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OPEC VERSUS A LARGE OPEN ECONOMY:

A STOCHASTIC EQUILIBRIUM MODEL

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## ABSTRACT

### OPEC VERSUS A LARGE OPEN ECONOMY: A STOCHASTIC EQUILIBRIUM MODEL

If nothing else, recent experience with oil prices has taught us that: (a) even though large, economies such as the U.S. are, nonetheless, open; and (b) even though "small", OPEC has successfully exploited her market power over oil. Yet, nowhere in the literature is there a model capable of adequately analyzing the macro-effects of OPEC's monopolistic behaviour on a large open economy. This paper provides such a model.

The model's main feature is that OPEC is an expected revenue-maximizing cartel who exploits the imperfectly-elastic oil demand curve emerging from the domestic economy's use of imported oil as an intermediate input into the production of manufactures. The system is subject to various stochastic shocks with OPEC's expectation on oil demand and domestic agents' price expectations being "rationally" formed.

It is found that, unlike small open economies, anticipated macro-policies will not alter domestic employment and GNP. In this regard, the large economy behaves as if it were closed. Moreover, micro-policies designed to reduce reliance on OPEC by encouraging adoption of oil-saving technologies are inflationary and permanently reduce GNP. While unanticipated macro-policies can temporarily alter domestic GNP, their impacts may be perverse depending both upon whether OPEC is a price or a quantity setter and upon the extent to which OPEC's revenues are recycled back to the domestic economy through the capital, rather than the trade, account. Lastly, the above results are shown also to hold when OPEC maximizes expected cartel profits, rather than revenues.

## OPEC VERSUS A LARGE OPEN ECONOMY: A STOCHASTIC EQUILIBRIUM MODEL

If nothing else, recent experience with oil prices has taught us two important lessons. First, even though large, economies such as the U.S. are, nonetheless, open. Second, even though "small", OPEC has successfully exploited her market power over oil. While there exists a number of small open economy models,<sup>1</sup> none of these is capable of adequately analyzing the macro-effects of OPEC's monopolistic behaviour on a large open economy.

This paper constructs a stochastic equilibrium model of a two-nation world which incorporates both a large domestic economy and rational monopolistic behaviour by OPEC. Its distinguishing feature is that OPEC is an expected-revenue-maximizing cartel who exploits the imperfectly-elastic oil demand emerging from the larger domestic economy's use of imported oil as an intermediate input into the production of manufactures. The system is subject to various stochastic shocks with OPEC's expected oil demand and domestic agents' price expectations being "rational".

It is found that, unlike small open economies, anticipated macro-policies are incapable of altering domestic employment and GNP. In this regard, the large economy behaves as if it were closed. Moreover, micro-policies which encourage adoption of oil-saving technologies are inflationary and permanently lower real GNP. While unanticipated macro-policies can temporarily alter GNP, their impacts may be perverse depending both on whether OPEC is a price or quantity setter and on the extent to which OPEC's revenues are recycled through the capital versus trade account.

Section I constructs the basic model of the domestic economy while II determines oil's price and output as OPEC's solution to the expected-revenue-maximization problem. The rational-expectations macro-equilibrium is computed and analyzed in III and IV when information is, respectively, complete and incomplete. Effects of change in the oil regime are treated in V while conclusions follow in VI.

# I. THE BASIC MODEL

Consider a simple two-good, two-nation world with complete specialization in production. The foreign economy produces a single, composite, traded, intermediate good called, for concreteness, "oil". Oil production is organized by a foreign-owned, revenue-maximizing cartel called OPEC. Domestic output comprises a single composite traded consumption good called "manufactures". These are produced under competitive conditions using variable amounts of imported oil and a composite bundle of non-traded primary inputs called, for simplicity, "labour".

The domestic production sector is characterized by Phelps-Friedman-Lucas-type behaviour. All production takes place at period's start while consumption takes place at period's end. The following price information is assumed available to agents at period's start when firms' output and households' labour supply decisions are made. First, each firm participates in both input markets and in the market for its own good. It can then be assumed that each knows the current nominal prices of labour, oil and its own output. Second, households participate in the labour market thereby observing the current nominal wage. However, no agent is able to observe, at period's start, the price index for manufactures. This will be revealed at period's end when the consumption decision is executed.

Define the following magnitudes, each referring to the domestic economy:

$Y$  = output of manufactures

$Q$  = real GNP

$N$  = employment of labour

$H$  = employment of oil

$P$  = price index for manufactures = domestic price level

$R$  = domestic currency price of oil

$Q^0$  = the real oil bill = OPEC's real GNP

3.

$\tau = R/P =$  oil's "real" price = the terms-of-trade

$W =$  nominal wage rate

$w = W/P =$  real wage rate

$p^e =$  expected current price level

$\theta = P/p^e =$  "unanticipated" current price level

$\alpha, \beta, \delta =$  stochastic shift parameters

#### (a) Domestic Production and Input Rewards

Let manufactures be produced according to a well-behaved, linearly-homogeneous production function with variable inputs being labour and oil. As well, let the quality (i.e., productivity) of inputs be subject to random shocks. The aggregate production function may be written, generally, as:

$$(1) \quad Y = Y(\alpha N, \beta H) \quad Y_1, Y_2, Y_{12} > 0; \quad Y_{11}, Y_{22} < 0$$

The stochastic parameters  $\alpha$  and  $\beta$  are both initially unity and represent technical shocks in production. For example, a rise in  $\alpha, \beta$  or  $\alpha = \beta$  would signify, respectively, a "Harrod-", "Solow-" or "Hicks-" neutral technical shock. For the moment, we leave unspecified the mechanism generating such shocks. Nonetheless, we shall assume that producers know their technology and the current quality of their inputs and, therefore, observe the current values of  $\alpha$  and  $\beta$ . Given constant returns to scale, the production function may also be written in intensive form as:

$$(2) \quad y = \alpha y(\beta h / \alpha) \quad y' > 0; \quad y'' < 0$$

$$(3) \quad y = Y/N \quad h = H/N$$

Given competitive behaviour by firms, it is well known that the real rewards to inputs will be given by:

$$(4) \quad \tau = \alpha y'(\beta h/\alpha)$$

$$(5) \quad w = y - \tau.h$$

That is, oil is paid the value of its marginal product while the real rewards to the two inputs exhaust output.

Next, we distinguish between manufacturing output,  $Y$ , and real domestic GNP,  $Q$ . Since the latter is simply domestic value-added while all oil is imported and there are constant returns to scale

$$(6) \quad Q = Y - \tau.H = wN$$

In short, real domestic GNP is simply the real wage bill. Further, since OPEC only produces oil, her GNP measured in terms of manufactures will be  $\tau H$  or:

$$(7) \quad Q^0 = Y - Q$$

Finally, all this implies that world GNP, measured in terms of manufactures, will be identical to domestic manufacturing output,  $Y = Q + Q^0$ .

#### (b) Domestic Labour Supply

At the beginning of the period, the typical household determines its labour supply by solving the usual one-period consumption-leisure choice problem, where consumption comprises only manufactures. Households observed the nominal wage,  $W$ , but not the current price level which they expect to take the value  $P^e$ . Of course,  $P$  will be revealed to them at period's end when consumption takes place.<sup>2</sup> It is a simple matter to show that solution to this problem yields a labour supply function of the general form:

$$(8) \quad N^S = N^S(W/P^e; \delta) \quad N_1^S, N_2^S > 0;$$

where:  $N^S$  is labour supply;  $W/P^e$  is the expected real wage;  $\delta$  is a shift parameter representing, say, a change in tastes; and we impose the required conditions on income and substitution effects to insure the labour supply curve is everywhere



upward-sloping. As well, for simplicity, we will assume the labour supply curve is (approximately) linear, so that  $N_{1j}^S \approx 0$ , for  $j = 1, 2$ .<sup>3</sup>

Lastly, to complete the domestic production sector, impose the usual equilibrium condition that, in every period, the labour market clears. Hence,

$$(9) \quad N^S = N.$$

### (c) Aggregate Supply and Oil Demand

We now derive the aggregate supply and oil demand functions emerging from the domestic production sector. Given the usual notions, aggregate supply must, of course, refer to domestic real GNP, not output, while oil demand must account for the existence of two variable inputs. Following standard procedures, derivation of these two functions involves fixing the exogenous variables, including the prices of oil and manufactures, to compute the profit-maximizing levels of, respectively, real GNP,  $Q$ , and oil use,  $H$ , when labour input has been optimally chosen. Since the observed nominal wage clears the labour market, this optimal labour input will be both profit-maximizing and market-clearing. Moreover, there will exist similarly-derived optimal levels of world GNP (i.e., manufacturing output),  $Y$ , and real oil use,  $Q^0 = \tau H$ .

For given values of  $R$ ,  $P$ ,  $P^e$ ,  $\alpha$ ,  $\beta$  and  $\delta$ , equations (2) through (9) are sufficient to solve for these optimal values of  $H$ ,  $Q$ ,  $Y$  and  $Q^0$ . The Appendix shows the resulting oil demand, domestic aggregate supply, world aggregate supply and real-oil-use functions take the forms, respectively:

$$(10) \quad H = H(\tau, \theta, \alpha, \beta, \delta) \quad H_1 < 0; H_4 \geq 0; H_j > 0, \text{ all other } j$$

$$(11a) \quad \begin{cases} Q = Q(\tau, \theta, \alpha, \beta, \delta) \\ Y = Y(\tau, \theta, \alpha, \beta, \delta) \\ Q^0 = Q^0(\tau, \theta, \alpha, \beta, \delta) \end{cases} \quad \begin{aligned} &Q_1 < 0; Q_j > 0, \text{ all other } j \\ &Y_1 < 0; Y_j > 0, \text{ all other } j \\ &Q_1^0, Q_4^0 \geq 0; Q_j^0 > 0, \text{ all other } j \end{aligned}$$

where, recall,  $\tau = R/P$  and  $\theta = P/P^e$ .

