ISSN 0316-5078

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1. SET-UP COSTS AND THEORY OF EXHAUSTIBLE RESOURCES

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2. A NOTE ON SET-UP COSTS FACING CONSUMERS

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DISCUSSION PAPER No. 412

November 1980

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ABSTRACT

Let there be several identical deposits of an exhaustible, non-reproducible resource, the working of a deposit entailing a set-up cost but no other costs. The deposits must be extracted in strict sequence with jump discontinuities of marginal benefit at transition points. Moreover the average rate of increase of marginal benefit is less than the rate of interest.

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1. Introduction

The opening of a mine may involve once-and-for-all "set-up" costs of exploration, clearing, tunneling, training, equipping; and its closure once-and-for-all costs of dismantling and cleaning-up. Indeed such non-recurrent costs may dominate the fixed and variable flow costs of resource extraction. It is puzzling then that in the formal economic analysis of exhaustible resources attention has been focussed on the latter to the almost complete neglect of the former. It is our purpose in the present note to repair this deficiency and, in particular, to set forth a generalized form of Hotelling's "r per cent rule" for the behaviour over time of the expected marginal current net benefit (MNB) derived from extraction Our most important finding is that if set-up costs are present then, along an optimal path, MNB must increase not continuously and exponentially at the current rate of interest but in discontinuous saw-tooth fashion at an average rate less than the rate of interest. In addition it is shown that, in general, the optimal path of extraction cannot be reproduced by competitive markets and that, in any case, set-up costs render the assumption of competitive spot markets highly implausible. Finally it is shown that under monopoly the rate of extraction is suboptimal, and that this is so even when the demand for the resource is of constant elasticity (too little being extracted).

Of course it is well known that the conditions under which Hotelling's rule is valid are quite strict. In particular it is known that the rule must be modified (or, at least, rephrased) if current extraction exerts a direct influence on expected benefit or cost of extraction. Thus if costs of extraction increase with cumulative extraction then any increase in the current rate of extraction adds to the expected cost of any planned future extraction; and if the resource-stock is of unknown extent then any addition to the current rate of extraction increases the probability that the stock will be exhausted before any assigned future point of time and therefore reduces the expected benefit associated with extraction planned for that time. In both sets of circumstances it is optimal for MNB to rise at a rate less than the current rate of interest. (For costs of extraction which depend on cumulative output, see Herfindahl (1967), Levhari and Liviatan (1977), Hartwick (1978), and Kemp and Long (1980a); for stock uncertainty, see Kemp (1976).) However in these circumstances it is optimal for marginal benefit to increase continuously; this is so even when the average cost of extraction jumps discontinuously from one deposit to another and even when the distribution function (defined on alternative stock sizes) contains jump discontinuities. Moreover in these circumstances the optimal path of extraction can be reproduced by laisser-faire competitive markets if only there are enough of them. (See Kemp and Long (1980b).)

2. Analysis of the simplest case

We begin with the simplest case. There are n identical deposits of some resource. Each deposit is of known initial extent R. The extracted resource is consumable but perishable; the rates of extraction and consumption therefore are always equal.

There is a second perishable commodity, leisure, available in a steady flow. Like the extracted resource, leisure may be consumed; but, as we shall see, it has other uses too.

Associated with each deposit is a set-up cost K, expressed in leisure. Since leisure comes as a finite flow, it is not possible for the community to incur finite set-up costs in a moment of time; they must be spread over a non-degenerate interval of time. We therefore suppose that, to prepare a deposit for extraction, leisure must be spent at some fixed rate (not greater than the rate at which it becomes available) over some fixed interval of time. The single number K is then the value of the expenditure flow compounded to the end of the interval. There are no other costs of extraction.

Social utility is a strictly concave function u of resource-consumption and a linear function of leisure. There is a positive and constant rate of time preference.

From the point of view of the community, if it is worth exploiting the deposits at all then it is optimal to exploit them in strict sequence, completely exhausting each deposit before moving on the the next; indeed, since the rate of time preference is positive, it is suboptimal to do otherwise. Of course it is a matter of indifference which of the n identical deposits is first exploited.

It is a necessary condition of optimality that, from the moment at which a deposit is first worked to the moment of exhaustion, MNB rise exponentially at the rate of interest; for, otherwise, it would be possible to increase the value, at any arbitrarily chosen point of time, of the flow of net benefits or rents derived by the community from the deposit. A more difficult question concerns the behaviour of MNB at the moment of transition from one deposit to another. We proceed to show that at all points of transition MNB drops discontinuously.

It suffices to consider the case n = 2. Let $q_i(t)$ be the rate of extraction from the ith deposit at time t, so that total extraction is $q(t) \equiv q_1(t) + q_2(t)$; let t_1 be the moment of transition from one deposit to the other; and let ρ be the rate of time preference. Then the social problem is to find

$$\text{(P.1)} \qquad \max_{t_1,\{q_1,q_2\}} \{\int_0^\infty \exp(-\rho t)\cdot u(q_1(t) + q_2(t))dt - K - K\cdot\exp(-\rho t_1)$$

$$\text{s.t.} \quad \int_0^\infty q_i(t)dt \leqq R \qquad i = 1, 2$$

$$q_1(t) \geqq 0 \text{ if } t \leqq t_1,\ q_1(t) = 0 \text{ if } t > t_1$$

$$q_2(t) \geqq 0 \text{ if } t \geqq t_1,\ q_2(t) = 0 \text{ if } t < t_1$$

Now to solve (P.1) it is necessary to solve the subproblem

$$\text{(P.2)} \qquad \max_{\{q_2\}} \{\exp(\rho t_1) \int_{t_1}^\infty \exp(-\rho t)\cdot u(q_2(t))dt - K\} \equiv V(t_1) - K$$

$$\text{s.t.} \quad \int_{t_1}^\infty q_2(t)dt \leqq R$$

$$q_2(t) \geqq 0$$

Hence the maximand of (P.1) can be re-written as

$$(1) \qquad \int_0^{t_1} \exp(-\rho t)\cdot u(q_1(t))dt - K - K\cdot\exp(-\rho t_1) + V(t_1)\exp(-\rho t_1)$$

Throughout our further calculations it will be assumed that $V(t_1)$ is greater than K, so that it will be optimal to exhaust the second deposit and suboptimal to fail to do so.

The Hamiltonian for (P.1), with the revised maximand (1), is

$$H(t) = u(q_1(t)) - \psi_1(t)q_1(t)$$

As necessary conditions of a maximum we then have

$$\text{(2)} \qquad \begin{aligned} & u'(q_1(t)) - \psi_1(t) \leqq 0 \quad (= \text{ if } q_1(t) > 0) \\ & \dot{\psi}_1 = \rho\psi_1 \end{aligned}$$

and the transversality condition associated with the choice of t_1,

$$\text{(3)} \qquad H(t_1^-)\cdot\exp(-\rho t_1^-) + \rho K\cdot\exp(-\rho t_1^+) = -\frac{d}{dt_1}[V(t_1^+)\cdot\exp(-\rho t_1^+)]$$

(See Hestenes (1966, theorem 11.1) and Long and Vousden (1977, theorem 1).) The left side of (3) may be interpreted as the marginal gain from delaying the exhaustion of the first deposit, and the right side may be interpreted as the marginal cost of doing so.

