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DUOPOLY IN EXHAUSTIBLE RESOURCE EXPLORATION AND EXTRACTION

by

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Abstract

Strategic considerations of exploration and extraction are investigated in a two player, two period, two stage perfect equilibrium framework. Relative to two "plant" monopoly, the duopolists explore more and extract more period by period. A mixed game in which there is co-operation "upstream" in exploration and Cournot competition "downstream" in quantities extracted is investigated. We also note that increasing returns to scale in exploration can introduce an unstable interior solution with a corner solution the presumed stable equilibrium.

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Introduction

Exploration can affect current output price when discoveries are relatively large. Large discoveries are associated with finds of new oil fields or major ore bodies. The phenomenon is inherently stochastic, regular steady search with many small or negative (dry wells) hits and the occasional large hit.[1] One discoverer's large hit inflicts capital losses on owners of known but unsold stock and this introduces strategic considerations into exploration. Successful explorers can also ultimately increase their market share of say oil. We investigate this strategic rivalry in a non-stochastic framework. Rivals anticipate the effects of competitor's exploration in a perfect equilibrium.[2] We deal with duopolist explorer-extractors in a two stage game. Each player explores in anticipation of playing a Cournot game in quantities extracted in a second stage. We observe under-exploration relative to pure price-taking rivalry and over-exploration relative to a "two-plant monopoly". We present an illustrative numerical example. The interesting dimension of resource exploration rivalry as distinct from say R & D rivalry is how success by a discoverer not only leads to encroachment on a rival's

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[1] Arrow and Chang [1982] consider an agent exploring in a stochastic framework in which only small finds can occur. See also Pindyck [1980]. Devarajan and Fisher [1982] and Lasserre [1985] make exploration costs rise with cumulative discoveries.

[2] Non-rivalrous exploration in a non-stochastic framework is analyzed in Pindyck [1978].

current and subsequent market but inflicts a capital loss on the rival's current known reserves.[3]

Two Period, Two Player Exploration and Extraction Rivalry

Each player opens with some stock or known reserves, knows its extraction costs and deterministic exploration technology, and market demand (either prices or demand schedules, depending on the competitive mode below). We will set out the competitive (price taking case) and two plant monopoly cases very briefly first in order to provide a comparison with the price-setting duopoly case of particular interest. Firm i's profits are

$$\pi^i = Q^i p\,(Q^1+Q^2) - c^i(Q^i) - w^i(x^i) + \beta\left\{\left(S^i + S^i(x^i) - Q^i\right) p \left[\sum_{i=1}^{2}\left(S^i + S^i(x^i) - Q^i\right)\right] - c^i\left(S^i + S^i(x^i) - Q^i\right)\right\} \qquad (i=1,2) \qquad (1)$$

where Q^i is firm i's output or extraction in period 1. $Q^i \leq S^i$.

S^i is firm i's reserves or known stock in period i.

$p(\cdot)$ is the stationary demand schedule for output extracted by the two firms.

$c^i(Q^i)$ is the total extraction cost for firm i.

x^i is physical resources devoted to extraction by firm i in period 1.

$w^i(\cdot)$ is the cost of exploration.

$S^i(x^i)$ is the discovery of stock made in period 1 to be extracted in period 2. $S^i(x^i)$ is assumed positive and concave in x^i with $S^i(0) = 0$. $w^i(x^i)$ is assumed to be positive and increasing in x^i. We will assume that $w^i(0) = 0$ and $\frac{d^2 w^i}{dS^{i2}} \geq 0$ in the normal case but comment on the case of increasing returns to exploration effort

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[3] A parallel in R & D rivalry would be success by player i forcing player j to write off part of his or her fixed capital in addition to having part of his or her market share reduced. In Spencer and Brander [1983] for example, the duopoly rivalry in R & D only affects current variable costs and market shares, not fixed costs. See the Concluding Remarks below.

i.e. the case of $\frac{d^2w^i}{dS^{i2}} < 0$.

β is the discount factor $\{$equal to $1/(1+r)\}$ where r is the discount rate say equal to the rate of interest.

Simplification is achieved by making the horizon exogenous. All stock is extracted over two periods. We focus on the exploration-extraction rivalry <u>per se</u>, not an attrition and withdrawal of a rival over an extended horizon, endogenously determined.

