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Measuring Resource Scarcity in Non-renewable Resources with Inequality Constrained Estimation

by

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Abstract

In this paper annual Canadian exploration data are used to estimate a multiple-output translog exploration cost function. A new definition of depletion is introduced and its estimated coefficient is found to be statistically significant. Another novel feature is the application of Monte Carlo integration to ensure the estimated cost function satisfies concavity and monotonicity. The fitted cost function parameters are then used to obtain estimates of the marginal costs of exploration for oil and gas. Our estimated marginal exploration costs are smaller than previous studies because we have allowed for technical progress which offsets the depletion effects. These marginal cost estimates are employed, along with previous estimates of exploration rents, to measure resource scarcity. We find some evidence for the increased scarcity of oil and gas in Alberta. For oil there is a 10.0% per year increase in scarcity along the trend line while for natural gas there is a 1.8% per year increase in scarcity along the trend line.

1. Introduction

A fundamental problem in resource economics is to measure resource scarcity. Conceptually the most appropriate measure of resource scarcity is resource rent, where the resource rent is just the difference between the price of a good produced and the unit cost of turning the natural resource into the good.[1] The unit costs include the value of the labour, capital, energy and materials needed to convert the natural resource into a finished product.[2] There are, however, several fundamental problems with using rent as a measure of resource scarcity. These include the fact that resource rents are affected by government tax policies and market imperfections. There is also the problem that resource rents are difficult to measure. This suggests that some suitable alternative method of measuring resource rents is required.

Theoretical results by Brown and Field [1978], Devarajan and Fisher [1982] and Lasserre [1985] suggest that marginal exploration costs may be used to measure resource scarcity. The measurement of resource scarcity is an empirical issue. One aim of this paper is to empirically measure oil and gas scarcity for the province of Alberta.

In this paper we follow Livernois [1988] and model an aggregate multiple-output exploration cost function by the multiple-output translog cost function. This paper differs from the existing literature in three important ways. First, we introduce a new measure of resource stock

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[1]This is discussed in Hartwick and Olewiler [1986].

[2]For non-renewable resources such as oil and gas, rising scarcity is indicated by rising resource rents.

depletion, which we call X, than the one commonly used. In connection with this new depletion variable we also include a time trend which allows for the measurement of technological change in exploration. Third we make use of Monte Carlo integration techniques to directly impose the monotonicity and concavity restrictions on the cost function. We discuss each of these issues in turn.

Most studies choose to measure the effect of depletion by specifying the variable X as a measure of cumulative drilling activity. For instance, X may represent the cumulative meters drilled to date or it may represent the total number of wells drilled to date. The intention is that as the resource is exploited it makes it harder and therefore more costly to discover any further amount. While it may seem reasonable that increases in exploration effort reflect measures of depletion, we would argue this is not necessarily true. Instead, increases in exploration drilling may just reflect bad luck on the part of the firm. By bad luck on the part of the firm we mean that there may be large reserves of the resource available but firms are drilling in the wrong place.[3] Instead, we prefer to measure resource depletion by the cumulative sum of proven reserves. If the rate of accumulation of reserves is slowing then we infer that resource depletion is being manifested.

The most direct way to incorporate technical change into exploration activity is to assume that exploration costs are a function of time, t. In this case, exploration costs can be written as $C = C(w,Q,X,t)$. Here, w is a vector of input prices, Q is a vector of outputs, X is a variable for

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[3] There are other problems with using cumulative drilling as a measure of depletion. When a large new deposit is first discovered, increased drilling activity occurs not because the resource is scarce but because the resource is plentiful.

depletion and t is a time trend. We would expect the marginal effect of technical change to be negative ($\partial C/\partial t < 0$) since the lapse of time reduces marginal exploration costs as new techniques are utilized. Thus, while depletion effects tend to raise exploration costs ($\partial C/\partial X > 0$), technical advance leads to lower exploration costs if $\partial C/\partial t < 0$. The net effect on exploration costs is ambiguous. Consequently, any empirical study which only includes depletion effects (but does not include technical change) will overestimate (underestimate) marginal exploration costs if there has been technical progress (regress).

Our third point of departure concerns a more general issue that arises when estimating any type of demand or cost system. It is often the case that after estimating a system of demand equations the researcher finds that the estimated coefficients violate the requirements for concavity and monotonicity. This means that the parameter estimates and associated elasticities of substitution are not economically meaningful. While traditionally one could reject the concavity conditions or estimate a more general flexible functional form, Chalfant and White [1987] have proposed a Bayesian inference procedure using inequality constrained Monte Carlo integration (Geweke [1986]), to impose curvature restrictions on demand systems. Following Geweke [1986], they use Monte Carlo integration and importance function sampling to obtain an inequality constrained estimate of the parameter vector for a demand system together with a probability that the restrictions hold. In this paper we adopt the Geweke procedure to ensure our estimated exploration cost function satisfies the requirements of monotonicity and concavity and this leads to direct economic interpretation.

This paper is organized as follows. Section 2 discusses the multiple-output translog cost function and presents parameter and

Allen-Uzawa partial elasticities of substitution. In section 3 the estimated cost function parameters are used to determine the marginal exploration cost for oil and gas in Alberta. In section 4 these marginal exploration costs are augmented with independently obtained measures of exploration rents to form a measure of resource scarcity. Section 5 concludes the paper.

2. The Multiple-Output Translog Cost Function

We assume that competitive input markets exist for the exploration industry. We do not, however, make any assumptions about the structure of output markets. Exploration activity is a multiple-output technology that uses three inputs; exploratory drilling, geophysical effort and land acquired for exploratory drilling to produce two outputs; oil discoveries and gas discoveries. We assume the multiple-output technology can be represented by the dual cost function.

(1) $C = C(w_D, w_G, w_L, Q_1, Q_2, X, t)$

where w_i, i = D,G,L are the input prices, (D=price of exploratory drilling, G=price of geophysical effort and L= price of land acquired for exploratory drilling) Q_1 = oil discoveries, Q_2 = gas discoveries and X is a variable that measures the effect of depletion. For (1) to be a proper cost function the following regularity conditions must hold: C(w, Q, X,t) is nondecreasing in w, concave in w, continuous in w and satisfies monotonicity. Here w equals the vector of input prices (w_D, w_G, w_L) and Q equals the vector of outputs, (Q_1, Q_2).

The variable X captures the effect of depletion. We would expect the largest pools are discovered first making it more difficult and therefore more costly to discover subsequent pools. This implies the cost function is increasing in X.

Letting C(w,Q,X,t) denote the total cost of joint exploration for oil and natural gas in Alberta, the three input (price of drilling, w_D, price of geophysical effort, w_G, and price of land acquisition, w_L) multiproduct translog cost function can be written as (2).[4]

$$
\begin{aligned}
(2)\ \ln C(w,Q,X,t) = {} & a_C + a_D \ln w_D + a_G \ln w_G + a_L \ln w_L + a_T t \\
& + 0.5\gamma_{DD}(\ln w_D)^2 + \gamma_{DG}\ln w_D \ln w_G + \gamma_{DL}\ln w_D \ln w_L \\
& + 0.5\gamma_{GG}(\ln w_G)^2 + \gamma_{GL}\ln w_G \ln w_L \\
& + 0.5\gamma_{LL}(\ln w_L)^2 + b_1 \ln Q_1 + b_2 \ln Q_2 \\
& + \beta_X \ln X + \beta_{XD}\ln X \ln w_D + \beta_{XG}\ln X \ln w_G + \beta_{XL}\ln X \ln w_L \\
& + \gamma_{TT} t^2 + \delta_D t \ln w_D + \delta_G t \ln w_G + \delta_L t \ln w_L
\end{aligned}
$$

where symmetry and homotheticity have been imposed.[5] Here we are considering the translog cost function as a second order approximation to a true cost function. Equation (2) also includes a time trend variable, t , to account for technological change. The translog cost function exhibits constant returns to scale if $b_1 + b_2 = 1$ and is linearly homogeneous in the input prices if the following conditions hold.

$$
(3)\quad
\begin{aligned}
a_D + a_G + a_L &= 1 \\
\gamma_{DD} + \gamma_{DG} + \gamma_{DL} &= 0 \\
\gamma_{DG} + \gamma_{GG} + \gamma_{GL} &= 0 \\
\gamma_{LD} + \gamma_{LG} + \gamma_{LL} &= 0 \\
\beta_{XD} + \beta_{XG} + \beta_{XL} &= 0
\end{aligned}
$$

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[4]The multiple-output translog cost function is discussed in some detail in Denny and Pinto [1978].