Let

$$J(t_1) \equiv \exp(-\rho t_1)V(t_1) \equiv \int_{t_1}^{\infty} \exp(-\rho t)u(q_2^*(t))dt$$

Then, following Hadley and Kemp (1971, pp. 117-120),

$$\text{(4)} \qquad \frac{d}{dt_1}[J(t_1)]\Big|_{t_1^+} = -[\exp(-\rho t_1^+)u(q_2^*(t_1^+)) - \psi_2(t_1^+)\exp(-\rho t_1^+)q_2^*(t_1^+)]$$

where $\psi_2(t_1^+) = u'(q_2^*(t_1^+))$. Hence (3) can be re-written as

(5) $u(q_1(t_1^-)) - \psi_1(t_1^-)q_1(t_1^-) \equiv H(t_1^-) + \rho K$

$$= H(t_1^+) \equiv u(q_2(t_1^+)) - \psi_2(t_1^+)q_2(t_1^+)$$

implying that the Hamiltonian values $H(t_1^-)$ and $H(t_1^+)$ differ by the amount ρK. Recalling (2), we can interpret (5) as requiring that at t_1 the consumers' surplus $u - qu'$ jumps up to compensate for the interest on the new set-up cost.

With (5) in our hands, it is easy to show that $q_1(t_1^-) < q_2(t_1^+)$, that is, that at t_1 the rate of extraction jumps up and, since u is concave, MNB jumps down. Recalling (2) and substituting for $\psi_1(t_1^-)$ and $\psi_2(t_1^+)$ in (5), we obtain

$$[u(q_1(t_1^-)) - u'(q_1(t_1^-))\cdot q_1(t_1^-)] + \rho K$$

(5') $$= u(q_2(t_1^+)) - u'(q_2(t_1^+))\cdot q_2(t_1^+)$$

The required conclusion then follows from the fact that $u(q) - u'(q)\cdot q$ is an increasing function of q.(1)

That completes our demonstration that optimal MNB follows a saw-tooth path with an average rate of growth less than the rate of interest. If there is some natural upper bound on MNB (stemming, for example, from the availability of a standby technology for the production of a resource-substitute or from the possibility of extinguishing demand at finite prices) then the n deposits must be exhausted in finite time, as in Figure 1(a); otherwise, the last deposit to be worked will be exhausted only asymptotically, as in Figure 1(b). Given the rate of time preference and the size of the deposits, the extent of the jumps at t_i (i = 1, .., n - 1) is determined by the amount of the set-up costs K. In the extreme case studied by Hotelling, the costs are zero and the jumps disappear.

Gathering together our findings to this point, we obtain PROPOSITION 1: Under the assumptions of this section, deposits must be extracted in strict sequence. Marginal net benefit follows the saw-tooth path of Figure 1, with an average rate of increase less than the market rate of interest. The optimal rate of extraction follows a complementary path, generally declining but jumping up at points of transition from one deposit to another.

3. Related cases

Nothing of any importance changes if, in addition to or instead of set-up costs, there are set-down costs, incurred at the end of the working life of a deposit and the same for each deposit. It remains true that the optimal MNB follows the saw-tooth path of Figure 1, that the optimal working life of a deposit is greater the longer its exploitation is delayed, and that the order in which the deposits are worked is a matter of indifference.

Similarly the analysis changes only in minor detail if there are constant average costs of extraction, the same for each deposit. One need only impose the additional assumption that something will be demanded at a price equal to the average costs of extraction, and recall that MNB is net of extraction costs.

Our conclusions change if the deposits are of different size. Two polar cases may be distinguished. (a) If the same set-up costs are incurred whatever the size of a deposit then it is optimal to work the deposits in descending order of size, for then the average delay in incurring costs is greatest. This finding has an interesting corollary. Suppose that the deposits are of equal size but with set-up costs payable not just once for each deposit, but recurrently, once for each R' (or part thereof) of resource extracted. Then each deposit can be viewed as a collection of n' + 1 subdeposits, all but one of size R', the remaining subdeposit of size R - n'R'. The initial endowment can then be viewed as consisting of n'n subdeposits each of size R' and n subdeposits each of size $R - n'R' \leq R'$. Applying (a), it is optimal to extract the n'n larger subdeposits before turning to the n smaller subdeposits.

We next note the possibility that the deposits are identical in size and set-up costs but are subject to different constant average variable costs of extraction. It is easy to see that in this case it is optimal to work the deposits in ascending order of average variable cost, a conclusion which generalizes a well-known result of Herfindahl (1967). (See also Hartwick (1978).)

Throughout this and the preceding section we have imagined that the resource-good can be obtained only from resource-stocks. Finally, we broaden our analysis by allowing for the possibility that there is available a relatively high-cost standby technology for the production of a flow substitute for the extracted resource. Specifically, we assume that the community possess a single homogeneous resource-stock with no set-up costs and no variable costs of extraction; but that after incurring set-up costs it can produce a perfect substitute for the extracted resource at constant average variable cost of extraction v. (In effect, the community has two deposits, one of infinite size.) Without a space-consuming reworking of Section 2 it can be accepted, perhaps, that the optimal path of marginal benefit is as depicted in Figure 2.(2) Depending on the size of the resource-deposit, the initial marginal benefit may be greater or less than v. Of course the standby technology may improve over time; in that case, marginal benefit declines after t_1.

4. Market outcomes

Having described the socially optimal path of extraction, we now consider whether the path can be supported by a system of competitive markets. Since the answer to that question will be NO, the argument can proceed in terms of a counter example.

EXAMPLE Let n = 2 and let the social utility function take the special constant-elasticity form

$$(6) \qquad u(q) = q^{\alpha} \qquad (0 < \alpha < 1)$$

The solution to (P.2) is then

$$q_2^*(t) = \frac{\rho R}{1-\alpha} \exp\left(-\frac{\rho}{1-\alpha}(t - t_1)\right)$$

so that

$$V(t_1) = \frac{1-\alpha}{\rho} \cdot \left(\frac{\rho R}{1-\alpha}\right)^{\alpha}$$

and

(7) $$H(t_1^+) = (1-\alpha)\left(\frac{\rho R}{1-\alpha}\right)^{\alpha}$$

Now consider (P.1), with maximand (1). If $(t_1^*, \{q_1^*(t)\})$ is the solution to (P.1) then $\{q_1^*(t)\}$ must solve

(P.3) $$\max_{\{q_1\}} \int_0^{t_1^*} \exp(-\rho t)u(q_1(t))dt \qquad (t_1^* \text{ fixed})$$

$$\text{s.t. } \int_0^{t_1^*} q_1(t)dt \leqq R$$

$$q_1(t) \geqq 0$$

Given (6), the solution to (P.3) is

(8) $$q_1^*(t) = \frac{\rho R}{1-\alpha} \cdot \frac{\exp(-\frac{\rho t}{1-\alpha})}{1 - \exp(-\frac{\rho t_1^*}{1-\alpha})}$$