(i) Price-taking behavior

With respect to quantity, firm i "follows" the r % rule in rent on the marginal ton

$$[p_1 - mc_1^i] = \left(\frac{1}{1+r}\right)[p_2 - mc_2^i] \qquad (i=1,2) \qquad (2)$$

where p_1 is the endogenously determined price in period 1 and p_2 is the endogenously determined price of output in period 2 and mc_k^i is firm i's marginal cost of extraction in period k. With respect to exploration activity, we have

$$\frac{dw^i}{dx^i} = \left(\frac{1}{1+r}\right)[p_2 - mc_2^i]\frac{dS^i}{dx^i} \qquad (i=1,2) \qquad (3)$$

This is simply the rule that the cost of the marginal unit of exploration should equal the value of its marginal product, suitably discounted and valued at "net price", $p_2 - mc_2^i$.

(ii) Two "plant" monopoly

A decision-maker maximizes the sum $\pi^1 + \pi^2$ by choice of Q^1, Q^2, x^1, and x^2. With respect to quantity, in plant i we have

$$[mr_1^i - mc_1^i] = \left(\frac{1}{1+r}\right)[mr_2^i - mc_2^i] - Q^j\frac{\partial p_1 \partial\Sigma_1}{\partial\Sigma_1 \partial Q^i} - \left(\frac{1}{1+r}\right)\frac{\partial p_2\ \partial\Sigma_2}{\partial\Sigma_2\ \partial Q^i}\left(S^j + S^j (x^j) - Q^j\right) \qquad (i=1,2) \quad (j=1,2;\ i \neq j) \qquad (4)$$

where mr_k^i is the marginal revenue in period k for plant i. With respect to exploration, we have

$$\frac{dw^i}{dx^i} = \left(\frac{1}{1+r}\right)\left[p_2 - mc_2^i\right]\frac{dS^i}{dx^i} + \left(\frac{1}{1+r}\right)\left(\frac{\partial p_2}{\partial(\Sigma_2)}\cdot\frac{\partial\Sigma_2}{\partial S^i}\cdot\frac{dS^i}{dx^i}\right)\cdot\left(S^i+S^i\left(x^i\right) - Q^i\right)$$

$$+\left(\frac{1}{1+r}\right)\left(\frac{\partial p_2}{\partial(\Sigma_2)}\cdot\frac{\partial\Sigma_2}{\partial S^i}\cdot\frac{dS^i}{dx^i}\right)\cdot\left(S^j+S^j\left(x^j\right) - Q^j\right) \quad (i=1,2;\ i\neq j) \qquad (5)$$

In comparison with the price taking case, the monopolist considers the effect of his or her discoveries on the second period price at the margin and the effect of this price decline on his or her own quantity extracted in plant i plus the effect of this price decline on the output in period 2 of plant j. Σ_2 is an abbreviation for $S^1 + S^1(x^1) - Q^1 + S^2 + S^2(x^2) - Q^2$. The monopoly solution involves the simultaneous choice of Q^1, Q^2, x^1 and x^2.

(iii) Non-cooperative subgame perfect duopoly

In this case each player knows and is committed to play a Cournot game in quantities extracted once exploration levels are agreed on. Given a Cournot game in quantities extracted, one can solve backwards to a game pure in exploration levels. If this game is Cournot in exploration levels, we label it non-cooperative. (Below we consider the exploration levels selected co-operatively or as in the two plant monopoly.) In the non-cooperative game, the players satisfy in the Cournot output game:

$$mr_1^i - mc_1^i = \left(\frac{1}{1+r}\right)\left[mr_2^i - mc_2^i\right] \qquad \begin{cases} i=1,2 \\ j=1,2;\ i\neq j \end{cases} \qquad (6)$$

