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INTERTEMPORAL COMMON PROPERTY EQUILIBRIA:

THE FISHERY

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Abstract

Intertemporal Common Property Equilibria: The Fishery

An agent adjusts its harvest in an intertemporal optimization problem taking other agents' harvests as fixed. With stock size affecting harvest costs, an intertemporal externality is present. Cooperative and competitive solutions are compared given an exogenously fixed number of agents. Then an entry-exit relation is introduced for each regime. Multiple equilbria obtain for each regime. One cannot say that the competitive solution has too many boats and too small a steady state stock of fish relative to the cooperative regime.

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Intertemporal Common Property Equilibria: The Fishery*

1. *INTRODUCTION*

No capital theoretic issues emerge explicitly in the classical treatments of common property in the fishery in for example Gordon [1954], and Dasgupta and Heal [1979] in spite of the obvious capital theoretic nature of optimal planning solutions in for example Clark [1976]. Intuition suggests that under common property, there will be too much fishing "today" because for any agent to restrict his take today in the interest of having a larger stock "tomorrow" will be frustrated because free entry and access by other agents will dissipate the planned savings. In Hartwick [1980] I indicated how one could formalize the essential capital theoretic "pathology" inherent in common property and free access[1].

Here I use those ideas to explore equilibria in an industry facing given demand conditions, a situation in which the number of agents or fishing "boats" must be determined simultaneously with stock size and catch per boat. In particular one is interested in the notion that under common property there are too many boats and too small a steady state stock relative to a socially optimal planning solution. For plausible sufficiency conditions or frequently used functional forms, multiple equilibria turn out to be the rule, some with equilibrium stocks lower under optimal planning and some with more boats under optimal planning. We find that we cannot rule out some equilibria by invoking stability considerations. There is no obvious basis for statements of the kind: "in a common property equilibrium, there will be too many boats and too small a stock relative to a socially optimal solution." We also observe that existence of any interior solution is sensitive to the relative magnitudes of the discount rate and the maximum rate of growth of the fish stock. We note at the end that controlled access to a stock will not eliminate the common property problem in the sense of leading to a first best solution. Each boat's catch must also be set if, for a particular number of boats, the common property inefficiency is to be eliminated.

The common property "pathology" arises in this model because each agent is so to speak peering over his shoulder, checking on what others are doing or will do, not because there are "too many" boats exploiting the stock. This was the essence of Hartwick [1980] and represents a quite different perspective than in the classical tradition of Gordon [1974], and Dasgupta and Heal [1979]. Here under common property, an agent can only reap the average "payoff" from foregoing at the margin some catch today, rather his marginal "payoff" (average being defined over all agents or boats). In the classical tradition, under common property each agent could only reap the average product of his *current* effort of fishing rather than his marginal product. The analysis was essentially atemporal. We in fact indicate the appropriate tax on fish caught which for a fixed number of boats will bring about a first best solution under common property assumptions. This tax involves time and the discount rate explicitly, reflecting the intertemporal nature of the common property externality.

In Section 4, we set out the analogous atemporal model of the fishery in which agents optimize only over the current period or optimize in a myopic fashion. This approach was well developed by Smith [1968] and our report is a concise statement of his model. From one perspective one can view our new model as an extension of Smith's analysis to an intermporal setting.

2. *THE INTERTEMPORAL EXTERNALITY*

We proceed to isolate the intertemporal externality associated with free access to the stock or with a common property regime. We have in the background, the familiar growth-exploitation dynamics, now in discrete time:

$$Z_{t+1}-Z_t = \phi(Z_t) - \sum_{i=1}^{N} q_t^i \tag{1}$$

where $\phi(Z_t)$ is a function indicating net births over deaths of identical fish. (We will later specialize to our familiar form $aZ_t\left(1-\frac{Z_t}{K}\right)$ where a and K are positive parameters)

Z_t is the stock of fish (in homogeneous members) of period t

q_t^i is the catch of boat i in period t.

Under the *common property* regime, each boat or agent maximizes discounted profit

$$\underset{\{q_t^i\}}{\pi^i} = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left\{pq_t^i - C^i\left(Z_t,q_t^i\right)\right\} \quad (i=1,\ldots,N) \tag{2}$$

where r is the rate of interest

p is the price per unit of fish delivered at the shore

$C^i\left(Z_t,q_t^i\right)$ is the cost per boat of catching q_t^i fish given stock size Z_t. $C^i_{q_t^i} > 0$ and $C^i_{Z_t} < 0$ since a larger stock implies lower costs or easier catching of fish.

An equilibrium occurs when (a) each agent is maximizing (2) and (b) entry has continued until each boat or agent is earning zero profits. Entry is taken up in Section 3.

Under *central control*, the boats are deployed by a manager or planner so as to maximize

$$\underset{\{q_t^i,\ldots,q_t^N\}}{W} = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left\{\sum_{i=1}^{N} \left[p\, q_t^i - C^i\left(Z_t,\, q_t^i\right)\right]\right\} \tag{3}$$

The first order conditions for problems (2) and (3) are respectively:

$$p - C^i_{q^i_t} - C^i_{Z_t}\left[\phi_{Z_t} + 1\right]^{-1} + \left(\frac{1}{1+r}\right) C^i_{Z_{t+1}} = 0 \qquad (i=1,\ldots,N) \tag{4}$$

and

$$p - C^i_{q^i_t} - \sum_{i=1}^{N}\left\{C^i_{Z_t}\left[\phi_{Z_t} + 1\right]^{-1} - \left(\frac{1}{1+r}\right) C^i_{Z_{t+1}}\right\} = 0 \qquad (i=1,\ldots,N). \tag{5}$$

If agents are the same (with the same costs and perceptions as the behaviour of other (i.e. Cournot))and we are in a steady state, equations (4) and (5) become respectively:

$$p - C^i_{q^i_t} - \left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right] C^i_Z = 0 \qquad (i=1,\ldots,N) \tag{6}$$

and

$$p - C^i_{q^i_t} - \left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right] NC^i_Z = 0 \qquad (i=1,\ldots,N) \tag{7}$$

Observe that under common property, the gap between price and marginal cost between p and $C^i_{q^i_t}$ is not as large as it "should" be (i.e. as it is under central management). The *appropriate tax per fish* caught in equilibrium for the common property regime is $(N - 1)\left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right] C^i_Z$. With q^i and Z adjusting to this tax, the two solutions, common property with tax and centrally managed, will be the same. By saving or foregoing some catch today in the hope of having a larger stock tomorrow, each agent can only reap the *average payoff*, $\left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right] C^i_Z > 0$ (average being taken over the number of agents (here,N)) rather than the *marginal payoff*, $N\left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right] C^i_Z > 0$. This result[2] permits us to make contact with the familiar common property result: the average product of the variable input equals its price rather than the marginal product equalling the price (e.g. Weitzman [1974]).

