{\partial a} \frac{\partial C}{\partial t} > 0$, as shown above.

The next section extends the model to two sectors.

IV A TWO-SECTOR MODEL WITH TECHNOLOGICAL CHANGE IN ONE SECTOR

A INTRODUCTION

This section considers the implications of the one-sector model above for a two-sector economy, with different rates of innovation between sectors. The main results are that skill-intensive inventions generate both changes in relative goods prices and inter-sectoral shifts in employment. At the same time, inequality in earnings increases.

We assume that:

A11 Sector 1 in the model is identical to the one sector in the above model.

The price of good 1 relative to that of good 2, the numeraire good, is P_1 .

A12 The second sector is characterized by constant returns to scale. The product of one person working in this sector is G .

A13 There is no technical change in sector 2.

A14 The training time required for workers in sector 2 does not vary with ability. The productivity parameter G incorporates this fixed training cost.

We simplify by assuming that only in sector 1 does the level of a

person's "ability" affect productivity. It is perhaps best to interpret a as some *sector-specific* skill. This simplification is not crucial for the results. What is crucial is that the two sectors differ in the rate of invention, and that the gains to a worker of an invention depend on his or her ability in at least one sector.¹⁰

A15 All workers have identical Cobb-Douglas utility functions:

$$(14) \quad U(C_1, C_2) = C_1^{\rho_1} C_2^{\rho_2}, \quad \rho_1, \rho_2 > 0$$

By virtue of this Cobb-Douglas assumption, the worker maximizes nominal income. The first step in the worker's optimization process is the choice of the optimal technology i^* in sector 1, conditional upon entry into sector 1. The optimization set out in equations (6) to (12) will continue to apply. The second step is the choice of the sector in which to work.

Since the endowment consists of one unit of labour, the worker's income is $P_1 \delta i^* (1 - T(a, i^*))$ if the worker chooses sector 1, and G if the worker chooses sector 2. Thus the worker chooses sector (1/2) as:

$$(15) \quad P_1 \delta i^* (1 - T(a, i^*)) \geq G$$

B THE GENERAL SOLUTION

Since lifetime output in sector 1 is strictly increasing in ability, but is independent of ability in sector 2, we can partition the work force into two groups, with those preferring to work in sector (1/2) having ability (above/below) a certain level. Accordingly, denote the unique ability level at which workers are equally productive in the two sectors by j . Then if we define $F(a)$ as the lifetime output in sector 1 of a worker of ability a ,

¹⁰ It can be shown that an innovation in sector 2, in the form of an increase in G , has only one effect: a change in the relative goods price.

$$P_1 F(a) \geq G \text{ as } a \geq j.$$

Thus a worker of ability a will choose sector (1/2) as $a \geq j$. That is, j determines the sectoral allocation of labour.

Note that by the definition of j , the equilibrium price must be:

$$(16) \quad \tilde{P}_1 = G/F(j).$$

Given this price equation, the condition that the goods markets clear determines j . By Walras' Law, we can consider either goods market. Consider the excess demand equation for the good produced in sector 2. Using the Cobb-Douglas demand function,¹¹ substituting for income of workers of each type (G in sector 2 and $P_1 F(a)$ in sector 1 for workers of ability a), integrating across abilities, and subtracting aggregate supply, we obtain:

$$(17) \quad \frac{\rho_2}{\rho_1 + \rho_2} \left\{ M(j)G + \tilde{P}_1 \int_j^{\bar{a}} F(k)m(k)dk \right\} - M(j)G = 0$$

Substituting for \tilde{P}_1 from (16) and re-arranging yields an implicit function for j in terms of the parameters:

$$(18) \quad -\rho_1 M(j)G + \rho_2 \frac{G}{F(j)} \int_j^{\bar{a}} F(k)m(k)dk = 0$$

If we define Z_j^k as the ratio of lifetime production in sector 1 of a worker of ability k to that of a worker of ability j , then we can write (18) as:

$$(18') \quad -\rho_1 M(j)G + \rho_2 G \int_j^{\bar{a}} Z_j^k m(k)dk = 0$$

Before deriving the dependence of j on the state of technology, it is useful to present the following result:

¹¹ For a derivation of these well known consumption functions, see, for instance, Varian (1984).

Result 1 The productivity ratio Z_i^k , for two workers of abilities $k > l$, changes with a skill-intensive invention $\partial t >$ as follows:

- If both k and l are technology-rationed, i.e. $t < (l+\beta)/2\alpha$, then

$$(19) \quad \partial Z_i^k / \partial t = \frac{\alpha \delta^2 t^2 (k-l)}{(l+\beta)(k+\beta)F(l)^2} > 0 \text{ where } F(l) = \delta t(1 - \alpha t/(l+\beta))$$

- If the worker of ability k is technology-rationed but the worker of ability l has reached an interior solution, i.e.:

$$(l+\beta)/2\alpha \leq t < (k+\beta)/2\alpha, \text{ then,}$$

$$(20) \quad \partial Z_i^k / \partial t = \frac{4\alpha(k+\beta-2\alpha t)}{(l+\beta)(k+\beta)} > 0$$

- If both types of workers are no longer technology-rationed, i.e. $t \geq (k+\beta)/2\alpha$, then $\partial Z_i^k / \partial t = 0$.¹²

Applying the implicit function theorem to (18'), we can obtain $\partial j / \partial t$. Denote the left-hand side of (18') by Ψ . Then

$$(21) \quad \partial \Psi / \partial t = \rho_2 G \int_j^{\bar{a}} \frac{\partial Z_i^k}{\partial t} m(k) dk > 0$$

as long as at least one worker in sector 1 is technology-rationed.¹³ The sign of the inequality follows directly from Result 1, holding j constant. Also,

$$(22) \quad \partial \Psi / \partial j = -\rho_1 m(j)G + \rho_2 \frac{G}{F(j)} [-F(j)m(j)] - \rho_2 \frac{G}{F(j)^2} \frac{\partial F(j)}{\partial j} \int_j^{\bar{a}} F(k)m(k)dk < 0$$

Applying the implicit function theorem, $\partial j / \partial t = \frac{-\partial \Psi / \partial t}{\partial \Psi / \partial j} > 0$ in the

¹² Proofs of all Results and Propositions are given in the appendix, unless explicitly included in the text.

¹³ We will neglect the special case in which technology is so far advanced that not even the most able workers would adopt a new technique. This case is unrealistic since it is a steady-state in which innovation ceases.

neighbourhood of the market-clearing value of j . Thus:

Proposition 1 *A skill-intensive invention ($\partial t > 0$) induces shifts of workers from sector 1 to the sector without technical change: i.e., $\partial j / \partial t > 0$.*

This important result is best interpreted jointly with the following:

Proposition 2 *A skill-intensive invention ($\partial t > 0$) causes the price of the good produced in the technically dynamic sector to fall relative to that of the good produced in the technically stagnant sector: $\partial \tilde{P}_1 / \partial t < 0$.*

Note that the price changes and sectoral shifts occur only between generations, since we are assuming that technology advances only when the technology set expands between generations.

The logic behind the changes in \tilde{P}_1 is clear: productivity in sector 1 rises with the technology index t , unlike that of sector 2. Hence, the relative price of good 1 must fall in order to avoid excess supply. Proposition 2 is thus a standard result of general equilibrium analysis.

