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INTEREST RATES AND DURABILITY IN
THE LINEAR EXPENDITURE FAMILY

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ABSTRACT

In this paper we demonstrate that the extended linear expenditure system with durables (DELES) satisfies the first-order conditions for local unidentifiability, and discuss alternative methods that might be used to achieve identifiability, as well as their implications. Estimates are then presented of the linear expenditure system (LES), the extended linear expenditure system (ELES) and DELES, under each of two alternative error specifications, using seasonally adjusted, quarterly Australian data from 1959(4) to 1976(2). The two main purposes of this exercise are: (i) to compare the effects of allowing for first-order autocorrelation in LES and ELES; and (ii) to gauge the contribution of DELES to the linear expenditure family by making corresponding comparisons with ELES. When we fix the decay rate for durables *a priori*, in an attempt to secure identifiability, the estimates of the remaining parameters are insensitive to the value chosen. This suggests a major difficulty in interpreting DELES and leads us to reconsider its limiting properties.

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- Gordon Fisher, Michael McAleer and Diana Whistler -

1. INTRODUCTION

The classical literature on complete systems of expenditure functions proceeds with the convenient fiction of a two-stage consumer maximization problem, in which no attempt is made to analyze the intertemporal consumption-savings choice. Since the saving decision is assumed to be predetermined, the allocation of total expenditure among different commodities is regarded as an atemporal utility maximization problem. When the utility function of Klein and Rubin (1947-48) is used, the resulting system of demand equations is given by the linear expenditure system (LES) of Stone (1954). Although Tintner (1938a, 1938b) considered the general theoretical problem of unifying the intertemporal and atemporal decision-making processes, the initial *operational* treatment of the trade-off between current and future consumption was not made until Lluch (1973), whose specialization of Tintner's general statement resulted in the extended linear expenditure system (ELES).

Savings and total consumption are made endogenous within ELES. The price and income streams, together with their expectations, are exogenous to the system. The expenditure equations are the initial solutions to a continuous time control problem in which the optimal paths for individual commodities are determined. It is assumed that there is continual replanning so that ELES may be regarded as the first-period solutions of continually revised optimal plans of indefinite length, when the instantaneous utility function is of the Klein-Rubin type.

In spite of its relative sophistication, however, even ELES does not

treat durable goods explicitly in the complete systems approach to applied demand theory. As is common with other studies of demand, no distinction is made between demand for stocks of durable goods, purchases of new durable goods and demand for the services from stocks of durables. Usually no theoretical distinction is made in explaining expenditures on durables and non-durables in the systems approach. However, ELES has been generalized by Dixon and Lluch (1977) to DELES, the extended linear expenditure system with durable goods, in which the consumer problem is formulated in an intertemporal setting with durability explicitly taken into account. In practice this means that DELES comprises more parameters than ELES and includes an extra 'shift-of-plan' term. Nevertheless, the two systems are the same in terms of data requirements, so the richer theory permits a richer interpretation of the data, but only by placing increased demands upon them; moreover, DELES is a much more cumbersome system to estimate. As far as the former is concerned, it is an empirical question whether the data are rich enough to tolerate the increased demands; in respect of the latter, it is a matter of judgement whether the greater computational burden is worth a richer interpretation of the data.

These questions may be answered by appeal to practical experience. In this paper we provide estimates of LES, ELES and DELES, under each of two alternative error specifications, using seasonally adjusted, quarterly Australian data from 1959(4) to 1976(2). The two main purposes of this exercise are: (i) to compare the effects of allowing for first-order autocorrelation in LES and ELES; and (ii) to gauge the contribution of DELES to the linear expenditure family by making corresponding comparisons with ELES. In respect of (i), we are led to reconsider the conclusion of Lluch and Williams (1975) who, using annual U.S. data, found LES sensitive and ELES insensitive to the type of error specification, using grid-search methods and weighted differencing (where appropriate).

In respect of (ii), we provide an empirical appraisal of the theoretical model developed by Dixon and Lluch.

On a theoretical level, it is demonstrated that DELES, as originally presented by Dixon and Lluch, is not identifiable and thus *a priori* parameter restrictions are required for estimation. One possibility is to incorporate prior information on the decay rate for durable goods. It turns out that when we fix the decay rate *a priori*, the estimates of the remaining parameters are insensitive to the value chosen. This suggests a major difficulty in interpreting DELES and leads us to question the numerical identification of the subjective time-preference and market interest rate parameters. On the basis of this, we are led in turn to reconsider the limiting properties of DELES.

2. IDENTIFIABILITY IN THE LINEAR EXPENDITURE FAMILY

With a few exceptions we shall use the notation of Dixon and Lluch (1977).