Without searching for generality we let $\phi(Z) \equiv aZ\left(1 - \frac{Z}{K}\right)$ and[3] $C(Z,q) \equiv q/Z$. Our two steady state equations, (6) and (7) become, respectively:

$$p - \frac{1}{Z} = \left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right]\left(-\frac{\phi}{NZ^2}\right) \tag{8}$$

$$p - \frac{1}{Z} = \left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right]\left(-\frac{\phi}{Z^2}\right) \tag{9}$$

Since the LHS is price minus marginal cost, if an economically admissable solution exists, then $\left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right] < 0$ in (8) and (9). We return to this below. $\phi_Z = a\left(1 - \frac{2Z}{K}\right)$ is positive (negative) as $2Z < K$ $(2Z > K)$ and $\phi_{ZZ} < 0$. For $Z = K$, $\phi = 0$ and the RHS's of (8) and (9) equal zero. The LHS's equal $p - \frac{1}{K}$. For $Z \to 0$, the RHS's approach $+a$ and the LHS's approach $-a$. The derivative with respect to Z of the RHS of (8) is

$$-\left(\phi_Z+1\right)^{-2} \phi_{ZZ} \left(-\frac{\phi}{2Z^2}\right) + \left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right]\left(-\frac{1}{2Z^3}\right)\left[\phi_Z Z - 2\phi\right]$$

which is negative if $\left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right]$ is negative which we asserted must be true if an equilibrium exists. However, $\phi_Z = 0$ at $Z = \frac{K}{2}$ so there is a region in which $\left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right] > 0$. In fact $\phi_Z = a\left[1 - \frac{2Z}{K}\right]$ ranges linearly between a and $-a$ as Z ranges between admissable values, namely between 0 and K. For $a \geq 1$, ϕ_Z can equal -1 yielding $\left(\frac{1}{\phi_Z+1}\right) = \alpha$. For $0 < a < 1$, $\left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right] > 0$ for $\frac{K}{2} \leq Z \leq K$ (this latter is not the complete region of $[\ \cdot\ ] > 0$, we note). Since the RHS's of equations (8) and (9) differ only by $1/N$, a constant, what we have said for the values of one RHS with respect to changes in Z is true for the other.

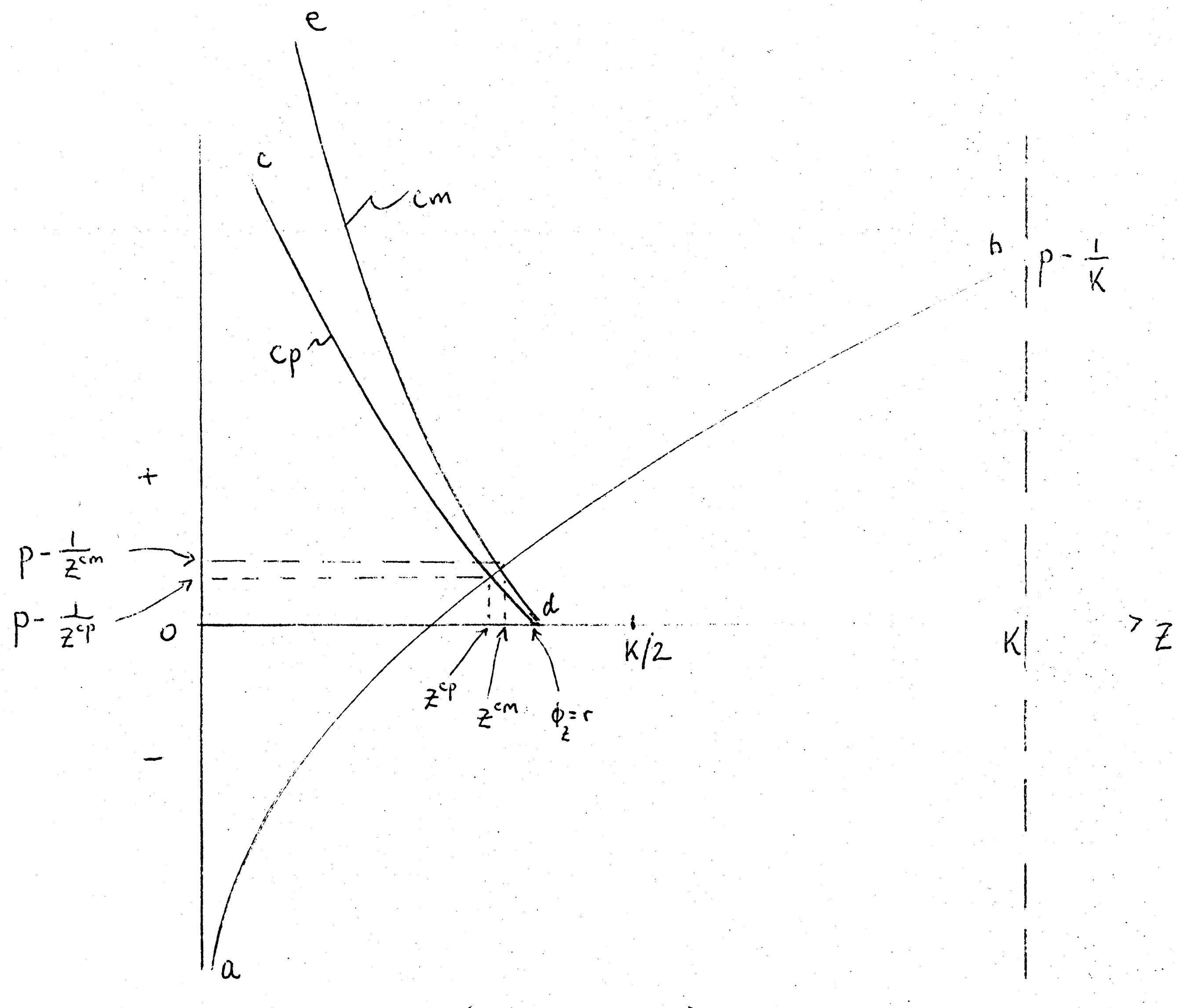
We sketch two cases in Figure 1. In Figure 1a, the growth parameter a and the interest rate r are related as in $0 < r < a < 1$ and in Figure 1b, we have $0 < r < a < \infty$ with $1 < a$. (Note for $0 < a < 1$ and $r > a$, $\left[\left(\frac{1}{\phi_Z + 1}\right) - \left(\frac{1}{1+r}\right)\right] > 0$ and an equilibrium with $p > C^i_{q^i}$ does not exist.) Whether a solution exists in Figure 1a depends on the values of the parameters as one can see. For example, the likelihood of a solution existing is less as p becomes smaller. Schedule ab in Figure 1a, is the LHS of equations (8) and (9). Schedule cd is the RHS of equation (8), relevant to the common property solution, and schedule ed is the RHS of equation (9), relevant to the central management solution. If solutions exist we have the

Proposition 1 (Lower steady state stock under common property): The solution stock Z^{cp} (cp for common property) is less than that under central management, namely Z^{cm} (cm for central management).

Clearly, if solutions exist for the case illustrated in Figure 1b, our proposition 1 remains valid for neighboring values of Z^{cm} and Z^{cp}. However, the region in which $\left[\left(\frac{1}{1+\phi_Z}\right) - \left(\frac{1}{1+r}\right)\right] > 0$ is now the central portion between 0 and K, namely for values of Z such that $-1 \leqq \phi_Z \leqq r$. Mulitple solutions cannot be ruled out by economically plausible sufficiency conditions. We also observe that for neighboring *solutions*, namely Z^{cm} and Z^{cp},

