Disc paper 418

ISSN 0316-5078

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Disscussion Paper No. 418

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Recent dramatic increases in world prices for many basic commodities and the consequent impact on employment and inflation have begun to generate theoretic interest in open economy models which explicitly consider the direct supply-side effects of these and other shocks on the domestic economy. Most such recent studies analyze the impact of a foreign price shock through its influence either on imported input prices within the context of a static, deterministic Fleming-Mundell-type model or on imported consumer goods and, thereby, expected consumer prices within the context of a dynamic (sometimes non-deterministic) "expectations-augmented Phillips curve" framework.[1] Nonetheless, all invoke the standard small-open-economy assumption whereby foreign prices are exogenously fixed. Consequently, all are severely hamstrung by the failure to attribute foreign price changes to their underlying source which may be a foreign real and/or monetary disturbance. Moreover, most such models also fail to distinguish between permanent versus transitory and expected versus unexpected foreign shocks. In short, such models may tell us much about the impact of domestic shocks, but little about that of foreign shocks, on the domestic economy.

This paper constructs a stochastic equilibrium model of the world economy which focuses on the impacts of various foreign and domestic disturbances. The basic model is a hybrid of the above two approaches. For simplicity, the world comprises only two nations: a large foreign and a small domestic economy. Nonetheless, several breaks are made from the existing small-open-economy model. First and foremost, foreign variables are not taken to be exogenous but, rather, the foreign economy is explicitly modelled. It is argued that one can not ask the impact of a foreign price (or any other) shock on the domestic economy without knowing the ultimate source of disturbance. In general, no such shock will occur in isolation but, rather, will be accompanied by change in other foreign variables, such as output and interest rates, relevant to the domestic economy. Second, the role of the terms-of-trade in both the supply and demand sides of the economy is explicitly modelled by permitting trade in both consumer and intermediate goods. Third, we carefully distinguish between gross output and real GNP; these two variables do not necessarily move together in response to shocks. Lastly, all shocks are generated by explicit stochastic processes with expectations corresponding to optimal forecasts based on a specified information set which includes knowledge of the economy's structure. In short, expectations are rational.[2]

Section I constructs the basic equilibrium, open-economy model paying special attention to the terms-of-trade and expectations. The model is then "closed" and applied to the large foreign economy in Section II where complete information solutions for foreign output, interest rates and prices are derived. These solutions then act as inputs into the determination of domestic real GNP, the terms-of-trade, prices and interest and exchange rates. The impacts of various foreign and domestic disturbances on the two economies are analyzed in Section III. Similar issues are addressed in Section IV where information is not complete.

I. THE BASIC MODEL

Consider a small open economy (SOE) which trades both goods and financial capital with the rest of the world. It produces a single composite commodity which is both exported and domestically consumed. This good is imperfectly substitutable for foreign goods so that the domestic price level is endogenous. Imports comprise another composite commodity which is an input into both consumption and production. Being small, the domestic economy is a price-taker with respect to the import good. The exchange rate is perfectly flexible, perfect competition prevails throughout the world and financial capital is perfectly mobile between nations.

The production sector is characterized by extension of Phelps-Friedman-Lucas-type behaviour to an SOE. All production takes place at the beginning of the period while all final demands are satisfied at period's end. The following price information is available to agents. First, at period's start, each (single-product) firm participates in both input markets and in the market for its own good. Given that imported inputs are commodity-specific, it can then be assumed that each firm knows the current nominal prices of labour, its imported input and its own output. Second, at period's start, households participate in the labour market and, thereby, observe the current nominal wage. However, at period's start, no agent is able to observe the foreign or domestic price indexes which will be revealed at period's end when final demands are executed. Third, being single prices, current exchange and interest rates are observed by all agents. Given this price information, we briefly consider the choice problems of the typical firm and household.

Define the following magnitudes:

y = gross output

q = real GNP

N = employment of labour

H = employment of imports

P = price index for domestic output

R = import price index = foreign price index

h = exchange rate = foreign currency price of domestic currency

τ = hP/R = terms-of-trade

W = nominal wage rate

For the moment, each variable is defined by its current level. However, all relevant functions will, for simplicity, ultimately be specified as log-linear expressions at which time these definitions will refer to natural logarithms. Throughout, the superscripts "e" and "*" will indicate, respectively, an expected and a foreign value.

(a) Firms

Assume an aggregate production function whose only variable inputs are labour and intermediate imports.

(1) $y = f(N,H) \qquad f_1, f_2 > 0;\ f_{11}, f_{22} < 0;\ f_{12} \gtreqless 0$

Since each firm knows all relevant current prices, profit maximization requires that each factor be hired to the point where its marginal product equals its real-product-price. This produces the following two inverse factor demand functions:

(2) $$\begin{cases} W = P.f_1(N,H) \\ \tau^{-1} = f_2(N,H) \end{cases}$$

We now need distinguish between gross production, y, and real GNP, q. The latter comprises only domestic value-added which, in turn includes all real factor rewards accruing to domestic residents. Given that all fixed factors are domestically-owned, real GNP will be gross production less real payments to imported inputs.

(3) $q = y - H/\tau$

(b) Households

At the beginning of the period, the typical household solves the usual one-period consumption-leisure choice problem where consumption comprises both domestic and foreign goods. This choice is made subject to the known nominal wage but unknown current foreign and domestic price indices. These latter two indices are expected to take the values, respectively, R^e and P^e. Solution to this problem will yield the usual labour supply and demand for domestically-produced consumer goods functions of the general form, respectively,[3]

(4) $N^s = g(W/P^e,\tau^e)$ $g_1,g_2 > 0$

(5) $C^d = C^d(W/P^e,\tau^e)$ $C_1^d > 0;\ C_2^d < 0$

where: N^s = labour supply; $\tau^e = hP^e/R^e$; and we have assumed that leisure, domestic goods and foreign goods are gross substitutes while the substitution outweighs the income effect in leisure demand so that the labour supply curve slopes upward.

If the utility function is separable in goods and leisure, it is well known that commodity demands can be made to depend upon expected real expenditures rather than upon the expected real wage. Since nominal household expenditures here equal known nominal income, Pq, expected real expenditures become Pq/P^e. This implies that household demand for domestic goods can be written as:

(6) $C^d = C(Pq/P^e,\tau^e)$ $C_1 > 0;\ \ C_2 < 0$

(c) Exports

We assume that foreign and domestic agents are behaviourally similar. Then foreign demand for domestic consumer goods, C^{d*}, is determined by the foreign analogue of expression (6): $C^{d*} = C^*(Rq^*/R^e,\tau^e)$, where q^* is foreign real GNP, $C_1^* > 0$ and $C_2^* < 0$. As well, domestic goods may also be intermediate inputs into foreign production. Without loss of generality, we may assume foreign technology is Cobb-Douglas in which case the foreign demand for domestically-produced intermediate goods, H^*, will be: $H^* = aq^*/\tau$, where a is the exponent on H^* in the foreign production function. Then, being the sum of consumer and intermediate goods exports, $C^{d*}+H^*$, total domestic physical exports can be written as:

(7) $X^d = X^d(Rq^*/R^e,\tau^e,aq^*/\tau)$ $X_1^d,X_3^d > 0;\ \ X_2^d < 0$

(d) Aggregate Supply and Intermediate Import Functions

The aggregate supply curve is derived in the usual fashion by computing the level of real GNP obtaining when the labour market clears, given that imported inputs are in infinitely-elastic supply to the domestic economy. Hence, set $N^s = N$ and solve expressions (1) to (4) for the labour-market-clearing levels of real GNP and intermediate imports.[4] If neither input is inferior we obtain, in general form:

(8a) $$q = q(P/P^e, \tau^e, \tau) \qquad q_1, q_2 > 0; \quad q_3 \gtreqless 0$$

(8b) $$H = H(P/P^e, \tau^e, \tau) \qquad H_1, H_2 \gtreqless 0; \quad H_3 > 0$$

Interpretation of these two functions is straightforward. First, with P constant, a rise in the terms-of-trade, τ, implies a fall in the domestic price of intermediate inputs, R/h. Imports rise. However, the demand for labour and, thereby, employment and wages rise if and only if labour and imports are gross complements in production.[5] On the other hand, standard production theory suggests that rents to fixed factors and entrepreneurs will rise. Consequently, being the sum of real rents and the real wage bill, real GNP will rise (may fall) if labour and imports are gross complements (substitutes) in production. Second, with τ constant, a domestic price hike lowers the real wage relative to the real price of intermediate inputs thereby raising labour demand, employment and real GNP but lowering imports if and only if they are gross substitutes for labour. Third, let expected foreign or domestic prices rise. Other things constant, labour supply falls as households shift from domestic or foreign goods into leisure thereby lowering employment and real GNP. Since this shift also raises the real wage relative to the terms-of-trade, imports rise if and only if they are gross substitutes for labour. Thus, a rise in P^e holding $\tau^e = hP^e/R^e$ constant or a fall in τ^e holding P^e constant each reduces real GNP while leaving imports indeterminate. Fourth, let P rise without holding τ constant. Since neither input is inferior, employment, intermediate imports and real GNP all rise.