[5]Ideally one would want to test the symmetry condition. In practice this is rarely done since the estimation period for these cost functions is usually only 30 or 40 observations. Without imposing symmetry, (2) has 24 estimated parameters but only 29 observations. Homotheticity implies no price output interactions (ie. all terms of the form: $\ln w_i \ln Q_j$,= 0, ∀ i,j i= D,G,L and j=1,2).

$$\delta_D + \delta_G + \delta_L = 0$$

The dual cost function in (2) will not exhibit any technical progress or regress if it is independent of time.

$$(3a) \quad \begin{aligned} a_T &= 0 \\ \gamma_{TT} &= 0 \\ \delta_D = \delta_G = \delta_L &= 0 \end{aligned}$$

The share equations can be easily derived from the cost function given in (2). Differentiating (2) with respect to the logarithm of w_D, for example, we have

$$(4) \quad \frac{\partial \ln C}{\partial \ln w_D} = a_D + \gamma_{DD}\ln w_D + \gamma_{DG}\ln w_G + \gamma_{DL}\ln w_L + \beta_{XD}\ln X + \delta_D t$$

Under the assumption of cost minimization and if the cost function is linearly homogeneous in prices then,

$$\frac{\partial \ln C}{\partial \ln w_D} = \frac{w_D}{C}\frac{\partial C}{\partial w_D} = \frac{w_D D}{C} = S_D$$

where S_D is the cost share of drilling. Shephard's Lemma and logarithmic differentiation with respect to each input price yields the following set of cost share equations.

$$(5) \quad \begin{aligned} S_D &= a_D + \gamma_{DD}\ln w_D + \gamma_{DG}\ln w_G + \gamma_{DL}\ln w_L + \beta_{XD}\ln X + \delta_D t \\ S_G &= a_G + \gamma_{GD}\ln w_D + \gamma_{GG}\ln w_G + \gamma_{GL}\ln w_L + \beta_{XG}\ln X + \delta_G t \\ S_L &= a_L + \gamma_{LD}\ln w_D + \gamma_{LG}\ln w_G + \gamma_{LL}\ln w_L + \beta_{XL}\ln X + \delta_L t \end{aligned}$$

where S_G is the share of geophysical effort and S_L is the share of land acquisition. This assumes that in each observation period there has been a full and complete adjustment of the input mix to the factor prices so that the minimum cost level is achieved.

In this study annual observations from 1955 to 1983 on the exploration industry in Alberta are used.[6] Exploratory drilling is measured by the total number of meters drilled annually while geophysical effort is measured by the total number of annual crew months. Land is total area of land acquired under lease and license annually for exploratory drilling. Input prices are calculated by dividing the annual expenditures on these activities by their respective quantities. The total cost of exploration is calculated as the sum of these three expenditures. The output variables (Q_1 = oil discoveries and Q_2 = gas discoveries) are the 1985 estimates of the values discovered in subsequent years.

All input prices are converted to constant 1981 dollar prices using the industrial selling price index (ISPI) which is available from Statistics Canada. Assuming an additive disturbance for the translog cost function leads to the following regression equation[7].

$$
\begin{aligned}
\text{(2a)}\quad \ln C = {} & a_C + a_D \ln w_D + a_G \ln w_G + a_L \ln w_L + a_T t \\
& + 0.5\gamma_{DD}(\ln w_D)^2 + \gamma_{DG}\ln w_D \ln w_G + \gamma_{DL}\ln w_D \ln w_L \\
& + 0.5\gamma_{GG}(\ln w_G)^2 + \gamma_{GL}\ln w_G \ln w_L \\
& + 0.5\gamma_{LL}(\ln w_L)^2 + b_1 \ln Q_1 + b_2 \ln Q_2 \\
& + \beta_X \ln X + \beta_{XD}\ln X \ln w_D + \beta_{XG}\ln X \ln w_G + \beta_{XL}\ln X \ln w_L \\
& + \gamma_{TT}t^2 + \delta_D t \ln w_D + \delta_G t \ln w_G + \delta_L t \ln w_L + u
\end{aligned}
$$

The results from OLS estimation of (2a) are shown in Table 1. From the parameter estimates in Table 1 we see that the cost function is increasing in

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[6] We are grateful to J. Livernois for the data.

[7] We assume the cost function C(w,Q,X,t) has errors distributed as lognormal. Taking natural logs yields (2a).

cumulative depletion, X, which is significant at the 10% level.[8] This would seem to indicate that the largest pools are discovered first and that subsequent exploration turns up small low quality reserves. In this case future exploration leads to smaller and smaller discoveries which not only add to existing reserves and lower extraction costs but also increase exploration costs. The cost function is decreasing in Q_1 and increasing in Q_2, but neither is statistically significant.

To evaluate the estimated cost equation we apply several diagnostic tests to the residuals from OLS estimation of (2a). The results are shown in Table 1a. Throughout this paper (unless otherwise noted) the 5% critical value is used for hypothesis testing. The null hypothesis of no autocorrelation can not be rejected by a Godfrey [1978] t statistic of +.478 (critical value = +2.45). Also shown in Table 1 are several tests for homoskedasticity. The null hypothesis of homoskedasticity cannot be rejected by either the Breush-Pagan [1979] test (test value of 14.003 compared with a critical value of 31.410) or the Harvey [1981] test (test value of 29.379 compared to a critical value of 31.410). The normality of residuals was tested using the Jarque-Bera [1980] asymptotic LM test. Since the computed value of 10.66 is greater then the critical value of 5.99 we can reject the null of normality.[9] Several Ramsey [1969] Regression Specification Error Tests (RESET) were computed using the powers of the fitted values from

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[8]All estimation was carried out using White's [1978] SHAZAM Version 6.0.

[9]This is an asymptotic test but our sample size is only 29. The small sample size may lead to inappropriate rejection of the null. The normality assumption is quite important since the Monte Carlo integration procedure assumes the data is distributed normally (at least asymptotically).

(2a). The computed F statistic values for RESET (2) (1.5 d.f.), RESET (3) (2,4 d.f.) and RESET (4) (3,3 d.f.) were 0.031, 0.012 and 0.014 respectively. Based on the results of these RESET tests the regression equation seems satisfactory. An Engle [1982a] test for autoregressive conditional heteroskedasticity (ARCH (1)) revealed a computed chi-squared test of 0.935. We can thus not reject the null of no ARCH (1).

Consistent estimation of (2a) requires the exogeneity of input prices w, output reserves Q and depletion X. While it seems reasonable to expect that input prices are exogenous it may be less reasonable to assume reserves Q_1 and Q_2 are exogenous. Following Davidson and Mackinnon [1987a], a joint test for the endogeneity of both Q_1 and Q_2 turned up an F statistic of 1.971 with 2 and 5 degrees of freedom. Since 1.971 is well below the 5% critical value of 5.79 we do not reject the null Q_1 and Q_2 are both exogenous.[10]

Since the computed F statistic for constant returns to scale is 78.191 with 1 and 8 degrees of freedom we can reject the null hypothesis of constant returns to scale. The computed F statistic for neutral technical change was 0.711 with 5 and 8 degrees of freedom which is less then the critical value of 3.69. Finally the computed F statistic for linear homogeneity in the input prices is 1.450 with 6 and 8 degrees of freedom which may be compared to the 5% critical F value of 3.58. Thus we cannot reject the null of linear homogeneity in prices at the 5% level. In light of these results we shall impose linear homogeneity.

With linear homogeneity in input prices, the share equations are given in (5). The cost function and the share equations form a system of seemingly

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[10]Further tests also revealed the exogeneity of the depletion variable, X.

unrelated regressions and we assume the errors are normally distributed with contemporaneous covariance matrix Σ. Since, by definition, the shares must sum to unity, one of the share equations is dropped to guarantee that Σ is positive definite. Given that the maximum likelihood estimation is invariant to which share equation is dropped we decided to exclude the share of land equation since the land share equation revealed the poorest set of diagnostic tests.[11] After imposing symmetry and linear homogeneity the 15 parameters for the three input cost function are

$$\theta = (a_C, a_D, a_G, a_T, \gamma_{DD}\ \gamma_{DG}, \gamma_{GG}, b_1, b_2, \beta_X, \beta_{XD}, \beta_{XG}, \delta_{TT}, \delta_D, \delta_G)$$

Estimation of the cost function was by the method of nonlinear maximum likelihood using the Davidson-Fletcher-Powell algorithm. Kmenta and Gilbert [1968], show that under the assumption of no heteroskedasticity or autocorrelation within equations the iterative Zellner method yields full information maximum likelihood estimates. Iterating until convergence on a system of the cost function and two of the share equations yields maximum likelihood estimates of the vector of translog parameters and the resulting covariance matrix Σ. The results appear in Table 2.

