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THE STRUCTURE OF MULTI-PERIOD EMPLOYMENT CONTRACTS WITH INCOMPLETE INSURANCE MARKETS

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Abstract

This paper examines the structure of multi-period employment contracts in an economy with identical workers when only incomplete insurance is provided against job-related contingencies. The incompleteness of insurance markets stems from an asymmetry of information between the worker and the insurer. In this situation, the employment contract will provide implicit insurance by paying workers less than their marginal product during some periods and more during others. The paper explores how the characteristics of such contracts are altered by increases in uncertainty, worker risk aversion, hiring costs, and the amount of specific and general training provided. A sequel to this paper examines the efficiency properties of such contracts.

The Structure of Multi-Period Employment Contracts with Incomplete Insurance Markets

A worker faces a host of uncertainties associated with his employment - will he be good at his job? will his worth be recognized? will he receive a favorable job offer from another firm? will he be fired? will he get along well with his co-workers? will he suffer from some job-related disability? and so on. He will be able to obtain full insurance against some of these risks, but not against most. Thus, the worker faces considerable residual risk.

This paper addresses the question: In what ways does the incompleteness of insurance markets affect the structure of multi-period employment contracts? The answer is of interest in a number of contexts. First, it may explain some hitherto puzzling features of employment contracts. Second, since the standard theorems of welfare economics fail to hold in an economy with incomplete markets, competitive employment contracts may be inefficient. If they are, what should the government do to correct the distortions? And third, an understanding of the structure of the employment contract may provide insight into various labor market phenomena associated with macroeconomic fluctuations.

The gist of the argument in the paper is that when insurance markets are incomplete, there is an incentive for the firm and the worker to share the residual risks associated with the worker's employment. This can be achieved by the firm paying workers more than their marginal products over some periods, and less during others. Thus, the employment contract provides implicit insurance against job-related contingencies. This general line of argument is familiar from the principal-agent literature (Ross [1973], and Stiglitz [1974], e.g.) but, to our knowledge, has not previously been applied to explain characteristics of multi-period employment contracts. We treat an economy of identical individuals in which the incompleteness of insurance markets stems from an asymmetry of information between the worker and the provider of insurance. In these circumstances, the provision of insurance, whether explicit or implicit, is characterized by moral hazard. As a result, only partial insurance is offered.

Most of the recent literature dealing with the employment contract examines those features of the contract, notably temporary layoffs and downward-sticky wages, which arise from workers and firms sharing the risk associated with fluctuations in demand (Azariadis [1975], e.g.). This paper is only tangentially related to this literature, and instead draws on a body of research extending back to the early sixties on the structure of multi-period employment contracts, of which the most notable contribution is Becker [1962]. While Becker's analysis was discursive, he clearly recognized the importance of risk-sharing as a determinant of the features of such contracts. Subsequent papers on the topic (Donaldson and Eaton [1976], Parsons [1972], and Hashimoto [1978] *inter alia*) have ignored this insight, treating all agents as being risk-neutral. Thus, this paper formalizes some aspects of Becker's classic paper that have been unjustifiably neglected.

This paper is the first in a series of three. Its emphasis is on *describing* the structure of competitive multi-period employment contracts when workers face residual employment-related risk. The second paper focuses on the efficiency properties of these contracts. And the third builds on the other two papers to provide a theory of unemployment.

The organization of the paper is as follows. Section I presents the basic model and derives the qualitative characteristics of multi-period employment contracts in economies with an incomplete set of insurance markets. Section II investigates the comparative static properties of such contracts. And concluding comments are presented in Section III.

I. Qualitative Properties of Multi-Period Employment Contracts

I.1 The Basic Model

The economy is very simple. A single numéraire commodity is produced according to a single technology which exhibits constant returns to scale to the only factor, labor. The risk-averse workers are equally productive and attach the same relative valuations to income and job satisfaction. There are, however, differences among firms and differences among workers, that determine the quality of a job match. This is measured by a worker's job satisfaction which remains constant during his period of tenure with a firm. Elements of job satisfaction include how much the worker enjoys his work environment - its temperature, lighting, decoration scheme, etc. - and how congenial he finds his co-workers. A worker's characteristics and his job satisfaction are unobservable (or observable only at prohibitively high cost) by the firm. And at the time of hiring, differences in firm characteristics are unobservable by the worker; thus, when a worker joins a firm, he is uncertain concerning his future job satisfaction. The differences among firms and workers are such that the probability distribution of a worker's job satisfaction is independent of both the firm and the worker. The quality of job matches is the sole source of uncertainty in the economy. Firms are competitive and risk-neutral.

