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PRICE DISCRIMINATION AND EQUILIBRIUM
IN MONOPOLISTIC COMPETITION[†]

by

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Abstract

Modern theories of monopolistic competition have borrowed extensively from techniques developed in location theory and the theory of spatial pricing: the monopolistically competitive firm is assumed to choose a "location" and price for its product. A subject that has been of increasing concern in this corpus of theory is that there exists no free-entry price-location equilibrium. In this paper we demonstrate that free-entry price-location equilibrium exists provided only that producers are allowed to price discriminate among customers in a reasonable manner. Equilibrium is modelled as a two-stage game using the Selten concept of subgame perfect Nash equilibrium. It is shown that the equilibrium discriminatory price system is one initially identified by Hoover. In addition, we show that equilibrium is not unique. The precise nature of equilibrium in a particular market will be dependent upon the history of that market.

1. Introduction

In this paper we demonstrate that a free-entry equilibrium in price-location models exists under quite general conditions, provided only that producers are allowed to price discriminate among customers in a reasonable manner.

This result is important since nonexistence of equilibrium in price-location models has concerned several authors in recent years. Gabszewicz and Thisse (1986) show that, given uniform mill pricing, a price-location equilibrium exists only under restrictive conditions. In particular, even introducing somewhat ad hoc additional restrictions such as the "no mill price undercutting" assumption proposed by Eaton (1972) does not, in general, solve the existence problem - as demonstrated by MacLeod (1985).

The standard model in which nonexistence proves to be a problem assumes that each firm charges a uniform mill price to all customers. In the context of spatial competition this is equivalent to f.o.b. pricing and may arise because:

- (i) transportation is under the control of customers, or
- (ii) institutional constraints (such as those proposed by proponents of the Robinson-Patman Acts and anti-dumping legislation) prevent price discrimination.

Price discrimination in the context of a spatial model has an obvious definition: it is manifested in some degree of freight absorption by the producer and is feasible so long as the producer controls transport of its good, i.e., has knowledge of customers' locations.

Allowing firms to price discriminate among their customers obviates most of the nonexistence problems noted above. Lederer and Hurter (1986) derive a similar conclusion in the context of the Hotelling duopoly model. The existence results presented in this paper expand significantly upon the Lederer and Hurter analysis in two major and one minor respects. The more significant extensions are first, that we allow firms to operate multiple plants, in contrast to the normal duopoly (and free entry) assumption that each firm is a single-plant firm. Secondly, we allow for free entry, so putting the analysis in a monopolistically competitive environment. By doing so we are able to discuss the welfare properties of the free-entry equilibrium. It turns out, in fact, that equilibrium may generate too much, too little, or socially optimal product variety. Our minor extension is to provide a product differentiation interpretation of what is essentially a spatial price discrimination model.

The model on which our analysis is based is detailed in Section 2. In Section 3 we discuss the properties of a price-location equilibrium. It is well known that the spatial model with uniform mill pricing can be given a differentiated product interpretation. We indicate in Section 4 how our discriminatory pricing model can also be interpreted as a model of product differentiation. Our conclusions are presented in Section 5.

2. The Basic Model

All firms in the "industry" produce a single homogeneous product but, in contrast to other models of this type, may decide to operate multiple plants. Production costs are identical for all plants¹, the plant production cost function being assumed to be of the form:

$$C = f + m Q \quad (1)$$

where f = fixed costs contingent upon entry;

m = constant marginal cost;

Q = total output of the plant.

Thus production is assumed to be characterised by economies of scale at the plant level.

The industry's customers are continuously distributed over a compact set S of the q -dimensional space R^q , $q \geq 1$. (S need not be connected.) The density of customers $v(\cdot)$ is assumed to be positive and continuous on S . All customers at location $s \in S$ are assumed to be identical and to purchase exactly one unit of the product.²

Transportation is under the control of the producer. Transport cost per unit of output transported from production location $x_{ik} \in S$ to customer location $s \in S$ is given by a function $t(\cdot)$ which is increasing, continuous and concave³ in distance, and such that $t(0) = 0$. The distance between x_{ik} and s is defined by $\|x_{ik} - s\|$ where $\|\cdot\|$ is a norm⁴ on R^q .

Producers are free to price discriminate among customers by means of freight absorption. In other words, a producer can charge different mill

prices to customers at different locations. We assume, however, that the same mill price is charged to customers at a given location: if this were not the case there is scope for arbitrage between customers at the same location.