The novelty of the model is Proposition 1, which states that innovation induces sectoral shifts of labour along with price changes. These sectoral shifts are best understood in the context of the following:

Result 2 *Denote the total output in sector i during the lifetime of a given generation of workers by Y_i , $i = 1, 2$. Then at all times,*

$$(23) \quad \frac{\tilde{P}_1 Y_1}{Y_2} = \frac{p_1}{p_2} = \text{constant}.$$

This result stems from the assumption of a Cobb-Douglas utility

function. It states that \tilde{P}_1 and the sectoral allocation of labour must adjust so as to leave the ratio of the total incomes earned by the two sectors equal.

Nevertheless, the ratio of incomes available to individual workers in the two sectors will change. A worker's potential income in sector 2 remains constant at G . But Result 1 indicates that output gains in sector 1 rise with ability: in general, $\partial Z_1^k / \partial t > 0$, where $k > \ell$. At the same time, the price of this output falls after an invention. Thus the income in sector 1 of types of workers above a certain ability level rises after an invention, while that of less able types of workers falls.¹⁴ Consequently, these least able types of workers in sector 1 will eventually be driven out of sector 1: $\partial j / \partial t > 0$.

C CHANGES IN THE INEQUALITY OF EARNINGS BETWEEN GENERATIONS

Changes in Inequality between Types of Workers in Sector 1

Result 1 above shows that inequality between any two types of workers of ability k and ℓ , $k > \ell$, will increase after an invention if at least one of those types of workers adopts the invention. That is, $\frac{\partial Z_1^k}{\partial t} > 0$ in such a case.

Changes in Inequality between Workers in the Two Sectors

The following considers inequality in terms of the average values of output per capita in the two sectors. It indicates that inequality in this average sense increases between generations.

Proposition 3 *A skill-intensive invention ($\partial t > 0$) causes the average value of output per capita in sector 1 to rise in absolute terms and relative to that of sector 2, which remains constant at G .*

¹⁴ The next section, which deals with inequality, states this result formally.

We can be more precise about changes in workers' incomes:

Proposition 4 *The income in sector 1 of a type of worker with ability a , both in absolute terms, and relative to the income of a worker in sector 2 (which is constant), changes as listed below after a skill-intensive invention:*

1) *Income falls for the least able types of workers, those for whom*

$$\frac{\frac{\partial F(a)}{\partial t}}{F(a)} < \frac{1}{F(j)} \left(\frac{\partial F(j)}{\partial t} + \frac{\partial F(j)}{\partial j} \frac{\partial j}{\partial t} \right).$$

2) *For the most able types of workers, those for whom the sign of the above inequality is reversed, income rises.*

In particular, the income of any types of workers in sector 1 who are at an interior solution with respect to technology i will decline following an invention $\partial t > 0$.

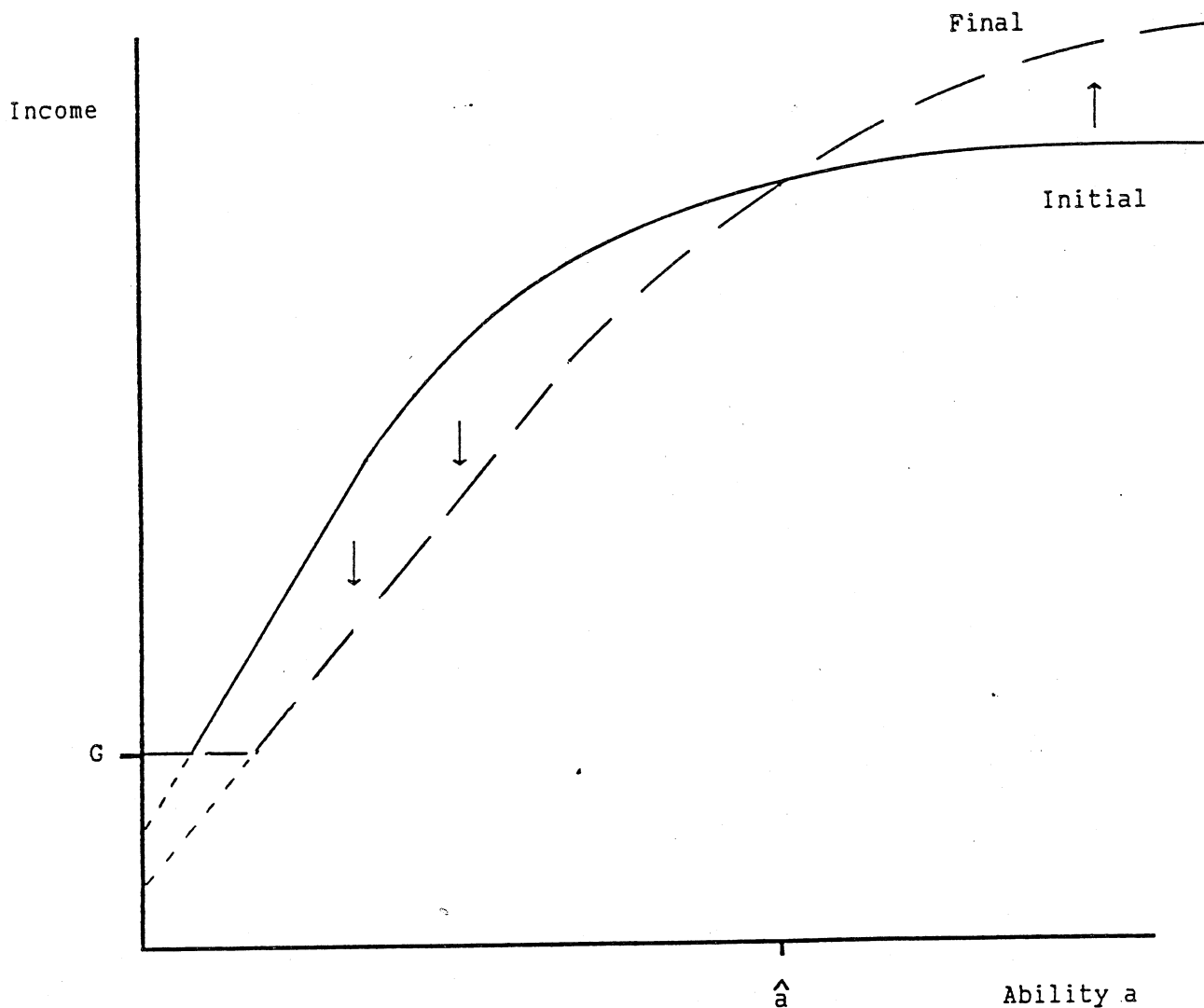
The right-hand side of the above inequality is the absolute value of the proportional change in \tilde{P}_1 . Thus income declines for those types of workers in sector 1 whose proportional increase in output following an invention falls short of the proportional decline in the price of good 1. Since the productivity gain resulting from an innovation rises with ability in sector 1, the income of the less able types of workers in sector 1 will fall when technology advances, and the income of the more able will rise.

Figure 2, over leaf, illustrates this occurrence. Before and after an invention, the income of workers in sector 2 remains constant at G . Define \hat{a} such that for this type of worker income $\tilde{P}_1 F(\hat{a})$ remains constant after an invention. Income rises for the more able types ($a > \hat{a}$), while the income of the less able types in sector 1 falls. The least able of the types of workers

Figure 2

The Effect of a Technological Advance on the Distribution of Income by Ability a in the Two-Sector Model. The numeraire is good 2.

Note: The dark line indicates the initial income:ability locus. The bolder dotted line (— — —) indicates what the income:ability locus would look like after an increase in the technology parameter t . The fainter dotted lines (---) indicate what the income of the less able would be if they did not have the alternative of earning G in sector 2.



whose income declines choose to work in sector 2 after the invention.

