QUEENS

Disc. paper 599

ISSN 0316-5078

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INTEREST RATE AND OUTPUT PRICE UNCERTAINTY AND INDUSTRY EQUILIBRIUM FOR NONRENEWABLE RESOURCE EXTRACTING FIRMS

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DISCUSSION PAPER NO. 599

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Abstract

We establish convexity of a nonrenewable resource extracting agent's value function in the future interest rate, a random variable. A preference by the agent for future interest rate uncertainty follows. A rational expectations, m identical firm industry equilibrium is characterized and the links between interest rate uncertainty and output price uncertainty are investigated.

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INTEREST RATE AND OUTPUT PRICE UNCERTAINTY AND INDUSTRY EQUILIBRIUM FOR NONRENEWABLE RESOURCE EXTRACTING FIRMS

We established that a non-renewable resource extracting firm with orthodox convex extraction costs preferred output price uncertainty in Hartwick and Yeung [1985]. Here we establish the same result for future interest rate uncertainty and move on to characterize a rational expectations industry equilibrium for m identical firms. Interest rate uncertainty induces output price uncertainty in an m-firm industry so the interest rate shock is a convenient avenue to introduce output price uncertainty at the level of the industry. Of note is the fact that a realized low rate of interest induces an upward jump in industry price and vice versa for a realized high rate of interest. In an industry model, the uncertainty is realized at the level of the firm as a pair of observations, a specific interest rate and a specific output price. In this case, appealing to rational expectations, the output price is a deterministic function of the realized interest rate.

Interest Rate Uncertainty and Non-renewable Resource Extraction

We begin with an investigation on the optimal strategy of a resource owner facing interest rate uncertainty. Let s denote the owner's initial endowment of a homogeneous non-renewable resource stock and r the current rate of interest. At time T_a, a new interest rate will be announced. The probability distribution of interest rate is as follows: the probability of the interest rate being equal to r_i is π_i and the expected mean rate is equal to $\bar{r}$. Hence

$$\sum_{i=1}^{n} \pi_i r_i = \bar{r} \qquad \text{and}$$

$$\sum_{i=1}^{n} \pi_i = 1 \tag{1}$$

Let the instantaneous objective function of the resource owner be denoted by

$$f(q(t)) \tag{2}$$

where $q(t)$ is the quantity of the resource extracted.

The function $f(q(t))$ becomes $[P(t)q(t) - c(q(t))]$ for a risk-neutral competitive resource firm facing a given price path $P(t)$ and a cost function $c(q(t))$. For a monopolistic resource firm, the function f becomes $[P(q(t)) \cdot q(t) - c(q(t))]$ where $P = P(q(t))$ is the market demand of the resource. Finally, for a social planner, the instantaneous objective may be represented by the net social welfare brought about by resource consumption in $u(q(t))$.

The objective of the resource owner is to maximize the expected present value of discounted instantaneous objective,

$$\int_0^{T_a} f(q(t))e^{-rt}dt + \sum_{i=1}^{n} \pi_i \int_{T_a}^{\tau_i} f(q_i(t))e^{-r_i t}dt \tag{3}$$

subject to the resource constraint

$$\int_0^{T_a} q(t)dt + \int_{T_a}^{\tau_i} q_i(t)dt = s \qquad \text{for} \quad i=1,2,\ldots n \tag{4}$$

where τ_i is the time when the resource is completely exhausted.[1]

For a given interest rate r and a certain level s, define the function

$$V(s;r) = \max_{q(t)} \int_0^{\tau} f(q(t))e^{-rt}dt \tag{5}$$

subject to

$$\int_0^{\tau} q(t)dt = s \tag{6}$$

Lemma 1: The function $V(s;r)$ is convex in r.