We would like to characterise equilibrium in this model when firms are free to choose first locations, and then prices. Formally, this is modelled as a two-stage game using the Selten (1975) concept of subgame perfect Nash equilibrium. In the first stage firms decide whether or not to enter the market and, given that they decide to enter, the number of plants to operate and where to locate them. In the second stage each firm chooses a price schedule for each of its plants, given the total number of plants in the market, their ownership and their locations.

Let us now describe this game in some detail.

At the entry stage the number of firms is unknown. Hence, let $\mathcal{I} = \{1, 2, \dots\}$ denote the set of possible firms. In the first stage, firm $i \in \mathcal{I}$ chooses the number of plants and their locations. Formally, a strategy for firm i is an infinite vector $x_i = (x_{ik})$ with

$$x_{ik} \in X_i \equiv S \cup \phi \quad (2)$$

where $x_{ik} \in S$ denotes that firm i has decided to enter and to locate plant k at the appropriate point in S , while $x_{ik} = \phi$ denotes a decision not to set up plant k . If $x_{ik} = \phi$ for all $k = 1, 2, \dots$ then firm i has decided not to enter the market. The existence of fixed costs and the compactness of the space S ensure that there will be a finite (unknown) number of plants operating in equilibrium. Without loss of generality, we may assume

that the set $\mathcal{X}_i$ of active plants for firm i corresponds to the first K_i elements in x_i .

Let the infinite vector $x = (x_i) \in X \equiv \prod_{i=1}^{\infty} X_i$ be the strategies

chosen by the firms in the first stage. We shall refer to x in subsequent analysis as a location configuration. Again without loss of generality, we may assume that the set $\mathcal{I}_e$ of firms that have actually entered the market corresponds to the first I_e elements of x . The total number of active plants in the market is

$$n = \sum_{i \in \mathcal{I}_e} K_i \quad (3)$$

In the second stage subgame, the strategy of firm $i \in \mathcal{I}_e$ is to choose for each plant $k \in \mathcal{X}_i$ a price function $p_{ik}(\cdot)$ which associates with each customer location s a delivered price $p_{ik}(s)$ at which the firm is willing to sell one unit of the commodity supplied from its plant k . Formally:

$$p_{ik}(\cdot) \in \mathcal{P} = \{p_{ik}(\cdot): s \rightarrow R_+, p_{ik}(\cdot) \text{ is measurable and } p_{ik}(s) > m + t(\|x_{ik} - s\|) \text{ a.e.}\}. \quad (4)$$

In other words, firms never price below marginal costs.

Let us now describe this subgame in some detail.

The only restriction we place on the price functions is that for each firm i the lowest delivered price to a customer location s is charged by that firm's nearest plant to s . Subsequent analysis is unaffected if this restriction is relaxed, but at the expense of some very awkward notation.

We want to specify the demand D_{ik} to plant k of firm i from customers at location s for any n -tuple $p(s) = \{p_{ik}(s)\}$. We assume that customers buy from the firm offering the product at the lowest price: from the above restriction on $p(\cdot)$ the supplying plant for this firm will be its plant nearest to s . However, there remains the familiar problem of the allocation of demand among firms when there is a price tie. We assume that customers act in a socially efficient manner and buy from the nearest low-price firm: without such an assumption equilibrium would not exist. This assumption can be rationalised on the basis that the nearest firm to s has a cost advantage over its rivals which would allow it to price ϵ below any admissible price quoted by these rivals.⁵ Price ties can, therefore, only occur at a customer location s if there are two or more nearest plants equidistant from s . In the event of such a price tie, we assume that demand $v(s)$ at s is shared equally among the nearest low-price plants equidistant from s . Let $b(s)$ denote the number of such plants (note that $b(s) \geq 1$). Then the demand to firm i 's plant k is given by:

$$\begin{aligned} D_{ik}[x, p(s); s] &= v(s)/b(s) \text{ if } i \text{ is a low-price firm with} \\ &\quad \text{a plant nearest to } s; \\ &= 0 \quad \text{otherwise.} \end{aligned} \quad (5)$$

Profit of plant x_{ik} is:

$$\pi_{ik}[x, p(\cdot)] = \int_S [p_{ik}(s) - m - t(\|x_{ik} - s\|)] D_{ik}[x, p(s); s] ds - f, \quad (6)$$

and profit of firm i is

$$\pi_i[x, p(\cdot)] = \sum_{k \in K_i} \pi_{ik}[x, p(\cdot)]. \quad (7)$$

We now define a Nash equilibrium for the second stage price subgame.