From Table 2, the estimates of b_1 and b_2 (corresponding to $\ln Q_1$ and $\ln Q_2$) are both positive, implying the cost function is increasing in outputs. The estimated coefficent on our new depletion measure, β_X, (the coefficient on lnX) is positive and significant demonstrating that the cost function is increasing in cumulative depletion.[12] A significant estimated β_X indicates the intercept of the cost function has shifted over time. From Table 2 we

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[11] Diagnostic tests on the residuals from the other two share equations indicated little evidence of either heteroskedasticity, first order seral correlation , ARCH(1) or absence of normality.

[12] In Livernois [1988] X was measured as cumulative drilling activity. He found a negative but insignificant coefficient on the variable lnX.

see that except for a_C, b_1 and b_2 all the paramater estimates are statistically significant. The coefficients β_{XD} and β_{XG} (by linear homogeneity $\beta_{XL} = -\beta_{XD} - \beta_{XG}$) are both negative and significant indicating that changes in depletion have been factor saving (cost reducing) for drilling and geophysical effort but factor using (cost increasing) for land. The effect of technological change is measured by the coefficients a_T, γ_{TT}, δ_D, and δ_G. The coefficient on the time trend variable a_T is positive and significant suggesting a positive shift over time in the cost function. The estimated coefficient γ_{TT} is negative and significant verifying the cost function is concave in time. The coefficients δ_D and δ_G are both statistically significant indicating technological change has not been Hicks neutral. The coefficient δ_D is positive and significant suggesting technological change in drilling has been cost increasing. Both δ_G and δ_L are negative indicating technological change has been cost reducing for geophysical and land inputs. Using the parameter estimates from Table 2 we computed the conditional mean functions for each of the three cost shares and found that all shares were positive in all the years.

As further evidence that technical change has not been neutral, consider Table 2a. Table 2a reports likelihood ratio (LR) test statistics for the hypothesis of constant returns to scale (CRS), no technical change (NTC) and the joint hypothesis of constant returns to scale and no technical change (CRS+NTC). The critical chi-squared values at the 5% level of significance for 1,4 and 5 degrees of freedom are 3.841, 9.488 and 11.070 respectively. Hence we can individually reject the null hypothesis of CRS, NTC and also reject the joint hypothesis of CRS+NTC. We again raise the point, however, that these LR test are asymptotic tests whereas our sample size is only 29x3 = 87.

The residuals from each of the three equations in the system were subjected to various diagnostic tests. Heteroskedasticity as measured by the Breusch-Pagan and Harvey tests was not detected in any of the three equations. Using a Gauss-Newton artificial regression a test for AR(1) turned up asymptotic t statistics (with probability values in parenthesis) of -0.7201 (0.600), 0.9299 (0.313) and 1.8558 (0.070) for the cost, drilling share and geophysical share equations respectively.[13] The reported chi-squared test statistics for ARCH(1) (with probability values in parenthesis) were 1.036 (0.308) , 0.191 (0.662) and 0.252 (0.616) for the cost, drilling and geophysical share equations respectively. For each of these three equations we cannot reject the null of no ARCH(1).

Imposing Concavity Conditions through Monte Carlo Integration

Negative semidefinitness of the matrix of second order partial derivatives of the cost function is a necessary and sufficient condition for a twice continuously differentiable cost function to be concave in prices, w, over the positive orthant. This requires $\nabla^2_{ww}C(w,Q,X,t)$ be negative semidefinite for $Q > 0$, $w >> 0$ and $t = 1,2,...,T$. Here $\nabla^2_{ww}C(w,Q,X,t)$ denotes the 3X3 matrix of second order partial derivatives of C with respect to the components of w. Denote $C_i = \partial C(w,Q,X,t)/\partial w_i$, $C_{ij} = \partial^2 C(w,Q,X,t)/\partial w_i \partial w_j$ and $\delta_{ij} = 1$ for $i = j$, and $\delta_{ij} = 0$ otherwise. By straightforward differentiation it can be shown that,

$$(6) \quad \frac{\partial^2 \ln C(w,Q,X,t)}{\partial \ln w_i \ln w_j} = \frac{\delta_{ij} w_i C_i}{C} - \frac{w_i w_j C_i C_j}{C^2} + \frac{w_i w_j C_{ij}}{C}$$

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[13] A good discussion of artificial regressions can be found in Davidson and Mackinnon [1984] and [1987b].

For the translog in (2), $\dfrac{\partial^2 \ln C(w,Q,X,t)}{\partial \ln w_i \partial \ln w_j} = \gamma_{ij}$

Using Shephard's Lemma and the definition of share functions, Diewert and Wales [1987] show that (6) can be written as

$$(7) \quad \frac{\mathrm{Diag}(w)[\nabla^2_{ww} C(w,Q,X,t)]\mathrm{Diag}(w)}{C(w,Q,X,t)} = \Gamma - [\mathrm{Diag}(s) - ss']$$

where s is a vector of factor shares (which is evaluated at some point in the sample), Diag(s) is a diagonal matrix formed from s, Diag(w) is a diagonal matrix formed from the vector of input prices (evaluated at some point in the sample) and $\Gamma = \gamma_{ij}$ (i, j=D, G, L) is the matrix of coefficients from the translog multiple output cost function. Provided that $C(w,Q,X,t) > 0$, $\nabla^2_{ww} C(w,Q,X,t)$ will be negative semidefinite if and only if,

$$\Gamma - [\mathrm{Diag}(s) - ss']$$

is negative semidefinite. A sufficient condition for concavity of the cost function C(w,Q,X,t) is that the matrix of coefficients, Γ, be negative semidefinite.

Using the parameter estimates from Table 2 the cost function was found to be concave in all but one year. Since concavity is a fundamental property of the dual cost function and if violated yields biased parameter estimates and biased elasticities, we decided to follow Chalfant and White [1987] and impose monotonicity and concavity through Monte Carlo integration. The Chalfant and White procedure for imposing monotonicity and concavity is described in detail in Chalfant and White [1987] and we will only sketch the steps involved.

The Bayesian procedure for estimating an inequality constrained cost function envolves first obtaining maximum likelihood estimates of the parameter vector θ and associated variance-covariance matrix Σ. Prior densities are specified for both Σ and θ. If there are no inequality restrictions the priors for Σ and θ are diffuse. While the joint posterior for Σ and θ is very complicated the marginal posterior for θ, which we will denote as $f(\theta|y)$ may be fairly simple. Here $y = (\ln C, s_D, s_G, s_L)$. Adding inequality constraints in the form of monotonicity and concavity restrictions implies a truncated marginal posterior for θ ($f^R(y|\theta)$). For convenience a quadratic loss function is used. Given that the restrictions are imposed the posterior mean is,

$$E(\theta) = \int_{\Omega} \theta k f^R(y|\theta) d\theta$$

where θ is restricted to lie in Ω, the region of the parameter space where the concavity and monotonicity conditions are imposed and k is a constant of proportionality. The probability that the restrictions hold under noninformative priors is given by p,

$$p = \int_{\theta} I(\theta) k f(\theta|y) d\theta$$

where $I(\theta)$ is the indicator function and $I(\theta) = 1$ if the restrictions hold ($\theta \in \Omega$) and $I(\theta) = 0$ otherwise. The numerical problem is that the integrals used in evaluating $E(\theta)$ and p are both too complicated to evaluate analytically. Consequently, Monte Carlo integration techniques are suggested. Moreover, $kf(\theta|y)$ is in general an unknown density and as such importance sampling is used because it is not possible to generate replications from the exact posterior density. The method of importance sampling substitutes a well known and simple density (say $g(\theta|y)$) for the unfamiliar true posterior density. In addition, an adjustment is made for the fact that $g(\theta|y)$ is not the true posterior density. If the data are normally distributed then two obvious choices for $g(\theta|y)$ are multivariate normal (for large samples) or multivariate t (for small samples) both with parameters $\hat{\theta}$ and $\hat{\Sigma}$, the maximum likelihood estimates from unrestricted estimation.