Interpretation of these functions is straightforward once it is noted that: (a) with constant returns to scale, labour and oil will be gross complements in production;<sup>4</sup> (b) with  $\theta$  constant, a domestic price change is fully anticipated; and (c) with  $\tau$  constant, a domestic price change only alters unanticipated prices,  $\theta$ . First, a rise in oil's price,  $R$ , or a fully-anticipated fall in domestic prices,  $P$ , both raise oil's real price,  $\tau$ , thereby reducing oil demand and, by gross complementarity, labour demand, employment and the real wage. This implies a fall in manufacturing output,  $Y$ , in the real wage bill and, thereby, in domestic GNP,  $Q$ . For the usual reasons, the real oil bill,  $Q^0 = \tau H$ , will rise if and only if the own-price-elasticity of oil demand is less than unity. Second, consider a hike in unanticipated prices,  $\theta = P/P^e$ . A rise in  $P$  would raise labour's VMP and, thereby, labour demand; a fall in  $P^e$  would raise labour supply, according to (8). In either case, employment and, by gross complementarity, oil demand will rise implying a rise in output and real oil use. Moreover, the Appendix shows that output will rise by more than the real oil bill so that, on balance, domestic GNP,  $Q = Y - Q^0$ , will rise. Third, for similar reasons to the previous case, an exogenous hike in labour supply,  $\delta$ , raises oil demand, real GNP, output and the real oil bill. Fourth, an oil-augmenting technical shock,  $\beta$ , has an ambiguous impact on oil demand and, therefore, on the real oil bill,  $Q^0$ . On the one hand,  $H$  tends to fall since oil is now more productive while its real price is fixed. On the other hand, gross complementarity implies a rise in labour's marginal product, labour demand and, thereby, in employment, the real wage and real domestic GNP. The real wage hike, which is greater the less wage-elastic is labour supply, may then cause sufficient substitution in favour of oil, ceteris paribus, that, on balance, oil demand rises. The Appendix shows oil use and, therefore, the real oil bill rises only when oil demand is own-price elastic. Fifth, a labour-augmenting technical shock,  $\alpha$ , raises the productivity of both inputs, given gross complementarity, thereby rais-

ing oil demand, the real oil bill, output and the real wage. While employment may fall, since workers are now more productive, the Appendix shows the real wage bill and, thereby, domestic GNP unequivocally rise. Finally, it can be shown that a Hicks-neutral technical shock (i.e.,  $d\alpha = d\beta$ ) unequivocally raises  $H$ ,  $Y$ ,  $Q$  and  $Q^0$ .

Expressions (10) and (11a) assume OPEC exogenously sets oil's price letting its output be demand-determined. Were oil produced under conditions of constant marginal cost while OPEC is a marginal-cost-pricer, oil would be competitively priced. Then, an oil price shock would stem, ultimately, from an exogenous hike in oil's marginal production cost. Lastly, we explicitly note the well-known result that, unlike similarly-constructed closed-economy models, the long-run domestic aggregate supply curve is not vertical. By altering oil's real price,  $\tau$ , a fully-anticipated domestic price hike will alter domestic output and real GNP.

Nonetheless, OPEC may equally-well choose to be a quantity-setter,<sup>5</sup> fixing oil's output and letting its price be demand determined. If this be the case, oil's price and, thereby, the terms-of-trade can be obtained by fixing  $H$  and inverting the oil demand function, expression (10). Thus

$$(12) \quad R/P = \tau = \tau(H, \theta, \alpha, \beta, \delta) \quad \tau_1 < 0; \tau_4 \geq 0; \tau_j > 0, \text{ all other } j$$

where, by the Implicit Function Theorem:  $\tau_1 = H_1^{-1}$ ;  $\tau_j = -H_j/H_1$ ,  $j = 2, \dots, 5$ . This value for  $\tau$  can now be substituted into (11a) to obtain domestic, world and OPEC real GNPs when OPEC is a quantity-setter. The Appendix shows that:

$$(11b) \quad \begin{cases} Q = \bar{Q}(H, \theta, \alpha, \beta, \delta) \\ Y = \bar{Y}(H, \theta, \alpha, \beta, \delta) \\ Q^0 = \bar{Q}^0(H, \theta, \alpha, \beta, \delta) \end{cases} \quad \begin{cases} \bar{Q}_1, \bar{Q}_4 > 0; \bar{Q}_j \geq 0, \text{ all other } j \\ \bar{Y}_j > 0, \text{ for all } j \\ \bar{Q}_1^0, \bar{Q}_3^0, \bar{Q}_4^0 \geq 0; \bar{Q}_2^0, \bar{Q}_5^0 > 0 \end{cases}$$

The output and real oil bill functions,  $\bar{Y}$  and  $\bar{Q}^0$ , are straightforward. Obviously, a rise in the fixed input,  $H$ , or in its productivity,  $\beta$ , will raise output. For

given  $H$ , any other positive shock raises output for exactly the reasons cited when OPEC was a price-setter. Moreover, for fixed  $H$ , any shock which raises  $\tau$ , in accord with (12), must raise  $Q^0 = \tau H$  and vice versa. For the usual reasons,  $Q^0$  rises with  $H$  if and only if oil demand is own-price-elastic. Impacts on domestic GNP,  $Q$ , are less clear. A rise in oil's input or in its productivity will, by gross complementarity, raise labour's marginal product, employment, the real wage and, thereby, real GNP. Any other positive shock raises both  $Y$  and  $Q^0$ , we have seen, thereby leaving  $Q = Y - Q^0$  indeterminate, a priori. The Appendix shows that  $Q$  rises with  $\theta$  and  $\delta$  if and only if the elasticity of substitution between oil and labour in production exceeds oil's share of output; for  $Q$  to rise with  $\alpha$ , labour and oil must be even more substitutable. Finally, the Appendix confirms that output and domestic GNP unequivocally rise with a Hicks-neutral technical shock (i.e.,  $d\alpha = d\beta$ ) while  $Q^0$  rises if and only if the own-price-elasticity of oil demand exceeds the elasticity of substitution between labour and oil.

Lastly, unlike the price-setting case, note that the long-run aggregate supply curve is vertical; a fully-anticipated domestic price hike will not alter domestic real GNP or, for that matter, OPEC and world GNPs. This is because oil's price is endogenous and rises, according to (12), in proportion to any domestic price hike thereby leaving the terms-of-trade,  $\tau$ , unaltered.

#### (d) Elasticity of Oil Demand

Were the price and/or output of oil truly exogenous, system (11a) or (11b) would be sufficient fully to characterize domestic and world aggregate supply when OPEC sets, respectively, oil's price or output. However, OPEC is assumed to be a revenue-maximizer in which case oil's price or output is not exogenous but, rather, depends upon domestic oil demand. Since revenues are maximized at the unit-elastic point on the demand curve, it will be necessary explicitly to consider the own-price-

elasticity of oil demand given by:  $\eta = |RH_R/H| = -\tau H_1/H$ .

Obviously,  $\eta$  will not be a constant but, rather, will be a function with arguments identical to those in the oil demand function, (10). Unfortunately, this elasticity function is hopelessly complicated without imposing a technical restriction. Consequently, assume domestic technology is CES with an elasticity of substitution,  $\sigma$ , less than unity. This is quite innocuous since, the Appendix shows, an (interior) solution to the revenue-maximizing problem exists only if  $\sigma < 1$ . The Appendix then shows that  $\eta$  is determined by one of the following two functions:

$$(13a) \quad \eta = g(R/P, \theta, \alpha, \beta, \delta) \quad g_1 > 0; \quad g_4, g_5 < 0; \quad g_2, g_3 \geq 0$$

$$(13b) \quad \eta = f(H, \theta, \alpha, \beta, \delta) \quad f_1, f_4 < 0; \quad f_2, f_3, f_5 \geq 0$$

These two expressions determine  $\eta$  at, respectively, a given price,  $R$ , and a given quantity,  $H$ , for oil. The signs on  $g_1$  and  $f_1$  reflect the fact that  $\eta$  declines throughout the length of the oil demand curve. Further, were the oil demand curve linear, it is well known that any shock which raises demand would reduce (raise) elasticity when price (quantity) is fixed. However, this is not necessarily the case here since each shock alters  $\eta$  by altering both the slope and the position of the (non-linear) oil demand curve. Lastly, a Hicks-neutral technical shock can be shown to have an ambiguous impact on elasticity, whether price or output is fixed.

#### (e) Aggregate Demand

The demand side of the economy is simply a Fleming-Mundell model of a large open economy. First, let there be perfect capital mobility with the world interest rate being determined in the larger domestic capital market. Second, let there be saving, but no investment, on the part of OPEC.<sup>6</sup> Third, assume domestic currency is not held in OPEC portfolios and vice versa. Then, the domestic demand sector can be summarized by:



$$(14) \left\{ \begin{array}{l} r = ip/lp^e \\ r^* = ip^*/lp^* \\ r^e = ip^e/lp^e \\ Q = D(Q, r, r^e) + X(Q^0, r^*) - Q^0 \\ M/P = L(Y, i) \end{array} \right. \quad \begin{array}{l} D_Q, X_{Q^0} > 0; D_r, D_{r^e}, D_{r^*} < 0 \\ L_1 > 0; L_2 < 0 \end{array}$$

where:  $i$  is one plus the nominal interest rate;  $p^e$  and  $p^*$  are the domestic price levels believed currently to prevail by, respectively, domestic and OPEC agents;  $lp^e$  and  $lp^*$  are the domestic price levels expected to prevail one period hence by, respectively, domestic and OPEC agents;  $M$  is the nominal money stock;  $L$  is the demand for real balances;  $Q^0 = \tau H$  is both OPEC GNP and domestic imports measured in terms of manufactures; and  $D$  and  $X$  are the real values of, respectively, domestic expenditures and exports. Note that, a priori, OPEC and domestic price expectations are not necessarily identical.