(6) can be solved (perhaps implicitly) to yield $Q^1 = f^1\left(x^1,x^2\right)$ and $Q^2 = f^2\left(x^1,x^2\right)$. If these equations are placed in (1) for Q^1 and Q^2, we have a pair of profit functions in exploration levels $\left(x^1,\ x^2\right)$ alone. The non-cooperative game in exploration levels is defined as the solution x^1 and x^2 to the pair $\dfrac{\partial\pi^1}{\partial x^1} = 0$ and $\dfrac{\partial\pi^2}{\partial x^2} = 0$. Given our specification of demand and costs, these two equations are

$$\frac{dw^i}{dx^i} = \left[\frac{1}{1+r}\right]\left[p_2 - mc_2^i\right]\frac{dS^i}{dx^i} + \left[\frac{1}{1+r}\right]\left[\frac{\partial p_2}{\partial(\Sigma_2)}\cdot\frac{\partial\Sigma_2}{\partial S^i}\cdot\frac{dS^i}{dx^i}\right]\cdot\left[S^i + S^i\left(x^i\right)-Q^i\right]$$

$$+\left[\frac{1}{1+r}\right]\left[\frac{\partial p_2}{\partial(\Sigma_2)}\cdot\frac{\partial\Sigma_2}{\partial Q^j}\cdot\frac{dQ^j}{dx^i}\right]\cdot\left[S^i + S^i\left(x^i\right)-Q^i\right] \quad (i=1,2;\ 1\neq j) \qquad (7)$$

where dQ^1/dx^2 and dQ^2/dx^1 are defined from the reaction functions from the Cournot game in quantities (or from the basic implicit equations, $Q^1 = f^1\left(x^1,x^2\right)$ and $Q^2 = f^2\left(x^1,x^2\right)\Big]$. The own effects dQ^i/dx^i vanish above because $d\pi^i/dQ^i$ are zero from (2). For our specification, one totally differentiates the pairs of equations in (6) to obtain

$$\partial\left[\frac{mr_1^1 - mc_1^1 - \beta\left(mr_2^1 - mc_2^1\right)}{\partial Q^1}\right]dQ^1 + {+\partial}\left[\frac{mr_1^1 - mc_1^1 - \beta\left(mr_2^1 - mc_2^1\right)}{\partial Q^2}\right]dQ^2$$

$$= -\partial\left[\frac{mr_1^1 - mc_1^1 - \beta\left(mr_2^1 - mc_2^1\right)}{\partial x^1}\right]dx^1 - \partial\left[\frac{mr_1^1 - mc_1^1 - \beta\left(mr_2^1 - mc_2^1\right)}{\partial x^2}\right]dx^2 \quad (8)$$

$$\partial\left[\frac{mr_1^2 - mc_1^2 - \beta\left(mr_2^2 - mc_2^2\right)}{\partial Q^1}\right]dQ^1 + \partial\left[\frac{mr_1^2 - mc_1^2 - \beta\left(mr_2^2 - mc_2^2\right)}{\partial Q^2}\right]dQ^2$$

$$= -\partial\left[\frac{mr_1^2 - mc_1^2 - \beta\left(mr_2^2 - mc_2^2\right)}{\partial x^1}\right]dx^1 - \partial\left[\frac{mr_1^2 - mc_1^2 - \beta\left(mr_2^2 - mc_2^2\right)}{\partial x^2}\right]dx^2 \quad (9)$$

and solves for $\frac{dQ^1}{dx^1}$, $\frac{dQ^2}{dx^1}$, $\frac{dQ^1}{dx^2}$, and $\frac{dQ^2}{dx^2}$. In (7), we make use of $\frac{dQ^1}{dx^2}$ and $\frac{dQ^2}{dx^1}$, to solve the Cournot game (non-cooperative game) in exploration levels.

Since (4) and (5) define the "two plant monopoly" case and (6) and (7) define the non-cooperative duopoly case, we can readily compare the two games. Equations in (5) and (7) differ only in the final terms on the RHS's.