For simplicity, we now assume that the aggregate supply function is (approximately) log-linear. Henceforth, let each of the above-defined symbols now refer to a natural logarithm. Then, expression (8a) may be written as:

$$(9) \qquad q = \alpha + \beta.\tau + \gamma.\tau^e + \delta.(P-P^e) \qquad \alpha,\gamma,\delta > 0;\ \beta \gtreqless 0;\ \delta+\beta > 0$$

where, being logs: $\tau = h+P-R$; $\tau^e = h+P^e-R^e$; and $(\beta+\delta)$ is the price-elasticity of aggregate supply.

(e) Aggregate Demand

The demand-side of the economy is a straightforward extension of the usual Fleming-Mundell model of an SOE. Define the new symbol ${}^1x^e$ to be the value of x currently expected to prevail one period hence. Then, the demand side of our SOE is summarized by the following log-linear expressions:

$$(10) \qquad i = i^* - ({}^1h^e-h)$$

$$(11) \qquad r = i - ({}^1P^e-P^e)$$

$$(12) \qquad M - P = L + L_1.q - L_2.i \qquad L,L_j > 0, \text{ for all } j$$

$$(13) \qquad q = E + E_o(P-P^e) - E_2.r - E_3.\tau^e + E_4.q^* - E_5.\tau + E_6(R-R^e)$$

where: i, i* and r are the log of one plus, respectively, the domestic nominal, the foreign nominal and the expected domestic real interest rates; M is the log of the nominal money stock; q* is log of foreign GNP; all other previously defined symbols are now logarithms; $E,E_2,E_4,E_6 > 0$; and $E_o,E_3,E_5 \gtreqless 0$.

Expression (10) captures the notion of perfect capital mobility whereby the domestic nominal interest rate equals the foreign rate adjusted for expected exchange appreciation. The expected real interest rate relevant to investment and, possibly, saving is given by (11), where $({}^1P^e-P^e)$ is the expected future rate of price inflation.[6] Expression (12) is the usual LM function where zero currency substitution is assumed. By standard assumption, transactions demand for nominal money is homogeneous in known nominal income, P+q. Thus, actual rather than expected prices are used to deflate nominal balances.[7] Finally, expression (13) is a log-linear IS function. The intercept, E, reflects exogenous components of aggregate demand including the effects of government expenditures and taxes. The expected real interest rate enters through its influence on investment and, possibly, on saving. Other variables enter via their effects on consumption of domestic goods, C^d, exports, X^d, and the real domestic value of intermediate imports, $H-\tau$. The sign of E_5 is ambiguous because, although τ has a negative impact on exports of intermediate goods, it has an ambiguous impact on the real domestic value of intermediate imports. The ambiguity of E_0 and E_3 results from the fact that (P^e-P) and τ^e both lower domestic consumption but have ambiguous impacts on intermediate imports. Nonetheless, we henceforth assume that the Marshall-Lerner condition holds so that, when the terms-of-trade are fully-anticipated, $E_1 = E_3+E_5$ is positive.[8]

(f) Stochastic Elements in the Domestic Economy

The nominal money stock, the monetary growth rate and the exogenous components of aggregate supply and demand are assumed to be generated by the following stochastic processes, where the subscript "-1" indicates last period's value and u is the log of one plus the monetary growth rate:

$$M = M_{-1} + u \tag{14}$$

$$u = (u_{-1}-n_{-1}) + n + m \tag{15}$$

$$\alpha = \alpha_{-1} + s \tag{16}$$

$$E = E_{-1} + d \tag{17}$$

where m, n, s and d are random terms independently generated by white noise processes (normal distribution, zero mean, serial independence, finite variance). The term m represents a permanent and n a transitory shock to u. Consequently, m permanently raises the monetary growth rate while n permanently raises the money stock. The terms s and d represent permanent shocks to, respectively, aggregate supply and demand. Since the model is already quite complicated, transitory shocks are not considered.

II. THE FOREIGN ECONOMY

We assume the two economies are behaviourally similar in which case the above model will equally-well apply (qualitatively) to the foreign economy. Nonetheless, the domestic economy is relatively small which, following the literature on SOEs, implies that domestic variables have no discernable impact on the foreign aggregate economy. Hence, in expressions (9) through (17), replace all domestic variables and coefficients with their foreign counterparts and vice versa while making the domestic economy relatively small by permitting q/q* and the coefficients attached to her other variables approach zero. One then obtains:

$$
(18)\quad \begin{cases}
q^* = \alpha^* + \delta^*.(R-R^e) & \alpha^*,\delta^* > 0 \\
M^* - R = L^* + L_1^*.q^* - L_2^*.(r^*+{}^1R^e-R^e) & L^*,L_j^* > 0, \text{ for all } j \\
q^* = E^* + E_0^*.(R-R^e) - E_2^*.r^* & E^*,E_j^* > 0, \text{ for all } j \\
M^* = M_{-1}^* + \overline{u}^* + n^* + m^* & \\
\alpha^* = \alpha_{-1}^* + s^* & \\
E^* = E_{-1}^* + d^* &
\end{cases}
$$

where: the "star" indicates a foreign magnitude; r* is the foreign real interest rate; $i^* = r^*+{}^1R^e-R^e$ is the foreign nominal interest rate; $\overline{u}^* = (u_{-1}^*-n_{-1}^*)$; and, recall, all variables are logarithms.

In short, the foreign economy behaves as if it were closed.[9] The first three expressions in (18) give the aggregate supply, LM and IS functions for, essentially, a closed economy. The last three define the stochastic processes generating the foreign state variables with the random terms, n*, m*, s* and d* being independent white noise processes (normal distribution, zero mean, serially independent).

Once expectations are specified, system (18) can be solved for the current values of foreign output, interest rates and prices. We assume rational expectations in the sense that agents make full use of all available information. Such information is assumed to include full knowledge of the economy's structure and, for the moment, the current levels of all state variables and, thereby, their past levels and current shocks. Thus, the information set will be given by:

$$(19) \qquad I^* = (M^*_{-1}, \bar{u}^*, E^*_{-1}, \alpha^*_{-1}, n^*, m^*, d^*, s^*)$$

The model is solved by the method of undetermined coefficients[10], with expectations being generated by agents' knowledge that the price level is determined by solution to system (18). It is apparent that, ultimately, the price level will be a log-linear function of the current values of the exogenous variables which, in turn, constitute the information set. Let C^* be an eight-element column vector with typical entry C^*_j. We postulate a solution for the price level of the form:

$$(20) \qquad R = C^*_o + I^*.C^*$$

Recalling that all entries in I^* are known, pass the expectations operator through (20) to find that the current price level is fully-anticipated; that is, $R^e = R$. Substitute this result into system (18) to find that:

$$(21) \qquad \begin{cases} q^* = \alpha^* = \alpha^*_{-1} + s^* \\ r^* = \lambda(E^* - \alpha^*) = \lambda(E^*_{-1} - \alpha^*_{-1} + d^* - s^*) \\ R = \{M^* - L^* - L^*_1.\alpha^* + L^*_2.{}^1R^e + \lambda L^*_2(E^* - \alpha^*)\}/(1+L^*_2) \end{cases}$$

where: $\lambda = 1/E^*_2 > 0$.

The price equation represents the "rational expectations problem" in that current prices depend upon expected future prices while the latter depend upon the same variables as the former. Nonetheless, shift expression (20) one period forward and take its expectation: ${}^1R^e = C^*_o + {}^1I^{*e}.C^*$, where, given the assumed white noise processes, ${}^1I^{*e} = (M^*, u^*-n^*, E^*, \alpha^*, 0,0,0,0)$. Now, substitute (20) and this expression for ${}^1R^e$

into the third equation of system (21). Collecting terms in the resulting expression, one obtains nine independent conditions on the C_j^*'s by equating to zero, in turn, the nine net coefficients on each magnitude in the information set and the constant. These net coefficients must be zero if R is to be determined by both (20) and (21). The resulting values for the C_j^*'s can be shown to imply that:

$$(22) \qquad R = M^*_{-1} + n^* - L^* + (1+L_2^*).(\overline{u}^*+ m^*) + L_2^*(E^*_{-1}+d^*) - C_4^*(\alpha^*_{-1}+s^*)$$

where $C_4^* = (L_1^*+\lambda L_2^*) > 0$.