First, recall that firms know their own current prices but the current price index is unobserved by either OPEC or domestic consumers; as well, OPEC consumes only domestic goods. Then, the magnitudes  $r$ ,  $r^e$  and  $r^*$  are the usual definitions of (one plus) the expected real interest rate relevant to, respectively, domestic investment, domestic saving and OPEC saving decisions. Second, the fourth expression in (14) is the usual IS function where exports are identical to OPEC's physical consumption which, for the usual reasons, depends on her real interest rate and GNP, both defined in terms of the consumption good.<sup>7</sup> Third, the last expression is the LM function where, for the sake of generality, money demand depends on domestic output,  $Y$ , rather than on GNP. Transactions balances may be held both by households, to finance nominal consumption which depends on  $PQ$ , and by firms, to finance the nominal wage and oil bills,  $PY = PQ + RH$ .<sup>8</sup> For the usual

reasons, nominal money demand is homogeneous of degree one in nominal output.

Recalling that  $Y = Q + Q^0$ , system (14) can now be solved for  $Q$  to yield the following domestic aggregate demand function:

$$(15) \quad Q = E(M/P, l_{P^e}/P, l_{P^e}/P^e, l_{P^*}/P^*, Q^0, z) \quad E_5 < 0; E_j > 0, \text{ all other } j$$

where  $z$  is a shift parameter reflecting changes in, say, government expenditures, taxes and other autonomous elements of aggregate demand. First, other things constant, a rise in real balances or a positive demand shock raise aggregate demand for the usual reasons. Second, by reducing some real interest rate(s), a hike in  $l_{P^e}/P$ ,  $l_{P^e}/P^e$  or  $l_{P^*}/P^*$ , other things constant, will raise, respectively, investment, consumption or exports and, thereby, aggregate demand. Third, the above two results imply aggregate demand falls with a current price hike, whether anticipated or not. Fourth, a rise in OPEC's GNP,  $Q^0$ , will unequivocally reduce aggregate demand since: (a) it lowers net exports,  $(X - Q^0)$  by the amount  $(X_{Q^0} - 1)$  which is minus OPEC's marginal propensity to save; and (b) it raises  $Y = Q + Q^0$  and, thereby, real money demand which raises the nominal and all expected real interest rates retarding investment, consumption and exports. This exemplifies the well-known "recycling problem": aggregate demand falls to the extent that any part of real oil expenditures are returned to the domestic economy through the capital, rather than the trade, account. Moreover, it can be shown that, when OPEC's marginal saving rate is zero,  $E_5 = -E_1 < 0$ : aggregate demand still falls because of the hike in real transactions balances needed to finance oil imports. Since the above results are straightforward, we leave it to the reader rigorously to verify the reported signs on the  $E_j$ 's.

The basic structure of the domestic economy is completely summarized by aggregate demand, (15) and aggregate supply, (11a) or (11b). Given fixed price expectations, these expressions are sufficient to determine the prices of oil and manufactures as well as domestic, world and OPEC GNPs for the cases where oil's

price or output is arbitrarily fixed by OPEC. Nonetheless, we have assumed that OPEC is a revenue maximizer so that oil's price (or output) is not arbitrary but, rather, derives from a rational decision on the part of OPEC. This decision must be explicitly modelled. Before doing so, however, matters will be greatly simplified by imposing some additional structure on the model.

(f) Log-Linearity

For simplicity but without loss of generality, we now assume that the aggregate demand function, (15), the two elasticity functions, (13a) and (13b), and the expressions for  $Q$  and  $Q^0$  contained in (11a) and (11b) are log-linear.<sup>9</sup> Henceforth, each previously-defined symbol will refer to the natural logarithm of the magnitude in question. Moreover, let the coefficients of these log-linear functions be represented by the above-defined partial derivatives of the respective underlying general functions. Thus, when OPEC is a price-setter:

$$(16a) \quad \begin{cases} Q = Q_0 + Q_1(R-P) + Q_2(P-P^e) + Q_3.\alpha + Q_4.\beta + Q_5.\delta \\ Q^0 = Q_0^0 + Q_1^0(R-P) + Q_2^0(P-P^e) + Q_3^0.\alpha + Q_4^0.\beta + Q_5^0.\delta \end{cases}$$

$$(17a) \quad \eta = g_0 + g_1(R-P) + g_2(P-P^e) + g_3.\alpha + g_4.\beta + g_5.\delta$$

where these log-linearize, respectively, (11a) and (13a) in which the coefficients  $Q_j$ ,  $Q_j^0$  and  $g_j$  are signed. Of course,  $Q_0$ ,  $Q_0^0$  and  $g_0$  are positive constants which define  $Q$ ,  $Q^0$  and  $\eta$  when all right-hand variables are zero. When OPEC sets output:

$$(16b) \quad \begin{cases} Q = \bar{Q}_0 + \bar{Q}_1.H + \bar{Q}_2(P-P^e) + \bar{Q}_3.\alpha + \bar{Q}_4.\beta + \bar{Q}_5.\delta \\ Q^0 = \bar{Q}_0^0 + \bar{Q}_1^0.H + \bar{Q}_2^0(P-P^e) + \bar{Q}_3^0.\alpha + \bar{Q}_4^0.\beta + \bar{Q}_5^0.\delta \end{cases}$$

$$(17b) \quad \eta = f_0 + f_1.H + f_2(P-P^e) + f_3.\alpha + f_4.\beta + f_5.\delta$$

where these log-linearize, respectively, (11b) and (13b) in which the coefficients  $\bar{Q}_j$ ,  $\bar{Q}_j^0$  and  $f_j$  are signed.<sup>10</sup>

Finally, log-linearize the aggregate demand function, (15):

$$(18) \quad Q = E_1(M-P) + E_2(P^e - P) + E_3(P^e - P^e) + E_4(P^* - P^*) + E_5.Q^0 + E_6.z$$

We now turn to OPEC's decision problem.

## II. OIL PRICING, OIL OUTPUT AND AGGREGATE SUPPLY

The usual textbook notion of the aggregate supply curve considers the own-price-response of domestic real GNP when all input markets clear. When oil's price or output are exogenous to the domestic economy such a supply curve would, we have seen, be defined in, respectively, (16a) or (16b). However, when OPEC is a revenue-maximizer, neither the price nor the quantity of oil is exogenous since both will depend upon domestic oil demand. This fact must, therefore, be reflected in the aggregate supply function if such function is to be based on clearance of both the oil and labour markets.

Assume, not unrealistically, that oil is priced in terms of the domestic currency (e.g., \$US).<sup>11</sup> To maximize oil revenues, it is well known that OPEC ought to choose a price (or output) corresponding to the unit-elastic point on the oil demand curve. Thus, since  $\eta$  is now a logarithm, set it equal to zero and solve (17a) for the revenue-maximizing price of oil:

$$(19) \quad R = G_0^+ + G_1^+ \cdot \eta + G_2^? (P - P^e) + G_3^? \cdot \alpha + G_4^+ \cdot \beta + G_5^+ \cdot \delta + P$$

where:  $\eta = 0$ ;  $G_1 = g_1^{-1}$ ; and  $G_j = -g_j G_1$ , for  $j = 2, \dots, 5, 0$ . The sign recorded for  $G_k$  comes from equation (13a). The revenue-maximizing oil price will rise in proportion to a fully-anticipated domestic price hike. It rises with an oil-augmenting or labour supply shock. Lastly, the Appendix shows  $R$  rises with an unanticipated price hike (a labour-augmenting technical shock) if and only if (only if) labour supply is wage-elastic.

Of course, at this price, the demand for oil would be just equal to the

revenue-maximizing quantity,  $H$ , as determined from (17b). Consequently, it no longer matters whether OPEC sets oil's price or its quantity since either action yields the same price-quantity combination. Substitute (19) into (16a) to obtain the new aggregate supply and real oil bill functions as, respectively:

$$(20) \quad \begin{cases} Q = \hat{Q}_0 + \hat{Q}_1 \cdot \eta + \hat{Q}_2^+(P-P^e) + \hat{Q}_3^+ \cdot \alpha + \hat{Q}_4^- \cdot \beta + \hat{Q}_5^? \cdot \delta \\ Q^0 = \hat{Q}_0^0 + \hat{Q}_1^0 \cdot \eta + \hat{Q}_2^0(P-P^e) + \hat{Q}_3^0 \cdot \alpha + \hat{Q}_4^0 \cdot \beta + \hat{Q}_5^0 \cdot \delta \end{cases}$$

where:  $\eta = 0$ ;  $(\hat{Q}_1, \hat{Q}_1^0) = G_1 \cdot (Q_1, Q_1^0)$ ; and  $(\hat{Q}_j, \hat{Q}_j^0) = (G_j Q_1 + Q_j, G_j Q_1^0 + Q_j^0)$ , for  $j = 0, 2, \dots, 5$ . At the unit-elastic point on the oil demand curve, the Appendix shows that:  $\hat{Q}_1^0 = 0$ ;  $\hat{Q}_1, \hat{Q}_4, \hat{Q}_4^0 < 0$ ;  $\hat{Q}_5 \geq 0$ ; and  $\hat{Q}_2, \hat{Q}_3, \hat{Q}_2^0, \hat{Q}_3^0, \hat{Q}_5^0 > 0$ .

The most striking feature of (20) is that a fully-anticipated domestic price hike no longer alters either foreign or domestic real GNP. Hence, the long-run aggregate supply curve is vertical. This is because the revenue-maximizing price of oil will rise proportionately leaving oil's real price,  $\tau = R-P$ , unaltered. Moreover, among supply shocks, only a labour-augmenting technical shock,  $\alpha$ , will now unequivocally raise domestic GNP. An oil-augmenting technical shock,  $\beta$ , unequivocally lowers GNP while, the Appendix shows, an exogenous hike in labour supply,  $\delta$ , raises GNP if and only if labour supply is sufficiently wage-inelastic. On the other hand, with the exception of a rise in oil's productivity,  $\beta$ , any positive domestic supply shock will raise OPEC's real GNP,  $Q^0$ . Further, it can be confirmed that a Hicks-neutral technical shock (i.e.,  $d\alpha = d\beta$ ) unequivocally raises  $Q$  and  $Q^0$ . Lastly, for future reference,  $Q$  will fall should OPEC, for some reason, raise her chosen value of  $\eta$ .