Workers live and are employed for two periods. If a worker finds out that he dislikes his job, he may quit the firm but only after working there one period. He will quit if doing so increases his expected utility. We may imagine that there are two types of firms, those that hire workers when they have just entered the labor market, and those that hire workers at the beginning of the second period of their working lives (workers who have just quit). The employment contract offered by the former type of firm (a type 1 firm) is characterized by a first- and a second-period wage, while that offered by the latter type (a type 2 firm) is characterized by a single wage. Competitive equilibrium requires that zero profits be made on <u>each</u> group of workers with a particular employment history at the time of hiring. If it were otherwise, profits could be made by hiring only workers with particular histories. Here, there are two possible employment histories at the time of hiring - "never worked before" and "quit after working with previous firm one period". Since all workers for type 1 firms have the former employment history at the time of hiring, then type 1 firms must make zero profits. By a similar line of argument, type 2 firms too must make zero profits.

Before presenting the model formally, we discuss some of its characteristics. The assumptions have been chosen so as to provide as simple as possible a general equilibrium model which contains both a multi-period employment contract and incomplete insurance markets. The qualitative results to be demonstrated will clearly generalize to more complex and realistic models.

The assumptions <u>imply</u> the incompleteness of insurance markets, in contrast to most papers which instead assume this. The insurer is unable to measure a worker's job satisfaction. If he were to provide insurance against job satisfaction and were to ask the worker what his level of job satisfaction is, the worker would have an incentive to dissimulate; specifically, he would state that his job satisfaction was the lowest possible consistent with his observed behavior. But the insurer can observe whether or not a worker quits. Thus, the insurance provided is against the act of quitting rather than job satisfaction _per se_. Since there is no market for insurance against job satisfaction, insurance markets are incomplete. If job satisfaction were observable, the worker would purchase complete insurance and determination of the competitive employment contract would be straightforward.

The incompleteness of insurance markets in the model results from an asymmetry in information - the worker knows his job satisfaction, but the insurer cannot observe it. In any economy in which the incompleteness of insurance markets derives from an asymmetry of information, two generic problems may arise in the provision of insurance. First, if individuals differ in their riskiness and if each knows better than does the insurer into what risk-class he falls, _adverse selection_ phenomena arise. We have excluded this possibility by assuming that individuals are, in relevant respects, identical. In so doing, we have eliminated a potentially important determinant of the structure of multi-period employment contracts.[1] Second, if the probability of the insured-against events can be influenced by the insured's actions which the insurer can observe only imperfectly or not at all, there is _moral hazard_.[2] More simply, moral hazard exists whenever the provision of insurance affects the probability of the insured-against event(s). In our model, the provision of insurance against quitting increases the probability of quitting; such insurance is therefore characterized by moral hazard. It is well-known that when moral hazard is present, typically only partial insurance is provided.

We have argued thus far that insurance against uncertain job satisfaction will take the form of partial insurance against quitting. Two questions remain. Who will provide this insurance, and in what form? In the context of the model, the same results are obtained whether the firm or an independent insurance agent provides such insurance. In a more complex economy, however, the firm has an informational advantage over an independent insurance agent. For instance, suppose the worker is paid different amounts depending on whether his separation from a firm is due to his having quit or having been fired. The firm can monitor this better than an external agent. Relatedly, an independent insurance agent might find it difficult to ascertain whether a worker's move to a subsidiary firm were just a transfer or a quit and rehire. Thus, we expect the firm to provide quit insurance.