First, foreign output and the real interest rate are both unaffected by monetary shocks, according to system (21). Only supply shocks alter output indicating that demand shocks are accompanied by complete crowding out. Only a positive shock to excess aggregate demand, $E^*-\alpha^*$, raises the real interest rate. Second, the foreign price level rises with monetary and demand shocks but falls with supply shocks, for the usual reasons. Moreover, the impact of a shock to the monetary growth rate on R will be larger when it is permanent than when it is transitory. From system (21), R is determined solely by the necessity for the money market to clear while it will be higher the greater is any shock's initial impact on net excess supply of real balances. Shift expression (22) one period forward and apply the expectations operator. It will then be found that, with the transtitory shock, n^*, $({}^1R^e-R) = 0$ while with the permanent shock, m^*, $({}^1R^e-R) > 0$. Hence, the permanent shock has the larger impact on the nominal interest rate, $i^* = r^*+{}^1R^e-R$, via its larger impact on expected inflation. Consequently, by raising i^* and, thereby, real money demand the more, the permanent shock has the larger initial impact on the excess supply of real balances and, ultimately, on the price level.

III. DOMESTIC EQUILIBRIUM AND RANDOM SHOCKS

The values of the foreign variables derived above can now be substituted into equations (9) through (13) which describe the domestic economy. First, however, we assume that foreign and domestic agents have identical information and, therefore, expectations. Second, from expressions (10) and (11) along with the definitions of the foreign nominal interest rate, $i^* = r^* + {}^1R^e - R^e$, and the terms-of-trade, $\tau = h + P - R$, the domestic real and nominal interest rates may be written as, respectively: $r = r^* - {}^1\tau^e + \tau^e$ and $i = r^* - {}^1\tau^e + \tau^e + {}^1P^e - P^e$, where ${}^1\tau^e$ is the terms-of-trade currently expected to prevail one period hence. Making use of these definitions and the above results on the foreign economy, the domestic economy can now be characterized by:

$$(23)\quad \begin{cases} q = \alpha + \beta.\tau + \gamma.\tau^e + \delta.(P-P^e) \\ M - P = L + L_1.q - L_2.(r^* - {}^1\tau^e + \tau^e + {}^1P^e - P^e) \\ q = E + E_0.(P-P^e) - E_2.(r^* - {}^1\tau^e + \tau^e) - E_3.\tau^e + E_4.q^* - E_5.\tau \\ q^* = \alpha^* \\ r^* = \lambda(E^* - \alpha^*) \end{cases}$$

The first three equations are the aggregate supply, LM and IS functions defined by, respectively, expressions (9), (12) and (13). In conjunction with the above-assumed stochastic processes generating M, E, α, E^* and α^*, system (23) is sufficient to solve for the values of domestic real GNP, prices and the terms-of-trade. The exchange rate is residually determined by the definition: $h = \tau + R - P$, where R is given by expression (22).

Again, expectations are rational with agents knowing the structure of both economies. As well, they know the current value of each foreign and domestic state variable and are capable of dividing monetary shocks into their permanent and transitory components. The relevant information set will be:

$$(24)\qquad I = (M_{-1}, \overline{u}, E_{-1}, \alpha_{-1}, m, n, d, s, I^*)$$

where I^* is the vector (19) and $\overline{u} = (u_{-1} - n_{-1})$. As with the foreign-economy model, the system is solved by the method of undetermined coefficients. It is apparent that, ultimately, the price level and the terms-of-trade will be log-linear functions of the magnitudes in the information set, I. Let C and T be 16-dimensional column vectors with typical entries, respectively, C_j and t_j. We postulate solutions for P and τ of the form:

$$P = C_o + I.C \qquad \text{and} \qquad \tau = t_o + I.T \tag{25}$$

Given full current information, the mathematical expectation of expression (25) yields $P^e = P$ and $\tau^e = \tau$ so that the current price level and terms-of-trade are fully-anticipated. We may also update the information set and take its expectation to obtain: ${}^1P^e = C_o + {}^1I^e.C$ and ${}^1\tau^e = t_o + {}^1I^e.T$ where, recall, the expected future value of any shock is zero. These expressions, along with (25), may now be substituted into (23) to solve for the "undetermined coefficients", C_j and t_j. Since one proceeds precisely as we did with the foreign economy, there is no need to belabour the derivation here. It can be shown that the solution will be:

$$(26) \quad \left\{ \begin{aligned} \tau &= t_3.(E_{-1} - \alpha_{-1} + d - s) + t_{11}.(E^*_{-1} + d^*) + t_{12}.(\alpha^*_{-1} + s^*) \\ q &= \alpha_{-1} + s + (\beta+\gamma).\tau \\ P &= M_{-1} + n - L + (1+L_2)(\overline{u} + m) + C_3.(E_{-1} + d) + C_4.(\alpha_{-1} + s) \\ &\quad + C_{11}.(E^*_{-1} + d^*) + C_{12}.(\alpha^*_{-1} + s^*) \end{aligned} \right.$$

The resulting expressions for the coefficients t_j and C_j are listed in the Appendix; each is, a priori, ambiguous in sign. This ambiguity stems from the indeterminacy of β, the impact of the terms-of-trade on aggregate supply. These parameters are discussed in more detail, shortly.

First, however, note that expression (26) implies that: $({}^1\tau^e - \tau) = 0$ and $({}^1P^e - P) = (1+L_2).m$. Consequently, with P and τ fully-anticipated, the domestic aggregate supply, IS and LM functions of system (23) reduce to, respectively:

$$(27)\quad \begin{cases} q = \alpha + (\beta+\gamma).\tau & (\beta+\gamma) \gtreqless 0 \\ q = E - E_1.\tau - E_2.r^* + E_4.\alpha^* \\ P = M - L - L_1.q + L_2.i \end{cases}$$

where: $E_1 = E_3+E_5 > 0$; $r^* = \lambda(E^*-\alpha^*)$; $q^* =\alpha^*$; and $i = r^*+(1+L_2)(\overline{u}+m)$.

It can be seen that domestic real GNP and the terms-of-trade are determined solely by reference to the aggregate supply and aggregate demand (i.e., IS) functions. The LM function serves only to determine the domestic price level while the exchange rate is then given by the definition: $h = \tau+R-P$, where R is given by (22). It is apparent from (27) that, other things constant, the impact of τ and, thereby of P on excess aggregate supply will be: $XS_p = (\beta+\gamma+E_1)$. We now impose the usual market stability condition that XS_p is positive. Given this condition, the Appendix shows that, in system (26): $t_3,t_{12} > 0$; $t_{11},C_4 < 0$; $C_3 \lesseqgtr 0$ as $(\beta+\gamma) \gtreqless 0$; $C_{11} > 0$ and $C_{12} < 0$ if $(\beta+\gamma) \geqq 0$. Of course, $(\beta+\gamma)$ and, therefore, XS_p would be negative <u>only if</u> labour and imports were gross substitutes in production (i.e., $\beta < 0$).

The aggregate supply and demand functions are, simply, curves in (q,τ)-space with slopes of, respectively, $(\beta+\gamma) \gtreqless 0$ and $-E_1 < 0$. Moreover, regardless of the slope of the aggregate supply curve, the above stability condition implies a rise in the terms-of-trade, ceteris paribus, always generates net excess aggregate supply. Consequently, geometry now suggests that a hike in aggregate supply will raise real GNP and lower the terms-of-trade while a hike in aggregate demand will unequivocally raise the terms-of-trade but will raise real GNP if and only if the aggregate supply curve is positively sloped. We are now in the position further to discuss the impacts of shocks. Since, according to system (26), lagged and current shocks have identical impacts, we need consider only current shocks.

(a) Domestic Demand and Supply Shocks (d,s)

We have already seen that a direct hike in aggregate supply raises GNP and lowers the terms-of-trade while leaving the real and nominal interest rates unaltered. As well, it reduces the domestic price level since the rise in GNP initially generates net excess demand for real balances. Of course, since both the terms-of-trade and price level are lower, the exchange rate, $h = \tau+R-P$ is indeterminate, a priori.