While the above exercise is instructive, it will not, in general, yield the short-run aggregate supply function. Unfortunately, except with perfect information, the oil demand curve is unobservable in which case OPEC will not be able to



determine, with accuracy, its unit-elastic point. Of course, this is because the current values of the arguments of the oil demand function are not, necessarily, observed. OPEC must, therefore, estimate the oil demand curve and its elasticity in order to maximize expected revenues.

(a) Expected Revenue Maximization

OPEC's expectation on the price-elasticity of oil demand is assumed to be rational in the sense that it is based on optimal use of all available information. Let OPEC know the structure of the oil demand function, (10), and of the (soon-to-be-specified) stochastic processes generating the values of its arguments. This knowledge further implies that OPEC knows the structure, but not the current argument values, of the two log-linear elasticity functions, (17a) and (17b). Consequently, for each OPEC-chosen oil price (or quantity), this known structure can be combined with mathematical expectations on the argument values to produce a rational expectation on the own-price-elasticity of oil demand.

First, let OPEC be a price-setter. Pass the expectations operator through expression (17a) to determine the expected-price-elasticity of oil demand at a given OPEC chosen oil price,  $R$ . Solve the resulting expression for  $R$ . It should be apparent that this procedure simply amounts to passing the expectations operator through expression (19).

$$(21) \quad R = G_0 + G_1^+ \cdot \eta^* + G_2^? (P - P^e)^* + G_3^? \cdot \alpha^* + G_4^+ \cdot \beta^* + G_5^+ \cdot \delta^* + P^*$$

where: the "star" indicates a mathematical expectation; since  $P^*$  and  $P^e$  are not necessarily identical,  $(P - P^e)^*$  measures the extent to which OPEC believes domestic workers have mis-anticipated the current domestic price level; since OPEC sets oil's price,  $R^* = R$ ; and, of course, explicitly solving (21) for  $\eta^*$  would recover the expected-elasticity function. Expressions (19) and (21) differ only in that, now, oil's price responds to OPEC-expected, rather than to actual, exogenous variables.

OPEC is now concerned to maximize expected revenues which, it is well known, requires setting oil's price such that the expected price-elasticity of oil demand is unity.<sup>12</sup> Since  $\eta$  is now a logarithm, set  $\eta^*$  to zero in expression (21) to produce the (log of) oil's expected-revenue-maximizing price. Given that oil output is endogenously determined as the quantity demanded at this price, (21) can be substituted into (16a) to produce a new domestic aggregate supply function and the corresponding real oil bill,  $Q^0$ .

$$\begin{aligned}
 (22a) \quad Q &= Q_0 + \bar{Q}_1 \cdot (G_0 + G_1 \cdot \eta^* + G_2 \cdot (P - P^e)^* + G_3 \cdot \alpha^* + G_4 \cdot \beta^* + G_5 \cdot \delta^*) \\
 &\quad + \bar{Q}_1 \cdot (P^* - P) + \bar{Q}_2 \cdot (P - P^e) + \bar{Q}_3 \cdot \alpha + \bar{Q}_4 \cdot \beta + \bar{Q}_5 \cdot \delta \\
 Q^0 &= Q_0^0 + Q_2^0 (P - P^e) + Q_3^0 \cdot \alpha + Q_4^0 \cdot \beta + Q_5^0 \cdot \delta
 \end{aligned}$$

where  $\eta^* = 0$  and all parameters have been evaluated at the certainty equilibrium where  $\eta^* = \eta = 0$ .

Since oil's price depends only upon OPEC's expectations, actual (unanticipated) shocks have precisely those impacts on  $Q$  and  $Q^0$  reported in (16a). As well, any expected shock which raises oil's expected-revenue-maximizing price and, thereby, oil's real price,  $\tau = R - P$ , will reduce domestic GNP,  $Q$ , and vice versa. However, since the system has been log-linearized around the certainty equilibrium where oil demand is unit-elastic, changes in oil's price induced by OPEC's expectations will leave nominal and, thereby, real oil revenues,  $Q^0 = \tau H$ , unaltered. For future reference note that, should OPEC choose a price where expected oil demand is greater (less) than unit-elastic,  $Q$  would be lower (higher), since oil's price would be higher (lower). Lastly, suppose OPEC fully-anticipates all exogenous variables; then system (22a) will collapse to system (20), above.

Second, let OPEC be a quantity-setter. The expected price-elasticity of oil demand, at a given quantity,  $H$ , is determined, as before, by passing the

expectations operator through expression (17b). Again, the expected-revenue-maximizing output of oil is determined by setting  $\eta^*$  to zero in this expected-elasticity function. Given that  $R$  is then determined as the demand-price for this quantity, the OPEC-optimal  $H$  can be substituted into system (16b) to yield a new domestic aggregate supply function and corresponding real oil bill. Such functions are readily shown to be:

$$\begin{aligned}
 (22b) \quad Q &= \bar{Q}_0 + \bar{Q}_1 \bar{f}_1^{-1} \cdot (f_0 + \eta^* - f_2(P-P^e)^* - f_3 \cdot \alpha^* - f_4 \cdot \beta^* - f_5 \cdot \delta^*) \\
 &\quad + \bar{Q}_2 \cdot (P-P^e) + \bar{Q}_3 \cdot \alpha + \bar{Q}_4 \cdot \beta + \bar{Q}_5 \cdot \delta \\
 Q^0 &= \bar{Q}_0^0 + \bar{Q}_2^0 \cdot (P-P^e) + \bar{Q}_3^0 \cdot \alpha + \bar{Q}_4^0 \cdot \beta + \bar{Q}_5^0 \cdot \delta
 \end{aligned}$$

where  $\eta^* = 0$  and, as before, all coefficients are evaluated where oil demand is unit elastic.

Again, since oil's output now depends only on OPEC's expectations, actual (unanticipated) shocks have exactly those impacts on  $Q$  and  $Q^0$  reported for system (16b). Moreover, any OPEC-expected shock which, other things constant, raises the expected price-elasticity of oil demand will raise the expected revenue-maximizing output of oil thereby raising domestic real GNP,  $Q$ . Of course, such an expectations-induced hike in  $H$  will not alter the real oil bill,  $Q^0 = \tau H$ , since all coefficients are evaluated at the unit-elastic point on the oil demand curve. Lastly, when all exogenous variables are fully-anticipated by OPEC, system (22b) must, obviously, collapse to system (20).

Finally, with uncertainty, it matters to domestic GNP whether OPEC is a price- or a quantity-setter because the two actions will generate different actual oil price-quantity combinations. The value of OPEC's policy tool, price or output, is determined by expected demand while the "free" variable, respectively, output or price, is determined by actual demand. Of course, expected and actual demand will be identical, in general, only when OPEC fully-anticipates all the

relevant variables.<sup>13</sup> We now explicitly specify the stochastic processes generating such variables.

(b) Stochastic Elements of the Model

Assume the exogenous variables of the system are generated by the following simple log-linear stochastic processes:

$$\begin{aligned}
 \alpha &= \alpha_{-1} + a \\
 \beta &= \beta_{-1} + b \\
 \delta &= \delta_{-1} + d \\
 (23) \quad z &= z_{-1} + e \\
 M &= M_{-1} + m \\
 m &= (m_{-1} - n_{-1}) + n + u
 \end{aligned}$$

where: an unsubscripted variable refers to a current value; the subscript "-1" refers to a value one period ago;  $m$  is the log of one plus the rate of monetary growth; and  $a$ ,  $b$ ,  $d$ ,  $e$ ,  $n$  and  $u$  are independently-distributed, serially-uncorrelated, random variables with zero means and finite variances (i.e., white noise).

Hence,  $a$ ,  $b$  and  $d$  are "supply shocks";  $e$  is a "demand shock"; and  $n$  and  $u$  are "monetary shocks". Since the model is already quite complicated, we assume, for simplicity, that all but monetary shocks are permanent. The magnitude  $n$  is a transitory shock to the monetary growth rate,  $m$ , and, therefore, can be viewed as a random deviation from, say, target growth as set by the Central Bank. Target growth,  $m^0 = (m_{-1} - n_{-1} + u)$ , is subject to periodic change at the whim of the Central Bank as indicated by the permanent monetary shock,  $u$ . Lastly, while supply and monetary shocks are solely of domestic origin, demand shocks may also originate in OPEC through, say, an exogenous hike in OPEC's saving which would be reflected in a fall in export demand.

### III. COMPLETE INFORMATION, REVENUE MAXIMIZATION AND MACRO EQUILIBRIUM

Once the mechanisms generating the various agents price expectations have been specified, the above system(s) are sufficient to determine the current values of  $P$ ,  $Q$  and  $Q^0$ . As before, assume expectations are rational, being based on optimal use of all available information. For the moment, let current information be complete and symmetric in the sense that OPEC and domestic agents both know:

- (a) the economy's basic structure; (b) the current levels of the state variables and the division of monetary shocks into their permanent and transitory components; (c) that OPEC is an expected-revenue-maximizer; and (d) that all agents share the above three pieces of information. Of course, the known basic structure of the economy will include aggregate demand, (15), and the stochastic processes, (23). What about aggregate supply? Since both OPEC and domestic agents have identical information, they will have identical price expectations, so that  $P^e = P^*$  and  ${}^1P^e = {}^1P^*$ . Moreover, knowing that OPEC is an expected-revenue-maximizer, both will expect the own-price-elasticity of oil demand to be unity (i.e.,  $\eta^* = 0$ ). In short, the known structure of aggregate supply will be given by expression (20), with  $\eta$  set to zero. Hence, in addition to this known structure, the information set common to all agents can be characterized as a vector of the observed elements of the state variables:

$$(24) \quad I = (M_{-1}, m_{-1}, n_{-1}, \alpha_{-1}, \beta_{-1}, \delta_{-1}, z_{-1}, n, u, a, b, d, e)$$

The system is solved by the method of undetermined coefficients.<sup>14</sup> It is apparent from expressions (15), (20) and (23) that, ultimately, the current price level will be a log-linear function of the current values of the state variables whose elements, in turn, make up the information set,  $I$ . Let  $C$  be a twelve-element column vector with typical entry  $C_j$ . Then, we postulate a solution for



the domestic price level of the form:

$$(25) \quad P = C_0 + I.C$$

Recalling that all elements of  $I$  are currently observed, pass the expectations operator through (25) to find that the current price level will be fully-anticipated by all agents: that is,  $P^* = P^e = P$ . Substitute this result and expression (23) into the aggregate supply functions, (20), and re-arrange terms to find:

$$(26) \quad \begin{aligned} Q &= Q_{-1} + \hat{Q}_3.a + \hat{Q}_4.b + \hat{Q}_5.d & \hat{Q}_3 > 0; \hat{Q}_4 < 0; \hat{Q}_5 \geq 0 \\ Q^0 &= Q_{-1}^0 + \hat{Q}_3^0.a + \hat{Q}_4^0.b + \hat{Q}_5^0.d & \hat{Q}_3^0, \hat{Q}_5^0 > 0; \hat{Q}_4^0 < 0 \end{aligned}$$

where  $\hat{Q}_j$  and  $\hat{Q}_j^0$  were signed in (20).