Denote by $p_i(\cdot) = (p_{i1}(\cdot) \dots p_{iK_i}(\cdot)) \in \mathcal{P}^{K_i}$ the vector of price

functions of firm i . Then the Nash equilibrium for the price subgame, given any location configuration x , is an n -tuple of price functions

$$p^*(\cdot|x) = \{p_{ik}^*(\cdot|x)\} \in \mathcal{P}^n$$

such that for all $i \in \mathcal{I}_e$:

$$\pi_i[x, p^*(\cdot|x)] \geq \pi_i[x, p_i(\cdot), p_{-i}^*(\cdot|x)] , \text{ for all } p_i(\cdot) \in \mathcal{P}^{K_i}. \quad (8)$$

This means that, given the price schedules of all other firms, firm i chooses the price at each customer location s that maximises its profit density at s . It follows, of course, that in equilibrium if firm i supplies customer location s it will do so from its plant nearest to s .

Now consider the payoffs in the first stage. Let $p^*(\cdot|x)$ be the price schedule equilibrium corresponding to x in the second stage subgame. The profit for firm $i \in \mathcal{I}$ is given by:

$$\pi_i^*(x) = \begin{cases} \pi_i[x, p^*(\cdot|x)] & , \quad \text{for } i \in \mathcal{I}_e \\ 0 & , \quad \text{for } i \in \mathcal{I} - \mathcal{I}_e \end{cases} \quad (9)$$

where $\pi_i[x, p^*(\cdot|x)]$ is as defined in equation (7).

An equilibrium for the first stage subgame is a location configuration x^* such that

$$\pi_i^*(x^*) \geq \pi_i^*(x_i, x_{-i}^*) , \text{ for all } x_i \in X_i \text{ and } i \in \mathcal{I}. \quad (10)$$

This means that, given the strategies x_{-i}^* of other firms, firm i decides

to enter (choose strategy $x_{ik}^* \in S$ for $k \in \mathcal{K}_i^* \neq \emptyset$) only if its after-entry profit is non-negative, and chooses the best possible number and locations of its plants.

The vector $(x^*, p^*(\cdot|x))$ describes a subgame perfect Nash equilibrium.

This equilibrium has the following properties:

- (i) no firm $i \in \mathcal{I}_e^*$ prefers any other vector of price functions to $(p_{i1}^*(\cdot), \dots, p_{iK_i^*}^*(\cdot))$ or any location vector to $(x_{i1}^*, \dots, x_{iK_i^*}^*)$;
- (ii) no firm $i \in \mathcal{I}_e^*$ prefers any other number of plants than K_i^* (this includes leaving the market, when $K_i = 0$);
- (iii) no firm $i \in \mathcal{I} - \mathcal{I}_e^*$ wishes to enter the market (given that it anticipates the post-entry price equilibrium).

Notice, finally, that prices and locations are not treated in the same manner in our equilibrium concept: the decision of a firm $j \in \mathcal{I} - \mathcal{I}_e^*$ not to enter the market and of a firm $i \in \mathcal{I}_e^*$ not to open additional plants is made conditionally upon the existing locations of the active firms ^{and} upon the price equilibrium that would emerge after entry of the new firm (or plant). This difference reflects the fact that, in the real world, locations are relatively difficult to modify whereas prices are relatively flexible: an approach that is standard in the literature.

3. Analysis

3.1 Equilibrium in the Price Subgame

Since transport costs to a point are unaffected by transport to other points and since m is constant, there is a separate Bertrand game at every point s . A standard Bertrand-like argument then runs as follows. Assume for some customer location s that the nearest plant is x_{ik} . Despite our assumption that plant production costs are identical, plant x_{ik} has a (transport) cost advantage which allows it to undercut any price set at s by its rivals. The price undercutting process will stop when no rival firm can further reduce its price, i.e., when price is equal to the marginal production and transport costs incurred by the nearest rival plant in supplying customers at s . Assume that the nearest rival plant to s is $x_{jl}(j \neq i)$. Then the equilibrium price at s is $p_{ik}(s) = m + t (\|x_{jl} - s\|)$. Returning to the allocation rule introduced above in defining demand D_{ik} , in equilibrium customers at s do not buy from the second nearest firm (plant x_{jl}) since we assume that customers act in a socially efficient manner and buy from the nearest low-price firm. The only set of customers for whom there will be a price and purchasing tie is the set of customers for whom the nearest competing firms are equidistant, and this set has zero measure provided only that the nearest competitors are not coincidentally located ($x_{ik} \neq x_{jl}$). We shall show below that equilibrium in the location game precludes coincident locations of rival plants.