It was also seen that a direct hike in aggregate demand, d, will raise the terms-of-trade but may lower domestic GNP if labour and imports are gross substitutes in production. As well, it has no effect on interest rates. Should the demand shock raise GNP real money demand will rise thereby generating a fall in the domestic price level and vice versa. Since the foreign price level is not affected, the exchange rate, $h = \tau+P-R$, will unambiguously rise only if P falls.

(b) Foreign Demand Shocks (d*)

As seen above, a foreign demand shock has no effect on foreign GNP but merely raises the foreign real interest rate and price level. In turn, this raises the domestic real and nominal interest rates thereby reducing aggregate demand. As argued above, a fall in demand unequivocally reduces the terms-of-trade while raising domestic GNP only if labour and intermediate imports are gross substitutes in production. If GNP falls, prices rise since lower GNP and higher interest rates conspire to generate net excess supply of real balances. If GNP rises, with interest rates higher, the domestic price level will be indeterminate. Finally, with foreign prices higher, the terms-of-trade lower and domestic prices indeterminate, the change in the exchange rate, $h = \tau+R-P$, will be ambiguous.

(c) Foreign Supply Shocks (s*)

A foreign supply shock raises foreign GNP while reducing the foreign real interest rate and price level, as seen above. In turn, this raises domestic exports, investment and, thereby, aggregate demand. Consequently, the terms-of-trade unambiguously rise while domestic GNP may fall if labour and intermediate imports are gross substitutes in production. Should GNP rise it will, combined with the lower interest rate, raise real money demand thereby lowering the domestic price level. Otherwise, domestic prices may rise should GNP fall sufficiently to offset the positive impact of lower interest rates on real money demand. In any case, lower foreign prices combined with the higher terms-of-trade will be enough to leave the exchange rate, $h = \tau+R-P$, indeterminate, a priori.

(d) Foreign and Domestic Monetary Shocks (m^*,n^*,m,n)

No foreign or domestic real variable is altered by a shock to either nation's monetary growth rate. A rise in the domestic (foreign) money stock merely generates a proportionate rise in the domestic (foreign) price level and, thereby, a proportionate fall (rise) in the exchange rate, $h = \tau+R-P$. Nonetheless, the permanent shocks, m and m^*, have larger impacts on prices and the exchange rate than do the transitory shocks, n and n^*. Since m is a permanent hike in the monetary growth rate it raises the nominal interest rate by raising the expectation of future inflation which, in turn, lowers the demand for real balances. But such effect is absent with the transitory shock, n. Hence, the permanent shock initially generates the greater excess supply of real balances which is eliminated by the greater domestic price hike and, therefore, exchange depreciation. For exactly the same reasons, we have seen, the foreign price hike and, therefore, the exchange appreciation will be greater with a permanent than with a transitory shock to the foreign monetary growth rate. In short, when generated by a nominal shock, a foreign price hike merely appreciates the domestic currency.

(e) Summary

Most of the above results are, in some measure, contrary to conventional wisdom. First, unlike the standard Fleming-Mundell prediction, domestic GNP is not neutral with respect to a domestic demand shock (e.g., fiscal policy) even though the exchange rate is flexible and capital is perfectly mobile.[11] While real GNP is indeterminate, a priori, it will fall only when technology is sufficiently "mis-behaved" that labour and imports are relatively close gross substitutes in domestic production. Moreover, should real GNP rise, it will be accompanied by a domestic price deflation and an exchange appreciation. Second, a hike in foreign demand is most likely to reduce domestic real GNP while raising domestic prices. By raising the world real interest rate, a foreign demand shock crowds out both foreign and domestic investment leaving foreign output unaltered but reducing domestic real GNP, unless labour and imports are close gross substitutes. Third, a foreign supply shock simply raises domestic aggregate demand by raising exports and reducing the real interest rate. Hence, its impact is qualitatively similar to that for a positive domestic demand shock. Fourth, as with other models, the flexible exchange rate serves perfectly to insulate all domestic variables from foreign monetary shocks. Moreover, despite the flexible exchange rate and perfect capital mobility, endogenous domestic prices insure that the system will be neutral with respect to domestic monetary shocks.

In short, it can not be determined a priori, what changes in the domestic economy will accompany changes in foreign variables, such as prices, output or interest rates, until the ultimate source of foreign disturbances is known. Such source lies in monetary, demand and/or supply shocks.

IV. INCOMPLETE INFORMATION AND THE SIGNAL EXTRACTION PROBLEM

The above results critically rely on the existence of complete current information. We now relax this assumption by permitting agents to observe the state variables and, thereby, all shocks only with a one period lag. However, we maintain that, being single prices, the current exchange and nominal interest rates are observed and that all agents have identical information including full knowledge of the structure of the world economy. Nonetheless, given that there are eight possible shocks to the system, the model is hopelessly complicated without imposing some additional restrictions. Consequently, we concentrate on the effects of monetary and demand shocks by assuming that, in both nations, no supply shocks occur over the period of analysis while the variance of such shocks is approximately zero. As well, for simplicity, assume that: (i) trade involves only intermediate goods ; and (ii) in both nations, a price hike, whether anticipated or not, will, other things constant, generate net excess aggregate supply.[12]

Given the above assumptions, systems (18) and (23) imply the world economy can be characterized by:

$$(28)\quad \begin{cases} q^* = \alpha^*_{-1} + \delta^*.(R-R^e) \\ R = M^*_{-1} + \overline{u}^* + n^* + m^* - L^* - L^*_1.q^* + L^*_2.i^* \\ q^* = E^*_{-1} + d^* + E^*_0.(R-R^e) - E^*_2.r^* \\ r^* = i^* - {}^1R^e + R^e \end{cases}$$

$$(29)\quad \begin{cases} q = \alpha_{-1} + \beta.\tau + \delta.(P-P^e) \\ P = M_{-1} + \overline{u} + n + m - L - L_1.q + L_2.(r^* - {}^1\tau^e + \tau^e + {}^1P^e - P^e) \\ q = E_{-1} + d + E_0.(P-P^e) - E_2.(r^* - {}^1\tau^e + \tau^e) + E_4.q^* - E_5.\tau \\ \tau = h + P - R \end{cases}$$

where: the first three equations of systems (28) and (29) are the aggregate supply, LM and IS functions for, respectively, the foreign and domestic economies; $\overline{u} = u_{-1} - n_{-1}$; r^* is the foreign real interest rate; and i^* and $i = (r^* - {}^{1}\tau^{e} + \tau^{e} + {}^{1}P^{e} - P^{e})$ are the observed current foreign and domestic nominal interest rates, as shown above.

As before, expectations are rational with agents knowing that the observed exchange and nominal interest rates, as well as the unobserved foreign and domestic price levels, are determined by simultaneous solution to (28) and (29). First, let X_j be the j^{th} shock such that $(X_1,\ldots,X_6) = (n,m,d,n^*,m^*,d^*)$. Second, hold price expectations constant and solve (28) and (29) for i^*, R, h and P. Third, pass the expectations operator through the resulting expressions to compute i^{*e}, R^e, h^e and P^e. From the above solutions one can then compute the following expectational errors on exchange and interest rates and foreign and domestic prices:

$$(30)\quad \left\{\begin{array}{llll} (i^*-i^{*e}) = \sum_{j=1}^{6} F_j.(X_j - X_j^e); & & (R-R^e) = \sum_{j=1}^{6} v_j.(X_j - X_j^e) \\ (h-h^e) = \sum_{j=1}^{6} h_j.(X_j - X_j^e); & & (P-P^e) = \sum_{j=1}^{6} B_j.(X_j - X_j^e) \end{array}\right\}$$

where, the Appendix shows: $F_j = v_j = 0$, for $j = 1,2,3$; $F_4 = F_5 < 0$; $F_6 > 0$; $v_1 = v_2 > 0$; $v_6 > 0$; $h_1 = h_2 < 0$; $h_4 = h_5 > 0$; $h_3, h_6 > 0$; $B_1 = B_2 > 0$; $B_5 = B_4$; and $B_3, B_4, B_5, B_6 \gtreqless 0$ as $\beta \lesseqgtr 0$.

Expression (30) indicates that, with expectations held constant: a domestic demand shock or any foreign shock would raise while a domestic monetary shock would lower the exchange rate; a domestic demand shock or any foreign shock would raise domestic prices only if labour and imports are gross substitutes in domestic production (that is, if $\beta < 0$); P would rise with a positive domestic monetary shock; any foreign shock would raise foreign prices while the demand (monetary) shock would raise (lower) the foreign nominal interest rate; and, of course, domestic shocks would not impact on foreign variables. Finally, note that the impacts of monetary shocks would be the same whether these be permanent or transitory.