Essentially, the maximum likelihood estimates of $\hat{\theta}$ and $\Sigma(\hat{\theta})$ are used to generate antithetic draws from the multivariate t distribution with mean vector $\hat{\theta}$ and covariance $\Sigma(\hat{\theta})$. For each draw and its antithetic replication concavity was checked at the mean budget shares for the three inputs and monotonicity was checked by computing the predicted shares for all 29 data points.[14] The mean of the posterior distribution can be obtained by using only those draws which satisfy both concavity and monotonicity. Finally, the probability that the restrictions hold can be obtained by using all the replications.

Table 3 reports the results from the calculation of the posterior mean using the multivariate t distribution to generate replications in importance sampling. The results in Table 3 are very similar to those of Table 2. Of the 4000 replications 97.05% of these satisfied the concavity and monotonicity constraint. Also shown in Table 3 is the standard error of proportion which is just the estimated standard error for the binomial random variable $\hat{p}$ (evaluated at .97050) and the numerical standard error of the parameter estimates (the square root of the variance divided by the number of replications). The numerical standard errors of the estimated parameters are measures of the precision of the estimated parameters. The posterior mean values for the parameters in Table 3 were then used to compute the

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[14]Concavity was also checked at the minimum and maximum of the budget shares. At the minimum of the budget shares the probability that the restrictions held was 5.57%. At the maximum of the budget shares the probability that the restrictions held was 0.0%. Thus we conclude that the probability of the restrictions holding is highly sensitive to where the budget shares are evaluated.

fitted shares (Figure 1) and the Allen-Uzawa partial elasticities of substitution (Figure 2). Summary statistics for each of Figures 1-4 are shown in Table 4.

Figure 1 reports the three sets of share estimates.[15] Clearly all shares are positive in all years which verifies that the monotonicity condition is satisfied. Over the period 1956 to 1983, the share of drilling has doubled while the share of geophysical effort has fallen from .29 in 1956 to .12 in 1983. The share of land has also fallen somewhat over the specified period.

Figure 2 reports the Allen-Uzawa partial elasticities of substitution for the estimates in Table 3. The Allen-Uzawa partial elasticity of substitution (PES) is a useful way of measuring the percent change in the ratio of the two factors resulting from a 1% change in their relative prices (holding output constant). The estimated Allen-Uzawa partial elasticities of substitution between input price i and j are given by

$$\hat{\sigma}_{ij} = \frac{\hat{\gamma}_{ij} + \hat{S}_i \hat{S}_j}{\hat{S}_i \hat{S}_j}, \quad i \neq j$$

$$\hat{\sigma}_{ii} = \frac{\hat{\gamma}_{ii} + \hat{S}_i^2 - \hat{S}_i}{\hat{S}_i^2}, \quad i = j$$

where $\hat{\gamma}_{ij}$ are the fitted coefficients from the cost function and $\hat{S}_i$ is the fitted share of input i, i = D,G,L (D = exploratory drilling effort, G = geophysical effort and L = land acquired for exploration). If $\hat{\sigma}_{ij}$ is less

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[15] Unfortunately the Monte Carlo integration procedure in SHAZAM does not provide covariance estimates between parameters so that we are unable to obtain standard errors for the estimated shares and elasticities.

than or equal to zero, the inputs are complements. If $\hat{\sigma}_{ij}$ is positive, the inputs are substitutes with a larger positive value of $\hat{\sigma}_{ij}$ indicating the inputs are better substitutes for one another.

From Figure 2 it is clear that all own price elasticities are negative and fairly stable over time. All cross price elasticities (except σ_{DG} for the years 1981-1983) are positive indicating that the three inputs are substitutes.

3. The Marginal Cost of Exploration for Oil and Gas

The marginal costs of exploration for oil and gas are easily obtained by straightforward differentiation of the cost function. The estimated marginal costs of exploration for oil (Q_1) and gas (Q_2) in each year is calculated as,

$$(8)\quad \partial\hat{C}_t/\partial Q_{1t} = \left\{ \hat{b}_1/Q_{1t} + \hat{\beta}_X/X_t + \hat{\beta}_{XD}\ln w_D/X_t + \hat{\beta}_{XG}\ln w_G/X_t + \hat{\beta}_{XL}\ln w_L/X_t \right\} \hat{C}_t$$

$$(9)\quad \partial\hat{C}_t/\partial Q_{2t} = \left\{ \hat{b}_2/Q_{2t} + \hat{\beta}_X/X_t + \hat{\beta}_{XD}\ln w_D/X_t + \hat{\beta}_{XG}\ln w_G/X_t + \hat{\beta}_{XL}\ln w_L/X_t \right\} \hat{C}_t$$

In the calculation of (8) and (9), the predicted values of C_t, $\hat{C}_t$, are used along with the estimated parameter vector, $\hat{\theta}$. The exponentiation of $\ln\hat{C}_t$ results in biased estimates of $\hat{C}_t$. Hence the transformation suggested in Goldberger [1968], p.469 is used in calculating $\hat{C}_t$. These marginal cost estimates are shown in Figure 3 where all estimates have been converted to 1981 dollars using the industrial selling price index (ISPI).

From Figure 3 it appears that the estimated marginal discovery costs seem to be rising over time. In 1955, the marginal finding cost of oil is estimated to be $.88 per cubic meter while in 1983 it is estimated at $2.93 per cubic meter. The estimated marginal finding cost of gas in 1955 is $1.23 per thousand cubic meters and $4.83 per thousand cubic meters in 1986.

To further investigate how the marginal exploration costs change over time we regressed the natural log of the estimated marginal cost against a time trend. The regression results are:

$$\ln\hat{MCo} = \underset{(41.72)}{-229.17} + \underset{(.021)}{.1167}\, t + \hat{u}$$

$$\bar{R}^2 = .5110 \qquad \text{s.e.e.} = .95465 \qquad DW = 1.451$$

$$\ln\hat{MCg} = \underset{(29.95)}{-126.69} + \underset{(.015)}{.0645}\, t + \hat{u}$$

$$\bar{R}^2 = .3756 \qquad \text{s.e.e.} = .08536 \qquad DW = 1.109$$

where the standard errors are in parenthesis and $\hat{MCo}$ is the estimated marginal cost of oil ($\partial\hat{C}/\partial Q_1$) and $\hat{MCg}$ is the estimated marginal cost of gas ($\partial\hat{C}/\partial Q_2$).[16] Both trend coefficients are significant at the 10% level. For the marginal cost of oil there is a 11.7% per year increase along the trend line while for natural gas there is a 6.5% per year increase along the trend line. Our results indicate trends that are roughly one-half as large as those reported in Livernois [1988] who found a 21.1% per year increase in the marginal cost of oil along the trend line and a 17.3% per year increase in the marginal cost of gas along the trend line. Our estimated marginal exploration cost measures are somewhat smaller than those of Livernois [1988] because we have explicitly allowed for technical progress which helps to offset the rise in exploration costs brought about by depletion effects. Our

[16] Since any measurement error is in the dependent variable there is no need to adjust the standard errors.

results, like those of Livernois, indicate some strong evidence for the increased marginal cost of oil and gas in Alberta.

4. The Measurement of Resource Scarcity

One outstanding issue is to determine how well these marginal cost estimates approximate the unobservable resource rents. In extending the Devarajan and Fisher [1982] two period exploration and extraction model to many periods, Lasserre [1985], has shown that resource rents can be expressed as the sum of the production cost of the expected marginal discovery unit, the scarcity rent on exploration discovery per expected marginal discovery unit and a term that accounts for uncertainty in the model. Lasserre refers to the sum of these first two terms as "full marginal discovery cost" or FMDC. If exploration rents and the effects of uncertainty are small compared to the marginal cost of exploration then we can expect our marginal exploration cost estimates to be good measures of resource rents and hence scarcity. Using data similar to ours, Lasserre showed that scarcity rent on exploration discovery can be large (as much as 20% of FMDC). Livernois compared his marginal discovery cost estimates to Eglington and Uffelmann [1983] esitmates of the shadow price of reserves in the ground and found the two to be unrelated. The evidence from Lasserre and Livernois suggests that marginal costs of exploration alone are not good measures of resource scarcity.

To get a better measure of resource scarcity we augment our estimated marginal cost estimates with estimated exploration rent. Following Lasserre we use the Eglington and Uffelmann data on bonus payments [17] to measure rent on exploration discovery.