Substituting (7) and (8) into (5'), we obtain

(5") $$(1-\alpha)\left(\frac{\rho R}{1-\alpha}\right)^{\alpha} \left[\frac{\exp(-\frac{\rho t_1}{1-\alpha})}{1 - \exp(-\frac{\rho t_1}{1-\alpha})}\right]^{\alpha} = (1-\alpha)\left(\frac{\rho R}{1-\alpha}\right)^{\alpha} - \rho K$$

from which the optimal t_1, if it exists, may be determined. It is easy to show that the optimal t_1 exists and is both unique and finite. Thus, taking logarithms in (5"), and defining $X = (1-\alpha)[\rho R/(1-\alpha)]^{\alpha}$, we obtain

(9) $$\alpha \ln[1 - \exp(-\frac{\rho t_1}{1-\alpha})] + \left(\frac{\rho\alpha}{1-\alpha}\right)t_1 - \ln\left(\frac{X}{X-\rho K}\right) \equiv g(t_1; \rho, \alpha, K, R) = 0$$

(Notice that, since $X/\rho = H(t_1^+)/\rho = V(t_1) > K$, $X/(X - \rho K) > 1$.) The desired properties then follow from the easily-verified facts that $g < 0$ if $t_1 = 0$, $g = \infty$ if $t_1 = \infty$, and $\partial g/\partial t_1 > 0$.

With the existence and uniqueness of an optimal program assured, we can sensibly ask whether the program could be reproduced by competitive markets. For such an outcome it is necessary that the resource sell at a current utility- or leisure-price of $u'(q(t))$. But it is also necessary that each deposit earn the same present value of profits or rents; for, otherwise, not all owners would be content with the optimal sequence of exploitation of the deposits. We now show that these two requirements may be incompatible, that if the prices $u'(q(t))$ prevail then the two deposits may differ in profitability; and we conclude that set-up costs may destroy the Pareto-optimality of competitive outcomes. Thus if the prices $u'(q(t))$ prevail then the total profit derived from the first deposit, referred to time t_1, is

$$\begin{aligned}\pi_1(t_1) &\equiv R\cdot u'(q_1^*(t_1^-)) - K\cdot\exp(\rho t_1)\\ &= \alpha R\left(\frac{\rho R}{1-\alpha}\right)^{\alpha-1}\left[\frac{\exp\left(-\frac{\rho t_1}{1-\alpha}\right)}{1-\exp\left(-\frac{\rho t_1}{1-\alpha}\right)}\right]^{\alpha-1} - K\cdot\exp(\rho t_1)\end{aligned} \tag{10}$$

and the total profit from the second deposit, also referred to time t_1, is

$$\begin{aligned}\pi_2(t_1) &\equiv Ru'(q_2^*(t_1^+)) - K\\ &= \alpha R\left(\frac{\rho R}{1-\alpha}\right)^{\alpha-1} - K\end{aligned} \tag{11}$$

It follows from (10) and (11) that

$$\left[\frac{\exp\left(-\frac{\rho t_1}{1-\alpha}\right)}{1-\exp\left(-\frac{\rho t_1}{1-\alpha}\right)}\right]^{\alpha-1} = \frac{\pi_1(t_1) + K\cdot\exp(\rho t_1)}{\pi_2(t_1) + K} \tag{12}$$

Choosing K large enough to make $\pi_2(t_1) = 0$, and setting $\pi_1(t_1) = \pi_2(t_1)$, (12) reduces to

$$[1 - \exp(-\frac{\rho t_1}{1-\alpha})]^{1-\alpha} = 1$$

But this is not possible because, as we have seen, t_1 is finite.

Thus in general $\pi_1(t_1) \neq \pi_2(t_1)$. This may be found surprising. However it must be remembered that the future seen from time 0 is not at all the same as the future seen from time t_1; for at time 0 there are two deposits to exploit, at time t_1 there is just one. But, surprising or not, the finding carries an important implication: Laisser-faire competitive markets cannot be relied on to reproduce the optimal program of extraction. To make this implication quite clear, let the set-up costs be such that $\pi_2(t_1) = 0$. Then, as will be verified, $\pi_1(t_1)$ must be negative, implying that, for the competitive outcome to be optimal, it is necessary that the owner of one deposit be subsidized, with payment of the subsidy conditional upon his extracting first. The verification is achieved by setting $\pi_2(t_1) = 0$ in (12), obtaining

$$1 = \left[\frac{\pi_1 + K\cdot\exp(\rho t_1)}{K\cdot\exp(\rho t_1)}\right]\left[\frac{1}{1 - \exp(-\frac{\rho t_1}{1-\alpha})}\right]^{1-\alpha} ,$$

and then noting that, since the second square-bracketed term is greater than one, the first must be less than one, which is possible if and only if π_1 is negative.

That completes our discussion of the counter example. The outcome of that discussion is

PROPOSITION 2: In general, the socially optimal path of extraction cannot be reproduced by laisser-faire competitive markets.

We now note an additional implication of the sequential nature of optimal extraction. Suppose that there is a complete set of spot markets and that one has calculated the set of taxes and subsidies which, if markets were competitive, would support the optimal program of extraction. Since that program is strictly sequential, the number of unexhausted deposits steadily declines until, after a finite interval of time, there remains only one. Why should the owner of the surviving deposit not exercise his new-found monopoly power? Indeed, why should competition not break down when only two or three deposits remain? Whether or not owners recognize at time zero the advantages of being last, the assumption of competition emerges as highly implausible.(3)

We are led therefore to consider markets containing elements of monopoly. Suppose that a single monopolist controls each of two identical deposits; and, to avail ourselves of the calculations of our Example, suppose again that the utility function is of the constant-elasticity type (6), so that the leisure-price of the resource is $u'(q) = \alpha q^{\alpha-1}$ and total revenue from sales is αq^{α}. If the monopolist seeks to maximize the present value of revenue less the present value of set-up costs, we obtain, instead of (5"),

$$(5''') \qquad \alpha(1-\alpha)\left(\frac{\rho R}{1-\alpha}\right)^{\alpha}\left[\frac{\exp\left(-\frac{\rho t_1}{1-\alpha}\right)}{1-\exp\left(-\frac{\rho t_1}{1-\alpha}\right)}\right]^{\alpha} = \alpha(1-\alpha)\left(\frac{\rho R}{1-\alpha}\right)^{\alpha} - \rho K$$

Evidently

$$(1-\alpha)\left(\frac{\rho R}{1-\alpha}\right)^{\alpha}\left[\frac{\exp\left(-\frac{\rho t_1}{1-\alpha}\right)}{1-\exp\left(-\frac{\rho t_1}{1-\alpha}\right)}\right]^{\alpha} = (1-\alpha)\left(\frac{\rho R}{1-\alpha}\right)^{\alpha} - (\rho K/\alpha)$$

$$\leqq (1-\alpha)\left(\frac{\rho R}{1-\alpha}\right)^{\alpha} - \rho K$$

Thus, if $K = 0$, the extraction path under monopoly is socially optimal; otherwise, the monopolist extracts the first deposit too slowly. The price paths under monopoly and perfect planning, with K positive, are displayed in Figure 3. (The superscripts m and s stand for monopoly and social optimum, respectively.)