Observation of the current exchange and nominal interest rates presents agents with an information-extraction problem since it indicates that some shock has (or has not) occurred. For example, a domestic shock would alter the exchange rate but not the foreign nominal interest rate which informs agents that only a domestic shock has occurred. On the other hand, a foreign shock would alter the foreign interest rate indicating that a foreign shock has occurred; but, since the exchange rate will also change, agents may infer that a domestic shock has also occurred. Nonetheless, rationality requires that agents' expectations on all current shocks be consistent with the observed exchange and nominal interest rates. In turn, this requires that (i^*-i^{*e}) and $(h-h^e)$ both be zero. If agents know the first two moments of the (normal) distributions generating shocks, they have all the information necessary to solve the classical linear least-squares signal extraction problem in the optimal way. Consequently, we assume that agents form expectations by choosing the expected values of all current shocks so as to maximize the likelihood of observing the known current exchange and nominal interest rates. Although tedious, solution to this problem is straightforward. Hence, we only briefly outline the derivation of agents optimal expectations on current shocks.[13]

Let $X_j \sim N(0,\sigma_j^2)$ be the j^{th} shock such that $(X_1,\ldots,X_6) = (n,m,d,n^*,m^*,d^*)$ and let $f_j(X_j)$ be the logarithm of its density function. Then, the logarithm of the joint density function of the six random shocks will be: $W = \sum_{j=1}^{6} f_j(X_j)$ where, recall, the X_j's are independently distributed. The expected values of all current shocks are then chosen so as to maximize W subject to the condition that these expectations be consistent with observed exchange and interest rates; that is, more formally, subject to the constraints that $(h-h^e) = 0$ and $(i^*-i^{*e}) = 0$.

Now, let $\sigma_N^2 = \sigma_1^2+\sigma_2^2$ and $\sigma_{N^*}^2 = \sigma_4^2+\sigma_5^2$, where $N = n+m$ and $N^* = n^*+m^*$. Next, define the magnitudes: $Z = h_1^2\sigma_N^2 + h_3^2\sigma_3^2 > 0$; $Z^* = F_4^2\sigma_{N^*}^2 + F_6^2\sigma_6^2 > 0$; $G = h_4F_6-h_6F_4 > 0$;

$Q = Z.Z^* + G^2\sigma_6^2\sigma_{N*}^2 > 0$; $G_4 = G_5 = GF_6\sigma_6^2/Q > 0$; and $G_6 = -GF_4\sigma_{N*}^2/Q > 0$; where F_j and h_j are given in (30).

It can then be shown that the expected value of X_k conditional on the j^{th} (unobserved) shock having actually occurred will be:[14]

$$(31) \qquad X_{kj}^e = w_{kj}.X_j \qquad k,j = 1,\ldots,6$$

where: $w_{kj} = h_j h_k \sigma_k^2/Z$, for $k,j = 1,2,3$; $w_{kj} = h_k\sigma_k^2 G_j$, for $k = 1,2,3$ and $j = 4,5,6$; $w_{kj} = 0$, for $k = 4,5,6$ and $j = 1,2,3$; $w_{44} = w_{45} = \sigma_4^2(ZF_4^2 + G^2\sigma_6^2)/Q > 0$; $w_{55} = w_{54} = w_{44}\sigma_5^2/\sigma_4^2 > 0$; $w_{64} = w_{65} = F_4F_6Z\sigma_6^2/Q < 0$; $w_{46} = w_{64}\sigma_4^2/\sigma_6^2 < 0$; $w_{56} = w_{64}\sigma_5^2/\sigma_6^2 < 0$; and $w_{66} = \sigma_6^2(ZF_6^2 + G^2\sigma_{N*}^2)/Q > 0$. Moreover, aside from already reported signs, it can be seen that: $0 < w_{kk} < 1$, for all $k = 1,\ldots,6$; $w_{13}, w_{23}, w_{14}, w_{24}, w_{15}, w_{16}, w_{26} < 0$; $w_{12}, w_{21}, w_{34}, w_{35}, w_{36} > 0$; and $\text{sign}(w_{kj}) = \text{sign}(w_{jk})$, for $j,k = 1,2,3$.

The magnitude X_{kj}^e is, simply, the maximum likelihood estimate of X_k given the observed impact of the actual but unknown shock, X_j, on the exchange and nominal interest rates. Thus, any observed shock leads agents to attribute the resulting observed impact on the interest and exchange rates to a weighted average of all possible shocks. There results a misinterpretation of and confusion among shocks because, essentially, agents must infer six exogenous shocks from the observation of only two endogenous variables. The extent to which the actual shock, X_j, is misinterpretted as the shock X_k depends, of course, on the weight w_{kj} which will be absolutely larger, other things constant, the larger is X_k's variance and/or its absolute impact on exchange and interest rates, respectively, $|h_k|$ and $|F_k|$. As well, the definition of w_{kj} reflects the above-noted fact that a foreign shock may be misinterpretted as a domestic shock but not vice versa. This implies that, even though the domestic economy is small, it is not irrelevant to the determination of foreign magnitudes since its structure enters into foreign agents' optimal predictions of foreign shocks. The optimal extraction of information about foreign shocks from the observed exchange rate makes use of the known structure of both economies.

(a) The Foreign Economy

Given the above expectations, it is a simple matter to solve for the equilibrium values of foreign output, prices and interest rates. Of greatest interest here is the impact of unobserved shocks on unanticipated prices, $(R-R^e)$. Substitute (31) into (30) and re-arrange terms:

$$(R-R^e) = \lambda_4^*.N^* + \lambda_6^*.d^* \tag{32}$$

where: $N^* = n^*+m^*$; from previous results, $\lambda_j^* = v_j - \sum_{k=4}^{6} w_{kj}.v_k > 0$, for $j = 4,5,6$; and v_j is defined in (30). In short, any positive foreign shock generates a rise in unanticipated foreign prices.

Now substitute (32) into the foreign aggregate supply and IS functions contained in system (28) and solve to find that:

$$(33) \quad \begin{cases} q^* = \alpha_{-1}^* + \delta^*\lambda_4^*.N^* + \delta^*\lambda_6^*.d^* \\ r^* = \lambda[E_{-1}^* - \alpha_{-1}^* - \delta^{*\prime}\lambda_4^*.N^* + (1-\delta^{*\prime}\lambda_6^*).d^*] \end{cases}$$

where: $\lambda = 1/E_2^* > 0$; $\delta^{*\prime} = (\delta^*-E_0^*)$ is positive by the assumption that an unanticipated price hike generates net excess aggregate supply; and, from prior results and definitions, $(1-\delta^{*\prime}\lambda_6^*) > 0$.[15] Next, substitute (33) into the foreign LM function in system (28) and solve for foreign prices, R. Using the method of undetermined coefficients, the Appendix shows the foreign price level is given by:

$$R = M_{-1} + (1+L_2^*).\overline{u}^* - L^* - B_1^*.\alpha_{-1}^* + B_3^*.E_{-1}^* + B_4^*.N^* + B_6^*.d^* \tag{34}$$

where: $B_1^*, B_3^*, B_4^* > 0$ while $B_6^* \gtreqless 0$.

Thus, by raising unanticipated prices, any positive foreign shock will generate a (totally unexpected) hike in foreign output, q^*. A demand shock raises while a monetary shock lowers the foreign real interest rate, r^*. Of course, since the nominal interest rate is observed, the change in r^* will be fully anticipated. A monetary shock, by initially generating excess supply of real balances, will raise actual prices, R. On the other hand, a demand shock has an ambiguous impact on R. While such a shock raises output and the real interest rate, it is also partly mistaken for a permanent negative shock to the monetary growth rate (i.e., $w_{56} < 0$) which reduces the expected future inflation rate leaving the nominal interest rate indeterminate. The rise in output and the ambiguous nominal interest rate conspire to leave real money demand and, therefore, prices indeterminate.[16] Lastly, since they are observed, lagged shocks have exactly those impacts on all foreign variables found for current shocks in the case of complete current information.