Proof: Define three interest rates r_1, r_2 and $\hat{r}$ such that

$$\hat{r} = \alpha r_1 + (1-\alpha)r_2 \qquad (0 < \alpha < 1) \tag{7}$$

Denote optimal extraction path which maximizes (5) given s and $\hat{r}$ subject to (6) by $\hat{q}(t)$. We have then

$$\int_0^{\hat{\tau}} f(\hat{q}(t))e^{-\hat{r}t}dt - \alpha \int_0^{\hat{\tau}} f(\hat{q}(t))e^{-r_1 t}dt - (1-\alpha) \int_0^{\hat{\tau}} f(\hat{q}(t))e^{-r_2 t}dt$$

$$= \int_0^{\hat{\tau}} f(\hat{q}(t))[e^{-\hat{r}t} - \alpha e^{-r_1 t} - (1-\alpha)e^{-r_2 t}]dt \tag{8}$$

For $t>0$ and r_1 and r_2 distinct and positive the term $[e^{-\hat{r}t} - \alpha e^{-r_1 t} - (1-\alpha)e^{-r_2 t}]$ in (8) is negative since e^{rt} is convex in r. For $t=0$, the term in square brackets is zero. Hence (8) is negative.

Let $q_1(t)$ and $q_2(t)$ denote the optimal extraction path when the interest rate is equal to r_1 and r_2 respectively. One can infer that

$$\int_0^{\tau_1} f(q_1(t))e^{-r_1 t}dt \geq \int_0^{\hat{\tau}} f(\hat{q}(t))e^{-r_1 t}dt$$

and

$$\int_0^{\tau_2} f(q_2(t))e^{-r_2 t}dt \geq \int_0^{\hat{\tau}} f(\hat{q}(t))e^{-r_2 t}dt$$

Using the result stated in equation (8), one obtains

$$V(s;\hat{r}) - V(s;r_1) - V(s;r_2) < 0.$$

Hence the function $V(s;r)$ is strictly convex in r. Q.E.D.

In the Appendix, we establish that for the case of our resource extracting firm facing simultaneous output price and interest rate uncertainty, the latter two being independently distributed, a preference for uncertainty obtains.

Theorem 1:

A resource extracting agent prefers facing a stochastic interest rate with mean $\bar{r}$ beyond T_a to facing a certain interest rate $\bar{r}$ beyond T_a.

Proof: In order to prove Theorem 1, one has to show that the maximized value function stated in (3) is greater than the maximized value function for a program with interest rate equal to r up to T_a and equal to $\bar{r}$ for time $t \epsilon (T_a, \infty)$.

Let $\bar{s}_1$ be the optimally chosen cumulative level of extraction up to time T_a for the program with interest rate equal to r and $\bar{r}$ for the time intervals $[0,T_a]$ and (T_a,∞) respectively. The maximized value function can be expressed as

$$V(s;r,\bar{r},T_a) = V(\bar{s}_1;r,T_a) + e^{-rT_a} V(s-\bar{s}_1;\bar{r}) \tag{9}$$

Since s_1 is optimally chosen, $\dfrac{\partial V(\bar{s}_1;r,T_a)}{\partial \bar{s}_1} = e^{-rT_a} \dfrac{\partial V(s-\bar{s}_1;\bar{r})}{\partial \bar{s}_1}$ (10)

Let s_1 be the optimally chosen cumulative level of extraction up to time T_a which maximizes the value function (3) and hence the maximized value can be expressed as

$$V(s;r,T_a,\underline{\pi}_i,\underline{r}_i) = V(s_1;r,T_a) + e^{-rT_a} \sum_{i=1}^{n} \pi_i V(s-s_1;r_i) \tag{11}$$

where $\underline{\pi}_i = (\pi_1,\pi_2,\ldots\pi_n)$ and $\underline{r}_i = (r_1,r_2,\ldots r_n)$

Once again, since s_1 is optimally chosen

$$\frac{\partial V(s_1;r,T_a)}{\partial s_1} = e^{-rT_a} \sum_{i=1}^{n} \pi_i \frac{\partial V(s-s_1;r_i)}{\partial s_1} \tag{12}$$