Domestic and OPEC real GNPs are neutral with respect to fully-anticipated monetary and demand shocks. Only supply shocks, of domestic origin, alter either  $Q$  or  $Q^0$ . A labour-augmenting technical shock,  $a$ , raises domestic GNP for the usual reasons. However, an oil-augmenting technical shock,  $b$ , lowers GNP. Consequently, attempts to reduce reliance on imported oil by introducing oil-saving technology serve only to lower domestic real GNP. At first glance, this result seems counter-intuitive. In fact, other things constant, a hike in  $b$  would raise manufacturing output, reduce the real oil bill and, thereby, raise domestic GNP. However, it also reduces the elasticity of oil demand in which case, to protect her (maximal) revenues, OPEC jacks up oil's price in accord with (19). This raises oil's real price sufficiently that, on balance, domestic real GNP falls. For similar reasons, a positive shock to labour supply,  $d$ , may also lower domestic GNP if, as the Appendix shows, labour supply is sufficiently wage-elastic. Lastly, as seen above, only an oil-augmenting technical shock will reduce OPEC's real revenues,  $Q^0$ .

What about the domestic price level? Given expressions (25) and (26),  $P$  is left to be determined by aggregate demand, expression (15). Recall that the common complete information implies that:  $P = P^* = P^e$  while  $l_{P^e} = l_{P^*}$ . Substitute

this into (15) and re-arrange terms.

$$(27) \quad (E_1 + E_0).P = E_1.M + E_0.^1P^* + E_5.Q^0 + E_6.z - Q$$

where:  $E_0 = E_2 + E_3 + E_4 > 0$ . This price equation illustrates the "rational expectations problem" in that the price level depends upon the expected future price level while the latter depends upon the same variables as the former. Nonetheless, expression (25) suggests the information set can be updated and its expectation taken to give the expected future price level as:  $^1P^* = C_0 + (I_{+1}).^*C$ , where the subscript "+1" refers to a value one period hence and  $(I_{+1}).^* = (M, m-n, \alpha, \beta, \delta, z, 0, 0, 0, 0, 0, 0)$ . Substitute this as well as (23) and (26) into (27). It is then possible to obtain thirteen independent conditions on the  $C_j$ 's by equating to zero, in turn, the net coefficients on the twelve magnitudes in the information set and the constant. These net coefficients must be zero in order for (27) to hold. Without belabouring the derivation, the solution for the price level is readily shown to be:

$$(28) \quad P = P_{-1} + n + \bar{C}_3.a + \overset{+}{C}_4.b + \overset{?}{C}_5.d + \overset{+}{C}_6.e + \overset{+}{C}_8.u$$

where:  $C_3 = C_9 = (E_5\hat{Q}_3^0 - \hat{Q}_3)/E_1 < 0$ ;  $C_4 = C_{10} = (E_5\hat{Q}_4^0 - \hat{Q}_4)/E_1 > 0$ ;  $C_5 = C_{11} = (E_5\hat{Q}_5^0 - \hat{Q}_5)/E_1 \geq 0$ ;  $C_6 = E_6/E_1 > 0$ ;  $C_8 = C_2 = 1 + E_1^{-1} > 0$ ;  $C_1 = C_7 = 1$ ;  $C_0 = (E_5Q_{-1}^0 - Q_{-1})/E_1 < 0$ ; and  $P_{-1} = C_0 + I_{-1}.C$ .

In short, demand or monetary shocks,  $e$ ,  $n$  and  $u$ , raise the price level for the usual reasons. Moreover, since  $u$  constitutes a permanent hike in the monetary growth rate, while  $n$  is only a permanent hike in the money stock,  $u$  will have the larger impact on prices. This is because it will raise the expected future proportionately more than the current price level while  $n$  raises the current and future price levels proportionately. Consequently,  $u$  but not  $n$  raises expected inflation thereby further stimulating aggregate demand by lowering, for each agent,

the expected real interest rate.<sup>15</sup> Moreover, since  $P^* = P^e = P$ , expression (19) suggests that demand and monetary shocks simply raise  $R$  and  $P$  proportionately so that oil's real price,  $R-P$ , is unaltered. Among supply shocks, only a labour-augmenting technical shock,  $a$ , unequivocally generates net excess aggregate supply and, thereby, reduces domestic prices. It does so by raising OPEC's GNP,  $Q^0$ , which, we saw, reduces aggregate demand while it also raises aggregate supply,  $Q$ . An oil-augmenting technical shock,  $b$ , has just the opposite effects and, therefore, raises  $P$ . Consequently, the introduction of oil-saving technology is inflationary! Domestic prices fall with a shock to labour supply,  $d$ , whenever this raises domestic GNP for which, we have seen, it will be sufficient that labour supply be relatively wage-inelastic. Finally, expression (19) indicates that oil's price will rise proportionately more than domestic prices with an oil-augmenting technical shock leading to that above-noted fall in GNP. As well, even should the labour-augmenting and labour supply shocks,  $a$  and  $d$ , lower domestic prices they may still raise oil's price, according to (19). However,  $R$  will rise with  $a$  only if labour supply is elastic.<sup>16</sup>

The above results are in sharp contrast to those obtained by previous writers for whom oil's price is arbitrarily set by OPEC. First, when  $R$  is fixed it is obvious, from expression (11a), that any shock which raises the domestic price level will reduce oil's real price,  $\tau$ , thereby raising domestic GNP and vice versa. Consequently, the system would not be neutral with respect to fully-anticipated demand and monetary shocks. Second, it matters to domestic GNP and prices what is the source of a domestic supply shock since the otherwise positive impacts of, say,  $b$  and  $d$  may be offset by OPEC's oil-price-response. Third, there can not exist an exogenous oil price (or output) shock since any observed oil-price hike is, ultimately, the result of a domestic shock which has altered the domestic demand for oil. There exists only one possible OPEC-originated exogenous oil shock:

an exogenous improvement in the quality of oil. This amounts to an oil-augmenting technical shock which, we have seen, is "stagflationary". Paradoxically, the domestic economy would be better off were OPEC to ship lower-grade oil.

#### IV. INCOMPLETE INFORMATION EQUILIBRIA

The above results critically depend upon the existence of complete current information. We now relax this assumption by permitting agents to observe all shocks only with a one-period lag. Nonetheless, we retain all other previous assumptions regarding the information structure. Further, we assume no agent is able to infer the nature of current shocks from the single prices he may currently observe.<sup>17</sup> Thus, aside from the known structure of the economy, the information set shared by all agents may be characterized as:

$$(29) \quad J = (M_{-1}, \bar{m}, \alpha_{-1}, \beta_{-1}, \delta_{-1}, z_{-1})$$

where  $\bar{m} = (m_{-1}, n_{-1})$ . As seen above, the structure of the economy will depend on OPEC's known choice of a policy tool since, now, there exists uncertainty about oil demand. Consequently, two cases must be considered.

##### (a) OPEC as a Price-Setter

When OPEC maximizes expected revenues by setting oil's price, the known structure of aggregate supply will be given by (22a) while aggregate demand will still be given by (15). Moreover, since OPEC and domestic agents share a common information set while both form expectations rationally, their respective expectations will be identical so that:  $p^* = p^e$ ;  ${}^1p^* = {}^1p^e$ ; and  $(p - p^e)^* = 0$ .

As before, the system is solved by the method of undetermined coefficients. Given the structures of aggregate demand, (15), and aggregate supply, (22a), the current price level must, in the end, be a log-linear function of both current and past values of the state variables since such past values form the information

base for price expectations. Of course, given (23), the current value of each state variable depends on its past value and its current shock. Consequently, we postulate a solution for the price level of the form:

$$(30) \quad P = D_0 + J.D + K.B$$

where:  $K = (n, u, a, b, d, e)$  is the vector of current shocks;  $J$  is the information vector, (29);  $D_0$  is a scalar constant; and  $D$  and  $B$  are six-element column vectors with typical entries, respectively,  $D_j$  and  $B_j$ . Of course, since only past shocks are observed while current and future shocks have expected values of zero, we have:  $K^* = (K_{+1})^* = 0$  and  $J^* = J$ . Consequently, passing the expectations operator through (30) reveals that:  $(P - P^*) = KB$ . As well, up-dating  $J$  and  $K$  in expression (30) and passing the expectations operator through the resulting equation gives the expected future price level as:  ${}^1P^* = D_0 + J_{+1}^*D$ . Making use of expression (23), this all implies that:

$$(31) \quad (P - P^*) = KB; \quad ({}^1P^* - P) = D_1 \cdot \bar{m} - K.B; \quad ({}^1P^* - P^*) = D_1 \cdot \bar{m}$$

Substitute (31) and the fact that  $J^* = J$  into the aggregate supply function, (22a), and re-arrange terms to obtain domestic real GNP as:

$$(32) \quad Q = Q_0 + \hat{Q}_3 \cdot \alpha_{-1} + \hat{Q}_4 \cdot \beta_{-1} + \hat{Q}_5 \cdot \delta_{-1} + (Q_3 + \bar{Q}B_3) \cdot a + (Q_4 + \bar{Q}B_4) \cdot b \\ + (Q_5 + \bar{Q}B_5) \cdot d + \bar{Q}B_6 \cdot e + \bar{Q}B_1 \cdot n + \bar{Q}B_2 \cdot u$$

where:  $\bar{Q} = (Q_2 - Q_1) > 0$ ; and the parameters  $Q_j$  and  $\hat{Q}_j$  are defined in, respectively, (11a) and (20). The corresponding expression for  $Q^0$  can be similarly derived and, it can be shown, will take the form of (32) with  $(Q_j^0, \hat{Q}_j^0)$  replacing  $(Q_j, \hat{Q}_j)$  for all  $j$ . It simply remains to derive the "undetermined coefficients",  $B_j$ .