(b) The Domestic Economy

It is now also a straightforward matter to determine the equilibrium levels of domestic real GNP, prices, the terms-of-trade and the exchange rate. First, we consider the impact of shocks on unanticipated domestic prices, $(P-P^e)$. Recalling that $(X_1,\ldots,X_6) = (n,m,d,n^*,m^*,d^*)$, substitute (31) into (30) to find:

$$(35) \qquad (P-P^e) = \lambda_1 . N + \lambda_3 . d + \lambda_4 . N^* + \lambda_6 . d^*$$

where: $N = n+m$; $N^* = n^*+m^*$; and $\lambda_j = B_j - \sum_{k=1}^{6} w_{kj} B_k$, for $j = 1,\ldots,6$. It can be shown that λ_1 and λ_3 are positive while λ_4 and λ_6 are indeterminate, a priori.[17]

The signs on the λ_j's reflect confusion among foreign and domestic shocks which may give rise to over- or under-anticipated price changes. The term B_j, as defined in (30), gives, for constant expectations, the j^{th} shock's impact on P while the term $\Sigma w_{kj} B_k$ gives its impact on P^e. First, regardless of its effect on expectations, a positive domestic monetary shock raises unanticipated prices. Second, a positive domestic demand shock also unequivocally raises $(P-P^e)$ despite the fact that B_3 is positive only when labour and imports are gross substitutes in production. This is because the observed rise in the exchange rate accompanying this shock will lead agents partly to mistake it for a negative domestic monetary shock (i.e., $w_{13}, w_{23} < 0$)

which, ceteris paribus, reduces P^e. In fact, when P falls, P^e always falls by more on account of this confusion among domestic shocks yielding an unequivocal rise in $(P-P^e)$. Third, as seen above, any positive foreign shock generates an observed change in world interest rates but a totally unexpected rise in foreign output, which must unequivocally raise unanticipated domestic aggregate demand. Hence, on account of actual and expected foreign shocks, we have, essentially, no more than a positive unanticipated domestic demand shock which, according to (30), would lower $(P-P^e)$ if and only if labour and imports are gross complements (i.e., iff $B_3 < 0$). At the same time, however, a foreign shock raises the observed exchange rate generating expectations on domestic shocks, in accord with (31), which would further alter P^e. This additional confusion among foreign and domestic shocks will, on balance, leave $(P-P^e)$ indeterminate, a priori.[18]

Next, briefly consider expected future appreciation in the terms-of-trade, $({}^1\tau^e-\tau^e)$, and expected future inflation, $({}^1P^e-P^e)$. First note that the expected value of any expectational error, $(X_j-X_j^e)^e$, is zero so that agents never expect to misanticipate the current levels of the state variables. Second, agents know the economy's structure, including the fact that, when all shocks are fully-anticipated, the terms-of-trade and domestic prices are given by expression (26). Consequently, τ^e and P^e must be determined by the expectation of (26) while ${}^1\tau^e$ and ${}^1P^e$ must be given by shifting the relevant equations in (26) one period forward and taking expectations of the resulting expressions where, recall, expected future shocks are zero.[19] Having done so, one finds that:[20]

$$(36) \qquad ({}^1\tau^e-\tau^e) = 0 \qquad \text{and} \qquad ({}^1P^e-P^e) = (1+L_2)(\overline{u} + m^e)$$

where: $\overline{u} = u_{-1}-n_{-1}$; $\overline{u}+m^e$ is the expected future growth rate of the money supply; and, according to (31), $m^e = w_{21}.n + w_{22}.m + w_{23}.d + w_{24}.N^* + w_{26}.d^*$.

Given the above results, the domestic economy, as characterized by system (29) can be greatly simplified to:

$$(37)\quad \begin{cases} q = \alpha_{-1} + \beta.\tau + \delta.(P-P^e) \\ q = E_{-1} + d + E_o.(P-P^e) - E_2.r^* + E_4.q^* - E_5.\tau \end{cases}$$

$$(38)\quad P = M_{-1} + (1+L_2)(\overline{u} + m^e) + n - L - L_1.q + L_2.r^*$$

$$(39)\quad h = \tau - P + R$$

where: q^* and r^* are given by (33); R is given by (34); $(P-P^e)$ is given by (35); and, according to (31), $m^e = \sum_{j=1}^{6} w_{2j}X_j$.

Equations (33), (35) and (37) are now sufficient to solve for domestic real GNP and the terms-of-trade. Given such solutions, (38) determines the domestic price level while (39) gives the exchange rate. Such solutions are formally presented in the Appendix and we simply discuss them here.

First, hold $(P-P^e)$ constant and solve (37) for q and τ:

$$(40)\quad \begin{cases} q = [E_5.\alpha_{-1} + \beta.(E_{-1} + d - E_2.r^* + E_4.q^*) + K.(P-P^e)]/\Delta \\ \tau = [E_{-1} + d - E_2.r^* + E_4.q^* - \alpha_{-1} - \delta'.(P-P^e)]/\Delta \end{cases}$$

where, by the above "stability" assumptions: $\Delta = (\beta+E_5) > 0$; $\delta' = (\delta-E_o) > 0$; and, thereby, $K = (\delta E_5+\beta E_o) > 0$.

Hence, holding unanticipated prices constant, expression (40) indicates the various shocks would have precisely those impacts on the domestic economy found above when information was complete. To review, domestic monetary shocks would have no impact on real GNP or the terms-of-trade; they would simply raise domestic prices and lower the exchange rate, in accord with (38) and (39). A hike in domestic aggregate demand, whether resulting from a domestic demand shock, d, a rise in foreign output, q^*, or a fall in the world real interest rate, r^*, would raise the terms-of-trade thereby raising real GNP if labour and imports are gross complements in production (i.e., if $\beta > 0$). Should real GNP rise from these shocks, the domestic price level would fall, in accord with (38). Nonetheless, unanticipated prices are not constant but, rather, they respond to shocks in accord with (35). Moreover, expression (40) indicates a rise in $(P-P^e)$ would, other things constant, unambiguously raise real GNP and lower the terms-of-trade. Consequently, any shock's ultimate impact on the domestic economy will differ from its full-information impact solely to the extent that it generates unanticipated price changes.

(i) Domestic Monetary Shocks ($N = n+m$): According to (40), domestic monetary shocks have no direct impact on aggregate demand and supply. However, given (35), they raise unanticipated prices which unambiguously raises domestic real GNP and lowers the terms-of-trade. With the terms-of-trade lower and domestic prices higher, the exchange rate must fall, according to (39), since domestic shocks do not alter the foreign price level, R. In short, money is no longer neutral with incomplete information.

(ii) Domestic Demand Shocks (d): With unanticipated prices constant, we have seen that a direct hike in aggregate demand would unambiguously raise the terms-of-trade while raising real GNP and lowering prices if labour and imports are gross complements (i.e., iff $\beta > 0$). However, according to (35), unanticipated prices will unambiguously rise. Consequently, real GNP will ultimately rise by more (or fall by less) than with a fully-anticipated demand shock. In fact, even when labour and imports are gross substitutes, the rise in unanticipated prices may be sufficient that, on balance, real GNP rises and the terms-of-trade fall. Now, we saw above that the positive domestic demand shock is partly mistaken for a negative domestic monetary shock in which case m^e falls lowering expected future inflation and, thereby, the domestic nominal interest rate. Thus, should q rise, the demand for real balances would unequivocally rise requiring a fall in the price level to restore money market equilibrium. Should q fall, the price level would be indeterminate depending upon the relative strengths of income and interest effects in the demand for real balances.

Finally, should the terms-of-trade rise and the price level fall, the exchange rate would appreciate. In short, when labour and imports are not "overly" close gross substitutes, a positive domestic demand shock will raise real GNP, the terms-of-trade and the exchange rate while lowering domestic prices.

(iii) Foreign Monetary Shocks ($N^*=n^*+m^*$): We saw in the previous section that a foreign monetary shock raises foreign output, q^*, and prices, R, while lowering the real interest rate, r^*. Hence, domestic exports, investment and, thereby, aggregate demand unequivocally rise. With unanticipated prices constant, such a hike in demand would unambiguously raise the terms-of-trade while lowering real GNP only if labour and imports are gross substitutes in production. In this respect, the foreign monetary shock amounts to little more than a positive domestic demand shock. However, the foreign monetary shock is partly mistaken for positive domestic demand and negative domestic monetary shocks which leads to an a priori ambiguous change in unanticipated prices, in accord with (35). As well, with r^* lower, m^e negative and, thereby, expected future inflation lower, the domestic nominal interest rate unequivocally falls. Consequently, should $(P-P^e)$ rise, it can be seen that the foreign monetary shock's impact on the domestic economy would be qualitatively identical to that discussed above for a positive domestic demand shock. On the other hand, suppose $(P-P^e)$ falls; two cases must be considered. First, let labour and imports be gross complements so that, with expectations constant, the foreign monetary shock raises q and lowers P. According to the LM function, (38), unanticipated prices could fall only with an unanticipated rise in real money demand; since the nominal interest rate is observed, this requires an unanticipated rise in real GNP. Hence, real GNP rises which, given the fall in $(P-P^e)$, implies the terms-of-trade also rise. Second, let labour and imports be gross substitutes such that, with expectations constant, q falls and P rises. The assumed accompanying fall in $(P-P^e)$ must, then, further accentuate the decline in q and, thereby, rise in P. In summary, regardless of what happens to unanticipated prices, a foreign monetary shock will raise real GNP, the terms-of-trade and the exchange rate while lowering domestic prices unless labour and imports are relatively close gross substitutes in production.