Figure 4 shows our estimated measures of resource scarcity for Alberta which are computed as the sum of estimated marginal exploration costs plus bonus payments (which are measures of exploration rents) where all values are in constant 1981 Canadian dollars.[18] Regressing the estimated natural log of $\hat{S}o$ (scarcity of oil) and $\hat{S}g$ (scarcity of gas) on a time trend yields the following results,

$$\ln\hat{S}o = \underset{(38.50)}{-196.55} + \underset{(.019)}{.1006}\,t + \hat{u}$$

$$\bar{R}^2 = .5366 \qquad s.e.e. = .62249 \qquad DW = 0.533$$

$$\ln\hat{S}g = \underset{(21.27)}{-34.561} + \underset{(.011)}{.0178}\,t + \hat{u}$$

$$\bar{R}^2 = .0726 \qquad s.e.e. = .34306 \qquad DW = 0.903$$

If any uncertainty in the exploration process is reflected by large year to year fluctuations in discoveries but not in the trend then our measure of resource scarcity should be a good approximation to the true scarcity. For oil there is a 10% per year increase in scarcity along the trend line and for natural gas there is a 1.8% per year increase in scarcity along the trend line. Again, both trend coefficients are significant at the 10% level. The above regression results for $\ln\hat{S}o$ and $\ln\hat{S}g$ indicate a fairly strong increase

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17Bonus payments include payments to the Director of Mineral Rights and the Mining Recorder, payments for Crown Drilling Reservations, Petroleum and Natural gas Drilling Reservations, Petroleum and Natural gas Leases, Natural Gas licenses and Leases, Block A permits and Leases, Petroleum and Natural Gas Permits, and Indian Lands and Federal Lands.

18The Eglington and Uffelmann [1983] measures of exploration rents only cover the period 1957-1979.

in the scarcity of oil in Alberta but a substantially less increase in the scarcity of natural gas.

5. Conclusions

The calculation of marginal exploration costs has received a great deal of attention in the literature recently because of the connection between marginal exploration costs and measures of resource scarcity. In this paper a multiple-output translog exploration cost function was estimated for Alberta. A new definition of depletion was introduced and its estimated coefficient was found to be empirically statistically significant. Concavity and monotonicity were imposed using Monte Carlo integration techniques.[19]

We found that the estimated cost function exhibits linear homogeneity in the input prices and to be increasing in both oil and gas discoveries. The cost function was also increasing in cumulative depletion. The estimated cost function did not exhibit constant returns to scale. Our results show that technological change over the period 1955-83, appears to have been biased. We found that changes in technology have been factor saving for geophysical effort and land but factor using for drilling. All of the own price elasticities are negative and except for the elasticity of substitution between drilling and geophysical effort in the years 1981,1982 and 1983 all cross price elasticities were positive indicating that the three inputs are

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[19] In our case the unrestricted and restricted cost function parameter estimates do not change much given that 97% of the restrictions are satisfied. However, in other applications this could change. Hence in studies where the unrestricted model can be potentially meaningless, this Monte Carlo integration technique is a good procedure for generating resticted estimates and at the same time delivers some measure of confidence in the restrictions.

substitutes.

There is strong evidence for the increased marginal cost of exploration for both oil and gas over time. Our marginal exploration cost estimates are, however, smaller than previous studies because we have explicitly allowed for technical progress which helps to offset the rise in exploration costs brought about by depletion effects. Augmenting these marginal cost estimates with measures of exploration rent we found evidence, although not as strong as other studies, for the increased scarcity of oil and gas in Alberta. It would be interesting to compare our marginal cost estimates from a multiple-output translog cost function with those that would be obtained from some other multiple-output flexible functional form like a Generalized McFadden cost function or a Generalized Barnett cost function.

References

Anders, et. al [1980], The Economics of Mineral Extraction, Prager.

Breusch, T.S. and A.R. Pagan [1979], "A Simple Test for Heteroskedasticity and Random Coefficient Variation," Econometrica, 47, pp.1287-94.

Brown, G.M. and B. Field [1978], "Implications of Alternate Measures of Natural Resource Scarcity," Journal of Political Economy, 86, pp.249-43

Burgess, D.F. [1975] "Duality Theory and Pitfalls in the Specification of Technologies," Journal of Econometrics 3:105-121.

Canadian Petroleum Association [1986], Statistical Handbook 1986.

Chalfant, J.A. and K.J. White [1987], "Estimation and Testing in Demand Systems with Concavity and Monotonicity Constraints," U.B.C. Discussion Paper 87-39.

Davidson, R. and J.G. Mackinnon [1984], "Model Specification Tests Based on Artificial Regressions," International Economic Review, 25, pp.485-502.

Davidson, R. and J.G. Mackinnon [1987a], "Testing for Consistency Using Artificial Regressions," Institute for Economic Research, Queen's University Discussion Paper # 687.

Davidson,R. and J.G. Mackinnon [1987b], "Double-Length Artificial Regressions," Institute for Economic Research Queen's University Discussion Paper #691.

Denny,M. and C. Pinto [1978], "An Aggregate Model with Multiproduct Technologies," in M. Fuss and D. McFadden, eds, Production Economics: A Dual Approach to Theory and Applications,vol 2(Amsterdam: North-Holland).

Diewert, W.E., M. Avriel, and I. Zang [1981], "Nine Kinds of Quasiconcavity and Concavity," Journal of Economic Theory 25:397-420.

Diewert, W.E. and T.J. Wales [1987], "Flexible Functional Forms and Global Curvature Conditions," Econometrica, 55, pp.43-68.

Devarajan, S. and A.C. Fisher [1982], "Exploration and Scarcity," Journal of Political Economy 90 (6), pp. 1279-90.

Engle, R.F. [1982a], "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation," Econometrica, 50, 82-1007.

Engle, R.F. [1982b], "A General Approach to Lagrange Multiplier Model Diagnostics," Journal of Econometrics, 20 pp. 83-104.

Eglington P. and M. Uffelmann [1983], "Observed Costs of Oil and Gas Reserves in Alberta, 1957-1979," Discussion Paper no. 235, Economic Council of Canada.

Geweke, J. [1986], "Exact Inference in the Inequality Constrained Normal Linear Regression Model," Journal of Applied Econometrics, 1, pp. 127-41.

Godfrey, L.C. [1978], "Testing Against General Autoregressive and Moving Average Errors When the Regressors Include Lagged Dependent Variables," Econometrica, 46, pp. 1293-1302.

Goldberger, A. [1968], "The Interpretation and Estimation of Cobb-Douglas Production Functions," Econometrica, 36, pp. 464-472.

Hall, R.E. [1973], "The Specification of Technology with Several Kinds of Output", Journal of Political Economy, 81, 878-92.

Halvorson, R. and T.R. Smith [1984], "On Measuring Resource Scarcity," Journal of Political Economy, 92, 51, pp. 954-964.

Hartwick, J.M. and N. Olewiler [1986], The Economics of Natural Resource Use, Harper and Row.

Harvey, A.C. [1981], The Econometric Analysis of Time Series, Phillip Allan, Oxford.

Howell, F.M. and H.A. Merklein [1973], "What it Costs to Find Hydrocarbons in the US," World Oil, 75-9.

Jarque, C.M. and A.K. Bera [1980], "Efficient Tests for Normality, Homoskedasticity and Serial Independence of Regression Residuals," Economics Letters, 6, pp. 255-59.

Johnston, J. [1986], Econometric Methods, (New York: McGraw Hill).

Kmenta, J. and R. Gilbert [1968], "Small Sample Properties of Alternative Estimators of Seemingly Unrelated Regressions," Journal of the American Statistical Association, pp. 1180-1200.

Lasserre, P. [1985], "Discovery Costs as a Measure of Rent," Canadian Journal of Economics, 18, pp. 474-483.

Livernois, J. [1988], "Estimates of Marginal Discovery Costs of Oil and Gas," Canadian Journal of Economics, 21, (2), pp. 379-93.

Livernois, J. and D. Ryan [1987], "Testing for Non-jointness in Oil and Gas Exploration: A Variable Profit Function Approach," International Economic Review, forthcoming.

Pagan, A. and A. Hall, [1983], "Diagnostic Tests as Residual Analysis," Econometric Reviews, 2, pp. 159-218.

Ramsey, J.B. [1969], "Tests for Specification Errors in Classical Linear Least-Squares Regression Analysis," Journal of the Royal Statistical Society B, 31, 2, pp. 350-371.

Slagorsky, Z.C. and J.L. Pasay [1985], "Estimated Petroleum Finding Costs in Alberta 1970-1983," Research Paper no. 85-1, Canadian Energy Institute, Calgary.

Spanos, A. [1986], Statistical Foundations of Econometric Modelling, Cambridge University Press, Cambridge.

Uhler, R. [1986], The Potential Supply of Crude Oil Reserves in the Alberta Basin (Ottawa: Economic Council of Canada).