Proposition 2 (Greater surplus per unit under central management):

$$\left(p - \frac{1}{Z^{cm}}\right) > \left(p - \frac{1}{Z^{cp}}\right).$$



$(0 < r < a < 1)$

Figure 1a

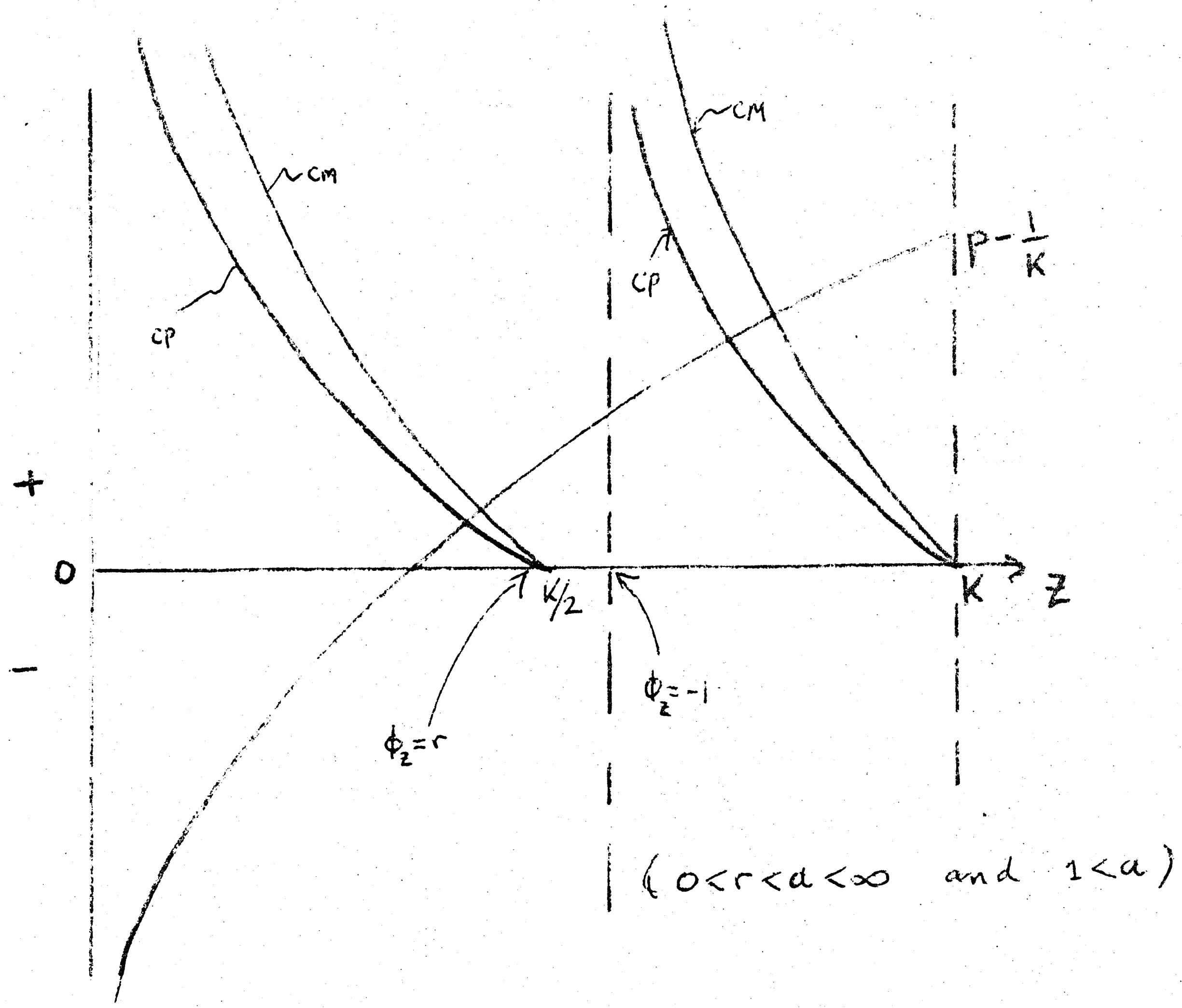


Figure 1b

We turn now to analyze industry equilibria with entry and exit of agents or boats. This is a key aspect of common property, namely free entry of competitors when profits are positive[4].

3. *ENTRY, EXIT AND INDUSTRY EQUILIBRIA*

Above, the number of agent or boats was fixed exogenously. A central feature of the common property model in that agents enter freely until net profit or rent per agent is zero. This gives us an additional equation with which to solve for N, the number of agents. The equation is

$$p(Q)q^i - C^i(Z,q^i) = w^i \qquad (i=1,\ldots,N)$$

where $Q \equiv \sum_{i=1}^{N} q^i$ and $p(Q)$ is the inverted industry demand schedule

w^i is the opportunity cost of a boat for a period and is treated as a constant. w^i being constant captures the notion that the resources flowing into the fishing industry are small relative to the size of the economy.

We will assume all boats are identical so that $C^i(\cdot) = C^j(\cdot)\ \forall\, i$ and j and $w^i = w^j\ \forall\, i$ and j. The superscript i can be omitted now, giving us

$$p(Q)q^i - C(Z,q^i) = w \qquad (i=1,\ldots,N). \tag{10}$$

Note that this equation holds at a steady state solution. We shall argue below, however that if $p(Q)q^i - C(Z,q^i) \gtrless w$, N will $\begin{Bmatrix}\text{rise}\\ \text{fall}\end{Bmatrix}$ or entry and exit will occur. Steady state versions of equations (1), and (6) and (10), constitute the three equations comprising *a common property equilibrium for the industry*. They can be solved for boat catch q^i, stock Z and number of boats N if a solution exits with q^i, Z, and N positive. p is assumed to be endogenous in equation (6) now.

The planner will regulate entry in order to maximize the present value of net surplus or discounted current welfare. We put this calculation in a two stage framework. For any N, we can calculate the optimal stock size Z and catch per boat q. These are the steady state values if an interior solution exists. With entry, we require that among these solutions, optimal for a given N, there is one that is optimal with respect to variation in N. In the first stage, a planner will optimize with respect to stock size, given N, and in the second stage, the optimal value of N will be chosen. Using equations (1) and (7), we have a steady state equation for Z given N. This permits us to compute the welfare value given N in

$$\Omega = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left\{ B\left(\sum_{i=1}^{N} q_t^i\right) - \sum_{i=1}^{N} C\left(Z_t, q_t^i\right) - Nw \right\}$$

Assuming that N can be treated as a continuous variable (it is really discrete), we have $\frac{d\Omega}{dN} = 0$ yielding

$$\left(\frac{1+r}{r}\right)\left\{pq - C(Z,q) - w + qC_q + \left\{[p - C_q]\,\phi_Z - NC_Z\right\}\frac{dZ}{dN}\right\} = 0$$

or $\quad pq - C(Z,q) - w = -[(p - C_q)\,\phi_Z - NC_Z]\,\frac{dZ}{dN} - qC_q \qquad (11)$

which differs from the entry condition under free entry or common property by the terms on the RHS being non-zero in general. dZ/dN is derived from (5), the equation characterizing a steady state, given fixed N, under a planning regime. The interpretation of (11) is as follows: In adding an extra boat at the margin, the gain directly is the marginal net surplus per boat, namely $pq - C - w$, plus indirectly, the cost saving of catching the same harvest with an extra boat (each boat catching marginally less (qC_q)); the indirect loss results from marginally lowering the stock size and thus inflating each boat's cost of catch, all in the neighborhood of the original steady state values. A <u>socially optimal or planning equilibrium</u> for the industry is defined by the steady state values of q, Z, and N (if they exist) of equations (1), (7), and (11), where it is understood that p above is endogenously determined.

Since multiple solutions are anticipated, it is necessary that second order conditions be satisfied in order to rule out local minima. Thus the derivative of (11) with respect to N must be negative.