(iv) Foreign Demand Shocks (d^*): As seen in the previous section, a foreign demand shock unambiguously raises foreign output, q^*, and real interest rate, r^*, while leaving foreign prices, R, indeterminate, a priori. In turn, this generates a rise in domestic exports, a fall in domestic investment and, thereby, an ambiguous change in domestic aggregate demand. As well, regardless of what happens to aggregate demand, confusion among foreign and domestic shocks will conspire to generate an a priori ambiguous change in unanticipated domestic prices, in accord with (35). Consequently, with respect to domestic real GNP and the terms-of-trade, the possible impacts are qualitatively identical to those for an ambiguous foreign monetary shock, discussed above. When domestic investment is sufficiently interest-inelastic that aggregate demand rises, domestic real GNP and the terms-of-trade will rise (may fall) if labour and imports are gross complements (substitutes) in production and vice versa. However, impacts on domestic prices and the exchange rate are less clear-cut. While raising the real interest rate, r^*, the foreign demand shock is also partly mistaken for a permanent negative shock to the domestic monetary growth rate (i.e., $w_{26} < 0$) which, by reducing expected inflation, will, on balance, leave the domestic nominal interest rate indeterminate, a priori. Hence, whether real GNP rises or falls, the demand for real balances and, thereby, the price level will be indeterminate, a priori, according to (38). Thus, real GNP and prices may readily move in the same or in opposite directions. Finally, whether the terms-of-trade rise or fall, the ambiguous change in both foreign and domestic prices conspire to leave the exchange rate, $h = \tau+R-P$, indeterminate, a priori.

In summary, with incomplete information, domestic real variables are no longer neutral with respect to current domestic and foreign monetary shocks. Of course, the economy _is_ neutral with respect to lagged monetary shocks, according to (40), so their current real effects are purely transitory. A domestic demand shock raises real GNP, the terms-of-trade and the exchange rate while lowering prices, unless labour and imports are relatively close gross substitutes in production. Lastly, a foreign demand shock has an ambiguous effect on all domestic variables largely because it has an indeterminate impact on domestic aggregate demand. Nonetheless, any rise in aggregate demand will be transitory since such a (positive) shock serves, in the end, only permanently to reduce domestic investment by raising the real interest rate; it has no permanent effect on foreign output or, thereby, domestic exports.

V. CONCLUSIONS

We have constructed a model of a small open economy (SOE) with four distinguishing features First, the larger foreign economy was explicitly modelled to analyze the impacts of those simultaneous movements in foreign output, prices and interest rates which must necessarily accompany any foreign shock. Second, as other writers have found, explicit consideration of the supply-side effects of the terms-of-trade effectively destroyed the notion of a "natural" level of real GNP in an SOE. Third, the impact of most shocks on an SOE was, **a priori**, ambiguous depending on the extent, if any, of gross substitutability between labour and intermediate imports in domestic production. Fourth, expectations were rational with agents making optimal use of all available information. This led, for both domestic and foreign agents, to confusion among and between foreign and domestic shocks capable of generating additional non-neutralities and/or ambiguities in the model.

With full current information, money is neutral in both large (or closed) and small open economies. Moreover, the flexible exchange rate serves perfectly to insulate the SOE from foreign monetary shocks. An indigenous supply shock expands real GNP and lowers prices in either nation; but the foreign shock may reduce the

SOE's real GNP and raise her prices if labour and imports are relatively close gross substitutes in domestic production. Fiscal policy is effective in an SOE but not in a large open (or closed) economy; its effectiveness in the SOE stems from the fact that it permanently raises the terms-of-trade. Nonetheless, if labour and imports are not overly-close gross substitutes in domestic production, a domestic fiscal expansion raises (lowers) while a foreign fiscal stimulus reduces (raises) domestic real GNP (prices). This is because the former stimulus raises while the latter lowers domestic aggregate demand.

With incomplete information, the effect of any shock on either economy may be opposite to that noted above. This results from confusion among and between domestic and foreign shocks when information is optimally extracted from observed interest and exchange rates. Such confusion generates (possibly perverse) movements in unanticipated prices which are absent with complete information. Money will no longer be neutral, in the short run, with monetary policy expanding real GNP and prices in either nation. By unequivocally raising indigenous prices, fiscal policy will now be effective in both nations; in the large economy a fiscal stimulus raises real GNP while, in the SOE, a fiscal stimulus will be more expansionary (or less contractionary) than when information is complete. Moreover, a foreign fiscal stimulus no longer unambiguously reduces domestic aggregate demand because, while raising the real interest rate, it also raises foreign output and, thereby, domestic exports. Consequently, with gross complementarity, it would tend to have an expansionary impact on the SOE if domestic investment is relatively interest-inelastic.

APPENDIX

A. Complete Information Coefficients

The parameters of system (26) are given below. For simplicity, define the terms: $A = E_4 + \lambda E_2 > 0$; $\beta' = (\beta+\gamma) \gtreqless 0$; and, recall, by the market stability condition, $XS_p = (\beta+\gamma+E_3+E_5) = (\beta'+E_1) > 0$.

$t_3 = -t_4 = t_7 = -t_8 = XS_p^{-1} > 0$; $t_{11} = t_{15} = -t_3\lambda E_2 < 0$; $t_{12} = t_{16} = At_3 > 0$.

$C_o = -L < 0$; $C_1 = C_6 = 1$; $C_2 = C_5 = (1+L_2) > 0$; $C_3 = C_7 = -\beta' L_1 t_3 \lesseqgtr 0$ as $\beta' \gtreqless 0$;

$C_4 = C_8 = -t_3E_1L_1 < 0$; $C_{11} = C_{15} = \lambda(L_2 - C_3E_2) > 0$ if $\beta' \geqq 0$; $C_{12} = C_{16} = C_3A - \lambda L_2 < 0$ if $\beta' \geqq 0$; $t_j, C_j = 0$, for all other j.

B. Unanticipated Prices, Exchange Rates and Interest Rates

Let $D^* = E_2^*(1+\delta^*L_1^*)+(\delta^*-E_o^*)L_2^*$ and $D = \beta+E_5+L_1(\delta E_5+\beta E_o)$ both of which are positive by the text's "stability" assumptions. Then, the parameters of (30) are:

$F_1 = F_2 = F_3 = v_1 = v_2 = v_3 = 0$; $F_4 = F_5 = (\delta^*-E_o^*)/D^* > 0$; $F_6 = (1+\delta^*L_1^*)/D^* > 0$;

$v_4 = v_5 = E_2^*/D^* > 0$; $v_6 = L_2^*/D^* > 0$; where, by assumption, $(\delta^*-E_o^*) > 0$.

$h_1 = h_2 = -(\beta+E_5+\delta-E_o)/D < 0$; $h_3 = (1+\delta L_1+\beta L_1)/D > 0$; $h_4 = h_5 = v_4(1+h_3E_4\delta^*) > 0$;

$h_6 = v_6(1+h_3E_4\delta^*) > 0$; $B_1 = B_2 = (\beta+E_5)/D > 0$; $B_3 = -\beta L_1/D \gtreqless 0$ as $\beta \lesseqgtr 0$; $B_4 = B_5 = B_3E_4\delta^*v_4 \gtreqless 0$ as $B_3 \gtreqless 0$; $B_6 = B_3E_4\delta^*v_6 \gtreqless 0$ as $B_3 \gtreqless 0$; where $(\beta+E_5) > 0$ and $(\delta-E_o) > 0$, by stability assumptions.

C. Foreign Prices and Incomplete Information

First note that the expectational error on any shock, $(X_j-X_j^e)$, has an expected value of zero; hence, agents never expect to mis-anticipate the current levels of the state variables. Second, agents know the structure of the economy, including the fact that, when all shocks are fully-anticipated, foreign prices are given by expression (22). Hence, R^e must be theexpectation of (22) while ${}^1R^e$ is given by shifting (22) one period forward and taking its expectation. This all implies that: $({}^1R^e-R^e) = (1+L_2^*)(\bar{u}^*+m^{*e})$ where, from (31), $m^{*e} = w_{55}N^* + w_{56}d^*$ with $w_{55} > 0$, $w_{56} < 0$. Substitute this and (33) into (28)'s LM function to obtain the coefficients in (34).