The same conclusion emerges if, following the final paragraph of Section 3, it is supposed that the second deposit is infinite. (A proof may be found in the Appendix.) Thus we have

PROPOSITION 3: In general, the extraction path under monopoly is suboptimal. This is so even if the utility function is of constant elasticity; in that case, with K positive, the rate of extraction is too slow. However if $K = 0$ and if the utility function is of constant elasticity then the extraction path under monopoly is optimal.

Appendix

Let there be two deposits, the first finite and the second infinite. For the first deposit there are neither set-up nor variable costs of extraction, and for the second deposit there is a set-up cost K and a constant average variable cost of extraction v. The utility function is of constant elasticity: $u(q) = q^{\alpha}$. If exploitation of the second deposit begins at t_1, society solves the problem

$$\max_{\{q_2\}} \int_{t_1}^{\infty} \exp(-\rho(t - t_1))\,(q_2^{\alpha} - vq_2)dt \equiv V(t_1)$$

The first-order condition

$$\alpha q_2^{\alpha-1} = v$$

yields

$$(13)\qquad q_2^* = (v/\alpha)^{1/(\alpha-1)}$$

whence

$$V(t_1) = \frac{1-\alpha}{\rho}(q_2^*)^\alpha$$

(It is assumed, of course, that $V(t_1) > K$.) On the other hand, (8) continues to yield the optimal value of $q_1(t)$ for $t < t_1$. Hence

$$(14)\qquad \begin{aligned} H(t_1^-) &= u(q_1(t_1^-)) - \psi_1(t_1^-)q_1(t_1^-) \\ &= u(q_1(t_1^-)) - u'(q_1(t_1^-))q_1(t_1^-) \\ &= (1-\alpha)(q_1^*)^\alpha \end{aligned}$$

and

$$(15)\qquad \begin{aligned} H(t_1^+) &= u(q_2^*) - vq_2^* \\ &= (1-\alpha)(q_2^*)^\alpha \end{aligned}$$

The transversality condition at t_1 is

$$(16)\qquad H(t_1^-) = H(t_1^+) - \rho K$$

Substituting from (8) and (13) into (14) and (15) and thence into (16), we obtain

$$(17)\qquad \begin{aligned} (1-\alpha)\left(\frac{\rho R}{1-\alpha}\right)^\alpha \left[\frac{\exp(-\rho t_1/(1-\alpha))}{1-\exp(-\rho t_1/(1-\alpha))}\right]^\alpha &= (1-\alpha)(q_2^*)^\alpha - \rho K \\ &= (1-\alpha)\left(\frac{v}{\alpha}\right)^{\alpha/(\alpha-1)} - \rho K \end{aligned}$$

which can be solved uniquely for t_1. Let the solution be t_1^s.

Consider now the monopolist's problem. If exploitation of the second deposit begins at t_1, the monopolist solves the problem

$$\max_{\{q_2\}} \int_{t_1}^{\infty} \exp(-\rho(t - t_1^m))(\alpha q_2^{\alpha} - v q_2)dt \equiv V_1^m(t_1)$$

The first-order condition

$$\alpha^2 q_2^{\alpha-1} = v$$

yields

$$q_2^* = (v/\alpha^2)^{1/(\alpha-1)}$$

whence

$$V^m(t_1) = \frac{\alpha(1-\alpha)}{\rho} (q_2^*)^{\alpha}$$

On the other hand, the monopolist's allocation of the first deposit, over the interval $[0, t_1]$, is again given by (8). Hence

$$H(t_1^-) = \alpha(1-\alpha)(q_1^*)^{\alpha}$$

and

$$H(t_1^+) = \alpha(1-\alpha)(q_2^*)^{\alpha}$$

Substituting into the transversality condition (16), we obtain

$$(18) \qquad (1-\alpha)\left(\frac{\rho R}{1-\alpha}\right)^{\alpha} \left[\frac{\exp(-\rho t_1/(1-\alpha))}{1 - \exp(-\rho t_1/(1-\alpha))}\right]^{\alpha} = (1-\alpha)(q_2^*)^{\alpha} - \rho K/\alpha$$

which can be solved uniquely for t_1. Let the solution be t_1^m. Now, if $K > 0$, the right-hand side of (17) is greater than that of (18); for

$$(v/\alpha^2)^{\alpha/(\alpha-1)} = (v/\alpha)^{\alpha/(\alpha-1)} \cdot \alpha^{\alpha/(1-\alpha)} < (v/\alpha)^{\alpha/(\alpha-1)}$$

and $-\rho K/\alpha < -\rho K$. Hence $t_1^m > t_1^s$ if $K > 0$; that is, if there are set-up costs, the monopolist works the first deposit at a slower rate and therefore switches to the second deposit at a later date than is socially desirable. Indeed it is possible that the monopolist will choose to withhold the second deposit for ever; for

$$V^s(t_1^s) = \frac{1-\alpha}{\rho}(q_2^*)^\alpha > V^m(t_1^m),$$

implying that

$$V^s(t_1^s) > K > V^m(t_1^m)$$

is possible. This generalizes a finding of Dasgupta and Stiglitz (1979) who assumed that $K = 0$.

FOOTNOTES

(1) Kemp and Long (1980d) have studied the bearing of (flow) fixed costs of extraction on the optimal path of extraction and have found that if two deposits are alike in all respects but fixed cost then (a) it is optimal to work one deposit at a time, with MNB increasing at the rate of time preference while any particular deposit is being worked and (b) at points of transition the rate of extraction jumps up or down according as the new deposit is associated with a higher or lower fixed cost. By interpreting the interest on earlier set-up expenditures as the fixed cost of current extraction, it can be seen that our present result is closely related to that of Kemp and Long.

It may be noted that equation (5) can be re-derived by introducing cumulative discounted set-up costs as a state variable subject to jumps and then applying the results of Vind (1967).

(2) The special case in which u(q) is of constant elasticity is examined in the Appendix.

(3) Of course the foregoing argument is inapplicable if there is a complete set of futures markets, so that all contracts are concluded at time zero.