Invoking the fact that s_1 is optimally chosen (for the uncertain program), one infers that

$$V(s;r,T_a,\underline{\pi}_i,\underline{r}_i) = V(s_1;r,T_a) + e^{-rT_a} \sum_{i=1}^{n} \pi_i \; V(s-s_1;r_i)$$

$$\geq V(\bar{s}_1;r,T_a) + e^{-rT_a} \sum_{i=1}^{n} \pi_i \; V(s-\bar{s}_1;r_i)$$

Recalling that V is strictly convex in r, one can infer that

$$V(\bar{s}_1;r,T_a) + e^{-rT_a} \sum_{i=1}^{n} \pi_i \; V(s-\bar{s}_1;r_i) > V(\bar{s}_1;r,T_a) + e^{-rT_a} \; V(s-\bar{s}_1,r)$$

$$= V(s;r,\bar{r},T_a)$$

Q.E.D.

An optimally chosen s_1 has to satisfy condition (12). This leads to the basic intertemporal zero arbitrage condition at T_a, the date of the onset of the uncertain output price.

Lemma 2: If $\{q(t)\}$ defines an optimal interior solution then

$$f'(q(T_a^-)) = \sum_{i=1}^{n} \pi_i f'(q_i(T_a^+)) \tag{13}$$

where $f'(q(T_a^-))$ is the left hand limit of $f'(\cdot)$ as t approaches T_a and $f'(q_i(T_a^+))$ is the right hand limit given that the interest rate is equal to r_i

Proof: We substitute in (12) to obtain this result. First we observe that for a candidate path $\{q(t)\}$

$$f'(q(t)) = f'(q(T_a^-))e^{-r(T_a-t)} \qquad t \in (0,T_a) \tag{14A}$$

If there exists a $f'(q_i(T_i))$ which satisfies transversality condition of $\lim_{t\to T_i} f'(q_i(T_i)) \cdot q_i(T_i) = 0$, then

$$f'(q_i(t)) = f'(q_i(T_i))e^{-r_i(T_i-t)} \qquad t \in (T_a,T_i) \tag{14B}$$

These are zero intertemporal arbitrage conditions for path $\{q(t)\}$ on either side of T_a.

The left hand side of (12) can be written as

$$\int_0^{T_a} e^{-rt} f'(q(t)) \frac{dq(t)}{ds_1} dt$$

which using (14A) becomes

$$f'(q(T_a^-))e^{-rT_a}\int_0^{T_a}\frac{dq(t)}{ds_1}\,dt \qquad (15)$$

and $\frac{dq(t)}{ds_1} = \frac{dq(t)}{df'(q(t))}\cdot\frac{df'(q(t))}{dq_{T_a^-}}\cdot\frac{dq_{T_a^-}}{ds_1}$ where $\int_0^{T_a}\frac{dq(t)}{ds_1}\,dt = 1.$

The term on the right hand side of (12) becomes using the same procedure

$$e^{-rT_i}f'(q_i(T_i))\int_0^{T_i-T_a}\frac{dq(t)}{d(s-s_1)}\,dt \qquad (16A)$$

Now $\frac{dq(t)}{d(s-s_1)} = \frac{dq(t)}{df'(q(t))}\cdot\frac{df'(q(t))}{d(T_i-T_a)}\cdot\frac{d(T_i-T_a)}{d(s-s_1)}$ where $\int_0^{T_i-T_a}\frac{dq(t)}{d(s-s_1)}\,dt = 1.$

Using (14B) one can express (16A) as

$$e^{rT_a}f'(q_i(T_a^+))\int_0^{T_a-T_i}\frac{dq(t)}{d(s-s_1)}\,dt \qquad (16B)$$

Substitution of (15) and (16B) into (12) yields (13).

Q.E.D.