Formally:

Proposition 1. Given any location configuration x with at least two firms there exists an equilibrium in price given by:

$$p_{ik}^*(s|x) = m + \max_{\substack{j \neq i \\ l \in \mathcal{N}_j}} \{t(\|x_{ik} - s\|), \min_{j \neq i} t(\|x_{kl} - s\|)\}, i \in \mathcal{I}_e. \quad (11)$$

Furthermore, this equilibrium is unique in that no other equilibrium exists in which a non-zero measure set of customers would be supplied at different prices.

The price subgame equilibrium is illustrated for three configurations of plants by the heavy line in Figure 1 when S is an interval of a one-dimensional space and transport costs are linear: firm i supplies all customers in (a,b) and shares with firm $i-1(i+1)$ demand from customers at location $a(b)$, while plant x_{i1} , for example, supplies all customers in (a,b') . It corresponds to a pricing policy first identified by Hoover (1937), rediscovered by Gee (1976) and formally investigated by Lederer and Hurter (1986) for the duopoly case.

The following remarks are in order. First, Proposition 1 guarantees the existence of a price equilibrium for any configuration of locations x . This is to be contrasted with the f.o.b. pricing case where such an equilibrium does not exist when some firms are "sufficiently close" to each other: see, e.g. d'Aspremont, Gabszewicz and Thisse (1979). Secondly, when two firms desire the same location marginal cost pricing occurs in the vicinity of the firms, as in Bertrand (1883). Thirdly, existence of a price equilibrium is guaranteed no matter the distribution of customers and equilibrium prices are independent of the customer density. Fourthly, the price equilibrium is such that neighbouring plants under common ownership do not compete on price. Finally, it is worth emphasising that, due to the assumed concavity of transport costs, the delivered price schedule of Proposition 1 precludes profitable retrading between customers (provided,

of course, that customers do not have access to a more efficient transport technology than that available to producers).

One further remark is in order. It would seem from a casual analysis of Figure 1 that delivered price always decreases with distance from the supplying plant. This holds, however, only in the one-dimensional case. Consider, by contrast, the two-dimensional case depicted in Figure 2. Assume initially that only two firms are located as shown in Figure 2(a), each operating a single plant. It is easy to show that price falls with distance from the supplying firm only for customers inside the circle with segment (x_{i2}, x_{j1}) as diameter: the shaded area illustrated. Outside this circle delivered prices increase with distance from the supplying firm.

Now introduce an additional firm as in Figure 2(b). Decreasing price with distance from the supplying firm now holds for a different subset of customers: those located in the shaded areas of Figure 2(b).

This implies that the behaviour of the equilibrium delivered price schedule may be very intricate in anything other than the standard, one dimensional case. Delivered price may fall in some directions while increasing in others. This is consistent, for example, with the empirical observations by Greenhut (1981) of pricing policy in West Germany.

It should also be recognised that we have confined this example to a simple, two-dimensional, two or three firm, Euclidean distance case. The behaviour of price in multi-dimensional space with more general distance norms remains to be investigated. There is certainly no reason to believe that equilibrium price schedules bear any simple relationship to distance.

(a)

(b)

(c)

Figure 1 : Equilibrium in the Price Subgame

(a)

(b)

Figure 2

3.2 Location Equilibrium

In proving the existence of a location equilibrium we need some intermediate results that of themselves offer interesting economic insights.

Let x be any location configuration containing n active plants, such that there are I_e active firms, with firm $i \in \mathcal{I}_e$ having K_i active plants. Define

$$F(x) = \int_S \min_{\substack{i \in \mathcal{I}_e \\ k \in \mathcal{K}_i}} t(\|x_{ik} - s\|) v(s) ds. \quad (12)$$

Clearly, $F(x)$ is the total transport cost in S associated with the configuration x when customers at location s are supplied from the nearest active plant (as will be the case in the price equilibrium of Proposition 1). Define also

$$C(x) = F(x) + \int_S mv(s)ds + nf \quad (13)$$

that is, the total transport and production cost associated with x .