In our economy, the employment contract is characterized by three parameters, first period remuneration, and second period remuneration if and if not the worker quit at the end of the previous period. Insurance is provided by setting these levels of remuneration above or below the corresponding value of marginal product. This can be achieved in a variety of ways - seniority increments that do not correspond exactly to productivity increases, pensions, severance pay, and so on. We treat the three parameters of the contract as three wage rates. It should be kept in mind, though, that they have many other possible interpretations.

The stage is now set to relate the structure of the employment contract to workers' tastes, the amount of risk, and the technology of production.

The following notation is employed:

w_i — the i^{th} period ($i=1,2$) wage of a (type 1) firm which hires workers when they enter the labor force

$\overline{w}$	the wage of a (type 2) firm which hires workers only for the second period of employment (i.e., those who quit another firm at the end of the first period)
m	marginal product of labor
j_i	a worker's job satisfaction during his i^{th} ($i=1,2$) period of employment
$f[j]$	the probability distribution of job satisfaction at the time workers enter a firm (square brackets are used to enclose the arguments of a function), $F[j] = \int_{-\infty}^{j} f[j']dj'$
$U[\cdot]$	utility (EU, expected utility)
q	the proportion of workers who quit at the end of the first period

Since firms act competitively, the employment contracts offered in long-run equilibrium will result in each firm making zero profits. A worker, meanwhile, knowing the employment contracts offered by all firms, will choose to work for that type 1 firm whose contract provides him with the highest level of expected utility. Thus, the equilibrium type 1 contracts will be those that maximize a worker's expected utility subject to yielding type 1 firms zero profit. Type 2 firms, meanwhile, have no choice but to pay workers their (net) marginal product.

We first employ the model to investigate a particularly simple issue: If insurance markets are incomplete and if hiring a new worker entails no cost, will type 1 firms pay workers their marginal products? The answer, perhaps surprisingly, is not necessarily. To see this, consider the extreme case where workers are so risk-averse that they choose the maximin type 1 contract, which maximizes the lowest possible <u>ex post</u> utility. Starting from a situation where workers are paid their marginal product, $w_1=w_2=\overline{w}=m$, raising w_1 and lowering w_2 such that type 1 firms continue to make zero profits increases the income of quitters and hence workers' expected utility.

To illustrate the application of the model, we examine this issue formally. We may view the determination of w_1 and w_2 as the solution to a two-stage maximization problem. In the first stage, workers decide on the subset of levels of job satisfaction corresponding to which they would quit, and the complementary subset corresponding to which they would not,[3] taking w_1, w_2, and $\overline{w}$ as given. This gives expected utility and the quit rate as functions of w_1, w_2, and $\overline{w}$. In the second stage, a type 1 firm chooses w_1 and w_2 to maximize workers' expected utility, subject to its making zero profits. In so doing, it takes into account the dependence of the quit rate on w_1 and w_2.[4]

A worker's utility if he quits is $U[w_1,\overline{w},j_1,j_2]$; he receives wages of w_1 and $\overline{w}$, and has different levels of job satisfaction during the two periods. Meanwhile, the utility of a worker who stays with the same firm for both periods is $U[w_1,w_2,j_1,j_1]$; his wages are w_1 and w_2 in the two periods, and his job satisfaction is the same during both periods with the firm. It will typically be the case that there is a critical level of job satisfaction θ; a worker whose job satisfaction exceeds θ stays, while other workers quit. For the worker with job satisfaction θ, the (expected, since second period job satisfaction is uncertain) utility from quitting equals the (certain) utility from staying:

$$\int_{-\infty}^{\infty} U[w_1,\overline{w},\theta,j_2]dF[j_2] = U[w_1,w_2,\theta,\theta].$$

(1) characterizes θ as a function of w_1, w_2, and $\overline{w}$:

$$\theta = \theta[w_1,w_2,\overline{w}]. \tag{1}$$

Since everyone with $j_1 < \theta$ quits, then

$$q = F[\theta]. \tag{2}$$

And (1) and (2) together imply

$$q = q[w_1,w_2,\bar{w}]. \tag{3}$$

Expected utility may be written as

$$EU = \int_{-\infty}^{\infty}\int_{-\infty}^{\theta} U[w_1,\bar{w},j_1,j_2]dF[j_1]dF[j_2]$$
$$+ \int_{\theta}^{\infty} U[w_1,w_2,j_1,j_1]dF[j_1]. \qquad (4)^5$$

