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INSTITUTE FOR ECONOMIC RESEARCH

QUEEN'S UNIVERSITY

Kingston, Ontario, Canada K7L 3N6

A Note on Multiproduct Economies of Scale and

Economies of Scope*

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Jack M. Mintz
Department of Economics
Queen's University
Kingston, Ontario, K7L 3N6
Canada

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Abstract

Let S denote a measure of (local) economies of scale for a multiproduct firm. Define "strong" economies of scope as the savings in costs arising from the joint production of all products rather than producing each product separately. Define "incremental" economies of scope as the savings in costs arising from the joint production of all products rather than producing one of the products separately from the others. We show that these two concepts of economies of scope along with a measure of product-specific economies of scale, are sufficient to define S for the general case. For economies of scale it is important for "incremental" economies of scope to dominate "strong" economies of scope. To demonstrate this point, we provide an example in which diseconomies of scale may exist for the multiproduct firm despite the presence of both "strong" and "incremental" economies of scope and constant product specific returns to scale for each product.

1. Introduction

In recent years, there has evolved considerable interest in studying the cost and technological structures of multiproduct firms. Willig's article (1979) provides a useful survey of various concepts underlying the existence of economies of scale in multiproduct firms. For the two product firm, he shows that a sufficient (but not necessary) condition for economies of scale to exist is for there to be (i) no product-specific diseconomies of scale (average costs fall or remain constant as one of the outputs is increased) for each of the outputs, and (ii) economies of scope (costs are less with joint production compared to independent production of the products).

The purpose of this note is to extend Willig's two-product economies of scale measure to a general case of J products. We show that the economies of scale measure can be expressed in terms of local measures of product-specific economies of scale and two types of economies of scope. The first is "strong" economies of scope which is the reduction in costs arising from the joint production of all products rather than producing each product in separate firms. The second is "incremental" economies of scope which is the reduction in costs arising from the joint production of products rather than producing one of the products separately from the others. In the two product case, "incremental" and "strong" economies of scope are equivalent. It is possible for "strong" and not "incremental" economies of scope to hold for a particular firm. In many industries, some firms may find it cheaper to produce some but not all of the products together because, for example, managers do not have the ability to handle certain product lines well. "Incremental" economies of scope allows for a measure of the reduction in cost realized in adding on one additional product line.

Once defining the above two concepts of economies of scope, we consider conditions under which (local) economies of scale will hold for a three or more product firm. We will show that if both "strong" and "incremental" economies of scope along with no product-specific diseconomies for each product line hold, that it may <u>not</u> be true that there are (local) economies of scale. Indeed it will be sufficient for economies of scale to hold if "incremental" economies of scope dominate "strong" economies of scope (with no product-specific diseconomies of scale for all outputs). In other words, economies of scale can hold if producing all products is cheaper than producing some but not all products separately.

2. Economies of Scope and Product-Specific Economies of Scale for the General Case

In Willig (1979) the multiproduct economies of scale, S, expressed in terms of product specific economies of scale (S_j) and economies of scope S_C was derived for the two product case as follows:

$$(1) \qquad S = \frac{\alpha_1 S_1 + \alpha_2 S_2}{1 - S_C}$$

where

$$(2) \qquad S = \frac{C(q)}{\sum_j \frac{\partial C}{\partial q_j} q_j} \qquad \text{(multiproduct economies of scale)}$$

$$(3) \qquad S_j = \frac{C(q_j, q_{i \neq j}) - C(0, q_{i \neq j})}{\frac{\partial C}{\partial q_j} q_j} \qquad \text{(average incremental cost divided by marginal cost)}$$

$$(4) \qquad S_C = \frac{C(0, q_2) + C(q_1, 0) - C(q_1, q_2)}{C(q_1, q_2)} \qquad \text{(economies of scope)}$$

(5) $$\alpha_j = \frac{\frac{\partial C}{\partial q_j} q_j}{\sum_j \frac{\partial C}{\partial q_j} q_j}$$

C = total costs

q_j = output of jth product, $j=1,2$

The above formula for the two product case is not easily generalized unless one allows for several definitions of economies of scope.[1] Define

a) Strong Economies of Scope:

(6) $$S_S = \frac{\sum_j C(q_j^*) - C(q)}{C(q)}$$

where q is a Jx1 vector of outputs of $j=1,\ldots,J$ products and q_j^* is the vector $\{0,0,\ldots,0,q_j,0,\ldots,0\}$

and

b) Incremental Economies of Scope:

(7) $$S_I^j = \frac{C(q_j^*) + C(q_j^0) - C(q)}{C(q)}$$

where q_j^0 is a vector where all outputs except the jth output is produced $\{q_1,\ldots,q_{j-1},0,q_{j+1},\ldots,q_J\}$.