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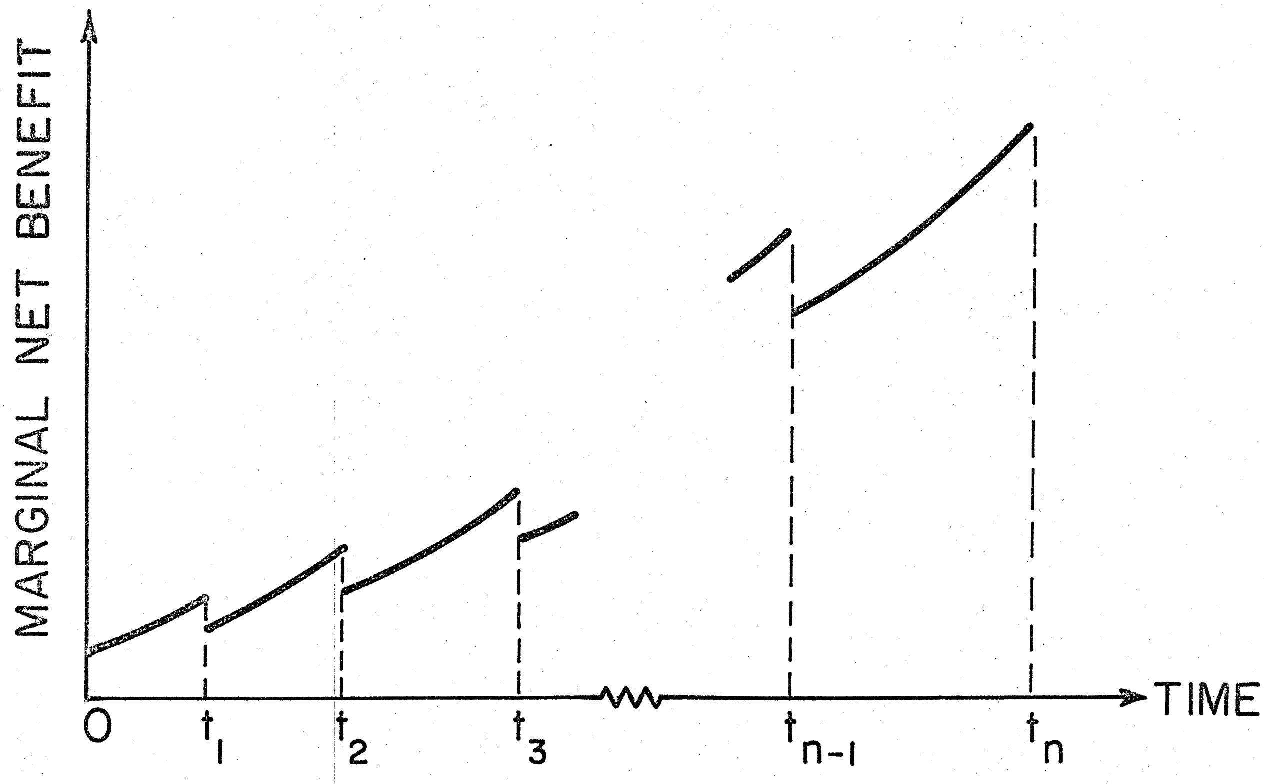

FIGURE 1(a)

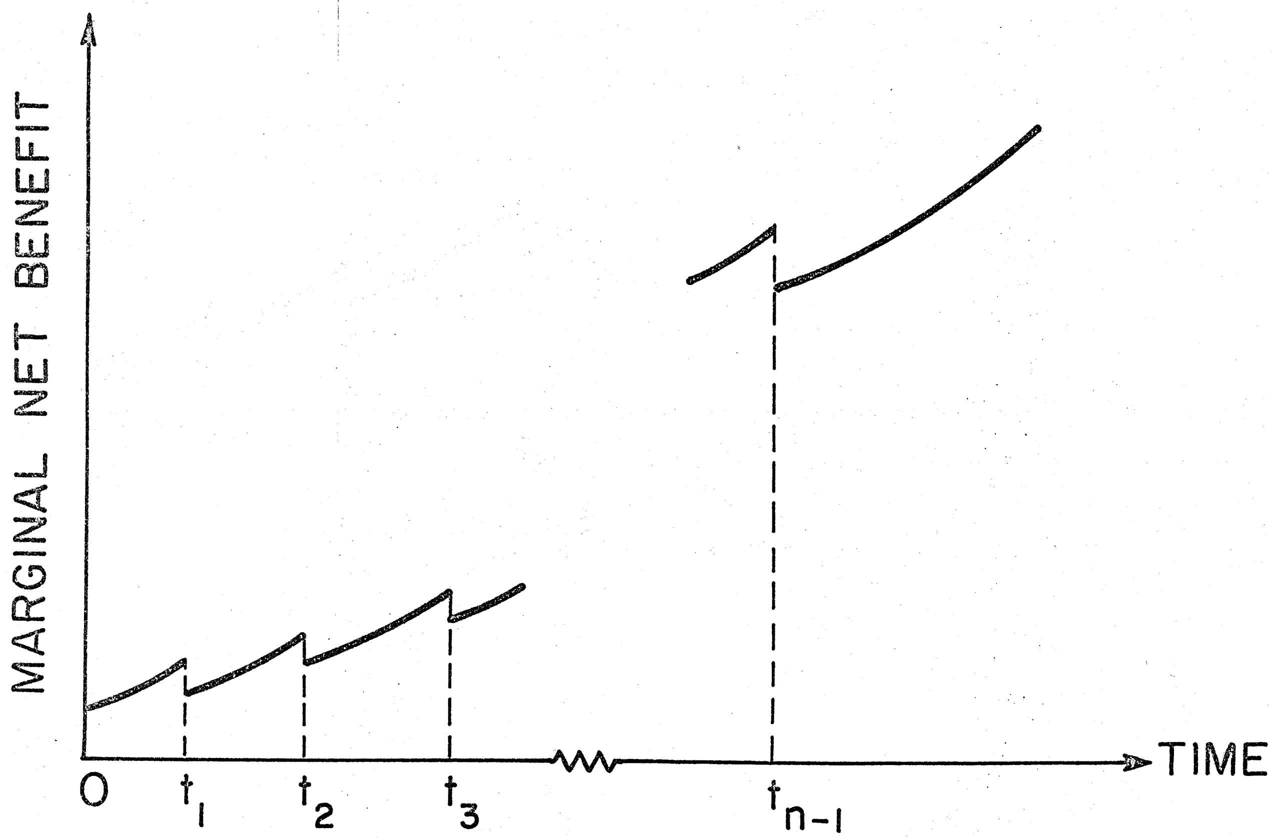

FIGURE 1(b)