Uhler, R. [1979], Oil and Gas Finding Costs, Study No. 7 Canadian Energy Research Institute, Calgary.

White, K.J. [1978], "A General Computer Program for Econometric Methods-SHAZAM," Econometrica, 46, p. 239.

White, K.J., S. Haun and N.G. Horsman [1987], SHAZAM The Econometrics Computer Program Version 6.0 Users Reference Manual, Vancouver, Department of Economics, University of British Columbia.

Table 1

OLS Regression Results from the Multiple Output Translog Exploration Cost Function

Parameter	estimated coefficient	t- ratio (8 d.f)
a_C	26318	1.0127
a_D	-126.65	-0.57042
a_G	421.25	1.3337
a_L	129.36	1.1023
γ_{DD}	1.4074	0.7607
γ_{GG}	0.4221	0.2983
γ_{LL}	0.4143	1.9120
γ_{DG}	1.4006	1.2734
γ_{GL}	-0.1730	-0.3032
γ_{DL}	-0.6247	-1.3238
b_1	-0.00853	-0.2128
b_2	0.003092	0.0310
β_X	21.338	1.8713
β_{XD}	-4.7521	-2.2195
β_{XG}	6.3203	1.6447
β_{XL}	1.3796	2.4050
a_T	-25.904	-0.9951
γ_{TT}	0.01258	0.9686
δ_D	0.09827	0.8098
δ_G	-0.2596	-1.4526
δ_L	-0.0743	-1.1988

$\bar{R}^2 = 0.9706$ s.e.e = 0.10519 DW = 1.761
LL = -150.938

Table 1A

Diagnostic Tests for the Regression Results Reported in Table 1

Test	Test Statistic	P-Value
$t_{SC}(6)$	0.478	0.649
$X^2_{BP}(21)$	14.003	0.830
$X^2_{H}(21)$	29.379	0.105
$X^2_{JB}(2)$	10.656	0.004
$F_{FF}(1,5)$	0.0310	0.867
$F_{FF}(2,4)$	0.0120	0.998
$F_{FF}(3,3)$	0.0140	0.997
$X^2_{ARCH}(1)$	0.9550	0.333
$F_{EXOG}(2,5)$	1.9710	0.234
$F_{CRS}(1,8)$	78.191	$0.211(10^{-4})$
$F_{NTC}(5,8)$	0.7110	0.632
$F_{HD1}(6,8)$	1.450	0.305

Notes:
The degrees of freedom for each test is shown in parenthesis.
t_{SC} is a Godfrey [1978] t statistic for AR(1).
X^2_{BP} is a Breusch-Pagan [1979] chi-squared test for heteroskedasticity.
X^2_{H} is a Harvey [1981] chi-squared test for heteroskedasticity.
X^2_{JB} is a Jarque and Bera [1980] chi-squared test for normality.
$F_{FF}(i,j)$ are Ramsey [1969] RESET F statistics for omitted variables.
X^2_{ARCH} is an Engle [1982a] chi-squared test for ARCH(1).
F_{EXOG} is a Davidson and Mackinnon [1987a] F test for exogeneity.
F_{CRS} is a F test for Constant Returns to Scale.
F_{NTC} is a F test for Neutral Technical Change.
F_{HD1} is a F test for Linear Homogeneity in input prices.

Table 2

Estimated Parameters, Translog Exploration Cost Function
MLE Method

Parameter	estimated coefficient	t- ratio (asymptotic)
a_C	0.975236	0.9748
a_D	-35.616	-9.8151
a_G	8.8747	2.4646
a_T	0.10895	7.7849
γ_{DD}	0.14873	6.8795
γ_{DG}	-0.06219	-3.6539
γ_{GG}	0.11387	6.2572
b_1	0.035836	1.3555
b_2	0.064434	1.3740
β_X	0.80744	5.1949
β_{XD}	-0.045892	-2.7485
β_{XG}	-0.048768	-2.6548
γ_{TT}	-0.000109	-7.3960
δ_D	0.018279	9.6975
δ_G	-0.004235	-2.4036

	Cost	Drilling Share	Geophysical Share
LL	124.554	124.554	124.554
R^2	.9422	.8427	.6297
s.e.e	.14499	.03707	.04176

Notes: The residuals for each equation are calculated as the dependent variable less the fitted values. The R^2 values are calculated for each equation separately as ESS_i/TSS_i where ESS_i is the explained sum of squares from equation i and TSS_i is the total sum of squares from equation i. The standard error of the estimate is just the standard error of the residuals from each equation.

Table 2a

LR Tests for CRS and NTC

Model	LL	Test Statistic	DF	P-Value
Unrestricted	124.554	---	--	
CRS	88.657	71.794	1	$.596(10^{-7})$
NTC	99.525	50.058	4	$.597(10^{-7})$
CRS+NTC	62.089	124.93	5	.0000

Notes: LL is the log of the likelihood function.
DF is the degrees of freedom and P-Value is the probability value.
CRS denotes constant returns to scale.
NTC denotes no technical change.
CRS+NTC denotes the joint hypothesis of CRS and NTC.

Table 3

Posterior Mean Values: Multiple-Output Translog Cost Function Parameters
Concavity and Monotonicity Imposed
Importance Function: Multivariate t

4000 Replications 3882 Satisfied
Proportion = 0.97050,
Numerical Standard Error of Proportion = 0.00268
Asymptotic Standard Error of Proportion = 0.00268

Parameter	Mean	St Dev	Var	Num Se
a_C	0.975236	1.0432	1.0883	.0167430
a_D	-35.486	3.6005	12.964	.0577880
a_G	8.8461	3.3495	11.219	.0537590
a_T	0.10859	.01423	.0002022	.0002284
γ_{DD}	0.14791	.021069	.0004439	.00033816
γ_{DG}	-0.062210	.017029	.0002900	.00027332
γ_{GG}	0.11437	.018150	.0003294	.00029131
b_1	0.035621	.027325	.0007466	.00043856
b_2	0.064306	.048300	.0023329	.00077521
β_X	0.80694	.161310	.026020	.00258900
β_{XD}	-0.045407	.016917	.0002862	.00027152
β_{XG}	-0.049205	.018300	.00033489	.00029371
γ_{TT}	-0.000109	.0000151	.22917E-9	.24297E-6
δ_D	0.018213	.0018749	.35152E-5	.30092E-4
δ_G	-0.004219	.0017585	.30923E-5	.28224E-4

Notes:

1. The variance of a particular coefficient is the square of its standard deviation (St Dev).

2. The numerical standard error (Num Se) for a particular coefficient is the square root of its variance (Var) divided by the number of replications.

Table 4

Summary Statistics for Figures 1 - 4

Figure 1 Fitted Shares

Parameter	Mean	St. Dev.	Min	Max
S_D	.38274	.08720	.26202	.60443
S_G	.24835	.05792	.12421	.36114
S_L	.36890	.08427	.19238	.49479

Figure 2 Allen-Uzawa Partial Elasticities of Substitution

Parameter	Mean	St. Dev.	Min	Max
σ_{DD}	-.58485	.12510	-.69020	-.24959
σ_{DG}	.29006	.16208	.02370	.61418
σ_{DL}	.35206	.11370	.00226	.48959
σ_{GG}	-1.0095	.34483	-1.1858	.36238
σ_{GL}	.35850	.26980	-.54754	.56652
σ_{LL}	-.64614	.12136	-.81321	-.45794