In deterministic form, the three models may be written as:

$$(1) \quad v_{it} = \gamma_i p_{it} + \beta_i (m_t - \sum_i \gamma_i p_{it}) \quad (\text{LES})$$

$$(2) \quad v_{it} = \gamma_i p_{it} + \mu \beta_i (z_t - \sum_i \gamma_i p_{it}) \quad (\text{ELES})$$

$$\left. \begin{aligned} (3.1) \quad v_{it} &= \gamma_i p_{it} + \mu \beta_i^{**} z_t^{**} \\ (3.2) \quad v_{jt} &= \theta_j \phi_j \pi_{jt} + \mu \eta_j^{**} z_t^{**} - \mu \eta_j^{**} \left\{ \frac{\pi_{jt}}{\pi_{jt}} z_t^{**} - z_t^{**} \right\} \end{aligned} \right\} \quad (\text{DELES})$$

in which the subscripts i and j refer to the i 'th non-durable and the j 'th durable good, respectively. Per capita expenditures on commodities are represented by v_{it} and v_{jt} , prices by p_{it} and π_{jt} , and per capital total expenditure and permanent income by $m_t = \sum_i v_{it}$ and z_t , respectively. The utility function

parameters for goods i and j are β_i and η_j , with $\sum_i \beta_i + \sum_j \eta_j = 1$, while the corresponding subsistence parameters are γ_i and ϕ_j . The decay rate for durable j is denoted by θ_j . Supernumerary permanent income in DELES is given by

$$(4) \quad z_t^* = z_t - \sum_i \gamma_i p_{it} - \sum_j \theta_j \phi_j \pi_{jt}$$

and the marginal propensity to consume (MPC) out of permanent income in both ELES and DELES is $\mu = \delta/\rho$, the ratio of the subjective time-preference discount rate (δ) to the market interest rate (ρ). If we let $\alpha = \sum_j \{\eta_j/(\theta_j + \rho)\}$, then $\beta_i^* = \beta_i/(1-\delta\alpha)$ is the marginal budget share (MBS) of non-durable i ; the MBS of durable j is $\eta_j^{**} = \eta_j^*(\theta_j + \rho - \delta)$, where $\eta_j^* = \eta_j/\{(\theta_j + \rho)(1-\delta\alpha)\}$ and $\mu\eta_j^*$ is the shift-parameter for durable j . The subscript t denotes a sample realization and a dot above a variable denotes its time derivative.

The specialization of DELES to ELES occurs in three steps. First, replace π_{jt} in equations (3.2) and (4) by $p_{jt}/(\theta_j + \rho)$, where p_{jt} is the price of a unit of *service* from durable j . Second, let all durables become non-durables by the device of requiring $\theta_j \rightarrow \infty$ for all j , when ρ and δ , and hence μ , are held fixed. In these circumstances, given values of η_j ensure $\alpha \rightarrow 0$ and hence $\eta_j^* \rightarrow 0$ and $\mu\eta_j^* \rightarrow 0$; similarly, given β_i , $\beta_i^* \rightarrow \beta_i$; the ratio $\{(\theta_j + \rho - \delta)/(\theta_j + \rho)\} \rightarrow 1$ and hence $\eta_j^{**} \rightarrow \eta_j$. Finally, redefine η_j and θ_j as β_j and γ_j , respectively. Durables in ELES are still included in the analysis, but they are treated as if they were non-durables.

The restrictions on each of the θ_j in DELES that lead to ELES cause δ and ρ to appear only as the ratio μ . Clearly, neither δ nor ρ is identifiable in ELES. To analyze the identifiability of DELES, equations (3.1) and (3.2) may be rewritten as:

$$(3.1') \quad v_{it} = \gamma_i p_{it} + a_i^* z_t^*$$

$$(3.2') \quad v_{jt} = b_j \pi_{jt} + c_j d_j z_t^* - c_j \left\{ \frac{\dot{\pi}_{jt}}{\pi_{jt}} z_t^* - \dot{z}_t^* \right\}$$

where

$$(3.3) \quad \begin{cases} a_i = \delta \beta_i^* / \rho \\ b_j = \theta_j \phi_j \\ c_j = \delta \eta_j^* / \rho \\ d_j = \theta_j + \rho - \delta. \end{cases}$$