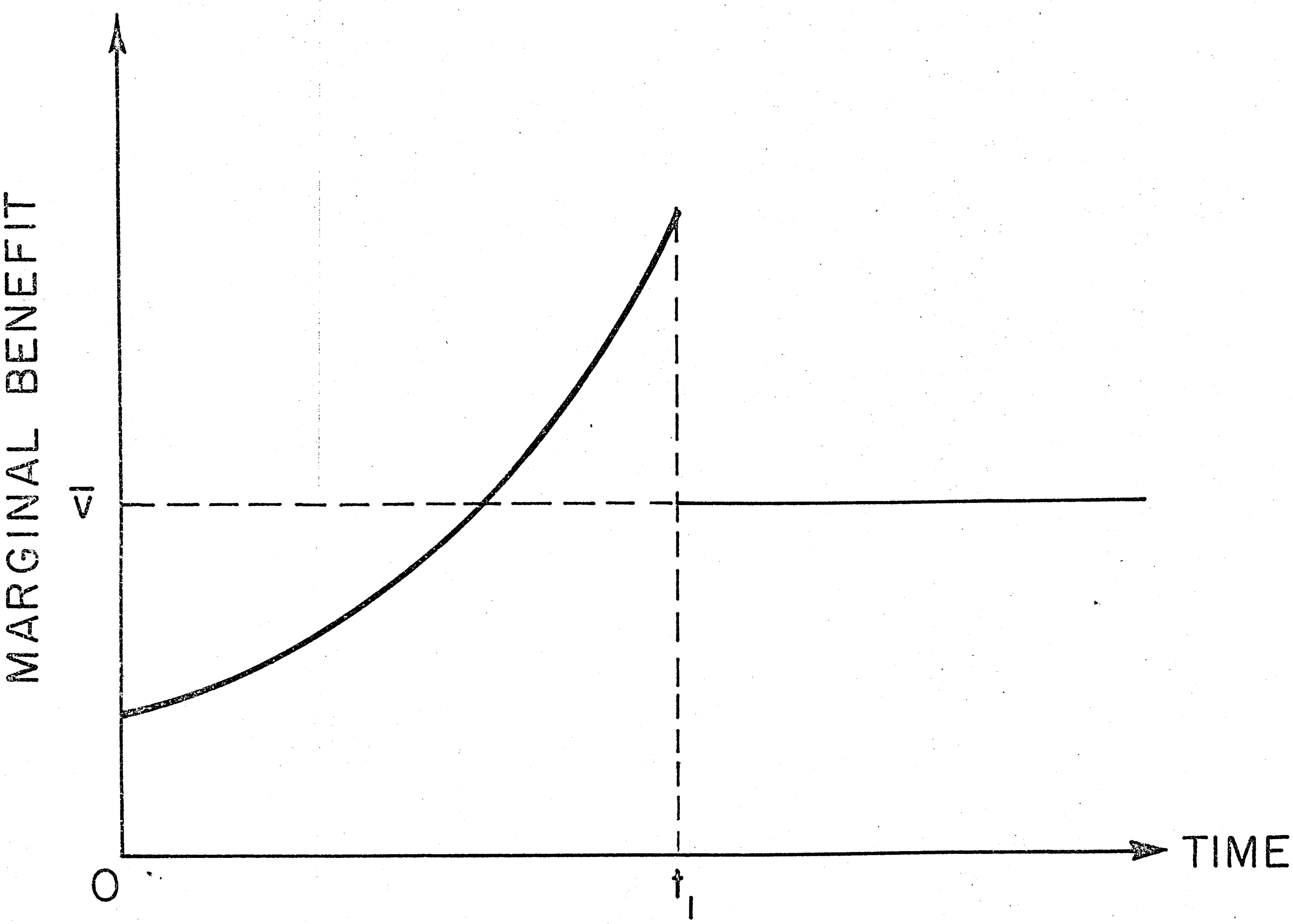

FIGURE 2

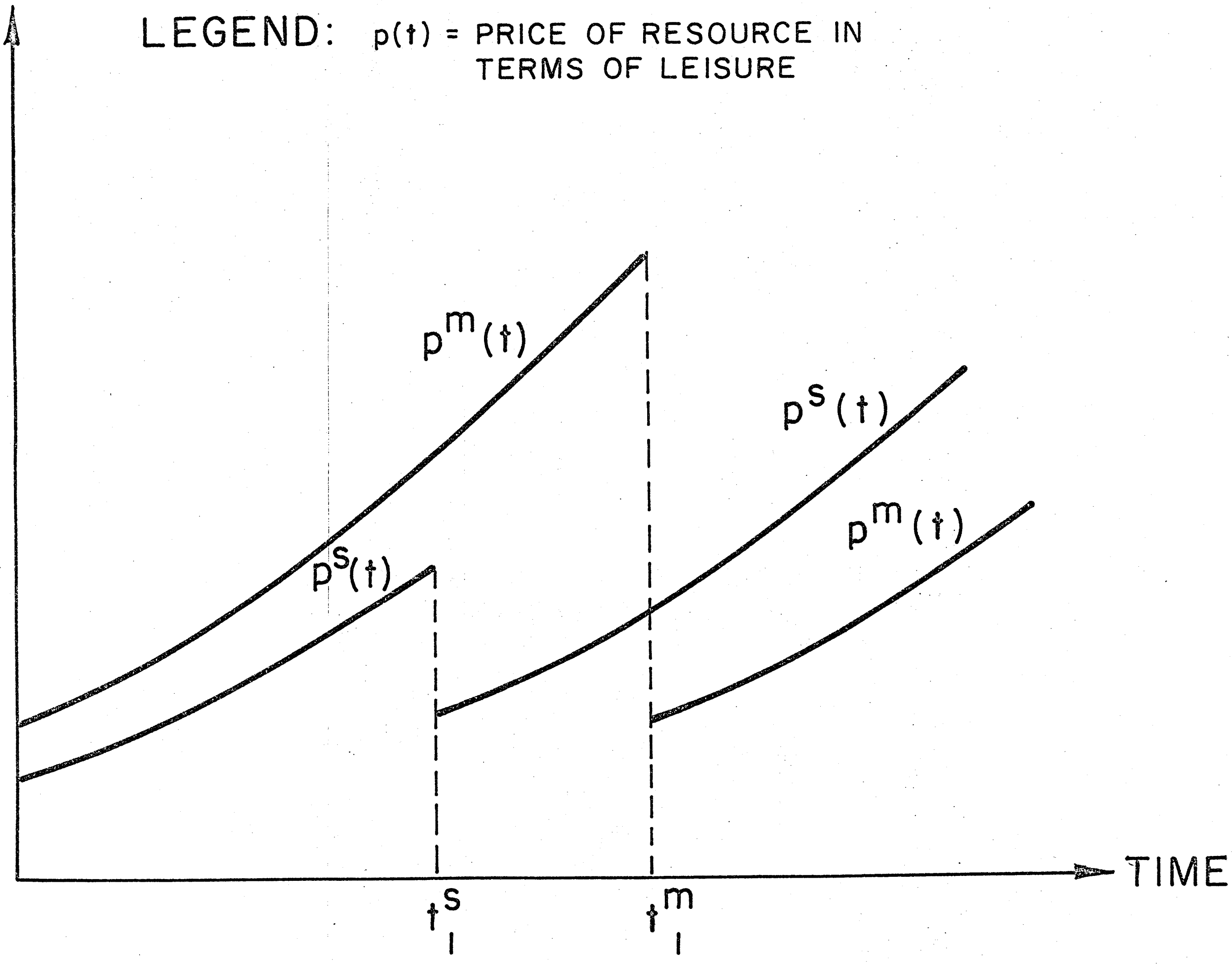
LEGEND: p(t) = PRICE OF RESOURCE IN TERMS OF LEISURE
$p^m(t)$
$p^s(t)$
$p^s(t)$
$p^m(t)$
TIME
t_1^s
t_1^m