Lemma 1:

$$\pi_i^*(x) = C(x_{-i}) - C(x) = F(x_{-i}) - F(x) - K_i f. \quad (14)$$

Proof ⁶

$$\text{Let } \alpha(s, x) = \min_{\substack{i \in \mathcal{I}_e \\ k \in \mathcal{K}_i}} t(\|x_{ik} - s\|),$$

$$\alpha_{-i}(s, x) = \min_{\substack{j \in \mathcal{J}_e - \{i\} \\ k \in \mathcal{K}_j}} t(\|x_{jk} - s\|),$$

$$\alpha_i(s, x) = \min_{k \in \mathcal{K}_i} t(\|x_{ik} - s\|).$$

Let $S_i = \{s \in S; \alpha_i(s, x) < \alpha_{-i}(s, x)\}$ be the market area of firm i . On S_i , we have $\alpha(s, x) = \alpha_i(s, x)$ while on $S - S_i$, $\alpha(s, x) = \alpha_{-i}(s, x)$. Then

$$\begin{aligned} \pi_i^*(x) &= \int_{S_i} [\alpha_{-i}(s, x) - \alpha_i(s, x)] v(s) ds - K_i f \\ &= \int_S \{\alpha_{-i}(s, x) - \alpha(s, x)\} v(s) ds - K_i f \\ &= F(x_{-i}) - F(x) - K_i f. \end{aligned}$$

Q.E.D.

In other words, the profit of firm i , given the locations of all other firms, is the amount by which the setting up of K_i plants at locations $x_{i1}, \dots, x_{iK_i}$ reduces total transport costs, less the corresponding fixed costs. This is illustrated in Figure 1. Prior to entry of firm i total transport costs to customers located between $x_{(i-1)1}$ and $x_{(i+1)1}$ are given by the area of the triangle BDF. If firm i enters with a single plant at x_{i1} then, as illustrated in Figure 1(a), total transport costs are reduced to the sum of the areas of the triangles BCA and AEF. Profit gross of fixed cost for firm i in this case is given by the shaded area ACDE. A similar argument applies to the multiple-plant entry cases illustrated in Figures 1(b) and 1(c).

It is then immediate that:⁷

Lemma 2:

Any configuration $\bar{x}$ such that

$$C(\bar{x}) \leq C(\bar{x}_{-i}, x_i) \quad , \quad \text{for all } x_i \in X_i \text{ and all } i \in \mathcal{I} \quad (15)$$

is a Nash equilibrium of the first-stage game.

It can be shown that (the proof is contained in the Appendix):

Lemma 3

There exists a location configuration x^0 such that

$$C(x^0) \leq C(x) \text{ for all } x \in X. \quad (16)$$

In other words, the location configuration x^0 for which total cost $C(x)$ is minimised satisfies Lemma 2 and is, therefore, a Nash equilibrium of the first stage game.

Let n^0 be the number of active plants in x^0 . Proposition 1 and Lemmas 2 and 3 imply:

Proposition 2: There exists a subgame perfect Nash equilibrium

$(x^*, p^*(\cdot|x))$ for the two-stage location-price game such that:

$$(i) \quad x^* = x^0 \text{ with } K_i^* = 1 \text{ for } i = 1, \dots, n^0$$

$$K_i = 0 \text{ for } i > n^0$$

$$(ii) \quad p^*(\cdot|x) \text{ is given by (11).}$$

Once more, some remarks are in order. Firstly, in contrast to standard models in monopolistic competition - see, e.g. Archibald, Eaton and Lipsey (1986), Lancaster (1979), Salop (1979) - we have relaxed the assumption that firms only operate single plants. Nevertheless, we have shown that there exists an equilibrium in which firms choose to run a single plant.⁸

Secondly, no subset of firms will agglomerate since agglomeration leads

to marginal cost pricing and negative profit. In other words, the principle of minimum differentiation does not hold in the present setting.

Thirdly, when fixed costs are very small relative to the market, the number of firms is large. As a result, transport costs arising at equilibrium become negligible (see the proof of Lemma 3 in the Appendix) and the free-entry equilibrium is approximately competitive.

Fourthly, and more importantly, the number and the location of plants obtaining at the free-entry equilibrium with the location configuration x^0 minimises total costs. As a result, this equilibrium location configuration is the socially optimal one.⁹ This final point should be qualified. We have shown in Proposition 2 that the socially optimal number and locations for firms is a free-entry equilibrium. But Lemma 2 implies that this equilibrium need not be unique. Other equilibria can be identified in which total transport costs are minimised for the number of active firms in those equilibria, but which contain a number of plants that may be greater or smaller than the optimal number.