We proceed to specialize to our particular functional forms already introduced above. We have $C(Z,q^i) = q^i/Z$ and $\phi(Z) = aZ(1-(Z/K))$. Our inverted demand function will be $p = AQ^{-\eta}$ where η is one divided by the constant elasticity of demand. In a steady state $\phi = Q$ and $q^i = \phi/N$. Using these results permits us to ignore equation (1) from here on when dealing with steady states. Using our functional forms, we observe that $\frac{\phi}{N}(p - C_{q^i})$ in (6) and (7) is the same as $pq - C(Z,q^i)$ for our functional forms, this latter equalling w in a steady state under common property. This permits us to write for a common property equilibrium, equation (8), multiplied by ϕ/N as

$$-\left[\left(\frac{1}{\phi_Z+1}\right) - \left(\frac{1}{1+r}\right)\right]\cdot\left(\frac{\phi}{NZ}\right)^2 = w \qquad (12)$$

The other steady relation for a common property equilibrum is

$$p(Q)q^i - C(Z,q^i) = w \ . \tag{13}$$

We can eliminate q^i and Q since $Q^i = \phi/N$ and $Q = \phi$ to have two equations in N and Z. Even with our particular functional forms, a wide variety of solutions is possible. We turn to a "well-behaved" numerical example, below.

For the socially optimal case, we use $Q = \phi = Nq$ from (1) and obtain upon substituting in (9) and (11) two equations in N and Z. Taking $q = \phi(Z)/N$ in (9) we obtain $dZ/dN = 0$ which when substituted in (11) yields

$$p(Q)q - w = 0 \tag{14}$$

The solutions of equations (9) and (14) characterize an equilibrium (or possibly equilibria) for the socially optimal fishery. We turn now to examining the equilibria under common property and optimal planning for a particular numerical example. (We observe that for our particular functional forms, there is a variety of equilibria and the analysis has not been extended to cases involving fairly general functions.)

Example:

We treat demand as constant elasticity so that $p = AQ^{-\varepsilon}$. Two cases are considered, the "normal" case of demand elastic ($A = 1$, $\varepsilon = 1/2$ giving the elasticity a value of 2) and demand inelastic ($A = 40$, $\varepsilon = 3/2$ giving an elasticity of 2/3).

We have also $w = r = 0.1$

$\phi = 5Z(1 - (Z/100))$

$C = q/Z$

In Table 1 we have the case of elastic demand. The values of N in column (2) satisfy equation (12) given the corresponding values of Z in column (1); the values of N in column (3) satisfy equations (1) and (9); the values of N in column (4) satisfy equation (13); and the values of N in column (5) satisfy equations (1) and (14).

In Table 2, we have the corresponding values for the case of inelastic demand.

TABLE 1

Demand Elastic ($\varepsilon = \frac{1}{2}$; $A = 1$)

(1)	(2)	(3)		(4)	(5)
Z	N^{cp}	N^{so}		N^{cp}	N^{so}
0	13.623732	18.0	73.0	-50.0	0.0
10	11.989853	18.0	73.0	22.082039	67.082039
20	10.269106	18.0	73.0	49.442719	89.442719
40	6.0677987	18.0	73.0	79.54451	109.54451
50	-56.818188*	18.0	73.0	86.8034	111.8034
60	∞**	18.0	73.0	89.54451	109.54451
70	6.5539717	18.0	73.0	87.46951	102.46951
80	3.753786	18.0	73.0	79.442719	89.442719
90	1.7624019	18.0	73.0	62.082039	67.082039
100	0.0	18.0	73.0	0.0	0.0

* At $Z = 49$, $\phi_Z = 5 \cdot \left(1 - \frac{98}{100}\right) = .1$ which equals r and $N^{cp} = 0.0$.

** At $Z = 60$, $\phi_Z = 5 \cdot \left(1 - \frac{120}{100}\right) = -1$ and $1/(\phi_Z + 1) = \infty$.

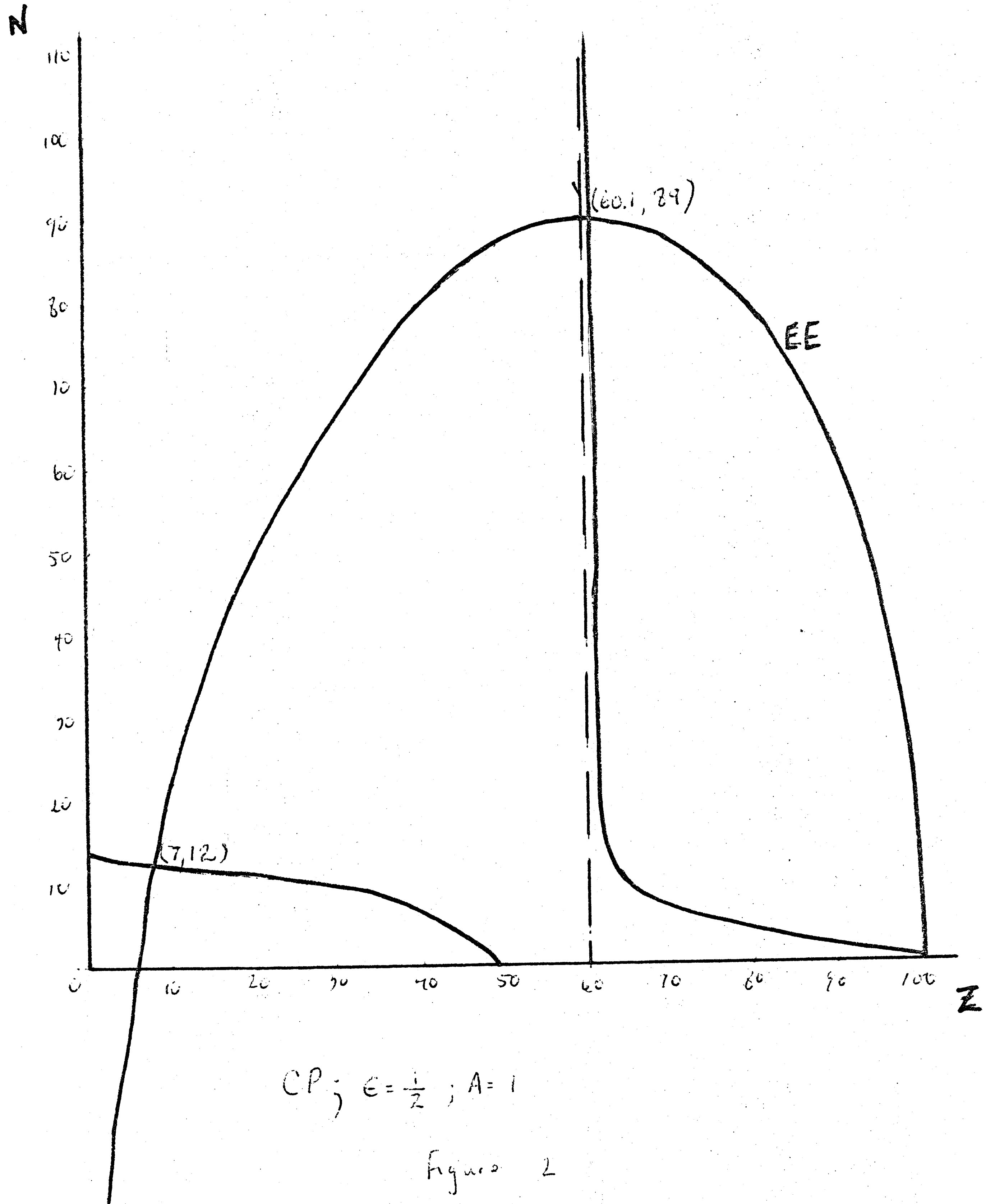
TABLE 2

Demand Inelastic ($\varepsilon = 3/2$; $A = 40$)