The overall conclusion from Propositions 3 and 4 is that a technological advance must always increase inequality, both within the technologically dynamic sector and between the two sectors. The following proposition establishes this result formally using the criterion of Atkinson (1972).

Proposition 5 After a skill-intensive invention inequality rises: the new Lorenz curve lies everywhere below the original Lorenz curve.

We can think of three groups of workers. The most able are those in sector 1 whose income share rises with an invention, the less able are those workers in sector 1 whose income share falls, and the least able are those workers in sector 2. The income shares of both the less able workers in sector 1 and the least able workers (those in sector 2) decline.

The result that inequality rises due to innovation is quite a strong one. We have assumed that within the technologically stagnant sector income distribution is completely equal, and that within the technologically dynamic sector it is inherently unequal. Even under these extreme assumptions, and with shifts of labour towards the sector with equal income distribution, inequality rises. While inequality does rise with innovation in the two-sector model, the existence of the second sector attenuates this rise. The reason is that workers have an alternative income of G in sector 2 which is not available in a one-sector world.

Changes in Utility Induced by Technological Change

The above results focus on inequality of earnings. One might also ask about the welfare implications of skill-intensive technical change. The following proposition establishes that only under certain conditions will a

new invention $\partial t > 0$ lead to a Pareto improvement.

Proposition 6 *An invention ∂t leads to a Pareto improvement iff for all types of workers in sector 1:*

$$\frac{\frac{\partial F(a)}{\partial t}}{F(a)} \geq \frac{-\rho_2}{\rho_1 + \rho_2} \left[\frac{\frac{\partial \tilde{P}_1}{\partial t}}{\tilde{P}_1} \right] \equiv \frac{\rho_2}{\rho_1 + \rho_2} \left[\frac{\frac{\partial F(j)}{\partial t} + \frac{\partial F(j)}{\partial j} \frac{\partial j}{\partial t}}{F(j)} \right]$$

Thus one necessary but not sufficient condition for there to be a Pareto improvement is that all workers in sector 1 must adopt the invention.

Thus, for there to be a Pareto improvement, the technology elasticity of output for all types of workers in sector 1 must exceed the absolute value of the technology elasticity of \tilde{P}_1 multiplied by $\rho_2/(\rho_1 + \rho_2)$. The reason why the term $\rho_2/(\rho_1 + \rho_2)$ appears in the inequality is that workers in sector 1 lose from a drop in \tilde{P}_1 as producers of good 1, but are partially compensated for this loss as consumers of good 1. Note that the conditions for a rise in t to cause a Pareto improvement are quite stringent. In particular, all workers in sector 1 must be technology-rationed for there to be a Pareto improvement.

V TECHNOLOGICAL CHANGES WHICH DO NOT INCREASE TRAINING REQUIREMENTS

A Introduction

Based on the stylized facts mentioned in the introduction, this paper has thus far emphasized *skill-intensive* inventions which increase labour productivity and also require a given worker to spend more time training. But one might also consider other types of innovation which do not increase training needs. This section explores two such types of innovation. The first, *skill-neutral* invention, has no effect on training requirements. A

good example might be electronic cashier systems which have increased cashiers' output without substantially altering training requirements. The second alternative type of invention, *skill-extensive* invention, reduces training requirements without increasing the productivity of the technique after training has been completed. An example of this is the evolution of word-processing programs in recent years: although their capabilities have not increased markedly, they have become dramatically easier to learn as "menus" have replaced complex command codes.

The findings of this section demonstrate that the earlier results concerning inequality and sectoral shifts depend crucially on the assumption that technical change increases the level of training required of workers. The conclusion to the section will draw inferences as to which type of invention a government which maximizes a standard social welfare function will be most likely to favour.

B Skill-Neutral Invention: Changes in the Shift Parameter δ

Since lifetime income is given by $\delta i(1 - T(a,i))$, we could model a technical change which increases the productivity of all techniques i , and which does not increase the training time $T(a,i)$, by an increase in δ .

It is true that in the two-sector model this different type of technical change still causes \tilde{P}_1 to fall. But innovation of this kind does *not* change the sectoral allocation of labour. Nor does it increase inequality of labour earnings in the one- or the two-sector context.

Starting with the one-sector model, it can be shown that following an increase in δ , equation (12) continues to describe workers' choice of technology. Since the $a:i$ mapping and hence the $a:T$ mapping is invariant to a rise in δ , we will refer to this as a skill-neutral invention.

A skill-neutral invention increases the income of workers regardless of whether they are technology-rationed. Differentiation of (13) with respect to δ shows that the income of technology-rationed workers rises:

$$\frac{\partial C_H}{\partial \delta} = t \left(1 - \frac{\alpha t}{a+\beta} \right) > 0$$

The more able gain more from the technological change, but by a decreasing amount as ability rises:

$$\frac{\partial^2 C_H}{\partial a \partial \delta} = t^2 \frac{\alpha}{(a+\beta)^2} > 0 \quad \text{and} \quad \frac{\partial^3 C_H}{\partial a^2 \partial \delta} = - \frac{2t^2 \alpha}{(a+\beta)^3} < 0$$

Income also increases for workers who are at an interior solution, with the income gain rising with ability, at a constant rate. From (9):

$$\frac{\partial C_L}{\partial \delta} = \frac{(a+\beta)}{4\alpha} > 0 \quad \text{and} \quad \frac{\partial^2 C_L}{\partial a \partial \delta} = \frac{1}{4\alpha} > 0$$

Thus a major difference between a skill-neutral innovation and a rise in t is that the former increases the income of all workers, rather than just the income of those who were initially technology-rationed.

A second result is that inequality does not rise with δ . The intuition is that an increase in δ increases the income of all workers proportionately. That is, $\partial Z_i^k / \partial \delta = 0$, because δ enters the production function multiplicatively. Thus inequality, whether measured by income shares of each ability group or ratios of income between any two groups, stays constant.

Application of this fact to the two-sector model yields the following:

Proposition 7 *In the two-sector model, an increase in δ leads to a decline in the price \tilde{P}_1 but not any sectoral shifts. Furthermore, the nominal incomes earned by all classes of workers are independent of δ . Thus the Lorenz curve is unshifted, and the degree of inequality does not change.*

One way of understanding why skill-neutral inventions do not cause sectoral shifts is to recall the reason why sectoral shifts do occur with changes in t . Inequality of earnings rises within the technologically dynamic sector, forcing the least able types of workers in that sector to opt for employment in sector 2. But in the present case, the output of all workers in sector 1 rises by the same proportion. Hence the price decline needed to maintain constant the ratio of the value of outputs in the two sectors also holds constant the ratio of incomes in the two sectors for all workers.

Finally, a welfare result:

Proposition 8 A skill-neutral invention ($\partial\delta > 0$) leads to a Pareto improvement in both the one- and two-sector contexts.

To summarize, the relative price of good 1 falls after a rise in δ , as was the case with a rise in t , to prevent excess supply of good 1. But there are five major differences between the two types of invention. First, only an invention which raises t increases average training times. Second, a rise in δ increases the output of all workers in sector 1, not just the output of technology-rationed workers, as is the case for a rise in t . Third, in either the one-sector or the two-sector model, the degree of inequality is invariant to a rise in δ , but increases after a rise in t . Fourth, only an invention which raises t induces sectoral shifts of labour. Finally, only a skill-neutral invention unequivocally leads to a Pareto improvement.