Substitute (31), (23) and the just-derived expressions for  $Q$  and  $Q^0$  into the aggregate demand function, (15). Re-arranging terms, we obtain the equilibrium



price level as a function solely of J and K.

$$(33) \quad P = F_0 + K.(A-A_0B) + J.F$$

where:  $E_0 = (E_2 + E_3 + E_4)/E_1 > 0$ ;  $\bar{Q} = Q_2 - Q_1 > 0$ ;  $\bar{Q}^0 = Q_2^0 - Q_1^0 > 0$ ;  $A_0 = (E_2 + \bar{Q} - E_5 \bar{Q}^0)/E_1 > 0$ ;  
 $A = (1, 1, A_3, A_4, A_5, C_6)'$ ;  $F = (C_1, F_2, C_3, C_4, C_5, C_6)'$ ;  $F_2 = (1 + D_1 E_0)$ ;  $F_0 = C_0 > 0$ ;  
 $A_j = (E_5 Q_j^0 - Q_j)/E_1$ , for  $j = 3, 4, 5$ ; and  $C_j$  is the typical parameter from the full-information price equation, (28). Moreover, from the underlying parameter definitions, it is readily confirmed that:  $A_3, A_5 > 0$  and  $A_4 \geq 0$ .

We now have two expressions for equilibrium prices, (30) and (33), both of which must hold. Substitute one into the other and re-arrange:

$$(34) \quad (D_0 - F_0) + K.(B + BA_0 - A) + J.(D - F) = 0$$

Since, in general, J and K are non-zero, this equation can hold if and only if each of its coefficients is zero. Consequently, the "undetermined coefficients" of the price equation, (30), will be:  $D_0 = F_0$ ;  $D = F$ ; and  $B = A/(1 + A_0)$ . In turn, this implies that  $D_2 = (1 + E_0) > 0$  while  $D_j = C_j$  for  $j \neq 2$ . In short:

$$(35) \quad \text{sign}(B) = (+, +, -, \pm, -, +); \text{sign}(D) = \text{sign}(C) = (+, +, -, +, \pm, +)$$

Given these values for B and D, equations (30) and (32) fully determine equilibrium prices and real GNP. Hence, the impact of past shocks on GNP and, with the exception of  $\bar{m}$ , on prices are exactly those found for current shocks when information was complete. This is because price expectations, expected oil demand and, thereby, oil's current price now respond to observed past shocks in precisely the fashion they used to respond to observed current shocks. Past monetary and demand shocks raise current prices but leave GNP unaltered; past labour-augmenting, oil-augmenting and labour supply shocks have, respectively, positive (negative), negative (positive) and ambiguous impacts on current GNP (prices). Nonetheless, while a past permanent shock to the monetary growth rate,  $\bar{m}$ , still leaves Q

unaltered, it will be more inflationary than such an observed current shock.

Next, consider current shocks. First, by raising aggregate demand, the unobserved demand and monetary shocks,  $e$ ,  $n$  and  $u$ , generate an unanticipated domestic price hike which, in turn, raises real GNP in accord with (16a). Since the price hike is not anticipated by OPEC (or by domestic workers), there will be no change in expected oil demand or, thereby, in the price of oil. Consequently, the rise in GNP is, in the end, the result of a fall both in the real wage and in oil's real price,  $(R-P)$ . Nonetheless, these impacts on  $Q$  are transitory since, other things constant, GNP will fall back to its previous level one period hence. This is confirmed by up-dating expression (32) to find, in the absence of current supply and all future shocks, that  $Q_{+1} = Q_{-1}$  and  $(Q_{+1} - Q) = -\bar{Q}KB < 0$ .

Second, the unobserved supply shocks,  $a$ ,  $b$  and  $d$ , also generate unanticipated domestic price changes; labour-augmenting and labour supply shocks have negative, while oil-augmenting technical shocks have ambiguous, impacts on the price level. This is contrary to the result obtained with complete information where, we saw, an oil-augmenting technical shock raises prices and a labour supply shock has an ambiguous impact. Now, to the extent that unanticipated prices fall, other things constant, real GNP will also fall and vice versa. On the other hand,  $a$ ,  $b$  and  $d$  would tend to raise GNP through their direct impacts on aggregate supply, respectively,  $Q_3$ ,  $Q_4$  and  $Q_5$ . In short, the otherwise expansionary effect of any supply shock may be offset to the extent that it generates a large enough fall in unanticipated prices, according to the relevant net coefficient in expression (32). Nonetheless, it can be verified that  $(Q_4 + \bar{Q}B_4)$  is unequivocally positive so that an oil-augmenting technical shock,  $b$ , raises current GNP. But the impacts of a labour-augmenting technical shock,  $a$ , or a labour supply shock,  $d$ , are ambiguous, a priori. Lastly, it can be verified that a Hicks-neutral technical shock,  $a = b$ , generates an unanticipated price fall which, for the above reasons, leaves real GNP

indeterminate.<sup>18</sup>

The fact that the supply shocks,  $a$ ,  $d$  and  $a = b$ , not only reduce prices but may also reduce real GNP is, ultimately, a reflection of the "recycling problem". Either shock will, we have seen, raise both aggregate supply and OPEC's real GNP,  $Q^0$ . However, the rise in  $Q^0$  will reduce aggregate demand (i.e.,  $E_5 < 0$ ). The rise in supply and fall in demand unequivocally reduce prices but leave real GNP indeterminate, a priori. Obviously, the (absolutely) smaller is  $Q^0$ 's impact on aggregate demand, other things constant, the less likely are these shocks to lower  $Q$ . Of course, as seen in our discussion of aggregate demand, this will be the case the lower is OPEC's marginal propensity to save or, alternatively, the greater is the extent to which OPEC recycles her oil revenues back to the domestic economy through the trade, rather than through the capital, account. For example, take the extreme case where  $E_5 = 0$  which implies OPEC's marginal saving rate is zero. Then, these supply shocks unequivocally raise GNP and lower unanticipated prices since each initially shifts only the aggregate supply curve.<sup>19</sup> Nonetheless, to the extent that money demand depends upon output,  $Y$ , not upon GNP,  $Q$ , the ambiguity in GNP will not disappear with the recycling problem. As seen above, even when OPEC's marginal saving rate is zero,  $E_5 = -E_1 < 0$  because a rise in the real oil bill,  $Q^0$ , raises domestic firms' transactions demand for cash balances. Hence, aggregate demand may still fall sufficiently that, on balance, real GNP falls.

Finally, the unambiguous rise in GNP as the result of an oil-augmenting technical shock and the fall, if any, in GNP as the result of a labour-augmenting or Hicks-neutral technical shock will be transitory. This because the former lagged shock reduces while the latter lagged shocks raise GNP. Consequently, these current impacts will be reversed one period hence, other things constant.

## (b) OPEC as a Quantity-Setter

When OPEC maximizes expected revenues by setting oil's output, the known structure of aggregate supply will be given by (22b) while aggregate demand will still be given by (15). Maintaining all other prior assumptions about the available information, the model can be solved in exactly the manner used for the case where OPEC sets oil's price. In fact, it is apparent that the solution will take the form of expressions (30) and (32) with the structural parameters of (22b),  $\bar{Q}_j$  and  $\bar{Q}_j^0$ , replacing those of (22a),  $Q_j$  and  $Q_j^0$ , in the various coefficient definitions. Nonetheless, given previous discussion and results, it will not require rigorous solution of the model qualitatively to derive the impacts of most shocks: aggregate supply and demand analysis will suffice.

First, we have seen that the structure of aggregate supply is independent of OPEC's choice of a policy instrument when shocks are observed: such structure is given by (20). Consequently, being observed, all lagged shocks will have impacts on current prices and real GNP identical to those derived above for the price-setting case. In turn, these are identical to the impacts of current shocks when information is complete. Second, as before, current demand and monetary shocks,  $e$ ,  $n$  and  $u$ , each raise aggregate demand, (15), with no direct impact on aggregate supply, (22b). Consequently, each must generate an unanticipated price hike thereby altering real GNP in accord with the parameter  $\bar{Q}_2$  in expression (22b). However, as demonstrated earlier,  $\bar{Q}_2$  is positive if and only if the elasticity of substitution between oil and labour in production exceeds oil's share of output (i.e., OPEC's share of world GNP). In short, while still unambiguously raising prices, monetary and demand shocks now have ambiguous impacts, a priori, on real GNP. If, as is often alleged, oil and labour are poor substitutes in production while OPEC's share of world GNP is rising, the likelihood of domestic real GNP falling is

enhanced. Third, the current labour-augmenting and labour supply shocks,  $a$  and  $d$ , have ambiguous impacts on aggregate supply while unequivocally raising OPEC's real GNP,  $Q^0$ , in accord with (22b). With aggregate demand down, through incomplete recycling, and aggregate supply indeterminate, the impact both on prices and on domestic GNP will be ambiguous, *a priori*. Moreover, unlike the price-setting case, this ambiguity will not disappear with the recycling problem; even were the rise in  $Q^0$  not to alter aggregate demand, the ambiguous impact on aggregate supply would leave  $P$  and  $Q$  indeterminate, *a priori*. As seen above,  $d$  raises aggregate supply if and only if the elasticity of substitution between oil and labour exceeds OPEC's share of world GNP while no simple condition exists whereby  $a$ 's impact can be signed. Fourth, an oil-augmenting technical shock,  $b$ , unequivocally raises aggregate supply and lowers aggregate demand, in accord with (22b) and (15). Nonetheless, it can be confirmed that real GNP unambiguously rises while, for the usual reasons, prices fall.<sup>20</sup> Lastly, a Hicks-neutral technical shock,  $a = b$ , raises aggregate supply and OPEC's real GNP which, in turn, reduces aggregate demand. Hence, prices fall and GNP is indeterminate. As we now know,  $Q$  will rise if the recycling problem is not "severe".