(1)	(2)	(3)		(4)	(5)
Z	N^{cp}	N^{so}		N^{cp}	N^{so}
0	13.623732	19.0	75.0	∞	∞
10	11.989853	19.0	75.0	14.62848	59.628479
20	10.269106	19.0	75.0	4.7213595	44.72136
40	6.0677987	19.0	75.0	6.5148377	36.514837
50	-56.818188*	19.0	75.0	10.777087	35.777088
60	∞**	19.0	75.0	16.514838	36.514837
70	6.5539717	19.0	75.0	24.036002	39.036003
80	3.753786	19.0	75.0	34.72136	44.72136
90	1.7624019	19.0	75.0	54.62848	59.628479
100	0.0	19.0	75.0	∞	∞

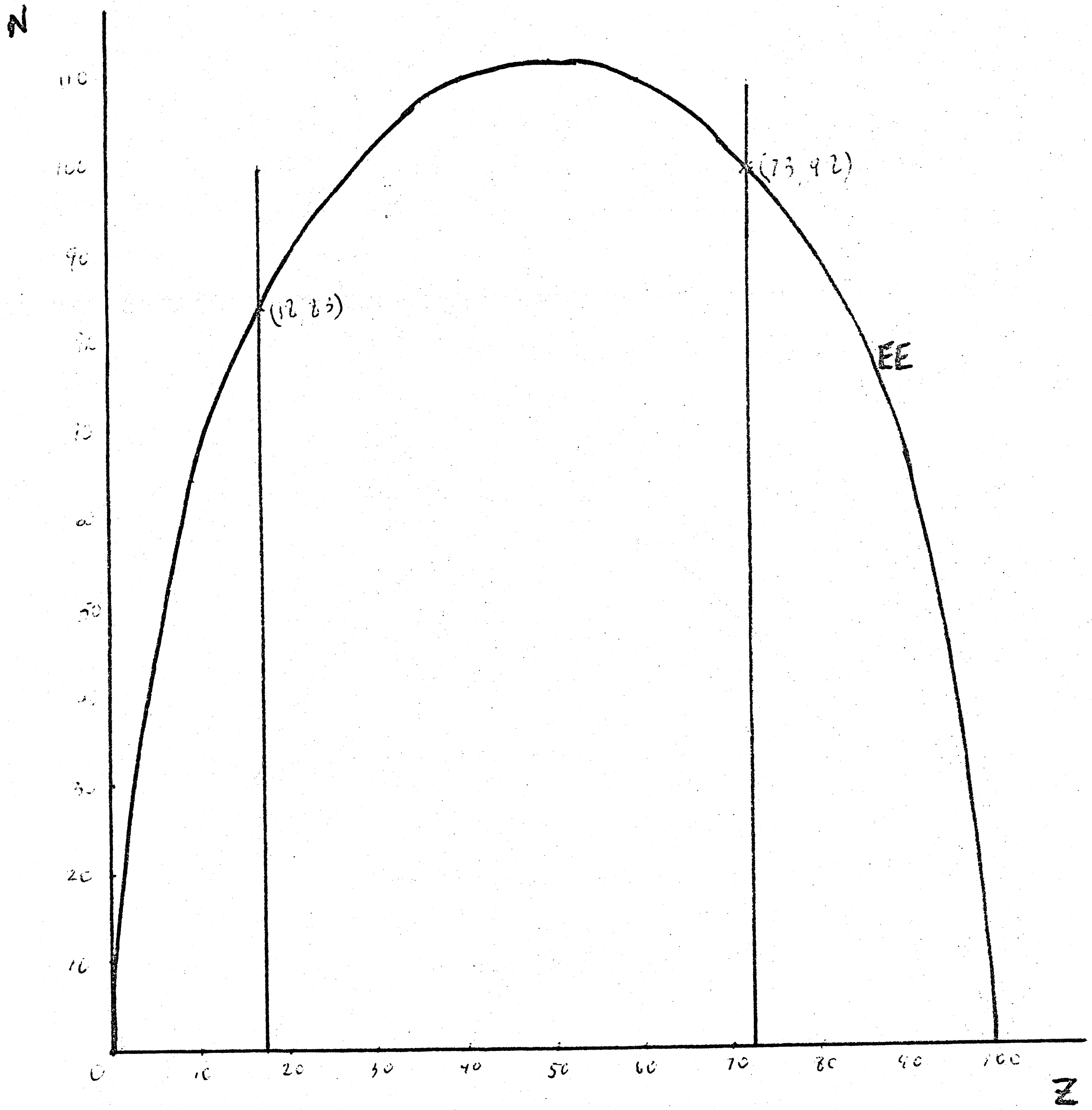
* See the footnote * to Table 1.