C Skill-Extensive Invention: Changes in α or β

An exogenous rise in β or fall in α reduces the time required for a worker to train to use a given technology i . In a sense, then, an innovation

which improved the "training technology" in this way would be expected to have the opposite effect to a rise in t , since it reduces the advantage of more able workers over less able workers.

Consider the one-sector model. For workers initially at an interior solution, (8) and (9) show that their choice of technology, and their income, rises following either type of skill-extensive invention:

$$(24a) \quad \frac{\partial i^*}{\partial(-\alpha)} = \frac{(a+\beta)}{2\alpha^2} > 0$$

$$(25a) \quad \frac{\partial C_L}{\partial(-\alpha)} = \frac{\delta(a+\beta)}{4\alpha^2} > 0$$

and

$$(24b) \quad \frac{\partial i^*}{\partial\beta} = \frac{1}{2\alpha} > 0$$

$$(25b) \quad \frac{\partial C_L}{\partial\beta} = \frac{\delta}{4\alpha} > 0$$

Equations (24a) and (24b) imply that the cross-over ability \tilde{a} at which workers become technology-rationed falls. From (12):

$$(26a) \quad \frac{\partial \tilde{a}}{\partial(-\alpha)} = -2t < 0$$

$$(26b) \quad \frac{\partial \tilde{a}}{\partial\beta} = -1 < 0$$

Among technology-rationed workers, income also rises. From (13):

$$(27a) \quad \frac{\partial C_H}{\partial(-\alpha)} = \frac{\delta t^2}{a+\beta} > 0$$

$$(27b) \quad \frac{\partial C_H}{\partial\beta} = \frac{\alpha\delta t^2}{(a+\beta)^2} > 0$$

Either type of skill-extensive innovation reduces the gap between the training times which workers of different abilities would require to master a given technology i . Thus inequality in earnings should fall after an innovation in β or $(-\alpha)$.

Result 3 After a rise in β or a fall in α , in general, $\frac{\partial Z_1^k}{\partial \eta} < 0$, where $\eta = \beta$

or $(-\alpha)$ and $k > l$. The one exception is when workers of both ability k and l are initially at an interior solution, in which case there is no change in the output ratio Z_1^k after a fall in α , since the income of the two types of workers rises proportionately.

What are the implications for the two-sector model? Again, since innovation increases productivity in sector 1, \tilde{P}_1 falls so as to maintain clearance of the goods markets. But Result 3 shows that inequality now decreases in sector 1. Therefore, sectoral shifts will occur, but from sector 2 and into sector 1. It can be shown that it is the least able workers in sector 1 whose nominal income rises after an innovation of the type $\partial\beta > 0$ or $\partial\alpha < 0$, drawing into the sector types of workers who in previous generations preferred sector 2. On the other hand, the nominal income of the most able types of workers in sector 1 falls after the innovation, because the increase in output for these workers falls short of the price decline. More formally:

Proposition 9 An innovation which increases β or decreases α causes \tilde{P}_1 to fall, and j to fall. The latter implies that some proportion of the work force shifts from the technologically stagnant sector and into the sector undergoing the innovation, sector 1.

Two other results are:

Proposition 10 After an innovation which increases β or decreases α , inequality in general declines within sector 1, in terms of ratios of income of any two types of workers. The one exception is that

$\partial Z_i^k / \partial(-\alpha) = 0$ if workers of both ability levels k and ℓ are at an interior solution. The average value of output per capita in sector 1 approaches that in sector 2. Furthermore, the Lorenz curve shifts up.

Proposition 11 A skill-extensive invention $\partial\eta$, $\eta = \beta$ or $(-\alpha)$, leads to a Pareto improvement in the two-sector model iff for all types of workers in sector 1:

$$\frac{\frac{\partial F(a)}{\partial \eta}}{F(a)} \geq \frac{-\rho_2}{\rho_1 + \rho_2} \left[\frac{\frac{\partial \tilde{P}_1}{\partial \eta}}{\tilde{P}_1} \right] \equiv \frac{\rho_2}{\rho_1 + \rho_2} \left[\frac{\frac{\partial F(j)}{\partial \eta} + \frac{\partial F(j)}{\partial j} \frac{\partial j}{\partial \eta}}{F(j)} \right]$$

It will be the more able types of workers in sector 1, if any, for whom utility falls.

Proposition 11 is highly analogous to Proposition 6. Neither a skill-intensive nor a skill-extensive invention necessarily leads to a Pareto improvement, because the proportional gain in output in sector 1 varies with ability. The only major difference between the two propositions is that whereas it is the least able types of workers in sector 1 whose utility is likely to decline after a skill-intensive invention, it is the more able workers who are likely to suffer this fate after a skill-extensive invention. Skill-neutral inventions are alone in unequivocally increasing the utility of all workers, because they benefit all workers equally in proportional terms.

D Policy Implications

The above shows that all three types of invention (skill-intensive, -neutral and -extensive) lead to increases in output per capita. However, skill-intensive inventions are alone in increasing the degree of inequality.

Skill-neutral inventions benefit all workers equally; skill-extensive inventions reduce the degree of inequality.

An immediate implication is that if a government wishes to subsidize inventive activity, there is a case for it to encourage skill-neutral or, even better, skill-extensive inventions. The latter type of invention leads to improvements in equity as well as efficiency. Examples of skill-extensive inventions include strengthening formal education, and improving teaching aids for on-the-job training. Both of these policies would induce given types of workers to choose more productive technologies (higher values of i^*).¹⁵

VI CONCLUDING REMARKS

The main contribution of this paper is to initiate a synthesis of the literatures on human capital and technological change. The model assumes that technologies differ not only in productivities but also in the time workers require to train to use them. Workers too are heterogeneous, in that there is a non-degenerate distribution of abilities.

Three different types of invention are considered. The first reflects the stylized fact that recent inventions appear to have increased training requirements. The second type of invention increases productivity without changing human capital needs, and the third reduces training needs.

¹⁵ This argument applies only if there is a case for government intervention in the direction of R and D in the first place. Two justifications come to mind. There is now a large literature arguing that because of spillovers, government should subsidize R and D. (See for instance Spence (1984).) A second argument that could be invoked for government subsidization of inventive activity is that some of its products are public goods. In particular, the two examples of skill-extensive inventions listed immediately above, both related to the educational infrastructure, could be considered public goods.

In the two-sector model, all three types of invention induce a fall in the price of the good produced in the technologically dynamic sector. This is a standard result of general equilibrium models which assume different rates of productivity growth between sectors.

When we also drop the assumptions that inventions don't alter human capital needs, and that workers are identical, we find that innovation can change the degree of inequality. These changes in inequality and goods prices in turn induce sectoral shifts of labour. These two results are novel aspects of the analysis.

The direction of the sectoral shifts and the changes in inequality depend on the nature of the invention. Skill-neutral inventions cause no change in inequality or the sectoral allocation of labour. Skill-intensive inventions increase the inequality of earnings, and induce shifts of labour out of the technologically dynamic sector. In contrast, skill-extensive inventions have the opposite effect: they reduce inequality in earnings, and cause shifts of labour toward the innovating sector.

Given the stylized facts outlined in the introduction, recent inventions related to micro-electronics appear to have most closely resembled the type of invention which increases training requirements. The model's predictions about the effects of such inventions conform to the stylized facts mentioned in Section II. First, income inequality in the U.S. and Canada have risen slightly in recent years. Second, the share of employment accounted for by the sector exhibiting the slowest rate of productivity growth, the service sector, has grown, as predicted by the model. Of course, we must recognize that other factors will have also contributed to these outcomes.