To see this, consider the following example. Assume that all customers are uniformly distributed along the circumference of a unit circle; let also $t(\|\cdot\|) = \|\cdot\|$ and $m = 0$. Clearly, for any integer n , the location configuration $x^0(n)$ which minimises total transport costs for n plants is given by the equidistant pattern of the n plants. Furthermore, a simple calculation shows that the number of plants minimising total costs is given by

$$n^0 = (4f)^{-1/2}$$

(17)

(ignoring integer constraints). From Proposition 2, it then follows that the location configuration

$$x^0 = x^0(n^0) \quad \text{with} \quad K_i^0 = 1 \quad \text{for} \quad i = 1, \dots, n^0$$

$$K_i^0 = 0 \quad \text{for} \quad i > n^0,$$

is a Nash equilibrium of the first-stage game.

Let now $n^{\min}$ be the smallest number of firms that can make a nonnegative profit at the equidistant locations and such that, given these locations, no active firm wants to increase its number of plants and no potential firm wants to enter the market:

$$n^{\min} = \min\{n; \pi_i^*[x^0(n)] > 0 \quad \text{for} \quad i = 1, \dots, n \quad \text{and}$$

$$\pi_j^*[x_{-j}^*(n), x_j] < \pi_j^*[x^*(n)] \quad \text{for all } x_j \in X_j \text{ and } j \in \mathcal{J}\}.$$
(18)

It then follows immediately from Lemma 1 and the definition of $x^0(n)$ that the location configuration

$$x^{\min} = x^0(n^{\min}) \quad \text{with} \quad K_i^{\min} = 1 \quad \text{for} \quad i = 1, \dots, n^{\min}$$

$$K_i^{\min} = 0 \quad \text{for} \quad i > n^{\min}$$

is a Nash equilibrium of the entry-location game. The value of $n^{\min}$ remains to be determined. To this effect, assume that firm i considers setting up two plants. It is clear from Figure 1 that these should be neighbouring plants, and that they should be located distance $2/3n$ from the rivals $(i-1)$ and $(i+1)$. Profit to Firm i , given the locations of all other firms, is then $2/3n^2 - 2f$. For the above single-plant configuration to be an equilibrium, we must have

$$\pi_i^*[x^0(n)] - \frac{1}{2n^2} - f \geq \frac{2}{3n^2} - 2f$$

which is equivalent to

$$n \geq (6f)^{-1/2}. \quad (19)$$

Note that the transport costs supported by firm i define a convex function of the number of plants established between $(i-1)$ and $(i+1)$. Thus, if (19) holds so that it does not pay for firm i to set up one additional plant, then neither will it pay firm i to set up two or more additional plants. Furthermore, since profit of an additional firm with one plant is $1/8n^2 - f$, (19) also guarantees that no potential firm wants to enter the market. As a consequence, we have

$$n^{\min} = (6f)^{-1/2}. \quad (20)$$

Finally, let $n^{\max}$ be the largest number of firms that can earn a nonnegative profit at the locations given by $x^0(n)$:

$$n^{\max} = \max\{n; \pi_i^*[x(n)] > 0 \text{ for } i = 1, \dots, n\}.$$

Since $\pi_i^*[x^0(n)] = \frac{1}{2n^2} - f$, it is straightforward that

$$n^{\max} = (2f)^{-1/2}. \quad (21)$$

Given that $n^{\max} > n^{\min}$, no firm (active or potential) wants to increase its number of plants so that the location configuration

$$x^{\max} = x^0(n^{\max}) \text{ with } K_i^{\max} = 1 \text{ for } i = 1, \dots, n^{\max}$$

$$K_i^{\max} = 0 \text{ for } i > n^{\max}$$

is also a Nash equilibrium of the first-stage game.

A simple comparison of (17), (20) and (21) shows that

$$n^{\min} < n^0 < n^{\max}. \quad (22)$$

This indicates that a prediction about excess or insufficient variety is impossible in the present setting: any integer n belonging to $[n^{\min}, n^{\max}]$ gives rise to a free-entry equilibrium in which n firms are equidistantly located.¹⁰

The discussion underlying equation (22) assumes a particular market "shape" and a uniform distribution of customers. It can be shown that the location configurations $x^0(n^{\min})$ and $x^0(n^{\max})$ are Nash equilibria of the entry-location game, and that (22) holds for any market S and any distribution of customers (proof from the authors on request). We offer the conjecture that the stronger result - existence of a free-entry equilibrium for any $n \in [n^{\min}, n^{\max}]$ - also holds in this more general setting.