Let there be m durable and n non-durable goods. Since β_i^* and η_j^* are functions of β_i , η_j , θ_j , δ and ρ , and since also $\sum_i \beta_i + \sum_j \eta_j = 1$, (3.3) represents a system of $(n + 3m)$ equations in $q = (n + 3m + 1)$ 'free' parameters. Thus if ξ represents the $q \times 1$ vector of these 'free' parameters, comprising $(n + m - 1)$ 'free' elements β_i and η_j , together with the $(2m + 2)$ elements θ_j , ϕ_j , δ and ρ , then (3.3) may be written as the system of $(q - 1)$ equations in q unknowns

$$(3.3') \quad E(\xi) = 0.$$

A necessary condition for local unidentifiability (F.M. Fisher, 1966; Rothenberg, 1971) is that the Jacobian matrix $\partial E / \partial \xi'$ be of rank less than q . This condition clearly holds for (3.3) since the number of equations $(q - 1)$ is less than the number of parameters (q) . While the rank condition for local unidentifiability is necessary, it is not alone sufficient. If we were to introduce another restriction on any of the elements of ξ , the order of $\partial E / \partial \xi'$ would become $(q - 1) \times (q - 1)$, and it may then be that the necessary condition for local unidentifiability would be violated. It then becomes an empirical question whether there is sufficient information for local identifiability. The larger the number of restrictions that is introduced, the more likely it will be that local identifiability is achievable. Where *a priori* information is

available, for example, on decay rates for durables, θ_j may be fixed at θ_j^* . This will generally facilitate local identifiability of the remaining parameters, including δ and ρ . However, it may happen, in view of lack of sufficiency, that the remaining parameters will not be locally identifiable. Some calculations we have done, which are not reported here, 'indicate' that this is not the case, since we can indeed generate 'precise' estimates.

3. FRAMEWORK FOR ESTIMATION

In stochastic form, any one of the equations (1), (2), (3.1) or (3.2) may be written as

$$(5) \quad v_t = f(x_t; \Psi) + u_t \quad (t = 1, 2, \dots, T)$$

where v_t , $f(\cdot)$ and u_t are $n \times 1$ vectors, n now being the number of equations. The function $f(\cdot)$ is linear in the vector-valued variable x_t and non-linear in the vector-valued parameter Ψ . A first-order autoregressive process is presumed to generate u_t , so that

$$(6) \quad u_t = \lambda u_{t-1} + \varepsilon_t$$

for all t , λ being the common coefficient for all equations in the system,¹ where ε_t is distributed independently over t according as $N(0, \Omega)$, Ω being the constant contemporaneous covariance matrix. Equation (6) comprises no cross-equation autocorrelation, and is the simplest possible vector autoregressive process when $|\lambda| < 1$; it also represents the serially independent case if $\lambda = 0$. Thus $-1 < \lambda < 1$ defines LES and ELES according to our two error specifications, although Lluch and Williams (1975) searched over the closed interval $[0, 1]$.

Using the above notation, the concentrated log-likelihood function used for estimating the parameters Ψ and λ may be written (up to a constant) as

$$(7) \quad L(\Psi, \lambda) = (n/2) \log(1-\lambda^2) - (T/2) \log \det[(1-\lambda^2)(v_1-f_1)(v_1-f_1)' + \sum_{t=2}^T \{(v_t-f_t) - \lambda(v_{t-1}-f_{t-1})\}\{(v_t-f_t) - \lambda(v_{t-1}-f_{t-1})\}']$$

where $f_t \equiv f(x_t; \Psi)$ for all t . All available observations are used in equation (7). As $|\lambda| \rightarrow 1$, $\log(1-\lambda^2) \rightarrow -\infty$, thereby restricting the error process to be stationary (see *e.g.* Beach and MacKinnon, 1979). When λ is set equal to zero, equation (7) becomes

$$(8) \quad L(\Psi) = -(T/2) \log \det[\sum_{t=1}^T \{v_t - f(x_t; \Psi)\}\{v_t - f(x_t; \Psi)\}'].$$

Full-information maximum-likelihood (FIML) estimates are obtained by locating a global maximum of the appropriate likelihood function with respect to the parameters. A maximum was determined by numerical optimization using a variable metric algorithm available in the computer package GQOPT (see Goldfeld and Quandt, 1972). Numerical derivatives were used since it was felt that calculating derivatives analytically for systems as complicated as DELES was more than counterbalanced by the computational advantages. Estimates of the asymptotic covariance matrix of estimated parameters were calculated from the Hessian of the log-likelihood function. The estimated asymptotic covariance matrix for functions of parameters (such as the MBS's in DELES) was obtained from Taylor-series linearizations. As the disturbances in LES are subject to an exact linear constraint, the covariance matrix is singular and the solution involves dropping one equation from the system. All estimates in the Aitken family are known to be invariant to the choice of equation deleted (see *e.g.* Barten, 1969). Since a feature of numerical optimization is that the algorithm may terminate at a local maximum, experimentation with several starting values was conducted so as to make the location of a global maximum more likely.