The first integral on the right-hand side is the expected utility of quitters times the probability of quitting; the second is the corresponding term for stayers. The budget constraint of a type 2 firm is

$$\bar{w} = m, \tag{5}$$

which determines $\bar{w}$. From the substitution of (5) and (1) into (4), and (5) into (3), we may write

$$EU = EU[w_1,w_2] \text{ and } q = q[w_1,w_2], \qquad (6a,b)$$

where m is suppressed to simplify notation. This characterizes the solution to the first stage of the maximization problem.

In the second stage of the maximization problem, a type 1 firm chooses w_1 and w_2 to maximize (6a) subject to (6b) and its budget constraint. A type 1 firm faces the budget constraint

$$m + \frac{m(1-q)}{1+r} = w_1 + \frac{(1-q)}{1+r} w_2, \tag{7}$$

where r is the discount rate. To make zero profit, it must on average make zero profit on each worker it hires. $m + \frac{m(1-q)}{1+r}$ is average discounted output per worker hired, while $w_1 + \frac{(1-q)w_2}{1+r}$ is the expected discounted payment. To simplify the analysis, it is assumed here and throughout the rest of the paper that capital markets are perfect,[6] and that the discount rate is zero. Thus,

$$U[w_1,\bar{w},j_1,j_2] = U[w_1+\bar{w},j_1,j_2],$$

$$U[w_1,w_2,j_1,j_1] = U[w_1+w_2,j_1,j_1], \text{ and}$$

$$m(2-q) = w_1+(1-q)w_2. \tag{7'}$$

The type 1 firm's maximization problem is therefore

$$\max_{w_1,w_2} \quad EU[w_1,w_2] \text{ s.t. } m(2-q[w_1,w_2]) = w_1+(1-q[w_1,w_2])w_2. \tag{8}$$

Where λ is the Lagrange multiplier on the budget constraint, the first-order conditions are:

$$w_1: \quad EU_1 - \lambda(1 - \frac{\partial q}{\partial w_1}(w_2-m)) = 0, \text{ and} \tag{9a}$$

$$w_2: \quad EU_2 - \lambda((1-q) - \frac{\partial q}{\partial w_2}(w_2-m)) = 0, \tag{9b}$$

where $EU_i = \frac{\partial EU}{\partial w_i}$. (9a) states that w_1 should be raised to the point where marginal benefit equals marginal cost. $\frac{1}{\lambda} EU_1$ is the expected benefit per worker in dollars from raising w_1 by one unit. There is a direct unit cost per worker, and also an indirect cost of $-\frac{\partial q}{\partial w_1}(w_2-m)$, which we now examine briefly. Consider the worker who is indifferent between quitting and staying. If he quits, his income rises by $\bar{w}-w_2 = m-w_2$. Offsetting this must be a fall in monetary-equivalent job satisfaction of the same amount. He therefore quits when doing so lowers his job satisfaction and has no effect on his productivity. Relative to the first-best optimum, this is clearly inefficient and involves excessive labor turnover. $-\frac{\partial q_1}{\partial w_1}(w_2-m)$ is the increase in the deadweight loss associated with excessive labor turnover from raising w_1 by one unit. This raise increases the proportion of quitters by $\frac{\partial q_1}{\partial w_1}$, and for each of these marginal quitters, the efficiency loss from quitting is $-(w_2-m)$. The interpretation of (9b) is the same, except that only stayers benefit from the rise in w_2.