4. Product Differentiation

There is a standard interpretation of the spatial model as a model of (horizontal) product differentiation in the uniform mill pricing case. Uniform mill pricing is equivalent to customers paying a full price consisting of a (constant) money price for the product plus the value of utility loss consequent upon the purchase of a product other than the customer's "most preferred" product - see, for example, Archibald et al (1986), Lancaster (1979).

Is there a similar nonspatial interpretation of the spatial model in the discriminatory case? The analogy between geographical and characteristic space in this case is not quite so straightforward. In the context of product differentiation, price discrimination arises when the producer begins with a "base product" and then redesigns this product to the customers' specifications. This means that the firm now produces a band of horizontally differentiated products instead of a single product as in the standard model. Transport cost is no longer interpreted as a utility loss, but as an additional cost incurred by the firm in adapting its base product to the customers' requirements. It should be emphasised that such customising is assumed to change the characteristics content of goods, but not their quality.¹¹ It is quite clear, however, that so long as product design is under the control of the producer - equivalent to the producer controlling transportation - he need not charge the full cost of design change. Arbitrage between customers can be ruled out so long as the producer does not offer the consumer the option of do-it-yourself customising; we further assume that consumers have access only to a less efficient customising technology.

There remains a slight complication in the product differentiation interpretation of our model when we consider firms competing to supply the same customers. The location of each firm is defined by the characteristics of its base product. Redesign of those base products to meet the stated requirements of a particular customer need not be interpreted as resulting in the firms offering the "same" product, but rather in offering products that yield the same utility level to the customer.

For this product differentiation interpretation to be meaningful we must modify, in particular, our assumption with respect to transport costs. Let us denote by a continuous function $t(x_{ik}, s)$ the total cost incurred by firm i when customising base product x_{ik} to the requirements of customers located at s in the characteristic space. Notice that $t(x_{ik}, s)$ represents not the marginal cost of redesigning product x_{ik} , but the total cost borne by the firm in meeting demand at s . We assume that $t(x_{ik}, s)$ exhibits non-decreasing returns to scale in quantity.¹² Furthermore, $t(x_{ik}, s)$ need no longer be a function of the "distance" between x_{ik} and s in the characteristic space. Any functional form is admissible.

Given this change, the profit earned from the product band derived from the base product x_{ik} can be written:

$$\pi_{ik}[x, p(\cdot)] = \int_S \{ [p_{ik}(s) - m] D_{ik}(s) - t(x_{ik}, s) \} ds - f \quad (23)$$

The equilibrium price at customer "location" s is now given by:

$$p_{ik}(s|x) = m + \max\{t(x_{ik}, s)/v(s), \min_{\substack{j \neq i \\ l \in \mathcal{K}_j}} t(x_{jl}, s)/v(s)\} \quad (24)$$

neglecting situations where $t(x_{ik}, s) = t(x_{jl}, s)$ holds for some $i \neq j$.

Generically, the set of locations for which this equality is satisfied is negligible.

It should be clear that this respecification of the profit and equilibrium price functions leaves unaffected the qualitative conclusions of section 3. What is the case, however, is that the product differentiation interpretation of the discriminatory model is likely to lead to much less connectedness of the market associated with any base product. Ongoing investigation of the policies of modern corporations in, for example, the manufacture of aircraft and soft drinks would appear to be consistent with our analysis.

5. Conclusions

Modern theories of monopolistic competition have borrowed extensively from techniques developed in location theory and the theory of spatial pricing. Firms are assumed to choose "locations" for their production, where these may be plant locations in the true spatial sense, or product differentiates in a Lancasterian sense, and to charge a uniform mill price to all customers. A subject that has been of increasing concern in this emerging corpus of theory is that, in general, there exists no free-entry price-location equilibrium.

In this paper we have demonstrated that all that is required to guarantee the existence of a free-entry price-location equilibrium is that firms be allowed to price discriminate among customers in a manner determined by competitive forces.

It remains to be explained why this result holds, whereas nonexistence appears to be inherent to uniform mill pricing systems, even if the customer distribution is constrained to be uniform. The essential point is that under constant marginal production and transport cost, the firm in deciding upon its discriminatory price schedule chooses a price for each market separately rather than a single price for the whole market. This effectively enlarges the strategy space and turns out to be sufficient to reduce the price-location instability that characterises uniform mill price systems.