In the special case of a linear "industry", examined in detail below, we have $dQ^1/dx^2 = dQ^2/dx^1 = 0$ and the last terms in the pair of equations in (7) vanish, suggesting that the monopolist takes account of the spillover of his or her exploration activity from one "plant" on the other whereas the duopolists completely ignore the spillovers. This is an extreme case illustrating our argument duopolists "over explore" because they ignore the "damage" their discoveries have on rival's market share and capital losses associated with known reserves of stock. Though formally, discovery can only drive down future prices in our first order conditions for exploration effort, future prices are tied to present prices through the first order conditions on quantities extracted. This is as it should be - namely discoveries to be mined in the future drive down current output price. These effects lead to the capital losses mentioned. Note in (7) that $dS^i/dx^i > 0$ and $\frac{\partial\Sigma_2}{\partial S^i} = -\frac{\partial\Sigma_2}{\partial Q^j}$. The sign of $\frac{dQ^j}{dx^i}$ turns on how the demand schedule is specified. The monopolist takes account of "plant" i's discovery on total profit whereas the non-cooperating duopolist takes account of his or her discovery on his or her market share (industry price) and on the effect of his or her discovery on his or her profit alone via the effect on his or her rivals extraction. This would suggest intuitively that the duopolist will be dealing with a smaller "cross effect" of marginal discovery since his or her profits will be a fraction of total profits and thus the duopolist will cut back exploration relative to the monopoly less given internalization of spillovers from exploration. Certainly in the example with a linear demand schedule, the monopolist explores in total less than the two duopolists combined.

(iv) **Co-operative exploration and Cournot output strategy**[4]

In this case, given $Q^1 = f^1(x^1,x^2)$ and $Q^2 = f^2(x^1,x^2)$ implicitly defined from the Cournot reaction function in (4), the duopolists maximize the sum of their profits by simultaneously choosing exploration levels x^1 and x^2. Thus the Cournot game in quantities is still defined by the Q^1 and Q^2 which solve (6) given x^1 and x^2. Equation (8) and (9) define $\frac{dQ^1}{dx^1}$, $\frac{dQ^2}{dx^1}$, $\frac{dQ^1}{dx^2}$, and $\frac{dQ^2}{dx^2}$.

Maximizing the sum of profits, π^1 and π^2 from (1) where Q^1 and Q^2 are implicit functions of x^1 and x^2 yields the same basic pair of equations in (7) plus new cross-effect terms indicated by braces. That is

$$\frac{dw^1}{dx^1} = \beta\left[p_2-mc_2^1\right]\frac{dS^1}{dx^1} + \left[S^1 + S^1\left(x^1\right) - Q^1\right]\beta\frac{\partial p_2}{\partial\Sigma_2}\left[\frac{\partial\Sigma_2}{\partial S^1}\cdot\frac{dS^1}{dx^1} + \frac{\partial\Sigma_2}{\partial Q^2}\cdot\frac{dQ^2}{dx^1}\right]$$
$$+\left\{\left[S^2 + S^2\left(x^2\right) - Q^2\right]\beta\frac{\partial p_2}{\partial\Sigma_2}\cdot\frac{\partial\Sigma_2}{\partial S^1}\cdot\frac{dS^1}{dx^1} + Q^2\frac{\partial p_1}{\partial\Sigma_1}\cdot\frac{\partial\Sigma_1}{\partial Q^1}\cdot\frac{dQ^1}{dx^1}\right\} \qquad (10)$$

$$\frac{dw^2}{dx^2} = \beta\left[p_2-mc_2^2\right]\frac{dS^2}{dx^2} + \left[S^2 + S^2\left(x^2\right) - Q^2\right]\beta\frac{\partial p_2}{\partial\Sigma_2}\left[\frac{\partial\Sigma_2}{\partial S^2}\cdot\frac{dS^2}{dx^2} + \frac{\partial\Sigma_2}{\partial Q^1}\cdot\frac{dQ^1}{dx^2}\right]$$
$$+\left\{\left[S^1 + S^1\left(x^1\right) - Q^1\right]\beta\frac{\partial p_2}{\partial\Sigma_2}\cdot\frac{\partial\Sigma_2}{\partial S^2}\cdot\frac{dS^2}{dx^2} + Q^1\frac{\partial p_1}{\partial\Sigma_1}\cdot\frac{\partial\Sigma_1}{\partial Q^2}\cdot\frac{dQ^2}{dx^2}\right\} \qquad (11)$$

This new game is defined by the Q^1,Q^2,x^1,x^2 which solve the two equations in (6), and (10), and (11) where $\frac{dQ^1}{dx^1}$, $\frac{dQ^2}{dx^1}$, $\frac{dQ^1}{dx^2}$, and $\frac{dQ^2}{dx^2}$ are defined in (8) and (9). One conjectures that less exploration will be done in this game relative to the non-cooperative duopoly game because here each internalizes