$B_1^* = (L_1^*+\lambda L_2^*) > 0$; $B_3^* = \lambda L_2^* > 0$; $B_4^* = 1 - (\delta^* L_1^* + \delta^{*\prime}\lambda L_2^*).\lambda_4^* + w_{55}(1+L_2^*) > 0$;

$B_6^* = L_2^*(1-\delta^{*\prime}\lambda_6^*) - \delta^* L_1^*\lambda_6^* + (1+L_2^*)w_{56} \gtreqless 0$; where, $\delta^{*\prime} = (\delta^*-E_0^*) > 0$, $\lambda = 1/E_2^* > 0$.

C. <u>Incomplete Information Solutions for Domestic Variables</u>

When $(X_1,\ldots,X_6) = (n,m,d,n^*,m^*,d^*)$ solutions take the form:

$$q = f_o + \Sigma f_j X_j;\quad \tau = t_o + \Sigma t_j X_j;\quad P = p_o + \Sigma p_j X_j;\quad h = g_o + \Sigma g_j X_j$$

Substitute (33) and (35) into (40) where, recall, $\Delta > 0$, $K > 0$, $\delta' = (\delta - E_o) > 0$.

(i) <u>Terms-of-Trade</u>: $t_o = [E_{-1} - \alpha_{-1} + (E_4+\lambda E_2).\alpha^*_{-1} - \lambda E_2.E^*_{-1}]/\Delta$

$t_1 = t_2 = -\delta'\lambda_1/\Delta < 0$; $t_3 = (1-\delta'\lambda_6)/\Delta > 0$; $t_4 = t_5 = [(\delta^* E_4+\lambda E_2\delta^{*\prime})\lambda_4^*-\delta'\lambda_4]/\Delta \gtreqless 0$;

$t_6 = [E_4\delta^*-\lambda E_2(1-\delta^{*\prime})]\lambda_6^*/\Delta - \delta'\lambda_6/\Delta \gtreqless 0$.

(ii) <u>Real GNP</u>: $f_o = \beta.t_o + (E_5/\Delta).\alpha_{-1}$; $f_1 = f_2 = \beta.t_1 + \delta.\lambda_1 = K\lambda_1/\Delta > 0$;

$f_3 = \beta.t_3 + \delta.\lambda_3 = (\beta+K\lambda_3)/\Delta > 0$ if $\beta \gtreqless 0$; $f_4 = f_5 = \beta.t_4 + \delta.\lambda_4$; $f_6 = \beta.t_6 + \delta.\lambda_6$.

Substitute these solutions into (38) and (39), where (31) and (33) give m^e and r^*.

(iii) <u>Prices</u>: $p_1 = p_2 = 1 - L_1.f_1 + (1+L_2).w_{22} > 0$; $p_3 = (1+L_2).w_{23} - L_1.f_3 \gtreqless 0$

$p_4 = p_5 = (1+L_2)w_{24} - L_1 f_4 - \lambda L_2\delta^{*\prime}\lambda_4^* \gtreqless 0$; $p_6 = (1+L_2)w_{26} + \lambda L_2(1-\delta^{*\prime}\lambda_6^*) - L_1 f_6 \gtreqless 0$;

$p_o = M_{-1} - L + (1+L_2).\bar{u} - L_1.f_o + \lambda L_2(E^*_{-1} - \alpha^*_{-1})$.

(iv) <u>Exchange Rate</u>: $h = \tau + R - P$, where R is given by equation (34).

$g_o = t_o - p_o + M^*_{-1} - L^* + (1+L_2^*).\bar{u}^* - B_1^*.\alpha^*_{-1} + B_3^*.E^*_{-1}$; $g_1 = g_2 = t_1 - p_1 < 0$;

$g_3 = t_3 - p_3$; $g_4 = g_5 = t_4 - p_4 + B_4^*$; $g_6 = t_6 - p_6 + B_6^*$.

FOOTNOTES

1. For examples, see Schmid (1975;1979); Findlay & Rodriguez (1977); Kingston & Turnovsky (1978); Laidler (1975;1977); or Burton (1980).

2. In this regard, we follow Barro (1978) and Burton (1980).

3. Maximize $U = U(C^d, C^f, 1-N^s)$ subject to $W.N^s = P^e C^d + R^e C^f/h$, where C^d, C^f and $(1-N^s)$ are demands for, respectively, domestic goods, foreign goods and leisure. Rewriting the budget constraint as $(W/P^e).N^s = C^d + C^f/\tau^e$, it is obvious that (4) and (5) will obtain. For simplicity, we ignore the possible effects of wealth, non-labour income and interest rates on the worker's decision. One may also assume the saving, consumption and labour supply decisions are separable. Note that this approach does not explicitly consider total consumption thereby avoiding the need artificially to construct a consumer price index.

4. Such solution is straightforward so details are left to the reader.

5. Two inputs are gross complements (substitutes) when f_{12} is positive (negative).

6. Given the later-assumed stochastic structure of the model, the inflation rate currently expected over <u>all</u> future periods will be constant and equal to $({}^1P^e - P^e)$. Hence, the fact that $r = i - {}^1P^e + P^e$ involves only one-period-ahead price forecasts is not to suggest a one-period planning horizon for investment.

7. See Scarth (1979) for the view that money demand may depend on y, not on q.

8. Note that the (nominal) demands contained in (12) and (13) are communicated to markets at period's start to insure, as required by an equilibrium model, that any observed wage, price, interest or exchange rate takes its market-clearing value.

9. Of course, the foreign economy is not, in fact, closed since domestic variables affect the decisions of individual foreign agents, such as importers, exporters and portfolio investors. Nonetheless, "largeness" implies such agents comprise an insignificant portion of aggregate foreign economic activity.

10. This method is detailed by Lucas (1972) and Barro (1976).

11. A result previously reported by Findlay & Rodriguez (1977).

12. Exclusion of consumer trade, implying $\gamma = E_3 = 0$, is innocuous and merely formalizes the empirical fact that most direct importers are firms, of one sort or another. Note, however, that a rise in τ is now all the more likely to reduce aggregate supply when labour and imports are gross substitutes in production. Assumption (ii) is simply the above market stability condition where, for fully-anticipated price changes, $XS_p = (\beta+E_5) > 0$ and, for hikes in unanticipated prices, $(\delta-E_o) > 0$ and $(\delta^*-E_o^*) > 0$. System (26) still gives the full-information solution but, now, $t_3 = (\beta+E_5)^{-1} > 0$.

13. For simplicity, assume agents are unable to extract any information from the individual product and/or factor prices they observe. Alternatively, see Lucas (1973).

14. Let the j^{th} shock occur. The first-order conditions for constrained maximization of the joint-normal density function are: (i) $X_{kj}^e/\sigma_k^2 = bh_k + cF_k$, for all k; (ii) $h_jX_j - \sum_{k=1}^{6} h_kX_{kj}^e = 0$; (iii) $F_jX_j - \sum_{k=1}^{6} F_kX_{kj}^e = 0$; where b and c are the Lagrangian multipliers on constraints (ii) and (iii). Solve these to obtain (31).

15. Given the definitions of h_j, F_j and, therefore, w_{kj} simplification reveals that: $(1-\delta^{*\prime}\lambda_6^*) = \sigma_6^2[G^2\sigma_{N*}^2 + ZF_6^2(E_2^*F_6-L_2^*F_4]/Q > 0$.

16. Obviously, prices unambiguously fall if the nominal interest rate falls.

17. From the definitions of D, h_j, F_j and, thereby, w_{kj} simplification yields: $\lambda_1 = h_3\sigma_3^2/ZD > 0$ and $\lambda_3 = -h_1\sigma_N^2/ZD > 0$. But, $\lambda_j = B_3(E_4\delta^*\lambda_j^*-w_{3j}) - B_1(w_{1j}+w_{2j})$, for j = 4,5,6 can not be signed, a priori.

18. Although a domestic demand shock unequivocally raises $(P-P^e)$, a foreign-induced hike in domestic demand will not. This is because the two shocks yield different price expectations. The domestic shock generates expectations based only on the observed exchange rate while, with the foreign shock, expectations are based on observation of both the exchange and world interest rates.

19. That is, if agents expect never to mis-anticipate shocks, they must act as if P and τ derive from a full-information model.

20. This result can be more formally derived using the method of undetermine coefficients with τ and P proposed to be functions of X_j <u>and</u> X_j^e.

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