APPENDIX

Proof of Result 1

The values of t used to define the three cases were obtained from (12). This equation substituted into the expression for net productivity,

$$F(a) = \delta i^*(1 - \alpha i^*/(a+\beta)),$$

leads directly to the results stated in the proposition.

• If workers of both abilities, k and ℓ , are technology-rationed, i.e. $t < (\ell+\beta)/2\alpha$, then

$$\partial Z_i^k / \partial t = \left[F(\ell) \partial F(k) / \partial t - \partial F(\ell) / \partial t F(k) \right] / F(\ell)^2$$

Substituting the production of a type-a worker who is technology-rationed, which is

$$\delta t(1 - \alpha t/(a+\beta)),$$

into the above expression, along with its rate of change, leads to (19). The sign of this equation depends on the following facts: $k > \ell$ by definition, and $\ell \geq 2t\alpha - \beta$ from (12).

• If k is technology rationed but ℓ has reached an interior solution, i.e. $(\ell+\beta)/2\alpha \leq t < (k+\beta)/2\alpha$, then from (9) and (13),

$$F(\ell) = \delta(\ell+\beta)/4\alpha, \text{ and}$$

$$F(k) = \delta t(1 - \alpha t/(k+\beta)).$$

Substituting these and the first derivative of the latter with respect to the technology parameter t gives:

$$\partial Z_i^k / \partial t = \frac{\partial F(k) / \partial t}{F(\ell)} = \frac{\delta(k+\beta-2\alpha t)}{F(\ell)(k+\beta)} = \frac{4\alpha(k+\beta-2\alpha t)}{(\ell+\beta)(k+\beta)} > 0$$

• If both types of workers are at an interior solution, ($t \geq (k+\beta)/2\alpha$), then $\partial Z_i^k / \partial t = 0$ since $Z_i^k = (k+\beta)/(\ell+\beta)$ from (9). ■

Proof of Proposition 2

Using the expression for \tilde{P}_1 from (16),

$$\partial \tilde{P}_1 / \partial t = - \frac{G}{F_j^2} \left(\frac{\partial F(j)}{\partial t} + \frac{\partial F(j)}{\partial j} \frac{\partial j}{\partial t} \right) < 0$$

(+/0) (+) (+)

The sign of the first term in brackets follows from the discussion in Section III. The fact that the second term is positive derives from the proof in Section III that $\partial F(a)/\partial a > 0$ regardless of whether the individual is technology-rationed or at an interior solution. The sign of the final term is

based on Proposition 1. ■

Proof of Result 2

From the Cobb-Douglas consumption functions, where W denotes nominal income, the ratio of the demand for good 1 to good 2 by a worker of ability a will be:

$$\frac{C^1}{C^2} \Big|_a = \frac{\frac{\rho_1 W}{(\rho_1 + \rho_2) \tilde{P}_1}}{\frac{\rho_2 W}{(\rho_1 + \rho_2)}} = \frac{\rho_1}{\rho_2} \frac{1}{\tilde{P}_1} \Leftrightarrow \frac{\tilde{P}_1 C^1}{C^2} \Big|_a = \frac{\rho_1}{\rho_2} \quad \forall a.$$

Since the ratio of expenditures on the two goods is equal for all workers, by integrating over the entire population we find that the ratio of expenditures on the two goods for the economy as a whole will also equal this ratio:

$$(23) \quad \frac{\tilde{P}_1 Y_1}{Y_2} = \frac{\rho_1}{\rho_2}$$

Proof of Proposition 3

Note that since $Y_2 = GM(j)$,

$$\partial Y_2 / \partial t = Gm(j) \partial j / \partial t > 0.$$

So from (23), $\partial(\tilde{P}_1 Y_1) / \partial t > 0$ too.

Now, the value of output per capita in sector 1 is $\frac{\tilde{P}_1 Y_1}{1-M(j)}$, so that its

first derivative with respect to t is:

$$\begin{aligned} & \frac{\partial(\tilde{P}_1 Y_1) / \partial t}{1-M(j)} - \frac{\tilde{P}_1 Y_1}{(1-M(j))^2} (-m(j)) \frac{\partial j}{\partial t} \\ &= \frac{\partial(\tilde{P}_1 Y_1) / \partial t}{1-M(j)} + \frac{\tilde{P}_1 Y_1}{(1-M(j))^2} m(j) \frac{\partial j}{\partial t} > 0 \end{aligned}$$

where the sign of the very last term follows from Proposition 1. This proves that the value of output per capita in sector 1 rises with a technological advance. It also rises relative to the average value of output per capita in sector 2, since the latter is constant at G . ■

Proof of Proposition 4

The sign of $\frac{\partial(\tilde{P}_1 F(a))}{\partial t}$ will be the same as the sign of

$$\frac{\partial \ln(\tilde{P}_1 F(a))}{\partial t} = \frac{\frac{\partial \tilde{P}_1}{\partial t}}{\tilde{P}_1} + \frac{\frac{\partial F(a)}{\partial t}}{F(a)}$$

\tilde{P}_1 $F(a)$
 $(-)$ $(+/0)$

where the signs follow from Proposition 2 and equations (9) and (13). Note that the above derivative will be negative for all workers in sector 1 who are at an interior solution, as in this case $\partial F(a)/\partial t = 0$.

Rewriting the above expression by substituting for \tilde{P}_1 from (16), we obtain the result that income of the type of worker with ability a rises or falls as

$$\frac{\frac{\partial F(a)}{\partial t}}{F(a)} \begin{matrix} > \\ < \end{matrix} \frac{1}{F(j)} \left(\frac{\partial F(j)}{\partial t} + \frac{\partial F(j)}{\partial j} \frac{\partial j}{\partial t} \right)$$

Finally, we show that the left-hand side of the inequality is increasing in ability among workers who are technology-rationed. Substituting for the specific values of $F(a)$ and its derivative with respect to t from (13),

$$\begin{aligned} \frac{\partial}{\partial a} \left(\frac{\frac{\partial F(a)}{\partial t}}{F(a)} \right) &= \frac{\partial}{\partial a} \left(\frac{a + \beta - 2\alpha t}{t(a + \beta - \alpha t)} \right) \\ &= \frac{\alpha}{(a + \beta - \alpha t)^2} > 0 \end{aligned}$$

which guarantees that only the most able types of workers in sector 1 will experience income rises after an invention. ■

Proof of Proposition 5

A sufficient but not necessary condition for the Lorenz curve after the innovation to lie everywhere below the Lorenz curve before the innovation ∂t is that all ability groups whose income share rises after the invention be clustered together at the top of the income distribution. This condition ensures that the two Lorenz curves will never cross. More formally, we want to show that if

$$\frac{\partial(\text{income share})}{\partial t} > 0 \text{ for workers of ability } \ell, \text{ then}$$

$$\frac{\partial(\text{income share})}{\partial t} \geq 0 \text{ for workers of ability } k > \ell. \text{ We also require}$$

that the income share of the least able group strictly falls.

We will prove the above sufficient condition by showing that all of the

less able groups experience a decline in their share of national income, and that among the remaining, more able, groups (more precisely, those in sector 1 which are technology-rationed both before and after the innovation), the change in the share of national income with a rise in t is increasing in ability, i.e.

$$\frac{\partial}{\partial a} \frac{\partial(\text{Share of national income})}{\partial t} > 0$$

This ensures that if the income share of group i rises, so must that of all more able groups.