Consider an example. For the case of a competitive firm facing a constant price p and a linear marginal extraction cost schedule $bq(t)$, we have in (14) and (15) respectively

$$p-bq(t) = [p-bq(T_a^-)]e^{-r(T_a-t)} \qquad t\ \varepsilon\ (0,T_a) \qquad (17)$$

$$p-bq_i(t) = pe^{-r_i(T_i-t)} \qquad t\ \varepsilon\ (0,T_i) \qquad (18)$$

We obtain $\frac{dmc(q(t))}{dq(T_a^-)} = be^{-r(T_a-t)}$ and $\frac{dmc(q(t))}{dT_i} = r\ p\ e^{-r_i(T_i-t)}$. We now define $s-s_1 \equiv \int_0^{T_a}q(t)dt$. Now using (17), we obtain $q(T_a^-) = -\frac{p}{b}\left[\frac{rT_a + e^{-rT_a}-1}{1-e^{-rT_a}}\right] +$

$\left[\dfrac{(s-s_1)r}{1-e^{-rT_a}}\right]$ whence $\dfrac{dq_{T_a^-}}{d(s-s_1)} = \dfrac{r}{(1-e^{-rT_a})}$. Using (17) and the fact that

$\int_0^{T_i} q_i(t)dt = s_1$, we obtain $s_1 = \dfrac{p}{br_i}[T_i r_i - 1 + e^{-r_i T_i}]$ whence

$\dfrac{dT_i}{ds_1} = \dfrac{b}{p}\left[\dfrac{1}{1-e^{-r_i T_i}}\right]$. Substitution in $\int_0^{T_a} \dfrac{dq(t)}{dmc(q(t))} \cdot \dfrac{dmc(q(t))}{dq_{T_i^-}} \cdot \dfrac{dq_{T_i^-}}{d(s-s_1)} dt$

yields the integral equal to unity. Similarly $\int_0^{T_i} \dfrac{dq_i(t)}{dmc(q_i(t))} \cdot \dfrac{dmc(q_i(t))}{dT_i} \cdot$

$\dfrac{dT_i}{ds_1} dt = 1$. For this linear marginal cost case aggregate value functions

$V(s_1;r_i,T_i) = \dfrac{p^2}{2br_i}\left[1-e^{-r_i T_i}\right]^2$ and $V((s-s_1);p,T_a) = \left[1-e^{-rT_a}\right]\left(\dfrac{.5}{br}\right)\left\{p^2 - e^{-rT_a}\left(p-bq(T_a)\right)^2\right\}$.

We computed an example. A firm faces a constant output price of \$105 and interest rate 0.015. It has 1000 tons to extract and marginal cost of extraction is simply $q(t)$ (from a total extraction cost of $.5q(t)^2$). At date $T_a = 3.5$ the interest rate is 0.15 with probability 0.5, or 0.05 with probability 0.5. The optimal solution has 727.729 tons extracted beyond T_a and this amount is extracted over interval 12.5886 if the interest rate is 0.15 or over interval 19.318 if the interest rate is 0.05. The optimized value for the program is \$50,156.25.

At a certain interest rate of 0.1 beyond T_a, the program's optimal value is \$48,405.184. Beyond T_a 716.704 tons are extracted over interval 14.474. It is not a coincidence that more output is extracted beyond T_a under an uncertain interest rate since more uncertainty (a mean preserving spread) in r makes the payoff on the average ton higher in the period of uncertainty relative to the period of certainty. Since stock extracted after T_a is the control in this problem we can prove our assertion by appealing to the results of Rothschild and Stiglitz [1970] on increases in riskiness.

Interest Rate Induced Price Uncertainty in a Competitive Resource Market

In this section, the effect of interest rate uncertainty on market price in a competitive resource market is examined. Assume that there are m (a large finite number) identical resource extracting firms in a competitive resource market. Each firm owns s amount of the homogeneous stock of initial resource endowment. The cost structures of these m firms are identical and specified as

$$c_j(t) = c_j(q^j(t)) \tag{19}$$

where $c_j' > 0$ and $c_j'' > 0$ for firms $j=1,2,\ldots m$

The demand curve for the resource is assumed to satisfy the following conditions.