We can express multiproduct economies of scale in terms of the expressions stated above:

(8) $$S = \frac{C(q)}{\sum_j \frac{\partial C}{\partial q_j} q_j}$$

Adding and subtracting $\sum_j C(q_j^*)$ in the numerator, we obtain

$$(9)\qquad S = \frac{C(q)-\sum_j C(q_j^*)+\sum_j C(q_j^*)}{\sum_j \frac{\partial C}{\partial q_j} q_j}$$

Using (6) and (8) and substituting into (9) yields

$$(10)\qquad S = -S_S S + \frac{\sum_j C(q_j^*)}{\sum_j \frac{\partial C}{\partial q_j} q_j}$$

Adding and subtracting $\sum_j C(q_j^0)-C(q)$, we obtain

$$(11)\qquad S = -S_S S + \frac{\sum_j [C(q_j^*)+C(q_j^0)-C(q)]-\sum_j [C(q_j^0)-C(q)]}{\sum_j \frac{\partial C}{\partial q_j} q_j}$$

Define $S_j=(C(q)-C(q_j^0))/\frac{\partial C}{\partial q_j} q_j$ which is the J good variant of (3). Substituting (5) and (7) into (11) and using the definition of S_j we obtain

$$S = -S_S S + \sum_j \alpha_j S_j + S \sum_j S_I^j$$

which implies

$$(12)\qquad S = \frac{\sum_j \alpha_j S_j}{(1-\sum_j S_I^j + S_S)}$$

In the two product case $S_S=S_I^j=S_C$ and hence, $S = \frac{\sum_j \alpha_j S_j}{1-S_C}$. With more than two products economies of scale depends additionally upon the relationship between "incremental" and "strong" economies of scope. For example, "strong" economies of scope may exist ($S_S>0$) but "incremental" economies may not hold for some multiproduct firm technologies ($S_I^j<0$). This may arise from the possibility that it is cheaper to produce some but not all products together. After all, in many industries, we witness a large array of multi-product firms some of which provide a whole range of products while others specialize in producing a smaller set of products. If some firms attempt to add on new product lines costs may rise (i.e.: $S_I^j<0$) because either management, labour or the kind of capital equipment used may not facilitate cheaper cost production compared to establishing a new firm or plant.

3. An Example

Below we provide an example in which there are no local product-specific economies of scale but there are both local "strong" and "incremental" economies of scope. The technology is such that it can be cheaper to produce all products other than only some products together at a small scale but for larger output levels it may be cheaper to produce only some products together (i.e.: there are no global "incremental" economies of scope).

Consider the following cost function

$$(12) \qquad C = q_1+q_2+q_3 - \frac{1}{3} q_1q_2 - \frac{1}{3} q_2q_3 - \frac{1}{3} q_1q_3 + aq_1q_2q_3$$

which is evaluated at $q_j=0$ or 1.[2] It can be seen that with independent production of each output $C(0,0,1) + C(0,1,0) + C(1,0,0) = 3$ in (13). If any two of the products are produced together and the third separately then $C = 2\frac{2}{3}$ for each combination of outputs. If all products are produced together, $C(1,1,1) = 2+a$.

The parameter, a, can be varied to take into account various cases. If $a<1$, then there are local "strong" economies of scope ($S_S>0$). If $a < \frac{2}{3}$ then local "incremental" economies of scope holds as well ($S_I^j>0$) in that it is cheaper to produce all outputs together rather than one of the outputs separately.

Solving for marginal cost and evaluating at $q_1=q_2=q_3=1$ $(\frac{\partial C}{\partial q_j} = \frac{1}{3} + a, \forall j)$, we obtain

$$S = \frac{C(1,1,1)}{\sum_i \frac{\partial C(1,1,1)}{\partial q_j}} = \frac{2+a}{1+3a}$$

For local multiproduct economies of scale (S>1), we require $a < \frac{1}{2}$. This suggests that for $\frac{1}{2} < a < \frac{2}{3}$ local diseconomies of scale could hold despite both local "incremental" and "strong" economies of scope and constant product-specific economies of scale holding.[3] When a is high enough, small increases in output will quickly result in "incremental" diseconomies of scope. For example if $q_1=q_2=q_3=2$ and $a = \frac{5}{8}$ $C(2,2,2) = 7$ and $C(2,2,0) + C(0,0,2) = 4\frac{2}{3}$. Thus it is important for "incremental" economies of scope to dominate "strong" economies of scope for multiproduct economies of scale.

Footnotes

1. These two definitions of economies of scope are two special cases of a large number of possible partition of products that may be used to define economies of scope (see Panzar and Willig (1979)).

2. When $q_1=q_2=q_3=1$, product-specific economies of scale measures, S_j, are equal to one and marginal costs are positive when a > -.5. For larger output levels other restrictions are needed to ensure positive marginal and total costs.

3. To consider cost complementarity as defined by Panzar and Willig (1979), we find $\frac{\partial^2 C}{\partial q_i q_j} = -\frac{1}{3} + a$ at $q_1=q_2=q_3=1$ and $\forall\ i \neq j$, j. If $a > \frac{1}{3}$, $\frac{\partial^2 C}{\partial q_i \partial q_j} > 0$ (which would be true in our example). Panzar and Willig show, however, that $\frac{\partial^2 C}{\partial q_i \partial q_j} < 0$ is a sufficient but not a necessary condition for economies of scope.

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