4. EMPIRICAL RESULTS

The three models were estimated using seasonally adjusted, quarterly Australian data, 1959(4) - 1976(2), for four commodity aggregates: Food, Rent, Other Non-durables and Durables. Thus, in terms of earlier notation, $i = 1, 2, 3$ and $j = 1$. Data sources and commodity definitions are given in McAleer *et al* (1980). In particular, it is assumed that the present value of expected future changes in labour income is zero for both ELES and DELES, so that measured (*i.e.* disposable) and permanent income are the same. This is not unreasonable in view of Powell (1973), Williams and Chang (1973) and Lluch *et al* (1977). First differences replace time derivatives (*viz.* $\dot{\pi}_t$ and $\dot{z}_t^*$) for estimation. Mean per capita expenditures and average budget shares of the four commodities are given in Table 1 for purposes of comparison.

Table 1: Mean Per Capita Expenditures and Average Budget Shares for Four Commodity Aggregates, 1959(4) - 1976(2), in current Australian dollars

Commodity	Mean Per Capita Expenditure	Average Budget Share
Food	72.72	0.199
Rent	46.33	0.127
Other	198.88	0.544
Durables	47.57	0.130
Total	365.50	1.000

4.1 Estimates for LES and ELES

FIML estimates for LES and ELES, obtained by maximizing (7) or (8), are given in columns 1 to 4 in Table 2.² It is of interest to compare these results with the approach taken by Lluch and Williams (1975) to analyze LES and ELES.

The transformation they used for serial correlation omitted the Jacobian term and the effect of the first observation. They conducted a grid-search at intervals of 0.1 over the range $[0,1]$, narrowing to 0.05 near the maximum of the likelihood function. Our estimates for ELES using this approach are presented in column 5.

Salient features of the estimates in Table 2 are:

(i) The estimates of λ in both LES and ELES are virtually the same (at near unity) and are significantly different from zero.

(ii) MBS's and their asymptotic standard errors are similar for both models when $\lambda = 0$ and when λ is estimated ($\lambda \neq 0$). However, the MBS's for Food and Rent change appreciably when $\lambda \neq 0$. The MPC decreases from 0.861 to 0.532 when λ is estimated.

(iii) Subsistence parameter (*i.e.* γ) estimates are different between LES and ELES, although their standard errors are similar. For each commodity, the γ -estimates are smaller for LES than for ELES. When $\lambda = 0$, all λ -estimates are less than their mean per capita expenditures. Except for the first few observations for Food, all subsistence estimates for LES and ELES are less than the minimum amounts consumed during the sample period. But when $\lambda \neq 0$, the γ -estimates for Rent (LES and ELES) and Durables (ELES) exceed their actual minimum consumption levels. Furthermore, the sum of the γ -estimates increases by 19.58 per cent for LES and 18.39 per cent for ELES compared with their values when $\lambda = 0$.

Some of these results are in marked contrast to the findings of Lluch and Williams (1975, 57) where γ -estimates for ELES were found to be relatively insensitive to the error specification. In addition, there was only a slight change in their estimate of μ after transformation for serial correlation ($\mu = 0.854$, $\lambda = 0$; $\mu = 0.863$, $\lambda = 0.85$). The grid-search method used by Lluch

and Williams may have led to these results, and a glance at column 5 of Table 2 would seem to support this contention. A grid-search over $[0,1]$ resulted in a λ -estimate of unity for ELES.³ The value of the likelihood ratio test statistic is 443.7 which clearly indicates that setting $\lambda = 0$ is not justified. Estimates of the MBS's and subsistence parameters are similar to the case where $\lambda = 0$ (compare columns 2 and 5). The sums of the γ -estimates are virtually identical. The estimated MPC at 0.759 is smaller for $\lambda = 1$ than for $\lambda = 0$, but considerably higher than when λ is freely estimated. These results certainly support the contention of Lluch and Williams that the ELES parameters are relatively insensitive to error specification, but they also suggest that LES is no more sensitive to error specification than is ELES. However, stationarity of the error process requires λ to be less than unity in absolute value *strictly*. Since Lluch and Williams search over the closed interval $[0,1]$, their method does not necessarily impose stationarity; but our method of maximizing (7) does.

The autocorrelated residuals for both LES and ELES are perhaps indicative of mis-specification. However, the autocorrelation may be removed by recognizing the possibility of habit formation. For instance, LES with habit formation was found to be superior to several other models considered, including the Rotterdam model and the Indirect Translog model (Klevmarken, 1979).

4.2 Estimates for DELES

Estimation of DELES was considered under two *a priori* values for the decay rate parameter. The first value of 0.0382 was calculated in McAleer *et al* (1980) as a quarterly decay rate for durables in Australia. This is equivalent to a quarterly depreciation rate of 3.75 per cent, or an annual rate of 15.86 per cent (*c.f.* footnote 4 below). The second value of 100 corresponds to full depreciation during the quarter. These extreme values permit appraisal

of the sensitivity of DELES to variations in θ . The most striking features of the results presented in Table 3 are:

(i) The near-absence of any differences in DELES, regardless of the value of the decay rate, is surprising. For example, comparing columns 3 and 4 of Table 3, we see that parameter estimates are the same, except for the utility function parameters -- and even these are almost the same. There is also a close similarity in standard errors.