Note that from (23) we can write national income in three different ways:

$$\tilde{P}_1 Y_1 + Y_2 = \tilde{P}_1 Y_1 \left(\frac{\rho_1 + \rho_2}{\rho_1} \right) = Y_2 \left(\frac{\rho_1 + \rho_2}{\rho_2} \right)$$

The final form of the equation shows most clearly that nominal national income rises with a change in t , since $\partial Y_2 / \partial t = G_m(j) \partial j / \partial t > 0$ based on Proposition 1. Consequently, the income share of all types of workers in sector 2, must fall: their income is constant at G , while national income rises. The same is true for the type of worker who before the invention was indifferent between the two sectors, since that type of worker would choose to work in sector 2 for the same income as before the invention.

The second form of the national income equation listed above can be used to assess changes in the income share of workers in sector 1 who remain in that sector after the invention. On a per capita basis, the type of worker of ability a has the following share of national income:

$$\frac{\tilde{P}_1 F(a)}{\tilde{P}_1 Y_1 \left(\frac{\rho_1 + \rho_2}{\rho_1} \right)} = \frac{\rho_1}{\rho_1 + \rho_2} \frac{F(a)}{Y_1}$$

Hence,

$$\begin{aligned} & \frac{\partial (\text{share of national income, worker of ability } a)}{\partial t} \\ &= \frac{\rho_1}{\rho_1 + \rho_2} \left[\frac{\begin{matrix} (-) & (+/0) \\ -\frac{\partial Y_1 F(a)}{\partial t} & + Y_1 \frac{\partial F(a)}{\partial t} \end{matrix}}{Y_1^2} \right] \end{aligned}$$

The second term is zero for workers who are not technology-rationed. We have thus proved that the income shares of all workers in sector 2, and workers

who are at an interior solution in sector 1, decline after a rise in t . The final step of the proof is to show that for technology-rationed workers in sector 1, the change in income share per capita is rising with ability, which would ensure that all types of workers whose income share rose after an invention are clustered together at the top of the ability distribution.

Differentiating the above expression with respect to ability level a :

$$\frac{\partial^2}{\partial a \partial t} \frac{F(a)}{Y_1 \left(\frac{\rho_1 + \rho_2}{\rho_1} \right)} = \frac{\rho_1}{(\rho_1 + \rho_2) Y_1} \left(-\frac{\partial F(a)}{\partial a} \frac{\partial Y_1}{\partial t} \frac{1}{Y_1} + \frac{\partial^2 F(a)}{\partial a \partial t} \right) \quad (*)$$

Substituting for the two terms involving $F(a)$ from (13) and simplifying,

$$\frac{\partial^2}{\partial a \partial t} \frac{F(a)}{Y_1 \left(\frac{\rho_1 + \rho_2}{\rho_1} \right)} = \frac{\rho_1}{(\rho_1 + \rho_2) Y_1} \frac{\delta t \alpha}{(a + \beta)^2} \left(-\frac{\partial Y_1}{\partial t} \frac{t}{Y_1} + 2 \right) > 0$$

The sign of this inequality derives from the fact the elasticity of Y_1 with respect to t is less than one. The proof is as follows. The form of Y_1 depends on whether there are any workers in sector one who are at an interior solution with respect to technology. Denote the ability level at which workers become technology-rationed as \tilde{a} .

Case 1: Not All Workers in Sector 1 Are Technology-Rationed: $\tilde{a} > j$

In this case:

$$Y_1 = \int_j^{\tilde{a}} \frac{\delta(a + \beta)m(a)da}{4\alpha} + \int_{\tilde{a}}^{\bar{a}} \frac{\delta t(a + \beta - \alpha t)m(a)da}{a + \beta}$$

In the derivation of $\partial Y_1 / \partial t$ below, the two terms in $\partial \tilde{a} / \partial t$ cancel because, as was shown in Section III, production is continuous in ability at the ability level \tilde{a} at which workers become technology-rationed.

$$\frac{\partial Y_1}{\partial t} = \frac{-\delta(j + \beta)m(j)}{4\alpha} \frac{\partial j}{\partial t} + \int_{\tilde{a}}^{\bar{a}} \frac{\delta(a + \beta - 2\alpha t)m(a)da}{a + \beta}$$

(+)

$$\equiv \lambda + \int_{\tilde{a}}^{\bar{a}} \frac{\delta(a + \beta - 2\alpha t)m(a)da}{a + \beta} \text{ where } \lambda \equiv \frac{-\delta(j + \beta)m(j)}{4\alpha} \frac{\partial j}{\partial t} < 0$$

(+)

Thus the elasticity of Y_1 with respect to t is:

$$\frac{\partial Y_1}{\partial t} \frac{t}{Y_1} = \frac{\lambda t}{Y_1} + \frac{\int_{\tilde{a}}^{\bar{a}} \frac{\delta t(a+\beta-2\alpha t)m(a)da}{a+\beta}}{\int_j^{\tilde{a}} \frac{\delta(a+\beta)m(a)da}{4\alpha} + \int_{\tilde{a}}^{\bar{a}} \frac{\delta t(a+\beta-\alpha t)m(a)da}{a+\beta}}$$

This is less than one by inspection, completing the proof for case 1.

Case 2: All Workers in Sector 1 are Technology-Rationed: $j \geq \tilde{a}$

Now,

$$Y_1 = \int_j^{\bar{a}} \frac{\delta t(a+\beta-\alpha t)m(a)da}{a+\beta}$$

So,

$$\frac{\partial Y_1}{\partial t} = \frac{-\delta t(j+\beta-\alpha t)m(j)\frac{\partial j}{\partial t}}{j+\beta} + \int_j^{\bar{a}} \frac{\delta(a+\beta-2\alpha t)m(a)da}{a+\beta}$$

(+)

$$\equiv \pi + \int_j^{\bar{a}} \frac{\delta(a+\beta-2\alpha t)m(a)da}{a+\beta} \text{ where } \pi \equiv \frac{-\delta t(j+\beta-\alpha t)m(j)\frac{\partial j}{\partial t}}{j+\beta} < 0$$

(+)

Proceeding exactly as above for case 1, we can prove that the elasticity of Y_1 with respect to t is less than one, completing the proof. ■

Proof of Proposition 6

If we define W to be nominal income, then by substituting the Cobb-Douglas consumption functions into the utility function (14), we can write the log of a worker's utility as

$$\ln(U) = C + (\rho_1 + \rho_2)\ln(W) - \rho_1 \ln(\tilde{P}_1)$$

where C is a constant. We will use this below.

Consider first workers in sector 2. From the above equation, a

sufficient condition for a type of worker to become (non-strictly) better off is that $\partial W/\partial t \geq 0$ and $\partial \tilde{P}_1/\partial t \leq 0$. The nominal income of these workers is constant at G. Furthermore, $\partial \tilde{P}_1/\partial t < 0$, so that workers in sector 2 become strictly better off.