FIGURE 3

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November, 1980

A Note on Set-up Costs Facing Consumers

In a recent paper (Hartwick, Kemp, and Long [1980]) there is the seeming anomaly: with identical exhaustible resource deposits <u>with identical set-up costs</u>, for an optimal program, each deposit "receives" a different net rent in present value. This is reminiscent of another seeming anomaly[1] reported in Mirrlees [1972]: in a socially optimal town identical households at different distances from the center "received" different utility levels, <u>ex post</u>. In this note, I indicate that the natural statement of the problem of allocation in the optimal town is in terms of set-up costs for consumption by consumers. Each consumer has a particular non-convexity in his budget set "caused by" the set-up costs and this non-convexity, investigated in some detail by Brown and Heal [1980] in the context of two part tariffs, "accounts for" the seeming anomaly of "equals being treated unequally" in the optimal town. In each case -- exploitation of many deposits of an exhaustible resource with set-up costs, and the allocation of commodities to households in the optimal town -- the set-up costs turn out to be treated essentially differently for different deposits or households by the planner. For the exhaustible resource model, the present values of two set-up costs must be different because they are incurred at different points in time and for the optimal town model, the <u>ex post</u> value of the set-up costs are different for any two households located at a different distance from the center. With Brown and Heal's analysis of two part tariffs in the background we reexamine allocation in the optimal town.

The following figure is found in Brown and Heal [1980].

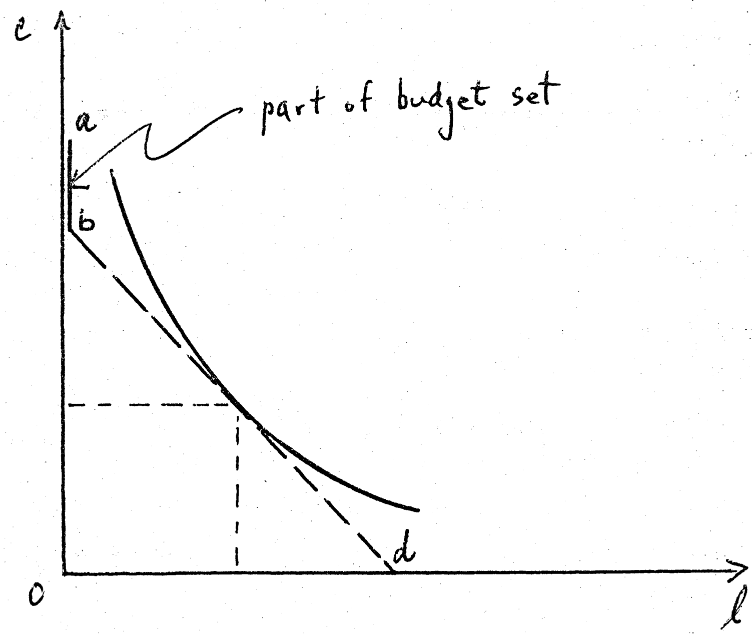

Figure 1

For our statement of the model of the optimal town, set-up costs are incurred if a positive amount of good ℓ is consumed. In Figure 1, the set-up costs ab will increase as a household is situated further from the center. The slope of bd is the implicit price ratio of commodity ℓ, land and the other commodity c ; this ratio will vary with distance from the center in the optimal town.

For a utilitarian social welfare function, an optimal town is the outcome of maximizing by choice of y and ℓ

$$\int_0^{\bar{x}} u(y-tx, \ell(x)) \frac{2\pi x}{\ell(x)} dx$$

Subject to $\int_0^{\bar{x}} \frac{2\pi x}{\ell(x)} dx = N$

$$\int_0^{\bar{x}} \frac{2\pi x}{\ell(x)} y \, dx = C$$

where $u(c,\ell)$ is the utility of a household consuming land ℓ and $c=y-tx$ of a composite commodity,

$\bar{x}$ is the radius of the boundary of the circular town,

t is the cost of moving a unit of radial distance to the center and back.

N is the exogenously given number of households

C is the exogenously given aggregate endowment of the composite commodity.

We can make our two points concerning set-up costs and consumption by moving to a numerical example involving a linear geography, two households, and Cobb-Douglas utility functions. Our first point is to illustrate the Mirrlees outcome -- ex post each identical household ends up with a different utility level in a socially optimal solution and secondly the set-up costs can lead to socially optimal outcomes involving only a single household receiving a non-zero utility.

Consider the example.[2] There are two individuals, labelled 1 and 2. There is a linear space, a line of length $\bar{x} = 2/3$, to be allocated to the two individuals. There is the endowment $C = 4/3$ to also be divided among the two individuals. The individual utility function is $(y^i x^i)^\alpha$ $(i=1,2)$ and $0<\alpha\leq 1/2$.

The utility function can be viewed as Cobb-Douglas with equal coefficients α on y^i and c^i. For α=.5, we have the utility function homogeneous of degree 1. $y^1 = c^1 - .5x^1$, $y^2 = C - c^1 - x^1 - .5(\bar{x} - x^1)$, $x^2 = \bar{x} - x^1$. Each individual travels from point 0 at one end of the line to the center of his space at cost in terms of C of \$1 per unit distance. For concreteness individual 1 is assumed to be located closest to 0. It turns out that maximizing u^1 subject to $u^2 = \bar{u}$ yields the result $[1+(u^2_{x^2}/u^2_{y^2})] = (u^1_{x^1}/u^1_{y^1})$ and in turn $(x^1)^2 - x^1 + (c^1/3) = 0$. In Table 1, we report of values of u^1, u^2 and Σu^i for different values of α, a measure of the concavity of the utility function.

Table 1

c^1	x^1	α	u^1	u^2	Σu_i
4/3	2/3	1/2	.81649658	0	.81649658
		.475	.82481519	0	.82481519
		1/4	.903602	0	.903602
		1/8	.95057982	0	.95057982
3/4*	1/2	1/2	.500000	0	.500000
		.475	.51763246	0	.51763246
		3/4	.70710678	0	.70710678
		1/8	.84089642	0	.84089642
2/3	1/3	1/2	.40824829	.23570226	.64395055
		.475	.42695112	.25336438	.6803155
		1/4	.6389431	.48549177	1.1244349
		1/8	.79933917	.6967724	1.4961116
1/2	$\frac{1-\sqrt{1/3}}{2}$**	1/2	.28867513	.42374329	.71241842
		.475	.30717715	.44233132	.74950847
		1/4	.53728496	.65095567	1.1882406
		1/8	.73299724	.80681824	1.5398155
1/3	$\frac{1-\sqrt{5/9}}{2}$	1/2	.18529766	.57028755	.75558521
		.475	.20159346	.5865286	.78812206
		1/4	.43046215	.75517385	1.185636
		1/8	.65609614	.8690074	1.5251035
1/6	$\frac{1-\sqrt{7/9}}{2}$	1/2	.08998497	.69886843	.7888534
		.475	.10149893	.71150123	.81300016
		1/4	.29997495	.83598351	1.1359585
		1/8	.54769969	.91432134	1.462021
1/24	$\frac{1-\sqrt{17/18}}{2}$	1/2	.02208497	.78790353	.8099885
		.475	.02672342	.79735073	.82407415
		1/4	.14861013	.8876393	1.0362494
		1/8	.38549984	.94214611	1.327646
0	0	1/2	0	.81649658	.81649658
		.475	0	.82481519	.82481519
		1/4	0	.903602	.903602
		1/8	0	.95057982	.95057982

* For values of c^1 between 3/4 and 4/3, individual 2 has insufficient c^2 to travel to his site. Hence individual 1 can attain any value between max u^1 in which he receives all the endowment and the value of u^1 at which u^2 first attains a positive utility level. A non-convexity occurs in the utility possibility schedule as one observes in Figure 1.