Finally, as in the case where OPEC sets oil's price, the impacts of current monetary and demand shocks on GNP, be they positive or negative, will be transitory with  $Q$  returning to its previous level one period hence. This because such lagged shocks have no effect on current GNP. As well, for reasons noted above, when any current supply shock moves GNP (or prices) in the opposite direction to that for the corresponding lagged shock, such impact will be reversed one period hence. For example, a current oil-augmenting technical shock raises current but reduces future GNP, other things constant.

# V. NATURAL GNP, EXPECTATIONS AND OPEC'S POLICY REGIME

A key feature of the above model is that, with OPEC maximizing expected revenues, the domestic economy can not be hit by exogenous oil shocks. Any hike (cut) in oil's price (quantity) is simply the outcome of OPEC's optimal response to domestic shocks. In this section, we briefly consider, without rigour, an oil shock in the form of a change in OPEC's policy regime.

Let information be complete so that OPEC fully-anticipates oil demand. Suppose the members of OPEC are, initially, marginal-cost-pricers with oil's constant marginal cost and, thereby, its price being less than that which maximizes revenues. This implies that oil demand is less than unit price-elastic. It is now 1973 and OPEC organizes as a revenue-maximizing cartel. This constitutes an exogenous shock in which, simply, OPEC raises the elasticity of oil demand to unity by choosing the appropriate hike in oil's price, in accord with (19). Domestic aggregate supply will fall while OPEC's real GNP,  $Q^0$ , will rise, according to (16a). But the rise in  $Q^0$  will reduce aggregate demand, (15). With aggregate supply and demand both lower, domestic GNP unequivocally falls while domestic prices are indeterminate. Consequently, the oil price hike may be deflationary! As seen above, this is less likely, the greater is the extent to which oil revenues are recycled through the trade, rather than through the capital, account. Nonetheless, other things constant, the oil price shock permanently raises oil's real price thereby permanently reducing complete-information (i.e., natural) real GNP.

This impact is, of course, no different than were OPEC arbitrarily to raise oil's price, for whatever reason. But the story does not end here since the shift in OPEC's policy regime alters the structure of the domestic economy; aggregate supply is now given by (20) not by (16a). This has two important implications. First, were the oil price hike arbitrary and, therefore, once-and-for-all, its impact on GNP could be "inflated away" by the appropriate monetary or fiscal expansion. This

because, with  $R$  fixed, the domestic price hike generated by a government-induced expansion of demand would reduce oil's real price,  $R-P$ , thereby permanently restimulating complete-information GNP, in accord with (16a). Nonetheless, since OPEC is now a revenue-maximizer, she will respond to this government-induced inflation by raising  $R$  proportionately, in accord with (19). Consequently, the initial rise in oil's real price and accompanying fall in "natural" GNP will be permanent. As with similarly-constructed closed-economy models, actual and the now-lower natural GNP could be made to differ only temporarily, through unanticipated macro-policies.<sup>21</sup>

Second, suppose domestic agents continue to view the initial and subsequent oil price changes as arbitrary. They, but not OPEC, will then be (rationally?) forming expectations on the basis of the wrong model! This is why, earlier, it was not assumed that OPEC and domestic agents necessarily have identical price expectations; OPEC may have superior knowledge of the domestic economy's structure if only she knows she is now a revenue-maximizer. Moreover, as a practical matter, even were OPEC's policy regime known to all agents, it may take time for both OPEC and domestic agents to learn the new structure since past observations upon which econometric estimates of structure might be based may be useless, having been thrown up by the "wrong model".<sup>22</sup> In short, aside from the above-noted initial impacts, the change in policy regime will have further persistent effects on GNP and prices, other things constant, for as long as ignorance about the economy's structure prevails.

Finally, suppose OPEC shifts to a policy of maximizing (expected) cartel profits. It is well-known that she would then choose a price-output plan where the oil demand curve is price-elastic. Consequently, whether she were initially a marginal-cost-pricer or a revenue-maximizer, OPEC would simply exogenously raise the elasticity of oil demand by jacking up oil's price. This would have a qualitatively similar impact to that noted above for an arbitrary oil-price hike. As well, it

constitutes a structural change which may not, for reasons above, immediately be incorporated into price expectations.<sup>23</sup> While the above model does not tell us precisely what this new structure will be, two things are clear. First, oil's (expected) profit-maximizing price will be endogenous, responding to (expected) shocks to oil demand. Since oil demand depends, ultimately, on oil's real, not nominal, price, R-P, it is apparent that a fully-anticipated domestic price hike will simply lead to a proportionate rise in oil's profit-maximizing price, leaving oil's real price and, thereby, GNP unaltered. Consequently, as before, domestic real GNP will be neutral with respect to anticipated macro policies so that the fall in GNP connected with OPEC's shift to profit-maximization cannot be permanently avoided. Second, unlike the revenue-maximization case, oil's price will also change with shock to oil's marginal production cost. This would, presumably, require explicit consideration of OPEC's indigenous factor markets.

## VI. CONCLUSIONS

We have constructed a stochastic equilibrium model of a large open economy with the distinguishing feature that the supplier of imported oil is a rational cartel with the objective of maximizing (expected) cartel profits. The critical implication of OPEC's rational exploitation of the oil market is that oil's price and quantity will be endogenous to the larger domestic economy. As a direct consequence of such endogeneity, monetary and demand shocks will, at best, have only transitory effects on domestic output, GNP and employment because such shocks will not permanently alter oil's real price. In this regard, the system behaves no differently than similarly-constructed "natural-rate-type" closed economy models. Hence, recognition of the fact that, say the U.S., is open in no way alters the models macro-policy implications. Moreover, while it was analytically convenient to assume revenue-maximization, this result is not sensitive to OPEC's objective which may



equally-well be to maximize cartel profits.

What is sensitive to OPEC's objective is the domestic economy's structure and its "natural" GNP which will be lower the higher is the own-price-elasticity of oil demand implied by the chosen objective. When the oil regime shifts into any form of rational cartel behaviour, macro-policies can prevent actual from moving with natural GNP only if they are unanticipated and/or agents have not yet, for whatever reasons, incorporated the implied structural change into their price expectations. It was argued that the 1973 oil price shock might be viewed as such a change in regime. Moreover, subsequent events suggest that OPEC, quite rationally, has been unwilling to permit the favourable turn in her terms-of-trade to be eroded by, say, U.S. inflation.

In assessing the impact of a supply shock, we saw that its source is critical. Most important, with OPEC maximizing revenues, micro-policies designed to reduce reliance on OPEC by stimulating oil-saving technical change will, at best, only temporarily raise GNP and reduce prices. Once OPEC's expectations have adjusted, such technical change will permanently lower GNP and raise prices. The model suggests the authorities ought, from the point of view of macro goals, to encourage labour-using technologies which, of course, may increase oil use.

Finally, with incomplete information, the model's policy implications are quite sensitive to OPEC's choice of a policy instrument. For example, when OPEC is a quantity-setter, monetary and demand shocks will be inflationary but may reduce, temporarily, real GNP by the intrusion of the "recycling problem". This "perverse" effect is more likely the greater is the extent to which OPEC's oil revenues are recycled through the capital, rather than through the trade, account. Nonetheless, with complete information, neither the manner in which oil revenues are recycled nor OPEC's choice of a policy tool will qualitatively change the model's predictions.

### FOOTNOTES

1. For examples, see: Bruno & Sachs (1979); Djajic (1980); Dornbusch (1979); Findlay & Rodriguez (1977); Harkness (1980); and Schmid (1976;1979).
2. That is, workers must commit themselves to a wage contract at period's start before the price level and, therefore, the real-consumption-wage are known.
3. This linearity has no fundamental impact on the model but is imposed to simplify later solution to OPEC's revenue-maximization problem. Also note that, for simplicity, wealth and inter-temporal substitution between leisure and future consumption have been ignored in the household choice problem.
4. Constant returns technology implies  $Y_{12} > 0$  in which case a rise in one input's price necessarily reduces the profit-maximizing use of both inputs.
5. Since we are concerned with an equilibrium model, we do not permit OPEC arbitrarily to set both the price and the quantity of oil.
6. As a revenue-maximizer, OPEC has no reason to invest unless she does not have the capacity to produce oil's revenue-maximizing output. Assume she does.
7. The consumption function implicit in  $D(\cdot)$  can be shown to be consistent with the labour supply function, (8), when utility is appropriately separable in leisure, current consumption and future consumption. Of course, the export function derives from OPEC's optimal allocation of her known nominal income,  $RH$ , between present and future consumption of manufactures. Again, for simplicity, wealth effects on consumption and exports are ignored.
8. Since she consumes only manufactures, OPEC may have a transactions demand for domestic currency providing an additional reason why, in the absence of distributional effects, money demand ought to depend on  $Y$ , not on  $Q$ .
9. This is not to imply that the primitive functions, such as technology or labour supply, are log-linear for this would imply a constant own-price-elasticity of oil demand and no (interior) solution to OPEC's revenue-maximization problem.