(ii) Estimates of MBS's, subsistence parameters, the MPC and the autocorrelation coefficient are exactly the same for DELES and ELES under both error specifications (compare column 2(4) of Table 2 with columns 1 and 2 (3 and 4) of Table 3). The standard errors for the MBS's in DELES are never larger than those of ELES. In addition, there are no differences in the values of the likelihood functions or in the summary statistics between ELES and DELES. The insensitivity of parameter estimates to chosen values of the decay rate involves a major difficulty in interpreting DELES.

(iii) The MBS's of DELES (β_i^* , η^{**}) do not equal the utility function parameters (β_i , η) when the decay rate (θ) is set equal to 100. This would appear to be a matter of concern because it was earlier demonstrated that $\beta_i^* \rightarrow \beta_i$ and $\eta^{**} \rightarrow \eta$ as $\theta \rightarrow \infty$, and the last requirement is well-approximated when $\theta = 100$.⁴ However, it may be argued that these conditions are met *statistically*: first, differences in the corresponding estimates of β_i^* and β_i , and η^{**} and η , are each insignificant; and second, the estimates of β_i^* and η^{**} from DELES (Table 3, columns 2 and 4) are exactly equal to the corresponding estimates of β_i from ELES (Table 2, columns 2 and 4). There is thus some conformity between the mathematical conditions under which DELES collapses to ELES, and the statistical relations between corresponding parameter estimates of the two models.

(iv) The shift parameter ($\mu\eta^*$) is statistically significant, even when $\theta = 100$. However, its value is so small as to be of no economic consequence. Unfortunately, the shift parameter-estimate is four times as *large* when $\theta = 100$ compared with its value when $\theta = 0.0382$, even though theory indicates it should *decrease* as θ increases, and be 'close to' zero when $\theta = 100$. Since the values of the likelihood functions are the same for both ELES and DELES, and the models differ essentially by the shift parameter, this term adds little or nothing to the explanatory power of the model.

The causes underlying these observations are not hard to find. Not surprisingly, they are principally attributable to numerical facts. For this reason, it is useful to note at the outset that the estimates we obtain for both ρ and δ , the market rate of interest and subjective time-preference parameters respectively, are each *numerically large*, so large in fact as to be economically meaningless. On the contrary, when θ is set equal to 100, it is set at a value which is *large economically* and meaningful; but its numerical value is of no consequence in relation to the numerical values of ρ and δ . It is also worth noting that the iterated values of ρ and δ varied considerably in moving toward the maximum of the likelihood function, and seemed sensitive to starting values. On the contrary, their ratio μ was more or less constant throughout, and is also constant for variations in θ .⁵ Of course, ρ and δ may each be multiplied by a constant without varying their ratio. For this and perhaps other reasons, we have not been able to pinpoint pairs of values of ρ and δ that are economically reasonable, but we can always obtain a reasonable value of μ corresponding to any pair. If we constrain ourselves to talk only about economically reasonable values of ρ , δ and μ , we can always locate numerical values of μ , but we can only pinpoint values of ρ and δ *up to a constant*. This naturally calls into question the numerical identification of ρ and δ and confirms the numerical

identification of μ .

The economic explanation of this state of affairs should be made clear. It is certainly a strong assumption that interest rates (ρ) and subjective time-preference rates (δ) are parameters, not variables (say ρ_t and δ_t) in a time-series system. As ρ_t and δ_t vary through time, they have no essential influence on expenditure decisions, *provided every movement in ρ_t is matched by an equi-proportional movement in δ_t in the same direction.* But if we try to find 'representative' values of ρ_t and δ_t to play the role of the parameters ρ and δ , there is a wide selection of possible values each of which has much the same influence on the likelihood function, since no one value is typical of the spread of the rates. Given the system is highly non-linear, economically bizarre values of ρ and δ are possible, even when μ is reasonable.

5. NUMERICAL LIMITING PROPERTIES

Dixon and Lluch (1977) suggest that a 'small' decay rate necessitates a re-interpretation of β_i and η as utility function parameters, rather than as MBS's. This is an *analytical* result and, as such, is perfectly satisfactory. But it does not recognize the possibility of purely *numerical* difficulties which may happen to be important in empirical work -- and which in the present case may even be termed 'perverse'.