** See the footnote ** to Table 1.



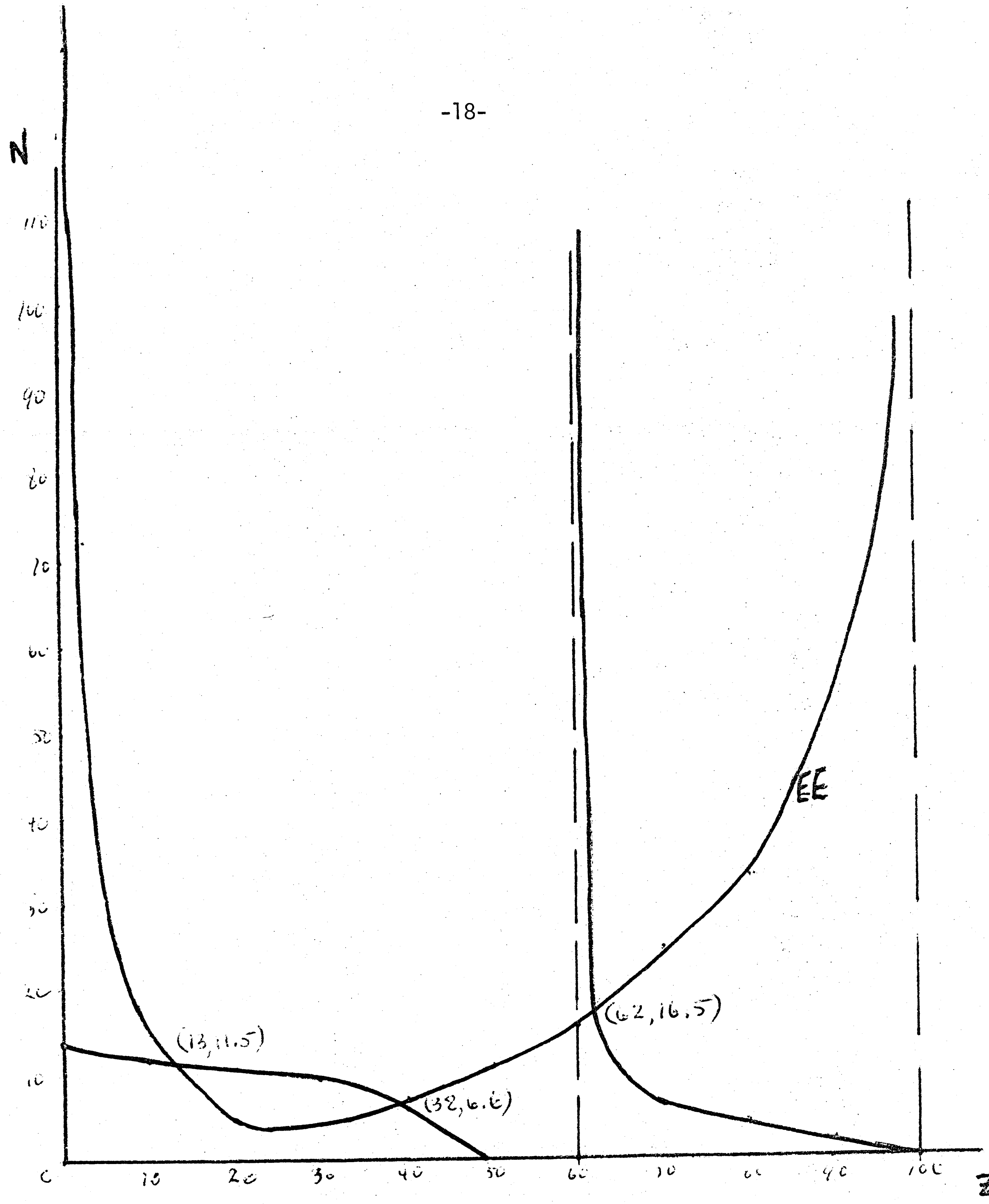
CP ; $\epsilon = \frac{1}{2}$; A = 1

Figure 2



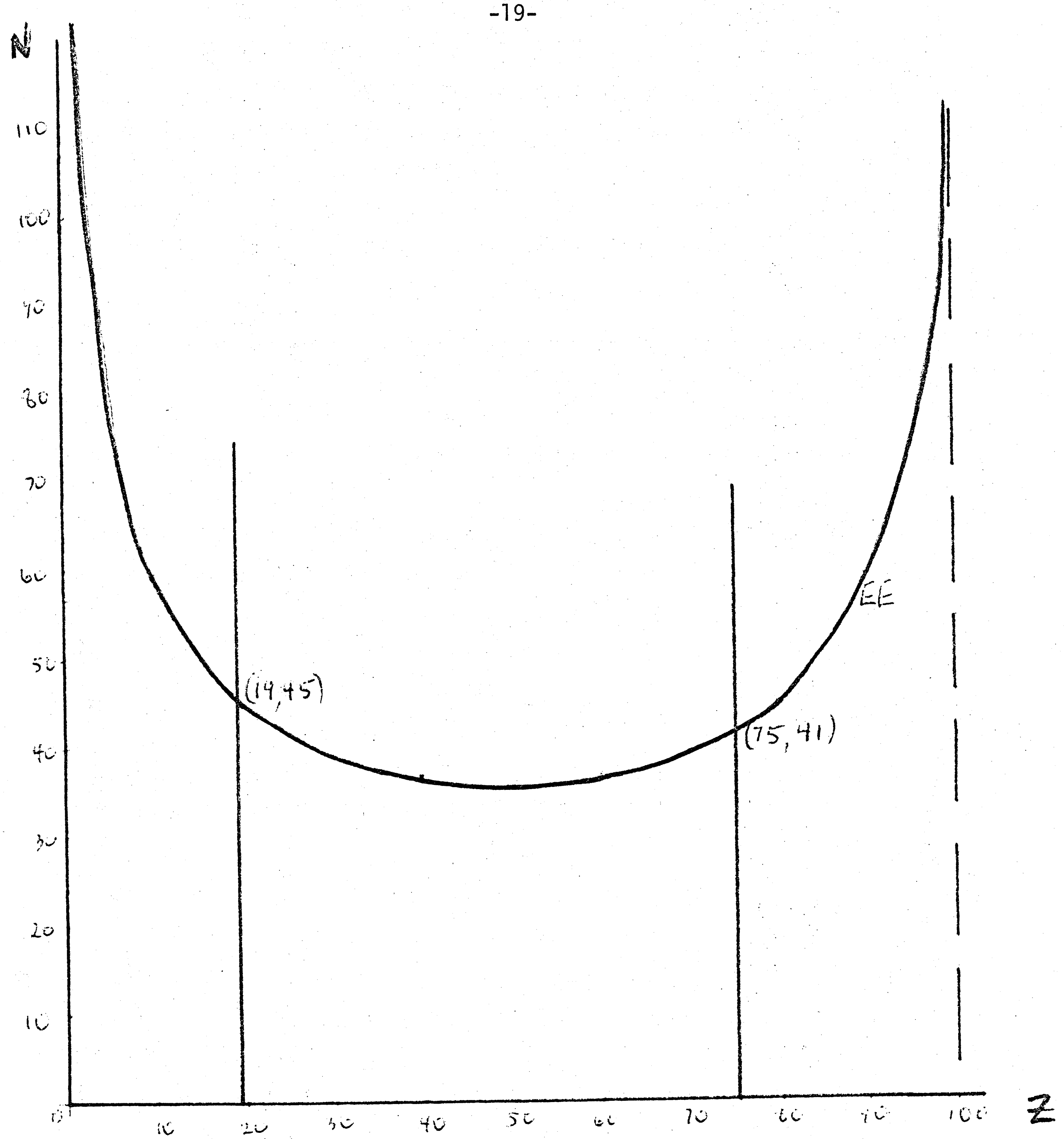
$SO;\ \epsilon = \frac{1}{2}\ ,\ A = 1$

Figure 3



CP; $c = \frac{3}{2}$; A = 40

Figure 4



SO; $\epsilon = 3/2$, A = 40

Figure 5

The schedules corresponding to the common property and socially optimal regimes under an elastic demand specification are set out in Figures 2 and 3 respectively. There are two interior solutions for each regime. (N should range over only integer values but we gloss over this complexity and assume that it can be treated as a continuous variable in the entry-exit analysis. Our results are thus approximate, given this caveat.)

The schedules corresponding to the common property and socially optimal regimes under inelastic demand specification are set out in Figures 4 and 5 respectively. There are three interior solutions for the common property regime and two for the socially optimal regime. For $N = 2$ exogenously fixed, there will be dynamics in Z, q^1 and q^2 in a three dimensional phase space. Since agents are optimizing one assumes that the solution is a type of saddle point in three dimensions and that there exists a stable arm or plane which will be followed by non-myopic agents. We proceed in any case to ask that if the system is at a steady state will, (a) under common property, profits turn negative by the entry of an additional boat and, (b) the entry condition define a local maximum with respect to N under a socially optimal regime?

The socially optimal case is easy in our example since $dZ/dN = 0$. That is, the entry condition in (14) was $p(\phi(z))(\phi/N) - w = 0$ and $d(\cdot)/dN < 0$ implying that each equilibrum is stable with respect to entry or exit under the socially optimal regime. (This approach to the analysis of stability of the model ignores the issue of whether in chainging N, our steady state may be disturbed in such a fashion that no new steady state solution "reappears".) Thus we cannot eliminate any equilibrium for the socially optimal regime in our example by the criterion of it being unstable.

Stability for the common property regime involves determining whether, at a steady state, profits in (10) "turn" negative as N is perturbed. Equation (10) for our example is $A\,\frac{\phi}{N}^{1-\varepsilon} - \frac{\phi}{NZ} - w = 0$ and $d\,\frac{(\cdot)}{dN}$

is

$$-\frac{A\phi^{1-\varepsilon}}{N^2} + \frac{\phi}{N^2 Z} + (1-\varepsilon)A\,\frac{\phi}{N}^{-\varepsilon}\,\phi_Z - \frac{\phi_Z}{NZ} + \frac{\phi}{NZ^2}\,\frac{dZ}{dN} \tag{15}$$

where dZ/dN is obtained from equation (8) and is negative as one observes in Figures 2 and 4. Direct computation of terms in brackets indicates that (15) has a negative sign for all common property equilibria except the one with ($Z = 12$; $N = 11.5$) in Figure 4.[5] We conjecture that since fish have ready substitutes the elastic specification of demand makes most sense from an empirical standpoint. For this elastic case, the common property regime is stable and we cannot rule out one of the equilibria as being unimportant because it is not stable.