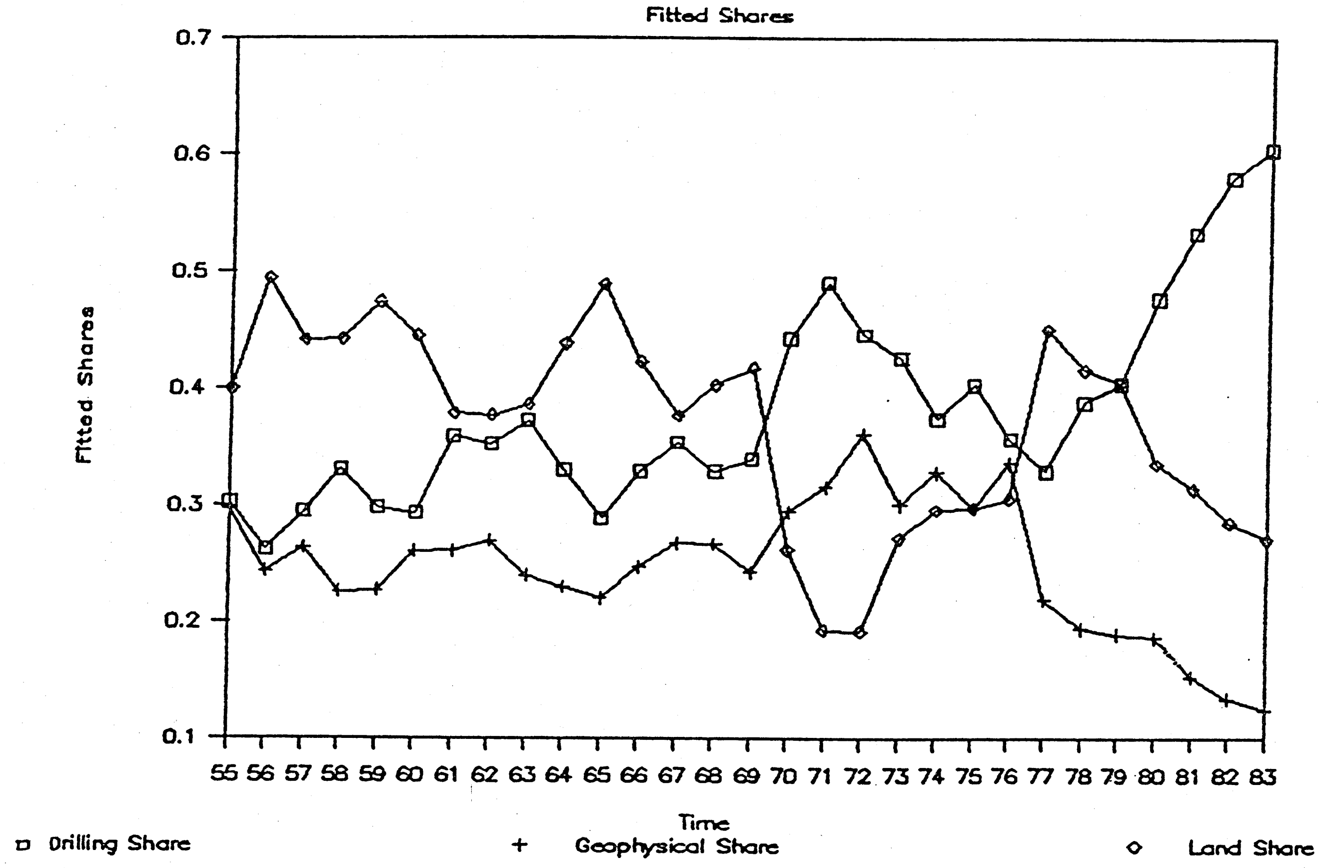
Figure 3 Predicted Marginal Discovery Costs

Parameter	Mean	St. Dev.	Min	Max
MCo	3.0223	4.1251	.08835	20.573
MCg	1.1556	0.9870	.06737	4.8272

Figure 4 Predicted Marginal Scarcity Costs

Parameter	Mean	St. Dev.	Min	Max
So	7.0381	6.1265	1.7720	26.699
Sg	1.6805	0.6449	.82227	3.0825

Fig. 1



Fitted Shares
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Fitted Shares
55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83
Time
Drilling Share
Geophysical Share
Land Share