We now illustrate some of the consequences for MBS's, the utility function parameters and the shift parameter in respect of the questions left unanswered in (iii) and (iv) above. The shift parameter may be rewritten as

$$(9.1) \quad \mu\eta^* = \frac{\mu\eta}{\theta + \rho(1-\mu\eta)}.$$

In line with our own results, let θ be 'small' relative to ρ and δ (which *includes* the case $\theta = 100$), and allow δ and ρ to increase while their ratio (μ)

is held constant. For given η such that $\mu\eta$ and $(1-\mu\eta)$ are positive fractions, $\mu\eta^*$ is dominated by the value of ρ , and hence as $\rho \rightarrow \infty$, $\mu\eta^* \rightarrow 0$ regardless of the value of θ . Moreover, if θ is now allowed to increase while the rest is constant, then of course $\mu\eta^*$ decreases, giving the standard result. But if, as θ increases, ρ decreases, and the proportional change in θ is much greater than the proportional change in ρ , then $\mu\eta^*$ will rise. Thus the perverse result in (iv) of Section 4.2 is explicable in terms of the corresponding movement in ρ consequent upon a given movement in θ . Of course, the actual estimates of ρ are identifiable only up to a constant which perhaps emphasizes the need to explicate perversity with an eye toward identifiability in non-linear systems.

The MBS's may be rewritten as

$$(9.2) \quad \beta_i^* = \frac{\beta_i}{1 - \frac{\delta\eta}{\theta + \rho}}$$

and

$$(9.3) \quad \eta^{**} = \frac{\eta(1 - \frac{\delta}{\theta + \rho})}{1 - \frac{\delta\eta}{\theta + \rho}}.$$

Under the same numerical conditions (θ small relative to δ and ρ which increase without limit, their ratio being constant), it is easy to see that the MBS's do not approach their utility function counterparts. In fact,

$$\beta_i^* \rightarrow \frac{\beta_i}{1 - \eta\mu}$$

which, if $(1 - \eta\mu)$ is a positive fraction as before, does not converge on β_i but something greater than that, *i.e.* $\beta_i^* > \beta_i$ (*c.f.* Table 3 for Food, Rent and Other Non-durables). Moreover

$$\eta^{**} \rightarrow \frac{\eta(1-\mu)}{1 - \eta\mu}.$$

Since $(1-\mu) < (1-\eta\mu)$ because both μ and η are positive fractions, η^{**} converges not on η but on something less than that, *i.e.* $\eta^{**} < \eta$ (*c.f.* Table 3 for Durables). Since the MBS's are insensitive to large variations in θ (their numerical values being dominated by the magnitude of ρ and δ), it follows that they do not approach their utility function counterparts when $\theta = 100$.

6. CONCLUSION

Dixon and Lluch (1977) have derived a system of expenditure equations that explicitly incorporates durable goods. A central theme of their analysis is that shift-terms should be included when the decay rate is not large; moreover, some ELES coefficients have to be re-interpreted. The empirical results reported here support the significance of the shift-term in the statistical sense, but estimates of the shift parameter are numerically unimportant. However, circumstances can arise in applications of the model such that it is difficult to identify two parameters numerically, and this means that the shift parameter itself may not be numerically identifiable; in fact, it may approach zero even when the decay rate is small. The identification problem for DELES has the implication that the MBS's do not converge to their utility function counterparts.

In spite of the additional parameters that are present in DELES, the data requirements are no greater than for ELES. The empirical results indicate that the aggregate time-series data used to estimate ELES are not sufficiently rich to meet the challenge of the increased demands of DELES. DELES is therefore unlikely to enhance understanding unless additional factual information is provided to evaluate it. There are essentially two ways of doing this.

(i) One method of tackling the problem is to fix *a priori* reasonable values for some parameters. For example, we could define the constant up to

which ρ and δ are identifiable by restricting ρ to equal the sample mean interest rate. However, an obvious limitation of this approach is that the market interest rate is treated as a constant over the sample period.

(ii) It is naturally better to face the problem squarely by providing observations on, for example, interest rates to enlarge the data set. McAleer *et al* (1980) used sample realizations on market interest rates to determine the range in parameter space within which to locate δ numerically.

The second point is particularly relevant in respect of the adequacy of the data to estimate DELES. It should be stressed that we are attempting to estimate interest rate and utility function parameters from observations on prices, expenditures and income. The market interest rate is *inter alia* a supply side variable, and hence a demand study requires it to be incorporated as an additional exogenous variable. For this reason, seeking *one* single representative value from the available data for what is inherently an exogenous *variable* is econometrically infeasible; it also happens in this case not to be a good approximation to economic reality. Consequently the fact we could not identify δ and ρ separately in DELES is to the credit of the model because a properly specified demand model *should*, in principle, exhibit an *inability* to identify δ and ρ . After all, identifying these two parameters in practice is tantamount to arguing the market interest rate plays no effective role in determining variations in consumer expenditures, including those on durable goods. This is certainly not a desirable property that a system of expenditure equations should have!