In general, multiple equilibria are the rule in this model of the fishery and in all cases, at least two equilibria are stable with respect to entry. We thus are unable to say that under common property there will be too many boats and too small a stock relative to the socially optimal equilibrium. There is no obvious pair of equilibria to compare. For the socially optimal case, one assumes that of the pairs of local optima, one is an optimum optimorum for the different specifications of demand. This could provide one point for comparison but we still have multiple equilibria under the common property solution.

We have labelled the condition for the equilibrium entry-exit as schedule EE in each figure.

Existence of solutions is sensitive to the parameter A in the inverted demand schedule $p = AQ^{-\varepsilon}$. The value of $1/A$ gives an indication of the "size" or "strength" of demand as a reflection of say the population of the demanders or the average income of the demanders. A smaller value of A in this demand function lowers the EE schedule in each of Figures 2a and 2b with some change in its "peakedness". For each demand situation, a small value of $A > 0$ can result in some solutions with N and Z positive being eliminated. Thus "large" demand cases can result in only unstable solutions being observed. For the case of an elastic demand, our plausible case, the only equilibria could occur for stock sizes above that corresponding to the maximum sustainable yield (i.e. $Z = 50$).

Above, we discussed the implications of different relative values of the "natural growth rate of fish", namely a and the interest rate, r. Those considerations are of course relevant for our industry equilibria. For $1 < a$ and $0 < r < a$, values of a closer to r correspond to points analogous to $N = 0$, $Z = 49$ and $Z = 60$ in Figure 4 moving apart. Thus the existence of solutions is sensitive to the relative values of the growth rate and the interest rate. We note, again, that for $r > a > 0$ and $a < 1$ there will be no equilibrium with $p - C_q > 0$. Economically meaningful solutions cease to exist for this latter case.[6]

4. THE FISHERY WITH MYOPIC AGENTS

The classic statement of the fishery under common property and socially optimal regimes is Smith [1968] who developed his analysis from the fundamental ideas in Gordon [1954]. Smith exploited the notion that the stock acts like a public input into the process of catching fish and observed that this public input would be evaluated differently under a socially optimal solution relative to a common property solution. Our analysis above also turned on the "publicness" of the fish stock in the process of harvesting fish. However under the myopic specification of the problem, each agent decides to change his level of fishing as <u>current</u> profits are positive or negative rather than on the basis of whether discounted future returns are positive or negative. There is a distinctly atemporal quality to the specification with myopic agents.

The myopic model has a growth-exploitation dynamics of the form

$$\dot{Z} = \phi(Z) - Nq(Z)$$

where $q(Z)$ is catch per boat and N is the number of boats and an entry-exit dynamics for a common property solution of the form

$$\dot{N} = k[p(\phi(Z))\cdot q - w]$$

where $p(\phi(Z))$ is the inverted industry demand curve

k is a positive constant

w is the opportunity cost of a boat for a "period".

Under this common property regime, an entrant is ignoring the cost he imposes on himself and others of depleting Z and making $q(Z)$ smaller, boat for boat, under the assumption of unchanged effort (implicitly defined) in fishing. Under a socially optimal regime, there will be a tax on entry and the dynamics will be

$$\dot{N} = k[p \cdot q - T - w]$$

where the tax per boat $T = pN \frac{\partial q}{\partial Z} \cdot \frac{dZ}{dN}$ and $\frac{dZ}{dN}$ derives from the steady state relation $\phi(Z) = Nq(Z)$ (the derivatives are evaluated at steady state values making T a parameter to the agents considering entry).

The steady state under a common property regime is defined by N and Z satisfying

$$\phi(Z) - Nq(Z) = 0$$

and

$$p(\phi) \cdot q(Z) - w = 0$$

The steady state under a socially optimal regime is defined by N and Z satisfying

$$\phi(Z) - Nq(Z) = 0$$

and

$$p(\phi) \cdot q(Z) - pN \cdot \left(\frac{\partial q}{\partial Z}\right) \cdot \left(\frac{dZ}{dN}\right) - w = 0$$

Using the fact that $\phi(Z) = q(Z) \cdot N$ in a steady state, we obtain

$\frac{dZ}{dN} = \frac{q}{\phi_Z - N\frac{dq}{dZ}}$ so that $T = \frac{pNq \frac{dq}{dZ}}{\phi_Z - N\frac{dq}{dZ}}$. Note that $q(Z)$ is the <u>average product</u> of the N boats engaged in fishing and $q(Z) + N \frac{dq}{dZ} \cdot \frac{dZ}{dN}$ is the <u>marginal product</u> of a boat in the industry. Thus in the common property regime current average product equals w , the opportunity cost of a boat, whereas under the socially optimal regime, current marginal product equals w . This is the basic insight that Gordon [1954] arrived at.

For $q(Z)$ linear in Z with $q(0) = 0$ and $\phi(Z)$ the logistic growth function, we can solve for N and Z under the alternate regimes as in Figure 6.

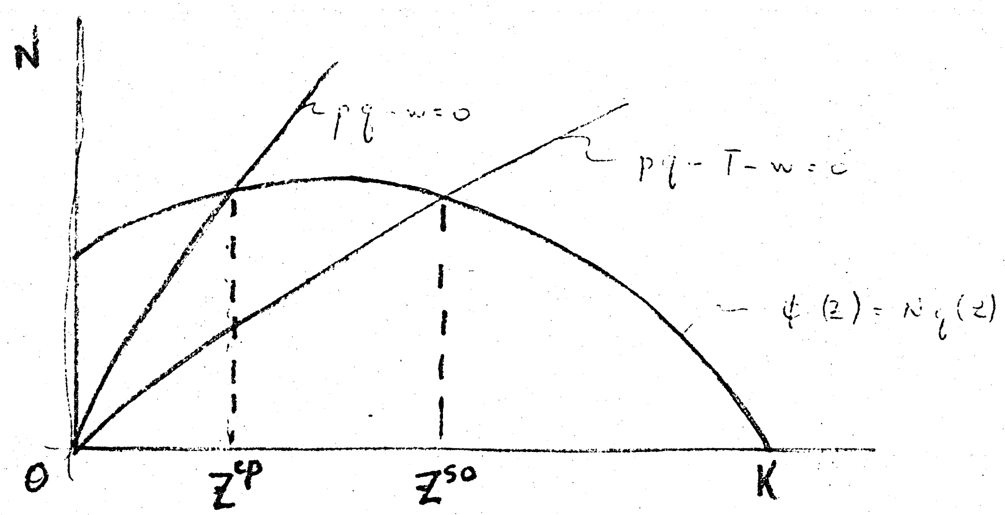


Figure 6

Note that ϕ_Z must be positive in the common property solution so that $Z^{cp} < (K/2)$. In Figure 6, we have illustrated the basic result: the steady state stock will in general be smaller under a common property regime relative to its value under a socially optimal regime. We cannot say whether there will be more or less boats under one regime or the other. The depiction of equilibria under alternate regimes is obviously much simpler under the myopic specification of agents' behavior.[7]

5. *CONCLUDING REMARKS*

We have reconstructed the model of the fishery under the assumption that agents optimize into the indefinite future and perform their calculations at time 0 under Cournot-Nash assumptions concerning the behavior of their rivals. Two sources of market failure emerge under a free-entry common property regime. Given a fixed number of competitors, an agent adjusts his catch until current marginal costs equal the current discounted average payoff from reducing catch rather than the current discounted marginal payoff. Secondly entrants undervalue the effect that their catch has on the cost of catch by others. Stock size behaves as a public input into production. This latter market failure was analyzed by Smith [1968] but the first failure is new (sketched in Hartwick [1980]). Multiple equilibria are the rule under both common property and socially optimal regimes given our specification of intertemporal optimization by agents. It is difficult to decide, if our model captures the essential features of the fishery as an aggregate, which equilibrium represents a real world situation.