** The larger root for x^1 results in a lower sum of utilities than is the case for the smaller root. Corresponding to values of c^1 less than 1/2, the larger root results in negative values for individual 2's amount of space.

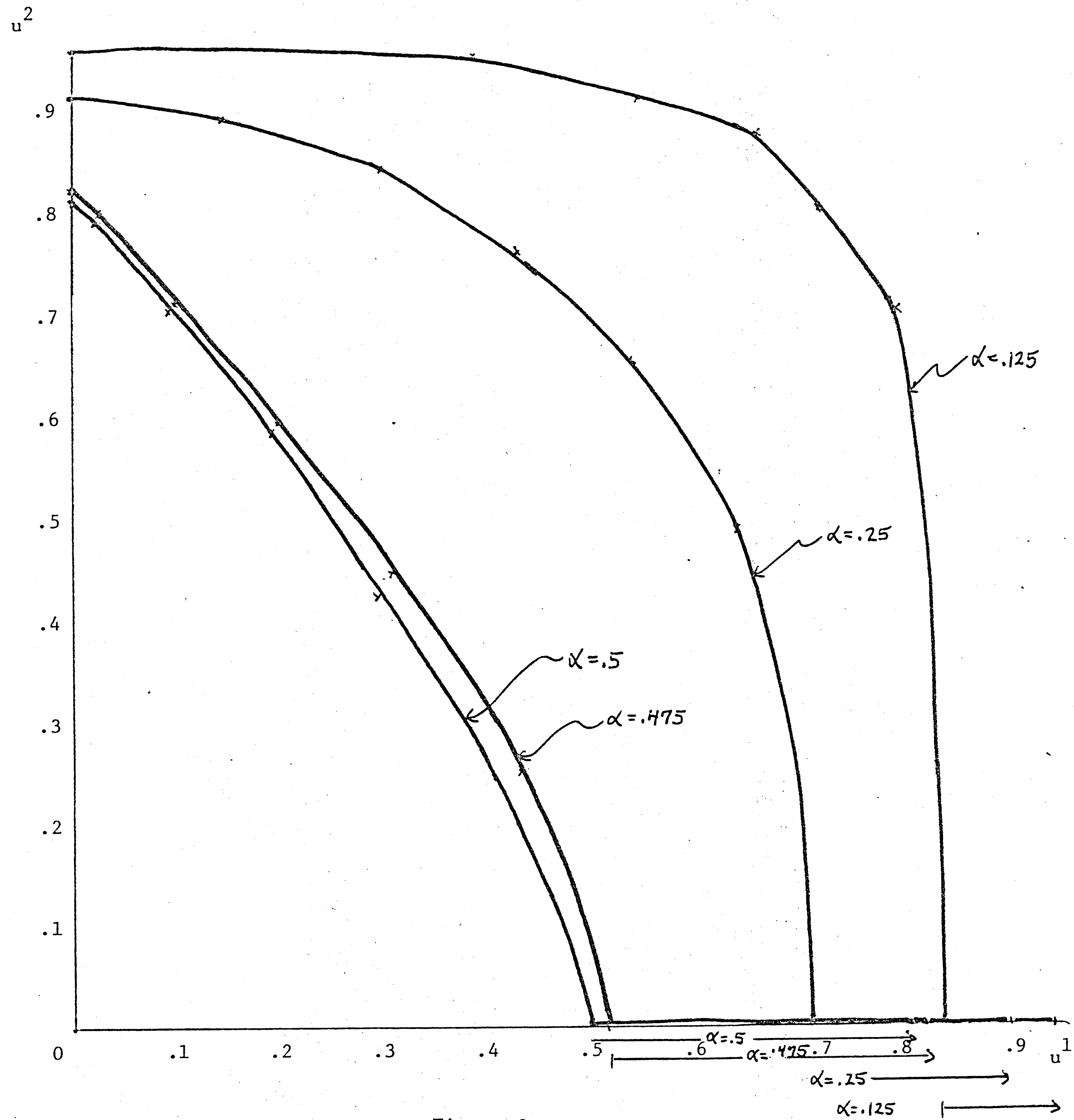

Figure 2

We plot the u^1-u^2 frontier for the four values of the marginal utility of "income" in Figure 2. A Benthamite social welfare function would have iso-value contours as straight lines with negative slopes in u^1-u^2 space, at 45° to the u^1 axis. Thus corner solutions occur for values of α equal to .5 and .475. Only one household should have positive utility at the social maximum because of the presence of set-up costs (here travel costs to the center). The optimal solution is interior for cases in which the value of α=.25 or .125. Set-up costs do not "loom as large" in the social allocation in these latter cases.[3]

We have then indicated how set-up costs in consumption can arise naturally in contexts other than those associated with two part tariffs and how in the context of the optimal town lead to seemingly anomalous outcomes. We also note that this is apparently the first occasion in which the optimal town problem has been examined explicitly from the point of view of set-up costs in consumption. The classic instance of set-up costs is transaction costs and the cost of overcoming geographical distance is an explicit type of transaction cost. Another type of set-up cost in consumer choice is the cost of shopping or searching -- that is the cost of acquiring information about price and quality.[4]

FOOTNOTES

1. Another anomaly associated with equilibria with set-up costs is that free entry or the competitive outcome in an industry will in general be associated with the socially suboptimal number of firms. See Stern [1972] for an analysis in the context of a Löschian, geographic setting and Meade [1974] and Spence [1976] for analyses in the area of monopolistic competition and product differentiation.

2. This example was set out in Arnot and Riley [1977] but no detailed investigation of its properties was reported.

3. For social welfare functions strictly quasi-concave in u^1 and u^2, the iso-value contours will appear as level curves convex to the origin in Figure 2. In the extreme case of a Rawlsian social welfare function, utilities will be equal ex post and the iso-value social welfare contours will be L-shaped.

4. See "Set-up costs in production of information" (p.423-25) in Dasgupta and Heal [1979]. Their analysis is based on Radner and Stiglitz [1975].

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