It has also been shown that even though we allow firms to operate multiple plants, there exists an equilibrium in which each firm chooses to operate only one plant. We leave for subsequent analysis the comparison of

multi-plant and single plant equilibria. It is clear, for example, that any single plant equilibrium of the multi-plant game is an equilibrium of the single plant game, but there are equilibria of the single plant game that are not equilibria of the multi-plant game. Similarly, there will be equilibria of the multi-plant game that are not equilibria of the single plant game: competition is less in the former case and prices higher, increasing the potential number of entrant plants. It would appear that the set of equilibria of the multi-plant game contains configurations with more plants than the single-plant game.

The free-entry price-location equilibrium that results from discriminatory pricing is not unique, but this should not cause concern. Rather, it generates the appealing implication that the precise nature of equilibrium in a particular market will be dependent upon the history of that market. In addition, we find that equilibrium may generate the socially optimal level of product variety, but may also produce more or less product variety than is socially optimal.

Footnotes

1. This simplifies the notation but is not necessary for the analysis.
Firm-specific cost functions are allowable.
2. This is equivalent to assuming that the reservation price - which may be location-specific - is sufficiently high as not to be binding in equilibrium. A binding reservation price would make the analysis more cumbersome while leaving the results qualitatively unaffected.
3. The concavity assumption is reasonable to the extent that actual transport costs of commodities generally increase less rapidly than in proportion to distance: see Hoover (1948).
4. See Love and Morris (1978) and Ward and Wendell (1985) for a discussion of different norm distances in location models.
5. One referee correctly pointed out that for equilibrium to exist price ties must be broken as described in the text. This does not mean that our equilibrium is "fragile". Indeed, the firm with the cost advantage at s can always profitably undercut any of its rivals, and in particular the second nearest firm. If price were treated as a discrete variable then the ϵ argument in the text could be used where ϵ is now the smallest unit of account (one cent, one penny). The technical problem that arises is that when p is treated as a continuous variable, no "smallest" ϵ can be identified: hence the need for our "socially optimal" assumption.
6. The proof draws on Lemma 4 of Lederer and Hurter (1986).

7. Our analysis resembles Spence (1976), Proposition 1 but differs from him in that, in equilibrium, firms do not perfectly price discriminate.
8. The price-location equilibrium of Proposition 2 is unaffected if firms run multiple plants provided that no neighbouring plants are under common ownership. Given the intensity of competition under price discrimination, concentrating on this kind of equilibrium seems justified by Judd's (1985) analysis.
9. Eaton and Lipsey (1979) derive a similar result for a special case of our model.
10. Note that, in common with the analysis of Capozza and van Order (1980) and Eaton and Lipsey (1978), free-entry price-location equilibrium is consistent with the persistence of positive pure profits.
11. Customising, in other words, leads to horizontal, rather than vertical product differentiation. Differentiation through the supply of "add ons" to the base product is not covered by our analysis.
12. Decreasing returns may lead firms to wish to ration customers at a given location.

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AppendixProof of Lemma 3

Let n be any positive integer. Since S is compact, $x^0(n)$ exists such that $F[x^0(n)] \leq F(x)$ for all location configurations x having n plants. As $f > 0$ it is sufficient for the proof to show that $F[x^0(n)] \rightarrow 0$ when $n \rightarrow \infty$. Let C be a cube of R^q with centre 0 and side of length L such that $S \subset C$. Define u as the largest integer smaller than or equal to $n^{1/q}$ so that $u \rightarrow \infty$ when $n \rightarrow \infty$. Let $\mathcal{C}(n)$ be the u^q -tuple of points c_h of C whose coordinates are

$c_h^i = -\frac{L}{2} + k_h^i \frac{L}{u}$ with $i = 1, \dots, q$ and $k_h^i = 0, 1, \dots, u$. Then, for all $s \in S$, we have

$$\begin{aligned} \min_{h=1, \dots, u^q} t(\|c_h - s\|) &= t \left(\min_{h=1, \dots, u^q} \|c_h - s\| \right) \\ &\leq t(\lambda \min_{h=1, \dots, u^q} \|c_h - s\|_e) \quad \text{where } \lambda \text{ is a positive constant} \\ &\quad \text{and } \|\cdot\|_e \text{ the Euclidean norm} \\ &\leq t(\lambda \sqrt{q} \frac{L}{u}). \end{aligned}$$

Consequently,

$$\begin{aligned} F[x^0(n)] &\leq \int_S \min_{h=1, \dots, u^q} t(\|c_h - s\|) v(s) ds \\ &\leq t(\lambda \sqrt{q} \frac{L}{u}) \int_S v(s) ds. \end{aligned}$$

The lemma then follows from the fact that $t(y) \rightarrow 0$ when $y \rightarrow 0$. Q.E.D.