Fig. 2

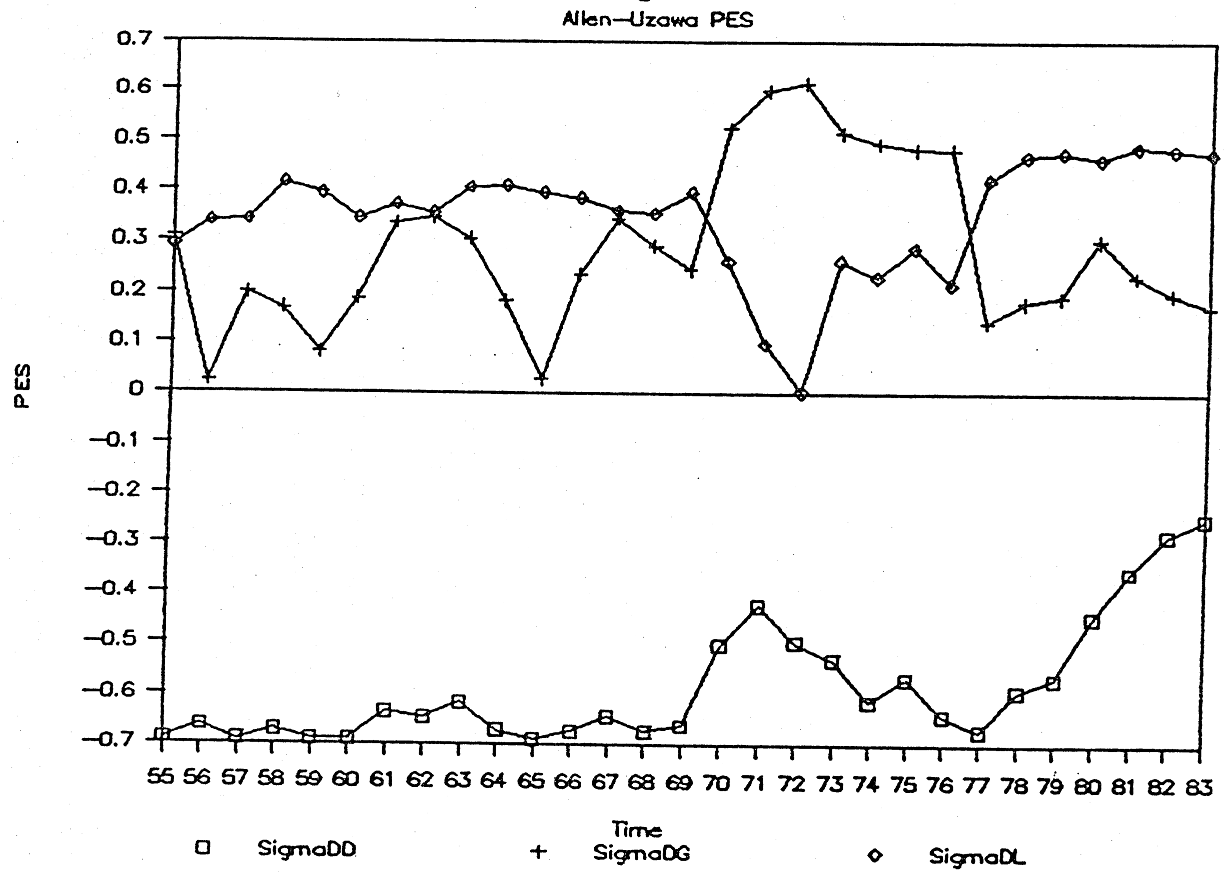

Allen–Uzawa PES
PES
Time
SigmaDD
SigmaDG
SigmaDL

Fig. 2a

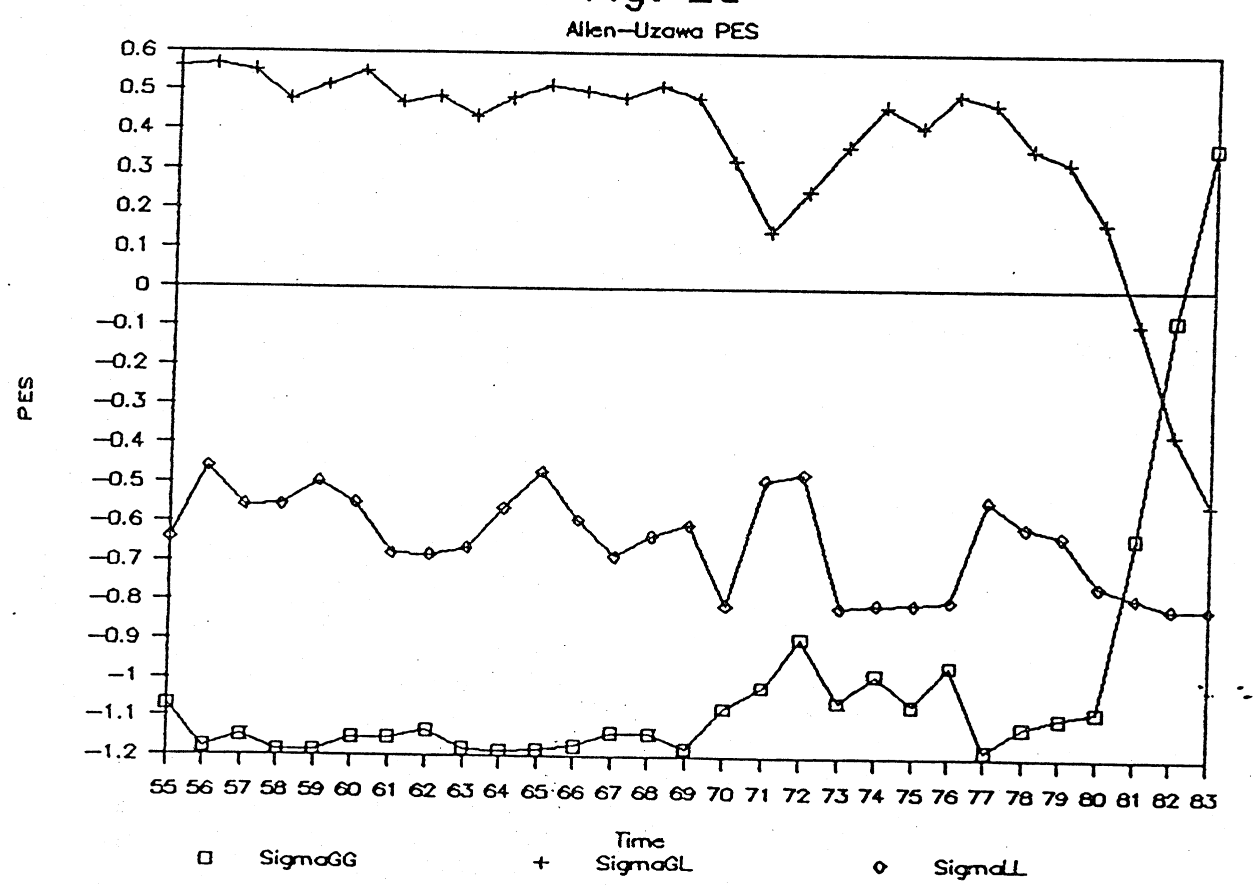

Allen–Uzawa PES
PES
Time
SigmaGG
SigmaGL
SigmaLL

Fig. 3

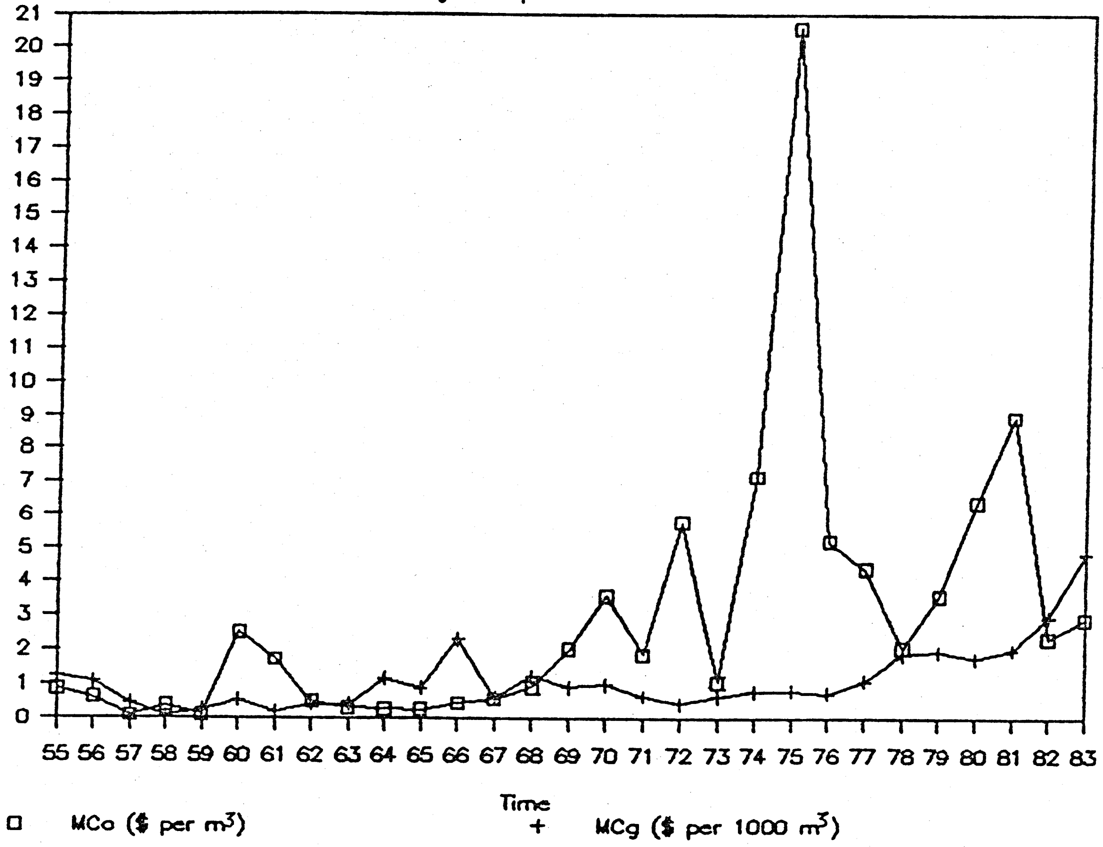
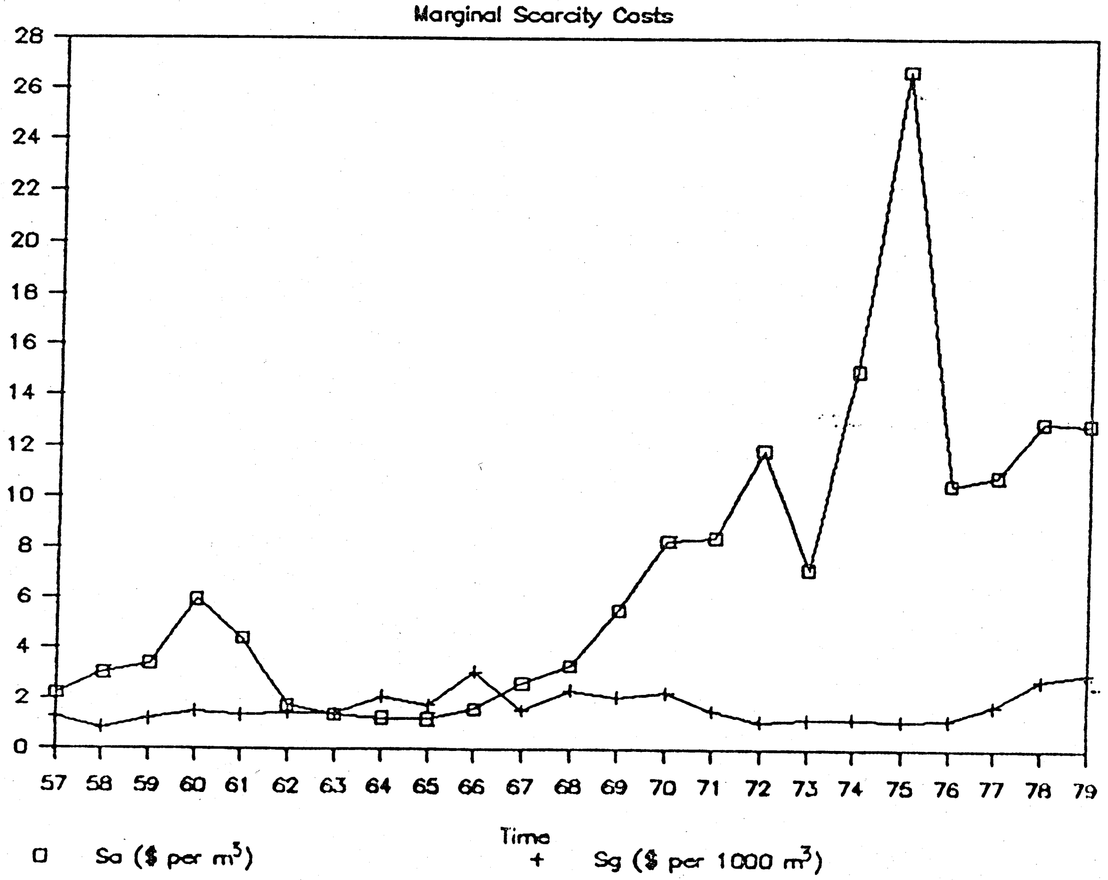

Marginal Exploration Costs
Time
MCo ($ per m³)
MCg ($ per 1000 m³)

Fig. 4


Marginal Scarcity Costs
Time
Sa ($ per m³)
Sg ($ per 1000 m³)