Now consider a worker of ability a in sector 1. Substituting for \tilde{P}_1 from (16), and also substituting $W = \tilde{P}_1 F(a)$,

$$\begin{aligned} \frac{\partial \ln(U)}{\partial t} &= \rho_2 \frac{\frac{\partial \tilde{P}_1}{\partial t}}{\tilde{P}_1} + (\rho_1 + \rho_2) \frac{\frac{\partial F(a)}{\partial t}}{F(a)} \\ &\quad (-) \quad \quad \quad (+/0) \\ &= -\rho_2 \frac{\frac{\partial F(j)}{\partial t}}{F(j)} + \frac{\partial F(j)}{\partial j} \frac{\partial j}{\partial t} + (\rho_1 + \rho_2) \frac{\frac{\partial F(a)}{\partial t}}{F(a)} \\ &\quad (-) \quad \quad (+) \quad \quad \quad (+/0) \end{aligned}$$

Since the second term will be zero for types of workers in sector 2 who are not technology-rationed, the utility of such workers must fall due to the price effect. For types of workers who are technology-rationed, their utility rises if the above expression is positive, which leads directly to the inequality stated in the proposition. ■

Proof of Proposition 7

Applying the implicit function theorem to (18'), the left-hand side of which we will call Ψ , we find that $\partial \Psi/\partial \delta = 0$ because Z_j^k is not a function of δ . Thus, using (22), $\partial j/\partial \delta = 0$.

\tilde{P}_1 falls with a rise in δ , though. From (16):

$$\frac{\partial \tilde{P}_1}{\partial \delta} = - \frac{G}{F(j)^2} \frac{\partial F(j)}{\partial \delta} < 0$$

(+)

Inequality does not rise within sector 1 because nominal incomes remain constant. Incomes also remain constant at G in sector 2. Thus, the Lorenz curve will not shift. For a worker of ability l , substituting for \tilde{P}_1 from (23):

$$\tilde{P}_1 F(\ell) = \frac{\rho_1}{\rho_2} \frac{GM(j)}{\bar{a} \int_j Z_{jm}^k(k) dk}$$

As argued in the text, Z_{jm}^k is independent of δ , as are all the other elements of the equation. Thus nominal incomes are constant. ■

Proof of Proposition 8

In the one-sector context, output of all workers increases, so utility must increase for all workers as well. In the two-sector context, utility is an increasing function of nominal income, which from Proposition 7 is constant for all workers, and a declining function of \tilde{P}_1 , which strictly declines. Therefore all workers are strictly better off. ■

Proof of Result 3

Case 1: Both types of workers are at an interior solution

Substitution of (9) into Z_{jm}^k and differentiation gives:

$$\frac{\partial Z_{jm}^k}{\partial(-\alpha)} = 0 \quad \frac{\partial Z_{jm}^k}{\partial\beta} = \frac{\ell-k}{(\ell+\beta)^2} < 0 \text{ since } k > \ell$$

Case 2: Only the worker of ability ℓ is at an interior solution

Substituting (9) and (13) into Z_{jm}^k and differentiating gives:

$$\frac{\partial Z_{jm}^k}{\partial(-\alpha)} = -\frac{4t(k+\beta-2\alpha t)}{(\ell+\beta)(k+\beta)} < 0 \text{ since } k \text{ is technology-rationed.}$$

(See (12).)

$$\frac{\partial Z_{jm}^k}{\partial\beta} = 4\alpha t \frac{(\ell+\beta)(k+\beta) - (k+\beta-\alpha t)(k+\ell+2\beta)}{(\ell+\beta)^2(k+\beta)^2} < 0$$

The proof that the sign of this expression is negative is as follows. Upon simplification, the equation can be shown to imply that:

$$\frac{\partial Z_{jm}^k}{\partial\beta} \gtrless 0 \text{ as } \alpha t(k+\ell+2\beta) - (k+\beta)^2 \gtrless 0$$

Since k is technology rationed, (12) implies that we can write:

$$k+\beta = 2t\alpha(1+\varepsilon) \text{ where } \varepsilon \text{ is some unknown strictly positive constant.}$$

Substituting for $k+\beta$ in the latter inequality, we obtain:

$$\begin{aligned} \alpha t(k+\ell+2\beta) - (k+\beta)2t\alpha(1+\varepsilon) &\gtrless 0 \Rightarrow \\ (\ell+\beta) - (k+\beta)(1+2\varepsilon) &\gtrless 0 \Rightarrow \end{aligned}$$

But the L.H.S. < 0 by inspection, since $k > l$ and $\epsilon > 0$. Therefore $\frac{\partial Z_l^k}{\partial \beta} < 0$.

Case 3: Both k and l are technology-rationed

In this case, (12) implies that $l > 2t\alpha - \beta$, $k > 2t\alpha - \beta$.

$$\frac{\partial Z_l^k}{\partial(-\alpha)} = \frac{l+\beta}{k+\beta} \frac{(l-k)t}{(l+\beta-\alpha t)^2} < 0 \text{ as } l < k$$

$$\frac{\partial Z_l^k}{\partial \beta} = \frac{[l+k+2\beta-\alpha t]}{(k+\beta)^2(l+\beta-\alpha t)^2} \alpha t(l-k) < 0 \quad \blacksquare$$

Proof of Proposition 9

To prove that j falls with either type of innovation, calculate $\partial \Psi / \partial \beta$ (and $\partial \Psi / \partial(-\alpha)$) from (18'), where Ψ again denotes the left-hand side of that equation:

$$\frac{\partial \Psi}{\partial \beta} = \rho_2 G \int_j^{\bar{a}} \frac{\partial Z_j^k}{\partial \beta} m(k) dk < 0$$

where the sign is based on Result 3. Similarly, from Result 3, as long as at least one person is technology-rationed, then:

$$\frac{\partial \Psi}{\partial(-\alpha)} = \rho_2 G \int_j^{\bar{a}} \frac{\partial Z_j^k}{\partial(-\alpha)} m(k) dk < 0$$

Applying the implicit function theorem to these results and (22), we obtain:

$$\partial j / \partial \beta < 0, \quad \partial j / \partial(-\alpha) < 0.$$

To prove that \tilde{P}_1 falls, examine (23). From this, we find that, for $\eta = \beta$ or $\eta = (-\alpha)$,

$$\frac{\partial \tilde{P}_1}{\partial \eta} = \frac{\rho_1}{\rho_2} \left[\underbrace{\frac{\partial Y_2}{\partial \eta}}_{(-)} \frac{1}{Y_1} - \frac{Y_2}{Y_1^2} \underbrace{\frac{\partial Y_1}{\partial \eta}}_{(+)} \right] < 0$$

where the first term is negative because the number of workers in sector 2 falls while their production per capita remains constant at G , and $\partial Y_1 / \partial \eta$ is positive because $\partial j / \partial t < 0$ and because productivity of all workers in sector 1 rises. \blacksquare

Proof of Proposition 10

Inequality declines within sector 1, with the one exception noted, as indicated by Result 3. Inequality between sectors, as measured by the ratio of lifetime income per capita in the two sectors, also declines. The proof is as follows. Y_2 falls after a rise in η ($\eta = \beta$ or $(-\alpha)$) because the mass of workers in sector 2 falls, and the productivity of the remaining workers is constant at G . The average value of output from sector 1 falls, since from (23) a fall in Y_2 implies a fall in $\tilde{P}_1 Y_1$ as well. Since at the same time the number of workers in sector 1 is rising, the value of output per capita in sector 1 is falling toward G , the constant value of output per capita in sector 2. Thus inequality is declining both within sector 1 and between sectors 1 and 2.

A sufficient but not necessary condition for the Lorenz curve after the innovation to lie everywhere on or above the Lorenz curve before the innovation $\partial\eta$, ($\eta = \beta$ or $-\alpha$), is that all ability groups whose income share rises after the invention be clustered together at the bottom of the income distribution.