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[4]The case of co-operation downstream and competition in exploration upstream is another case. We have not reported on it since it seems less empirically relevant.

his or her spillover from undertaking exploration in the exploration phase of competition for profits. The sign of the first "new term" in (10) is negative since $\frac{\partial p_2}{\partial \Sigma_2} < 0$ which should induce less exploration and the sign of the second "new term" turns on the sign of $\frac{dQ^1}{dx^1}$ which for the case of a linear demand schedule is positive making the sign of this "new term" for a linear demand schedule positive.

Linear Demand and Quadratic Extraction Costs

This example with discovery concave in effort yields mostly expected results. Duopolists extract period by period more than the corresponding two plant monopolist and duopolists explore (and discover) more than the corresponding two plant monopolist. The mixed case of "downstream" duopoly and "upstream" co-operation in exploration yields the striking result that exploration levels are the same as those for the two plant monopoly. Downstream quantities are larger for the mixed case. In other words, fully anticipated rivalry downstream has no effect upstream on the levels of exploration under co-operation (two plant monopoly).

The profit functions for the two players (firms or plants) are

$$\pi_1 = \left[A-B\left(Q^1+Q^2\right)\right] Q^1 - c^1\left(Q^1\right)^2 - w^1x^1 + \beta\left[A-B\left(S^1 + S^1(x^1) - Q^1 + S^2 + S^2(x^2) - Q^2\right]\right. \left\{S^1 + S^1(x^1) - Q^1\right\} - \beta c^1 \cdot \left[S^1 + S^1(x^1) - Q^1\right]^2 \qquad (12)$$

$$\pi_2 = \left[A-B\left(Q^1+Q^2\right)\right] Q^2 - c^2\left(Q^2\right)^2 - w^2x^2 + \beta\left[A-B\left(S^1 + S^1(x^1) - Q^1 + S^2 + S^2(x^2) - Q^2\right]\right. \left\{S^2 + S^2(x^2) - Q^2\right\} - \beta c^2 \cdot \left[S^2 + S^2(x^2) - Q^2\right]^2 \qquad (13)$$

(i) Two plant monopoly

In this case we solve $\frac{\partial \pi}{\partial Q^1} = \frac{\partial \pi}{\partial Q^2} = \frac{\partial \pi}{\partial x^1} = \frac{\partial \pi}{\partial x^2} = 0$ where $\pi = \pi^1 + \pi^2$. We can solve the first two equations for Q^1 and Q^2 in terms of x^1 and x^2 and parameters. These are then:

$$Q^1 = \left(\frac{\beta}{1+\beta}\right)Z^1 + \left[\frac{c^2}{2Bc^2 + 2Bc^1 + 2c^1c^2}\right]\left(\frac{1-\beta}{1+\beta}\right)A \tag{14}$$

$$Q^2 = \left(\frac{\beta}{1+\beta}\right)Z^2 + \left[\frac{c^1}{2Bc^2 + 2Bc^1 + 2c^1c^2}\right]\left(\frac{1-\beta}{1+\beta}\right)A \tag{15}$$

where $Z^i = S^i + S^i(x^i)$. If Q^1 and Q^2 from (14) and (15) are substituted into

$\frac{\partial\pi}{\partial x^1} = \frac{\partial\pi}{\partial x^2} = 0$ we obtain

$$-w^1\frac{dx^1}{dS^1} + \frac{2\beta A}{(1+\beta)} - \left(\frac{2\beta}{1+\beta}\right)\left(B+c^1\right)Z^1 - \left(\frac{2\beta}{1+\beta}\right)BZ^2 = 0 \tag{16}$$

$$-w^2\frac{dx^2}{dS^2} + \frac{2\beta A}{(1+\beta)} - \left(\frac{2\beta}{1+\beta}\right)\left(B+c^2\right)Z^2 - \left(\frac{2\beta}{1+\beta}\right)BZ^1 = 0 \tag{17}$$

This pair of nonlinear equations defines the x^1 and x^2 for the pure monopoly case. Given x^1 and x^2, we obtain the solution values for Q^1 and Q^2 above. For the case $S^i(x^i) \equiv E^i x^{0.5}$ we solve some examples. Results are reported in Table 1. We discuss them below.