$P(t) = D[Q(t)]$, the demand for the resource is assumed to be time invariant

$D(0) = F$, F is a choke price above which demand is zero

$D' < 0$, the demand curve is downward sloping.

Although the demand curve is downward sloping, the decision of an individual firm is assumed to have insignificant effect on market price.

The market supply curve is given by

$$Q_S(t) = \sum_{j=1}^{m} q^j(t) \tag{21}$$

where $q^j(t)$ is the output of the j^{th} firm which is determined through the maximization behaviour of the firm.

From (20), the market demand can be expressed as

$$Q_D(t) = D^{-1}(P(t))$$

A rational expectations equilibrium for the resource market is a price process $P(t)$ such that

$$Q_D(t) = Q_S(t)$$

The firms' expectations are rational in that the anticipated price process coincides almost surely with the actual price process generated on the market by their maximization behaviour.

Given an anticipated price process, $P(t)$, the k^{th} firm will maximize the expected present value of future discounted profit. First order conditions require

$$P(t) - c_k'(q^k(t) - \lambda^k(t) = 0 \qquad \text{for } t \in [0,T_a] \tag{22}$$

$$P(t) - c_k'(q_i^k(t)) - \lambda_i^k(t) = 0 \qquad \text{for } t \in (T_a,\infty) \tag{23}$$

for all interest rate realisations i=1,2,...n.

where λ^k is the costate variable

$$\dot{\lambda}^k(t) = r\lambda^k(t) \qquad \text{for } t \in [0,T_a]$$

$$\dot{\lambda}_i^k(t) = r\lambda_i^k(t) \qquad \text{for } t \in [T_a,\infty]$$

$$\text{and all } i=1,2,...n \tag{24}$$

$$\lambda^k(0) = e^{-rT_a} \sum_{i=1}^{n} \pi_i \lambda_i^k(T_a) \tag{25}$$

At terminal time,

$$\lambda_i^k(\tau_i) = P(\tau_i) \qquad \text{for } i=1,2,\ldots n \tag{26}$$

Condition (25) is the zero profit arbitrage condition at the date T_a of the arrival of the uncertain interest rate. Condition (26) is a basic end point condition (see Gelfand and Fomin [1963, p. 56]) and we make use of $c'(0) = 0$.

Since the m firms are identical, we can obtain industry marginal extraction cost at each date by summing the relevant marginal extraction cost schedules for the firms. Alternatively, we can solve the industry problem in Theorem 1, ignoring the firms, and then obtain the extraction paths for the identical firms by dividing the industry output at each instant by m and obtain firm k's marginal extraction cost at each instant by dividing industry marginal cost by m. See the example with a linear, negatively sloped industry demand curve and linear marginal costs below.

The analysis of a rational expectations equilibrium for competitive firms above can be reduced to a much simpler analysis using the techniques developed by Lucas and Prescott (1971) and extended by Brock and Magill [1979].

Define the problem of finding the optimal extraction paths of the m firms which maximizes the integral

$$\int_0^{T_a} \left\{ \int_0^{\sum_{j=1}^{m} q^j(t)} D(y)dy - \sum_{j=1}^{m} c_j(q_i^j(t)) \right\} e^{-rt}dt$$

$$+ \sum_{i=1}^{n} \pi_i \int_{T_a}^{\tau_i} \left\{ \int_0^{\sum_{j=1}^{m} q_i^j(t)} D(y)dy - \sum_{j=1}^{m} c_j(q_i^j(t)) \right\} e^{-r_i t}dt \tag{27}$$

Subject to

$$\int_0^{T_a} q^j(t)dt + \int_{T_a}^{\tau_i} q_i^j(t)dt = s$$

for all i=1,2,...n

j=1,2,...m (28)

as the extended integrand problem.