The new externality which emerges in the explicitly intertemporal model implies that regulating common property cannot be managed solely by regulating entry to the fishery. Once the socially optimal number of boats has been allowed in, taxes must be imposed in order to assure that the socially optimal amount of fish is harvested by each agent. A second best approach would be to use only entry controls to approximate a socially optimal solution. This may be the policy option which is practicable but which has not been analyzed in a full-blown model as far as I know.

FOOTNOTES

*An earlier version of this paper was presented at a workshop at the University of Michigan, October 1980, organized by Ted. Bergstrom.

1. Levhari and Mirman [1980] have considered what are essentially dynamic oligopoly models of renewable resource exploitation in a slightly different setting. Khalatbari [1977] and Kemp and Long [1980] have considered common property problems associated with seepage of oil from under one extractors well to below a rival's well. Khalatbari [1977] develops an "over exploitation" argument for a decentralized or *quasi* common property regime and Kemp and Long [1980] explore aspects of the demand specification in such exhaustible resource oligopoly problems.

2. Note that if extraction of exploitation costs per boat only depend on the *current amount caught* and not on the density of fish or the stock size, then there is no difference between outcomes under our so-called common property and centrally managed regimes. This is the essence of Kemp and Long's [1980] critique of Khalatbari [1977]. This same point appears in a different guise in Hartwick [1980] where I noted that in a many person Ramsey savings-growth model, each agent with an infinite horizon, there is no difference between the centrally managed plan and the plan emerging from optimization by individual agents, provided the static social welfare function was of the Benthamite additive sort. In our renewable resource problem, the act of saving by an agent or boat, foregoing some q_t^i at the margin not only affects the stock size next period directly but affects the cost of catch also. It is this affect

of "saving" on next period's cost of catch which gives this model is distinctive quality, i.e. leads to the emergence of a difference between the centrally managed solution and the solution emerging under common property.

3. This cost function has average equal to marginal cost equal to $1/Z$ for each boat. $1/Z$ is taken as parametric to each agent. To avoid indeterminacy, we take the number of boats as given exogenously at this point, and "fit" the boats and catch per boat to the stock of fish which is bounded above by k. Below we make the number of boats endogenous by introducing an entry-exit relation for boats. Some "stickiness" is also introduced then by postulating an exogenously given and fixed opportunity cost per boat per period. (In Hartwick [1980], each boat was assumed to have an exploitation cost function of the form $(q^i)^2/Z$ which made a cost *per fish* function, q^i/Z.) The general problem below is dividing a market among small but not infinitesimal firms. Some small fixed cost is required to avoid the presence of an infinity of firms. (Difficulties associated with an infinite number of competitors is a general matter in industrial organization, broadly defined, and is not peculiar to our renewable resource situation.) Thus an agent and his "boat" are defined by the cost function $C(Z,q^i)$ and one cannot have half a boat although one can have any sized catch, q^i. Also once a boat is launched, it must remain for the period incurring a minimum opportunity cost of w per boat per period even if no fish are caught. Periods are of fixed length *a priori*. These assumptions rule out situations such as each boat catching

one fish in a huge rush of boats and having the stock fall to zero. There are some crucial indivisibilities built into the model *a priori*.

4. The model above was cast in discrete time. It is straightforward to set it up in continuous time as colleagues have done. In either case, one can perform stability analyses around the *steady state values*. One treats the N agents in terms of a representative agent, defined by its characteristics at the steady state. We leave this for a reader to pursue if he is inclined. The interesting dynamic analysis involves $N + 1$ simultaneous equations, one equation for each agent, "away from" the steady state solution. We have not analyzed this dynamic system. Finally we note that when we let the number of agents be endogenous below the non-steady state behavior of the system becomes most difficult to analyze since the non-steady state behavior of an agent "in" the model must take account of potential entrants "outside" the model. In Smith [1968] and Hartwick [1981], the non-steady state behavior of an individual agent was neglected and the system was viewed in terms of two dynamic relations, industry size and biological-exploitation dynamics.

5. We did not calculate $\frac{dZ}{dN}$ for this last equilibrium owing to the complexity of the expression and the approximate nature of the numerical values. We conjecture that this equilibrium is unstable since the two schedules intersect at quite different angles compared with the other equilibria.

6. A result of this kind is well-known for the single-boat planning solution in which there are no stock effects on demand or exploitation costs. The key necessary condition there is $\phi_Z = r$. We have a non-existence result in a quite different framework; in particular with essential stock effects. Clark [1976; p. 47] made the result ϕ_Z unable to equal r the basis of a discussion of extinction for the case of the model without stock effects. $r > a$ implies that largest value which the gross marginal product of the fish stock can assume (i.e. ϕ_Z can at most equal a) is less than the marginal product of other capital goods yielding a prevailing rate of return r. Thus from an economic standpoint there is no point in maintaining the stock intact.

7. In Hartwick [1981] some dynamics of the above myopic version of the common property fishery were reported. See also Smith [1968]. Under plausible assumptions on demand elasticities (i.e. elastic specification) and catch elasticities, the equilibrium is stable (both eigen values negative).

References

Clark, C., [1976], Mathematical Bioeconomics, New York: Wiley.

Dasgupta, P. and G. Heal, [1979], Economic Theory and Exhaustible Resources, Cambridge: Cambridge University Press.

Gordon, H.S., [1954], "The Economic Theory of a Common Property Resource: The Fishery", Journal of Political Economy, Vol. 62, pp. 124-142.

Hartwick, J.M., [1980], "The Intertemporal Externality in the Dynamic Common Property Renewable Resources Problem", Economics Letters, (forthcoming).

Hartwick, J.M., [1981], "Free Access and the Dynamics of the Fishery" in L. Mirman and D. Spulber, eds., Essays in the Economics of Renewable Resources, Amsterdam; North Holland.

Kemp, M.C., and N.V. Long, [1980], Exhaustible Resources, Optimality and Trade, Amsterdam: North-Holland.

Khalatbari, F., [1977], "Market Imperfections and Optimal Rate of Depletion of an Exhaustible Resource", Economica, 44, pp. 409-414.

Levhari, D. and L.J. Mirman, [1980], "The Great Fish War: An Example Using a Dynamic Cournot-Nash Solutions", Bell Journal of Economics, Vol. 11, No. 1, Spring, pp. 322-334.

Smith, V.L., [1968], "Economics of Production from Natural Resources", American Economic Review, Vol. 58, pp. 409-431.

Weitzman, M., [1974], "Free Access vs. Private Ownership as Alternative Systems for Managina Common Property", Journal of Economic Theory 8 pp. 225-234.