(ii) Duopoly

In this case the "downstream" Cournot game is played in Q^1 and Q^2 and the "upstream" game is played in x^1 and x^2 given that each player knows and is committed to play a downstream game in Q^1 and Q^2. The first order conditions or reaction functions $\frac{\partial\pi^1}{\partial Q^1} = 0$ and $\frac{\partial\pi^2}{\partial Q^2} = 0$ (corresponding to the equations in (6) are:

$$Q^1 = \frac{A}{2B+2c^1}\left(\frac{1-\beta}{1+\beta}\right) + \left(\frac{\beta}{1+\beta}\right)Z^1 + \left(\frac{\beta}{1+\beta}\right)\left(\frac{B}{2B+2c^1}\right)BZ^2 - \left(\frac{B}{2B+c^1}\right)Q^2 \tag{18}$$

$$Q^2 = \frac{A}{2B+2c^2}\left(\frac{1-\beta}{1+\beta}\right) + \left(\frac{\beta}{1+\beta}\right)Z^2 + \left(\frac{\beta}{1+\beta}\right)\left(\frac{B}{2B+2c^2}\right)BZ^1 - \left(\frac{B}{2B+c^2}\right)Q^1 \tag{19}$$

Observe that $B/(2B+c^1) < 1$, implying that the reaction functions cross in a way compatible with stability of convergences to the solution Q^1 and Q^2. Also, $\frac{d^2\pi^i}{d(Q^2)^2} < 0$ or the second order conditions are satisfied. The solution to (18) and (19) is

$$Q^1 = \left(\frac{\beta}{1+\beta}\right)Z^1 + \left[\frac{B+2c^2}{3B^2+4c^1c^2+4Bc^1+4Bc^2}\right]\left(\frac{1-\beta}{1+\beta}\right)A \tag{20}$$

$$Q^2 = \left(\frac{\beta}{1+\beta}\right)Z^2 + \left[\frac{B+2c^1}{3B^2+4c^1c^2+4Bc^1+4Bc^2}\right]\left(\frac{1-\beta}{1+\beta}\right)A \tag{21}$$

These equations imply

$$\frac{dQ^1}{dx^1} = \left(\frac{\beta}{1+\beta}\right)\frac{dS^1(x^1)}{dx^1} \quad \text{and} \quad \frac{dQ^2}{dx^2} = \left(\frac{\beta}{1+\beta}\right)\frac{dS^2(x^2)}{dx^2} \tag{22}$$

and $\frac{dQ^1}{dx^2} = \frac{dQ^2}{dx^1} = 0$. If one substitutes (20) and (21) in the profits functions in (14) and (15) one has two non-linear equations in x^1 and x^2. The first order conditions are

$$-w^1\frac{dx^1}{dS^1} + \left(\frac{2\beta}{1+\beta}\right)A - \left(\frac{2\beta}{1+\beta}\right)\left(B+c^1\right)Z^1 - \left(\frac{\beta}{1+\beta}\right)BZ^2 = 0 \tag{23}$$

$$-w^2\frac{dx^2}{dS^2} + \left(\frac{2\beta}{1+\beta}\right)A - \left(\frac{2\beta}{1+\beta}\right)\left(B+c^2\right)Z^2 - \left(\frac{\beta}{1+\beta}\right)BZ^1 = 0 \tag{24}$$

Comparison of (16) and (17), the monopoly case, with (23) and (24), the duopoly case reveals that only the "off-diagonal" terms are larger for the monopoly case. (23) and (24) can be viewed as the reduced-form reaction functions for the duopoly game. We solved for Q^1, Q^2, x^1 and x^2 for $S^i(x^i) \equiv E^i\cdot\left(x^i\right)^{.5}$. The results are reported in Table 1. Note that the slope of the reaction function in (23), for example, is

$$\frac{dx^1}{dx^2} = \frac{\left(\frac{\beta}{1+\beta}\right)B\ \frac{dS^2}{dx^2}}{w^1\left|\frac{dS^1}{dx}\right|^{-2}\frac{d^2S^1}{{}^{1\ 2}} - \left|\frac{2\beta}{1+\beta}\right|\left(B+c^1\right)\frac{dS^1}{{}^{1}}}$$