Let $\tilde{q}^j(t)$ denote the optimal extraction path of firm j for time $t \varepsilon [0,T_a)$ and $\tilde{q}_i^j(t)$ denote the optimal extraction path of firm j for time $t \varepsilon (T_a,\infty)$ and interest rate equal to r; of the extended integral problem. The price path process

$$D(\sum_{j=1}^m \tilde{q}^j(t)) \quad \text{in time} \quad t \varepsilon [0,T_a] \quad \text{and}$$

$$D(\sum_{j=1}^m \tilde{q}_i^j(t)) \quad \text{in time} \quad t \varepsilon [0,T_a] \qquad \text{for } i=1,2,...n$$

is a rational expectations equilibrium for the competitive resource market.

To see this, note that first order conditions for an interior optimum of the extended integrand problem requires

$$D(\sum_{j=1}^m \tilde{q}^j(t)) - c_k'(q^k(t)) - \lambda^k(t) = 0 \qquad \text{for } t \varepsilon [0,T_a] \tag{29}$$

k=1,2,...m

$$D(\sum_{j=1}^m \tilde{q}_i^j(t)) - c_k'(q_i^k(t)) - \lambda_i^k(t) = 0 \qquad \text{for } t \varepsilon (T_a,\infty)$$

k=1,2,...m

i=1,2,...n (30)

$$\dot{\lambda}^k(t) = r\lambda^k(t) \qquad \text{for } t \in [0,T_a] \quad k=1,2,\ldots m \tag{31}$$

$$\dot{\lambda}_i^k(t) = r_i\lambda_i^k(t) \qquad \text{for } t \in (T_a,\infty) \quad k=1,2,\ldots m \quad i=1,2,\ldots n \tag{32}$$

$$\lambda^k(0) = e^{-rT_a} \sum_{i=1}^{n} \pi_i\lambda_i^k(T_a) \qquad \text{for } k=1,2,\ldots m \tag{33}$$

At terminal time

$$\lambda_i^k(\tau_i) = D(\sum_{j=1}^{m} \tilde{q}_i^j(\tau_i)) \qquad \text{for } i=1,2,\ldots n \quad k=1,2,\ldots m \tag{34}$$

Equations (29)-(34) inclusive are sufficient conditions for each firm to maximize the present value of discounted expected profit.

The existence of a market equilibrium for each state of interest rate realisation given a certain level of s_1 can be proved using the technique developed by Eswaran and Lewis (1984). Hence a rational expectations equilibrium is established.

Lemma 3: (i) $\tau_1 > \tau_2$ and (ii) $\lambda_1^k(T_a) > \lambda_2^k(T_a)$ for $r_2 > r_1$

Proof of (i):

Rewrite condition (29) as

$$\lambda_i^k(t) = \varphi_i = D[\sum_{j=1}^{m} \tilde{q}_i^j(t)] - c_k'(\tilde{q}_i^k(t)) \qquad \text{for } i=1,2 \tag{35}$$

Since all the m firms are identical, equation (35) can be expressed as

$$\lambda_i^k(t) = \varphi_i = D[m\tilde{q}_i^k(t)] - c_k'(\tilde{q}_i^k(t)) \qquad \text{for } i=1,2, \tag{35'}$$

Differentiating the right hand side of (35') with respect to $\tilde{q}_i^k$ one obtains

$$\frac{\partial \varphi_i}{\partial \tilde{q}_i^k} = D'[\cdot]m - c_k''(\tilde{q}_i^k(t)) < 0 \tag{36}$$

Equation (36) demonstrates that there exists a unique $\tilde{q}_i^k(t)$ for any $\lambda_i^k(t)$. In particular, a higher $\lambda_i^k(t)$ is associated with a lower $\tilde{q}_i^k(t)$.

At terminal time τ_i, $\lambda_i^k(\tau_i) = F$ for i=1,2. By (32) $\lambda_2^k(t)$ decreases more rapidly from time τ_2 back to T_a and since $\int_{T_a}^{\tau_2} \tilde{q}_2^k(t)dt = \int_{T_a}^{\tau_1} \tilde{q}_1^k(t)dt = s - s_1$, τ_1 must be greater than τ_2.