10. Of course, the coefficients in (16) and (17) are now constants evaluated at the point around which the system is log-linearized. For most later purposes, this will be where oil demand is unit-price-elastic.
11. This conforms to real-world practice and, happily, eliminates the need explicitly to consider the exchange rate.
12. This strategy also maximizes expected real revenue,  $Q^0$ , which is identical to maximizing the expected utility of real revenue when OPEC is risk-neutral.
13. We treat OPEC's choice of a policy tool as arbitrary. However, with stochastic oil demand, there may be an optimal instrument if OPEC is not risk-neutral.
14. Such method is detailed by Lucas (1972) and by Barro (1976).
15. Were agents unable to distinguish between permanent and transitory monetary shocks, each would have the same initial "inflationary" impact.
16. That is, only if  $G_3 = -g_3/g_1 > 0$  since, by (19),  $dR/da = G_3 + C_3$  where  $C_3 < 0$ .
17. For alternate views, see Lucas (1976), Burton (1980) and Harkness (1980).
18. Qualitatively, the typical supply shock's impact on GNP will be:  $\text{sign}(Q_j + \bar{Q}B_j) = \text{sign}(E_1 + E_2)Q_j - E_5(Q_j\bar{Q}^0 - \bar{Q}Q_j^0)$ , for  $j = 3, 4, 5$  or shocks, respectively, a, b, d. This is positive for b but ambiguous for a and d since:  $Q_3^0, Q_5^0 \geq 0$ ;  $E_5, Q_4^0 < 0$ ; and  $E_1, E_2, \bar{Q}, \bar{Q}^0, Q_3, Q_4, Q_5 > 0$ . All three effects are positive when  $E_5 = 0$ .
19. See footnote 18.
20. In footnote 19, replace  $(Q_j, Q_j^0)$  with  $(\bar{Q}_j, \bar{Q}_j^0)$  to obtain the effects of supply shocks when OPEC is a quantity-setter.
21. In the absence of supply-side effects to macro-policies, such as taxes.
22. This is strictly analagous to Lucas's (1976) argument on the structural effects of a shift in the government's macro-policy regime.
23. Note that the sign of many parameters in the basic GNP equations, (11a) and (11b), depend on oil's demand-elasticity being greater than, less than or equal to unity. Hence, the oil regime's effect on structure may depend most critically upon its implication for the value of  $\eta$ .

## APPENDIX

Define the following positive magnitudes: labour's share =  $k = wN/Y = Q/Y$ ; wage-elasticity of labour supply =  $\epsilon = WN_1^S/N$ ; elasticity of substitution between labour and oil =  $\sigma = -y'w/hyy''$ ; and own-price-elasticity of oil demand =  $\eta$ . By prior assumptions,  $k$  and  $\sigma$  are each less than unity while, by "largeness" of the domestic economy we assume  $k > 1/2$ . Then, solution to equations (1) through (9) will produce the following parameters where, initially,  $\alpha = \beta = \delta = 1$ .

(i) Oil Demand, (10)

$$\eta = \{(1-k)\epsilon + \sigma\}/k > 0; \quad H_1 = -\eta H/\tau < 0; \quad H_2 = \epsilon H/\theta > 0; \quad H_3 = H\{k(1+\epsilon) + \sigma\}/k > 0; \\ H_5 = hN_2^S > 0; \quad H_4 = H\{k(\eta-1) - \sigma\}/k = ?; \quad H_3 + H_4 > 0; \quad \text{and } H_4 < 0, \text{ when } \eta \leq 1$$

(ii) Output and Real GNPs, (11a)

$$Q_1 = -H(1+\epsilon) > 0; \quad Q_2 = \epsilon Q/\theta > 0; \quad Q_3 = Q(1+\epsilon) > 0; \quad Q_4 = Q^0(1+\epsilon) > 0; \quad Q_5 = wN_2^S > 0 \\ Q_1^0 = H(1-\eta) = ?; \quad Q_2^0 = \epsilon Q^0/\theta > 0; \quad Q_3^0 = \tau H_3 > 0; \quad Q_4^0 = \tau H_4 = ?; \quad Q_5^0 = \tau H_5 > 0 \\ Q_3 + Q_4 > 0; \quad Q_3^0 + Q_4^0 > 0; \quad Y_j = Q_j + Q_j^0, \text{ for all } j; \quad Y_1 < 0; \quad Y_j > 0, \text{ all other } j$$

(iii) Output and Real GNPs, (11b)

$$\bar{Q}_1 = \tau(\sigma + \epsilon)/\eta > 0; \quad \bar{Q}_2 = \epsilon Y(\sigma + k - 1)/\eta\theta = ?; \quad \bar{Q}_3 = (1+\epsilon)Y\{(2k-1)\sigma - k(1-k)\}/\eta k = ?; \\ \bar{Q}_4 = (1-k)(\sigma + k)(1+\epsilon)Y/\eta k > 0; \quad \bar{Q}_5 = yN_2^S(\sigma + k - 1)/\eta = ?; \quad \bar{Q}_3 + \bar{Q}_4 > 0 \\ \bar{Q}_1^0 = \tau(\eta - 1) = ?; \quad \bar{Q}_2^0 = (1-k)\epsilon Y/\eta\theta > 0; \quad \bar{Q}_3^0 = Y\{(1-k)(1+\epsilon) + (2k-1)(\eta + 1 - 2\sigma)\}/\eta k = ? \\ \bar{Q}_4^0 = (1-k)Y\{(1-k)\epsilon - k\}/\eta k = ?; \quad \bar{Q}_5^0 = (1-k)yN_2^S/\eta > 0; \quad \bar{Q}_3^0 + \bar{Q}_4^0 > 0 \\ \bar{Y}_j = \bar{Q}_j + \bar{Q}_j^0 > 0. \text{ As well, if } \eta = 1, \bar{Q}_1^0 = 0, \bar{Q}_3^0 > 0 \text{ and } \bar{Q}_4^0 < 0; \text{ and } \bar{Q}_2, \bar{Q}_5 \geq 0 \text{ as } \\ \sigma \geq (1-k) = Q^0/Y. \text{ Finally, the smaller is } \sigma \text{ the more likely is } \bar{Q}_3 \text{ to be negative.} \\ \text{In fact, } \bar{Q}_3 > 0 \text{ only if } \sigma \gg (1-k).$$

## (iv) Elasticity of Oil Demand, (13a) and (13b)

From (i) above,  $\eta = \{(1-k)\epsilon + \sigma\}/k > 0$ . Given their definitions,  $k$ ,  $\epsilon$  and, thereby,  $\eta$  can be solved for in terms of the state variables using equations (1) through (9) and the fact that technology is CES. First, however, it is well-known that the first- and second-order conditions for (oil) revenue-maximization are, respectively,  $\eta = 1$  and  $d\eta/dR > 0$ . From the definition of  $\eta$ , this implies that:  $(1-\sigma) = (1-k)(1+\epsilon) > 0$  and  $d\eta/d\tau = g_1 > 0$ . Hence, OPEC can be a revenue-maximizer only if  $\sigma < 1$  and  $\eta$  rises with  $\tau$ . It can now be shown that:

$$g_1 = (1-k)\phi/\tau k^2 > 0, \text{ where } \phi = (1-k)\epsilon(\epsilon-1) + k(1-\sigma)(\eta+\epsilon) > 0, \text{ by the 2}^0 \text{ condition}$$

$$g_2 = (1-k)\epsilon(1-\epsilon)/\theta k \geq 0 \text{ as } \epsilon \leq 1; \quad g_4 = (k-1)\{\phi + k\sigma(\eta+\epsilon)\}/k^2 < 0; \quad g_5 = (k-1)\epsilon N_2^S/kN < 0;$$

$$g_3 = (1-k)\{\epsilon(1-\epsilon) + (\eta+\epsilon)\sigma\}/k \geq 0; \quad (g_3+g_4) = (1-k)\{\epsilon(1-\epsilon) + (\eta+\epsilon)k(\sigma-1)\}/k^2 \geq 0$$

As well,  $g_3 < 0$  and  $(g_3+g_4) > 0$ , only if  $\epsilon \gg 1$ .

$$f_1 = -\tau g_1/\eta H < 0; \quad f_2 = \epsilon(1-k)\{\sigma(1-\epsilon) + (\eta+\epsilon)k(1-\sigma)\}/\eta k^2 \theta \geq 0 \text{ but } f_2 > 0 \text{ if } \eta \geq 1.$$

$$f_4 = -\{(k+\sigma)k^2 g_1 + (1-k)(\eta+\epsilon)(1-\sigma^2) + (1-k)^2(\eta+\epsilon)\epsilon\}/\eta k^3 < 0;$$

$$(f_3+f_4) = \theta f_2 > 0 \text{ if } \eta \geq 1; \quad f_3 = (f_3+f_4) - f_4 > 0 \text{ if } \eta \geq 1$$

## (v) Complete Information Aggregate Supply, (20)

The typical parameter of (20) is given as:  $(\hat{Q}_j, \hat{Q}_j^0) = (G_j Q_1 + Q_j, G_j Q_1^0 + Q_j^0)$  where  $G_j = -g_j/g_1$  and each parameter is evaluated at  $\eta = 1$ . Consequently,  $\hat{Q}_1^0 = 0$  and, therefore,  $\hat{Q}_j^0 = Q_j^0$  as defined in (ii), above. Finally, the appropriate manipulations reveal that:

$$\hat{Q}_1 = -Q_1(1+g_1^{-1}) < 0; \quad \hat{Q}_2 = H(1-k)\epsilon\{(2k-1)\epsilon + 2k + \epsilon^2\}/kg_1 > 0; \quad \hat{Q}_4 = -\epsilon H(1-k)(1+\epsilon)^2/g_1 k^2 < 0;$$

$$\hat{Q}_5 = H(1-k)\{-2\epsilon + k(1+\epsilon)^2\}/kN \geq 0 \text{ with } Q_5 < 0 \text{ only if } 1 > \epsilon > 0$$

$$\hat{Q}_3 = Q(1+\epsilon)(1-k)^2\{(2k-1)\epsilon + k^2(1+\epsilon)^2 + (1-k)\epsilon^2\}/g_1 \tau k^2 > 0; \quad \hat{Q}_3 + \hat{Q}_4 \geq 0$$

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