Stability of reactions requires that $\frac{dx^1}{dx^2}$ above, corresponding to player 1's reaction function, be less than $\frac{dx^1}{dx^2}$ corresponding to player 2's reaction function i.e. in the neighborhood of the solution (x^1, x^2)

$$\frac{\dfrac{dS^2}{dx^2}}{w^1\left(\dfrac{dS^1}{dx^1}\right)^{-2}\dfrac{d^2S^1}{d(x^1)^2} - \left(\dfrac{2\beta}{1+\beta}\right)\left(B+c^1\right)\dfrac{dS^1}{dx^1}} < \frac{\dfrac{dS^1}{dx^1}}{w^2\left(\dfrac{dS^2}{dx^2}\right)^{-2}\dfrac{d^2S^2}{d(x^2)^2} - \left(\dfrac{2\beta}{1+\beta}\right)\left(B+c^2\right)\dfrac{dS^2}{dx^2}}$$

For the symmetric case and $\frac{d^2S^1}{d(x^1)^2} < 0$, stability obtains. Clearly however for $\frac{d^2S^1}{d(x^1)^2} > 0$ it is possible to have the above inequality reversed and observe unstable reactions in the neighborhood of the solution. In other words, increasing returns to scale in exploration can bring about instability of "the equilibrium". Ultimately, a stable solution may be obtained at a boundary with one player not exploring, a "natural monopoly".

(iii) Co-operation in exploration and downstream duopoly

This case displays the striking property that <u>the same level of exploration is undertaken as with pure monopoly though downstream quantities extracted differ</u>. Equations (20) and (21) solve the downstream game in quantities given x^1 and x^2. One inserts for Q^1 and Q^2 in $\pi = \pi^1 + \pi^2$ from (20) and (21) and solves for $\frac{\partial \pi}{\partial x^1} = 0$ and $\frac{\partial \pi}{\partial x^2} = 0$. The resulting equations turn out to be (16) and (17). Hence our result on equal exploration efforts in the two different games. In Table 1, we report on some numerical solutions.

A comparison of the outcomes under three different strategic modes is contained in Table 1. T is the two plant monopoly case; D the two stage Cournot duopoly case and M the mixed case of downstream duopoly in quantities and upstream monopoly or co-operation in extraction. The base case is symmetric, marginal extraction cost is increasing in quantity and discoveries are concave in effort. Demand is linear in price. For the base case we observe the duopolists each deliver more in period 1 than for T and M and explore almost twice as much as under T and M. Competition thus promotes greater output and exploration (and discovery since the link is non-stochastic). We see numerically our result established earlier, namely the T and M modes each explore at the same intensity though quantities delivered in period 1 differ, being larger for the M mode.

Consider the comparative static results in Table 1. A rise in the interest rate (the discount factor changes from .9 to .85 in the second row group) shrinks quantities and exploration levels.

A decline in initial stock reserves for firm 2 or plant 2 (S^2 is reduced from 2.4 to 2.3) induces more exploration by both players (firms or plants), reduces Q^2 output in period 1 slightly and induces Q^1 to rise relative to the base case. Increased initial scarcity of one player's stock induces greater search activity and greater production in period 1 by the other player.

A rise in player 2's extraction costs (parameter c^2 increases from .10 to .11) induces a decline in 2's research effort and first period production but an expansion in firm 1's research effort and production in period 1.

A rise in player 2's exploration costs (parameter w^2 rises from .55 to .56) induce less exploration by 2 and less production in period 1 while inducing more exploration by player 1 and more production in period 1.

Finally a decline in the productivity of research effort for firm 2 (E^2 declines from 1.0 to 0.99) induces less exploration by player 2 and less production in period 1 while at the same time inducing more exploration by player 1 and more production in period 1.