Proof of (ii) (by contradiction):

Assume that $\lambda_1^k(T_a) \leq \lambda_2^k(T_a)$, from (32) and part (i) of Lemma 3, $\lambda_2^k(t)$ would be greater than $\lambda_1^k(t)$ in the time interval $(T_a, \tau_2]$. Hence $\int_{T_a}^{\tau_2} \tilde{q}_2^k(t)dt < \int_{T_a}^{\tau_2} \tilde{q}_1^k(t)dt < \int_{T_a}^{\tau_1} \tilde{q}_1^k(t)dt$. The resource constraint is not satisfied.

Q.E.D.

Let $p(r_i, T_a)$ stand for market price of the resource at time T_a if interest rate is equal to r_i.

Theorem 2: Different interest rate realizations lead to different market prices at time T_a. In particular $p(r_1, T_a) > p(r_2, T_a)$ for $r_2 > r_1$.

Proof:

Using the results from part (ii) of Lemma 3 one can show that $\lambda_1^k(T_a) > \lambda_2^k(T_a)$. Since (36) states that a higher $\lambda_i^k(t)$ associates with a lower $\tilde{q}_i^k(t)$, $p(r_1, T_a) > p(r_2, T_a)$.

Q.E.D.

We computed an example. There is a linear industry demand curve $p(t) = 105 - 0.5Q(t)$. The sum of the marginal cost curves of m identical firms gives us an industry marginal extraction cost of $0.5Q(t)$. Each firm is a price taker, has rational expectations, and has $1000/m$ tons to extract. Up to date $T_a = 3.5$ the interest rate is certain at 0.015. At T_a the interest rate rises to 0.15 with probability 0.5 and to 0.05 with probability 0.5. The price jumps[2] to 72.484 at T_a if the low uncertain interest is realized and 727.729 tons are extracted by the industry (m firms) beyond T_a over interval 19.318. The price becomes 60.445 at T_a if the high uncertain interest rate is realized and 727.729 tons are extracted beyond T_a over interval 12.589. The present value of industry rent is \$50,156.246 at time 0. Expected price at T_a is then \$66.464 and the variance is 36.234.

For the case of a certain interest rate 0.1 at T_a. The price at T_a is \$64.847 and the total industry rent is \$48,405.184. The m firms extract 716.704 tons beyond T_a over interval 14.4738.

We recalculated the case of an uncertain interest rate with a mean preserving spread of r. (We let r at T_a become 0.17 with probability 0.5 and 0.03 with probability 0.5.) The mean value of price at T_a rose to 68.45 and the variance increased to 87.53. (The value of the program was \$52,301.43 and extraction beyond T_a was 741.27.) Hence in this particular sense, increased uncertainty in the interest rate leads to increased uncertainty in output price.

APPENDIX: A Competitive Resource Extracting Firm Facing Simultaneous Price Uncertainty and Interest Rate Uncertainty (independently distributed random variables)

In this appendix, we examine the case of a competitive firm in a non-renewable resource industry facing price uncertainty and interest rate uncertainty simultaneous. Price and the interest rate are assumed to be independent random variables.

Let the probability distribution of the interest rate be the same as before. The probability distribution of price at time T_a is as follows: the probability of price being equal to P_j is t_j and the expected mean price is equal to $\overline{p}$. Hence

$$\sum_{j=1}^{w} \psi_j p_j = \overline{p} \tag{A-1}$$

$$\sum_{j=1}^{w} \psi_j = 1 \tag{A-2}$$

Lemma A: A risk neutral firm prefers to facing a stochastic price with mean $\overline{p}$ and a stochastic interest rate with mean $\overline{r}$ to facing a certain price equal to $\overline{p}$ and a certain interest rate equal to $\overline{r}$.