We will prove the above sufficient condition by showing that all of the less able groups experience a rise in their share of national income, and that among the remaining more able groups, (more precise definitions of which will be given later), the change in the share of national income with a rise in η is falling in ability, i.e.

$$\frac{\partial}{\partial a} \frac{\partial(\text{Share of national income})}{\partial \eta} < 0$$

This ensures that if the income share of a type of worker of ability i rises, so must that of all less able types of workers. The left-hand side of the above equation can be written as follows, after substituting the second expression for national income given in the proof of Proposition 5:

$$(\chi) \quad \frac{\partial}{\partial a} \frac{\partial(\text{Share of national income})}{\partial \eta} = \frac{\rho_1}{\rho_1 + \rho_2} \frac{1}{Y_1^2} \left[Y_1 \frac{\partial^2 F(a)}{\partial a \partial \eta} - \frac{\partial F(a)}{\partial a} \frac{\partial Y_1}{\partial \eta} \right]$$

$\begin{matrix} (+) & (+) \end{matrix}$

First, note that nominal GNP is falling: as shown in the proof of Proposition 5, nominal GNP can be written as a positive function of one variable: Y_2 . Since workers shift out of sector 2 after an innovation $\partial\eta$, that is, $\partial j / \partial \eta < 0$, and since output per capita is constant at G in that sector, $\partial Y_2 / \partial \eta < 0$ and hence nominal GNP falls also. Thus the share of

nominal GNP per capita for types of workers who choose sector 2 before and after the innovation must rise, as their output is constant.

The next steps in the proofs for $\eta = \beta$ and for $\eta = (-\alpha)$ proceed along separate lines.

The Case $\partial\eta \equiv \partial\beta$

A sufficient condition for

$$\frac{\partial}{\partial a} \frac{\partial(\text{Share of national income})}{\partial \eta} < 0$$

is simply that $\frac{\partial^2 F(a)}{\partial a \partial \beta}$ be non-positive, as can be seen from (χ) above. For the case in which the worker of ability a is at an interior solution, from (9),

$$\frac{\partial^2 F(a)}{\partial a \partial \beta} = 0.$$

For the case in which the worker of ability a is technology-rationed, from (13):

$$\frac{\partial^2 F(a)}{\partial a \partial \beta} = \frac{-2\delta t^2 \alpha}{(a+\beta)^3} < 0$$

This proves that the income shares of only the most able types of workers will fall after the innovation in β .

The Case $\partial\eta = \partial(-\alpha)$

First we show that the per capita income share of all types of workers in sector 1 who are at an interior solution rises after a fall in α . Since nominal national income drops after a fall in α , we need only show that the change in the nominal income of all such types of workers is non-negative.

Taking the natural log of income of such a worker of ability a , we see that the worker's nominal income rises or falls as:

$$\frac{\frac{\partial \tilde{P}_1}{\partial(-\alpha)}}{\tilde{P}_1} + \frac{\frac{\partial \delta(a+\beta)}{4\alpha}}{\frac{\partial(-\alpha)}{\delta(a+\beta)}} > 0$$

Substituting $\tilde{P}_1 = G/F(j)$ and then substituting for $F(j)$ from (9), the left-hand side equals:

$$\frac{-1}{F(j)} \left[\frac{\partial F(j)}{\partial(-\alpha)} + \frac{\partial F(j)}{\partial j} \frac{\partial j}{\partial(-\alpha)} \right] + \frac{1}{\alpha}$$

$$= \left\{ \frac{-1}{\alpha} - \frac{\partial j}{\partial(-\alpha)} \frac{1}{j+\beta} \right\} + \frac{1}{\alpha}$$

(-)

> 0 by inspection.

This proves that the income share of all groups up to ability \tilde{a} rises after the innovation in α . Hence we need only show that equation (χ) is negative for types of workers in sector 1 who are technology-rationed. From (13),

$$\frac{\partial^2 F(a)}{\partial a \partial(-\alpha)} = - \frac{\delta t^2 \alpha}{(a+\beta)^2} < 0$$

This is sufficient to show that (χ) is negative. Thus the income shares of only the most able types of workers decline after a fall in α . ■

Proof of Proposition 11

The utility of workers in sector 2 must rise, since \tilde{P}_1 falls, while the nominal income of these workers remains constant at G.

Now consider workers who are in sector 1. Using the expression for the log of utility given in the proof of Proposition 6, and substituting $W = \tilde{P}_1 F(a)$,

$$\frac{\partial \ln(U)}{\partial \eta} = \underbrace{\rho_2 \frac{\frac{\partial \tilde{P}_1}{\partial \eta}}{\tilde{P}_1}}_{(-)} + \underbrace{(\rho_1 + \rho_2) \frac{\frac{\partial F(a)}{\partial \eta}}{F(a)}}_{(+)}$$

where $\eta = (-\alpha)$ or β .

Substituting for \tilde{P}_1 from (16), the worker's utility will (rise/fall) as

$$(Θ) \quad \frac{\frac{\partial F(a)}{\partial \eta}}{F(a)} > \frac{\rho_2}{(\rho_1 + \rho_2)} \frac{\frac{\partial F(j)}{\partial \eta} + \frac{\partial F(j)}{\partial j} \frac{\partial j}{\partial \eta}}{F(j)}$$

We now show that the left-hand side of the above equation, which we denote below by LHS, is decreasing in a , so that if any types of workers become worse off, it is the most able types. We consider the cases $\eta = \beta$ and $(-\alpha)$ separately.

The Case $\eta = (-\alpha)$

i) Workers of ability a are at an interior solution

This case is unique in that it is the only case in which the LHS of

equation (Θ) is independent of a. In other cases, the LHS is strictly decreasing in a. From (9) and (25a),

$$\text{LHS} = 1/\alpha \text{ so } \partial \text{LHS} / \partial a = 0.$$

However, it can be shown that utility of these types of workers increases unequivocally, so that there remains the possibility of a Pareto improvement. If workers of ability a are at an interior solution, so are workers of ability j. Substituting from (9) and (25a) into the RHS of (Θ) and simplifying, that inequality becomes: workers are (better/worse) off as:

$$\begin{array}{ccc} \frac{p_1}{\alpha} & > & \frac{p_2}{j+\beta} \partial j / \partial (-\alpha) \\ (+) & & (-) \end{array}$$

Thus all such types of workers are better off after the invention. (It can be shown that this is the only case about which the change in utility is unambiguous.)

ii) *Workers of ability a are technology-rationed*

For workers who are technology-rationed, from (13) and (27a),

$$\text{LHS} = \frac{t}{a + \beta - \alpha t} \text{ so } \frac{\partial \text{LHS}}{\partial a} = \frac{-t}{(a + \beta - \alpha t)^2} < 0$$

The Case $\eta = \beta$

i) *Workers of ability a are at an interior solution*

For workers who are at an interior solution, from (9) and (25b),

$$\text{LHS} = \frac{1}{a+\beta} \text{ so } \frac{\partial \text{LHS}}{\partial a} = \frac{-1}{(a+\beta)^2} < 0$$

ii) *Workers of ability a are technology-rationed*

For workers who are technology-rationed, from (13) and (27b),

$$\text{LHS} = \frac{\alpha t}{(a+\beta)(a+\beta-\alpha t)} \text{ so } \frac{\partial \text{LHS}}{\partial a} = \frac{-\alpha t(2a + 2\beta - \alpha t)}{(a+\beta)^2(a+\beta-\alpha t)^2} < 0,$$

as $a + \beta > 2\alpha$ given that the worker is technology-rationed. (See (12).) ■

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