The empirical results give the strong impression that without additional information, DELES is not a good vehicle for estimating interest rate parameters. In addition, since DELES involves a relatively heavy computational burden, it may be sensible to favour the simpler LES or ELES, although it is entirely a

matter of judgement which of the two systems should be preferred. In this context, it is worth noting that the LES parameter estimates are no more sensitive to error specification than are the ELES parameter estimates, a result which contrasts with the findings of Lluch and Williams (1975).

Table 2: FIML Estimates for LES and ELES Under Alternative Error Specifications

Commodity Group	Marginal Budget Shares (β_1)					Subsistence Parameters (γ_1)					Summary Statistics					
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	
Food	.076 (.002)	.080 (.002)	.132 (.013)	.126 (.017)	.063 (.010)	53.28 (1.137)	56.40 (1.412)	37.72 (3.288)	43.12 (3.904)	57.39 (.902)	R ²	.9985	.9951	.9997	.9988	.9994
											d	.2542	.1906	1.1802	.7012	1.056
											z	-6.25	-6.00	-3.00	-3.75	-1.75
Rent	.211 (.007)	.194 (.007)	.075 (.010)	.065 (.012)	.155 (.014)	7.49 (4.295)	18.93 (3.925)	44.67 (2.764)	48.96 (2.789)	23.33 (1.323)	R ²	.9985	.9919	.9999	.9998	.9989
											d	.2450	.2139	1.0667	.5128	.4766
											z	-5.50	-5.75	-3.00	-3.50	-3.50
Other Non-durables	.538 (.004)	.533 (.005)	.562 (.028)	.557 (.032)	.499 (.012)	90.18 (8.894)	116.05 (9.358)	92.21 (15.77)	123.07 (16.114)	115.91 (.997)	R ²	.9995	.9956	.9999	.9993	.9994
											d	.1577	.1910	1.0532	.6510	.9284
											z	-6.25	-5.75	-0.50	-2.25	-2.25
Durables	.175 (.007)	.193 (.009)	.231 (.028)	.252 (.036)	.283 (.024)	14.73 (2.145)	21.29 (2.572)	23.52 (10.03)	36.64 (10.219)	15.75 (1.515)	R ²	.9872	.9829	.9985	.9964	.9960
											d	.1582	.2351	1.0917	.8418	.8404
											z	-6.75	-5.25	-1.75	-1.75	-1.75
TOTAL	1.000	1.000	1.000	1.000	1.000	165.68	212.67	198.12	251.79	212.38						

Note: Asymptotic standard errors are in parentheses.

1. LES, $\lambda = 0$.
2. ELES, $\lambda = 0$, $\mu = .861$ (.017).
3. LES, $\lambda = .99984$ (.00014).
4. ELES, $\lambda = .99977$ (.00017), $\mu = .532$ (.090).
5. ELES, $\lambda = 1$, $\mu = .759$ (.0335).

Table 3: FIML Estimates for DELES Under Alternative Error Specifications and A Priori Decay Rates

Commodity Group	Marginal Budget Shares (β_1^*, η^{**})				Utility Function Parameters (β_1, η)				Subsistence Parameters ($\gamma_1, \theta\phi$)				Summary Statistics				
	1	2	3	4	1	2	3	4	1	2	3	4		1	2	3	4
Food	.080 (.002)	.080 (.002)	.126 (.017)	.126 (.017)	.037 (.002)	.037 (.002)	.098 (.015)	.099 (.015)	56.40 (1.412)	56.40 (1.412)	43.12 (3.907)	43.12 (3.907)	R ²	.9951	.9951	.9988	.9988
													d	.1906	.1906	.7012	.7012
													z	-6.00	-6.00	-3.75	-3.75
Rent	.194 (.007)	.194 (.007)	.065 (.012)	.065 (.012)	.088 (.003)	.089 (.003)	.051 (.010)	.051 (.010)	18.93 (3.924)	18.93 (3.925)	48.96 (2.793)	48.96 (2.788)	R ²	.9919	.9919	.9998	.9998
													d	.2140	.2140	.5128	.5128
													z	-5.75	-5.75	-3.50	-3.50
Other Non-durables	.533 (.005)	.533 (.005)	.557 (.032)	.557 (.032)	.242 (.014)	.244 (.014)	.433 (.046)	.437 (.045)	116.05 (9.356)	116.05 (9.358)	123.07 (16.056)	123.07 (16.041)	R ²	.9956	.9956	.9993	.9993
													d	.1910	.1910	.6510	.6510
													z	-5.75	-5.75	-2.25	-2.25
Durables	.193 (.009)	.193 (.009)	.252 (.036)	.252 (.036)	.633 (.019)	.630 (.019)	.418 (.057)	.413 (.056)	21.29 (2.572)	21.29 (2.572)	36.64 (10.081)	36.64 (10.112)	R ²	.9829	.9829	.9964	.9964
													d	.2353	.2353	.8418	.8418
													z	-5.25	-5.25	-1.75	-1.75
TOTAL	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	212.67	212.67	251.79	251.79					

Note: Asymptotic standard errors are in parentheses.