Table 1

inverse demand	$p = 10-(\cdot)$	discovery function	$E^i(x^i)^{0.5}$ $E^1=E^2=1.0$
extraction cost	$c^i\cdot(\cdot)^2$, $c^1=c^2=.1$	$\beta = \frac{1}{1+r} = 0.9$	
exploration cost	$w^i x^i$, $w^1=w^2=.55$	T for 2 plant monopoly	
reserves	$S^1 = S^2 = 2.4$	D for duopoly	
		M for the mixed case	

		Q^1	Q^2	x^1	x^2
	T	1.982607	1.982607	2.313299	2.313300
Base	D	2.358098	2.358098	4.977300	4.977296
Case	M	2.021767	2.021768	2.313298*	2.313300
	T	1.986952	1.986950	2.263144	2.263128
$\frac{1}{1+r} = .85$	D	2.367954	2.367954	4.850182	4.850183
	M	2.047278	2.047277	2.263135	2.263128
	T	1.995980	1.952369	2.399977	2.424612
$S^2 = 2.3$	D	2.363753	2.332523**	5.030712	5.184702
	M	2.035141	1.991528	2.399983	2.424603
	T	1.992768	1.966214	2.342229	2.248544
$c^2 = .11$	D	2.361190	2.346082	4.998472	4.882116
	M	2.027105	2.009748	2.342230	2.248545
	T	1.986263	1.974340	2.336840	2.260519
$w^2 = .56$	D	2.360370	2.347825	4.988724	4.880999
	M	2.025422	2.013501	2.336833	2.260524
	T	1.986684	1.973388	2.339557	2.300254
$E^2 = .99$	D	2.360632	2.346638	5.001197	4.968818
	M	2.025845	2.012547	2.339559	2.300251

* Values for x^1 and x^2 are the same for the T (monopoly) and M (mixed) cases (see text).

** This is not economically feasible since Q^2 exceeds the initial reserves of size 2.3 in the first period.

Concluding Remarks

We have considered a market failure in exploration for exhaustible resources arising from the large size of players relative to the market. We considered duopoly outcomes. In a two stage duopoly game we observed more exploration and production early on than occurs in a pure monopoly or partial monopoly. Duopoly in output markets did not affect exploration levels in a mixed model relative to a pure monopoly model for a particular example. Though our analysis may not have clear policy implications it is of interest in comparing R & D activity and exploration activity since the two stage game framework has been made use of recently in R & D economics (eg. Spencer and Brander [1983] and d'Aspremont and Jacquemin [1987]).

The Spencer-Brander R & D game can be readily reformulated and made similar to our game of exploration and extraction. In the R & D game, there will be one open-ended period. Firm i's output is produced with a variable input L^i and knowledge capital K^i as in $Q^i = f^i(L^i,K^i)$. Variable costs are w^i per unit of L^i and knowledge capital is augmented by investment I^i costing s^i per unit and increasing K^i by $g^i(I^i)$. Let there be an industry inverse demand schedule $p(Q^1+Q^2)$. Firm i's profit is then $p(Q^1+Q^2)\ f^i\left[L^i,K^i+g^i\left(I^i\right)\right] - w^iL^i - s^iI^i$. In a sense of priorness, a R & D game is played conditional on a game in output levels being played downstream. We suppose implicitly that investment in new knowledge is not leaked to the rival although such a model could be investigated (d'Asperemont and Jacquemin). Thus property rights problems are not essential here. Two differences in this model and the exploration-extraction model are (i) in the latter even in perfect competition these are rents to be used to pay for exploration activity whereas in the R & D game there is a shadow price corresponding to the value of additional knowledge. Output price will lie above current operating or variable costs. Exploration and R & D investment are paid for in somewhat different ways in the two models. (ii) In the R & D game there is only an implicit capital loss on existing knowledge as competition increases i.e. the shadow price of K^i changes as competition increases or decreases whereas in the exploration-extraction game the capital losses on existing reserves from more competition are directly observed as output price changes. Presumably the K^i in the R & D game is marketable as say patents and its value depends on the marginal product of the knowledge <u>and</u> the price of output produced with the patents. The shadow price $p \, \partial f^i / \partial \left(K^i + g^i(I^i)\right)$ will change with the intensity of competition (duopoly vs monopoly for example.) These differences in the models are really of a second order significance. The Spencer-Brander formulation focuses on cost functions rather than the production function and contains no potential capital loss term.

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