Proof:

Let $q_{ij}(t)$ stand for the optimal extraction path for the case when the interest rate is equal to r_i and price is equal to p_j.

$$i=1,2,\ldots n$$
$$j=1,2,\ldots w$$

$q_{-j}(t)$ for the case when the interest rate is equal to $\overline{r}$ and price is equal to p_j

$q_{i-}(t)$ for the case when the interest rate is equal to r_i and price is equal to $\overline{p}$.

Using Lemma 1, one can show that

$$\sum_{i=1}^{n} \pi_i \int_0^{\tau_{ij}} [P_j \cdot q_{ij}(t) - c(q_{ij}(t))]e^{-r_i t} dt > \int_0^{\tau_{-j}} P_j \cdot q_{-j}(t) - c(q_{-j}(t))]e^{-rt} dt$$

for all prices j=1,2,...3 (A-3)

Recalling the result extablished by Hartwick and Yeung (1985), one can demonstrate that

$$\sum_{j=1}^{w} \psi_j \int_0^{T_{-j}} [P_j q_{-j}(t) - c(q_{-j}(t))]e^{-\bar{r}t} dt > \int_0^{\bar{\tau}} [\bar{P}\bar{q}(t) - c(\bar{q}(t))]e^{-\bar{r}t} dt \qquad \text{(A-4)}$$

where $\bar{q}(t)$ is the optimal extraction path for prices equal to $\bar{p}$ and the interest rate equal to $\bar{r}$.

From (A-3) and (A-4)

$$\sum_{i=1}^{n} \sum_{j=1}^{w} \pi_i \psi_i \int_0^{T_{ij}} P_j q_{ij}(t) - c(q_{ij}(t))]e^{-r_i t} dt$$

$$> \int_0^{\bar{T}} [\bar{P}\bar{q}(t) - c(\bar{q}(t))]e^{-\bar{r}t} dt \qquad \text{(A-5)}$$

The left hand side of (A-5) is the expected maximized value function of the stochastic program while the right hand side is the maximized value function of the certainty program. Hence Lemma A is proved.

Q.E.D.

Following the analysis of Theorem 1, one can prove that a risk neutral firm prefers to facing a stochastic price and a stochastic interest rate beyond T_a to facing a certain price and a certain interest rate beyond T_a.

Footnotes

* We thank Rick Harris for suggesting that we investigate interest rate uncertainty.

1. Existence of a path (not a spike of output at say time 0) requires $f(q(t))$ concave for q and t positive. Thus for the case of a firm facing a constant output price p over time, $\frac{d^2c}{dq^2}$ must be positive.

2. It is not a coincidence that the numerical values for this problem are for the most part the same as those for our earlier example. Subject to the correct adjustment of the slopes of the industry demand curve and the sum of the firms marginal extraction cost curves, the problems are of identical formal structure. For example a problem with a constant price \$105 and a marginal extraction cost $q(t)$ is the same as a problem with a linear demand curve $p(t) = 105 - 0.5Q(t)$ and a marginal extraction cost $0.5Q(t)$.

References

Brock, W.A. and M.J.P. Magill, [1979], "Dynamics under Uncertainty", _Econometrica_ 47, pp. 843-868.

Eswaran, M. and T. Lewis, [1984], "Ultimate Recovery of an Exhaustible Resource under Different Market Structures", _Journal of Environmental Economics and Management_ 11, pp. 55-69.

Gelfand, I.M. and S.V. Fomin, [1963], _Calculus of Variations_, (Englewood Cliffs: Prentice Hall).

Hartwick, John M. and David Yeung, [1985], "Preference for Output Price Uncertainty by the Nonrenewable Resource Extracting Firm", _Economics Letters_, (forthcoming).

Lucas, R.E. and E. Prescott, [1971], "Investment under Uncertainty", _Econometrica_ 39, pp. 659-681.

Rothschild, M. and J.E. Stiglitz, [1970], "Increasing Risk: I. A Definition", _Journal of Economic Theory_ 2, pp. 225-243.