- 1: $\lambda = 0$, $\theta = .0382$, $\mu = .861(.017)$, $\delta = 48,517.3(546.25)$, $\rho = 56,348.7(497.05)$, $\mu\eta^* = 2.126 \times 10^{-5}$ (2.52×10^{-6}).
- 2: $\lambda = 0$, $\theta = 100$, $\mu = .861(.017)$, $\delta = 48,516.9(546.40)$, $\rho = 56,348.2(496.78)$, $\mu\eta^* = 2.099 \times 10^{-5}$ (2.45×10^{-6}).
- 3: $\lambda = .99977(.00017)$, $\theta = .0382$, $\mu = .532(.090)$, $\delta = 11,414.1(1,519.7)$, $\rho = 21,456.0(764.77)$, $\mu\eta^* = 1.334 \times 10^{-5}$ (5.17×10^{-6}).
- 4: $\lambda = .99977(.00017)$, $\theta = 100$, $\mu = .532(.091)$, $\delta = 2,811.6(327.7)$, $\rho = 5,285.7(282.6)$, $\mu\eta^* = 5.203 \times 10^{-5}$ (2.057×10^{-5}).

FOOTNOTES

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1. The excuse for using this excessively contrived form is that the LES requires it (see *e.g.* Berndt and Savin, 1975). Clearly, it also simplifies the calculations and, if we adopt this latter viewpoint, we must regard equation (6) as a convenient approximation. Moreover, it makes the estimation of ELES and DELES symmetrical with LES. Furthermore, as we shall see below, the implied values of the autocorrelation coefficients in each equation of the system are very much the same and this seems to justify our use of equation (6).
2. For Tables 2 and 3, the subsistence parameters are measured in dollars per capita in 1966-67 prices. One observation is lost in computing the estimates of column 5 in Table 2 since the grid search for λ resulted in an estimate of unity, entailing the use of first-differences of the data. By R^2 is meant $\{1 - (\sum \hat{u}_{it}^2 / \sum (v_{it} - \bar{v}_i)^2)\}$, where $\bar{v}_i$ is the mean of v_{it} , the observed value of the i 'th dependent variable during period t ; hence $\hat{u}_{it} = v_{it} - \hat{v}_{it}$, and the summation is over $t = 1, 2, \dots, T$. Goodness-of-fit statistics are indicative rather than definitive measures, so R^2 is to be interpreted solely as an index of the predictive power of the model. The Durbin-Watson statistic (d) gives an indication of patterns in the residuals, but its statistical properties are unknown in this context. z is the two-tailed non-parametric sign test for serial correlation, under the null hypothesis of independently distributed residuals; if r is the number of changes of sign of residuals, the ratio $z = \{r - 0.5(T-1)\} / \{0.25(T-1)\}^{\frac{1}{2}}$

is asymptotically normally distributed with zero mean and unit variance. After allowing for serial correlation, R^2 still refers to the original dependent variables, but d and z relate to the estimated errors after transformation.

3. This is not surprising when we consider that the FIML estimate of λ using weighted first-differences, rather than the grid-search method, was 1.057. Since this also resulted in negative MBS's for Food and Rent, in addition to an absurdly low estimate of the MPC of 0.132, the other estimates are not presented here.
4. If d is the depreciation rate corresponding to the decay rate θ , then $d = (1 - e^{-\theta})$ or $\theta = -\{\log_e(1-d)\}$; as $d \rightarrow 1$ so $\theta \rightarrow \infty$, and *vice versa*. In fact, for quite modest values of θ , d is very close to 1: *e.g.* when $\theta = 10$, $d = 0.9999546$. Hence d is very close indeed to unity when $\theta = 100$ so that, judged by depreciation rates, the tendency of DELES to behave more and more like ELES as $\theta \rightarrow \infty$ ought to be well approximated by DELES when $\theta = 100$.
5. For example, in an earlier run with $\lambda = 0$, estimates for δ and ρ went as high as 3.942×10^{11} and 4.578×10^{11} respectively, although their ratio remained at 0.861. At the same time, the estimated shift parameter was 2.616×10^{-12} , even though θ was constrained at 0.0382.

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