To examine whether firms will pay workers their marginal product, we set $w_2=m$ (which implies that all workers are paid their marginal product) and determine under what circumstances (9a) and (9b) are satisfied. (9a), (9b), and $w_2=m$ together imply $EU_1 = \frac{1}{1-q} EU_2$, which may be rewritten as

$$\frac{1}{q}(EU_1-EU_2) = \frac{1}{1-q} EU_2. \tag{10}$$

The term on the left-hand side of (10) is the average marginal utility of income for quitters, while that on the right is the corresponding magnitude for stayers. (10) therefore implies that workers should be paid their marginal products, only when doing so results in the average marginal utility of income of quitters equalling that of stayers.[7] If income and job satisfaction (or equivalently consumption and job satisfaction) are strongly separable in demand, by which we mean

$$U[y,j_1,j_2] = U[y] + v[j_1,j_2],$$

where y is income, this condition holds since a worker's income and hence his marginal utility of income is the same whether he quits or stays. If, however, job satisfaction is a substitute for income, i.e.

$$U[y,z[j_1,j_2]],\ U_{yz} < 0,$$

then with income the same for quitters and for stayers, the average marginal utility of income of quitters exceeds that of stayers. In this case, expected utility can be increased by transferring income from stayers to quitters. This can be achieved by setting $w_1 > m$ and $w_2 < m$. Doing this, however, causes excessive turnover. Because there is an efficiency loss associated with this, redistribution from quitters to stayers will stop short of equalizing the average marginal utility of income of the two groups. More precisely, at the equilibrium, the sum of the deadweight loss associated with excessive labor turnover and that associated with incomplete equalization of workers' marginal utilities of income is minimized. The analogous argument for the case where income and job satisfaction are complements is left to the reader.

The results of the above argument are summarized in Proposition 1.

Proposition 1: Even when there is no cost to hiring a worker and no training, multi-period employment contracts will not in general pay workers the value of their marginal product at each point in time.

However, in the special case where capital markets are perfect and consumption is strongly separable in demand from the outcome of the random variable which determines whether or not a worker quits, multi-period employment contracts will pay workers the value of their marginal product at each point in time.[7]

It should be evident that Proposition 1 applies very generally. There can be many time periods, many factors of production, and many sources of uncertainty which influence a worker's decision whether or not to quit, and the proposition still holds.

The next two subsections extend the argument to treat hiring and training costs.

I.2 Hiring costs

There are typically fixed costs to hiring a worker. These include the costs of advertising and interviewing, as well as the costs of training the worker in the basic skill he is to perform. We term these fixed costs, turnover costs, and distinguish them from variable or specific training costs, which are associated with the upgrading of firm-specific skills. To simplify the analysis, it is assumed that the turnover cost per worker is T for both type 1 and type 2 firms. If workers are paid their marginal product less the turnover costs they cause (net marginal product), then stayers receive an income over the two periods of 2m-T, while quitters' income is 2m-2T. Since quitters receive a lower income, then unless job satisfaction and income are strongly complementary, they will have a higher marginal utility of income, on average, than stayers. In this case, therefore, one expects quitters to be paid more than their net marginal product and stayers to be paid less.

A type 2 firm has no choice but to pay workers their net marginal product, $\bar{w}$=m-T. A type 1 firm faces the maximization problem

$$\max_{w_1,w_2} EU[w_1,w_2] \text{ s.t. } m(2-q[w_1,w_2])-T = w_1+(1-q[w_1,w_2])w_2, \tag{11}$$

which is the same as (8) except for training costs. The first-order conditions are the same in (9a) and (9b). It will be useful to provide an alternative interpretation of those equations for the case where training costs are positive.

We term the worker's loss in income from quitting, the turnover penalty, and denote it by p. And we term the difference between what the turnover penalty would be if workers were paid their net marginal product and the actual turnover penalty, the turnover subsidy; we denote this by τ. In the case being treated, the turnover penalty is $w_2-\overline{w}$, while the turnover subsidy is $m-w_2$. Intuitively, one expects the turnover subsidy to be larger, c.p., the more risk-averse are workers, the greater the uncertainty associated with job satisfaction, and the less sensitive is the quit rate to the turnover penalty. This intuition is checked in the next section. (9a) and (9b) may be combined to give

$$(1-q)\,\frac{EU_1}{\lambda} - \frac{EU_2}{\lambda} = \left((1-q)\,\frac{\partial q}{\partial w_1} - \frac{\partial q}{\partial w_2}\right)\tau \tag{12a}$$

or equivalently

$$\frac{(1-q)q}{\lambda}\left(\frac{EU_1-EU_2}{q} - \frac{EU_2}{1-q}\right) = \left((1-q)\,\frac{\partial q}{\partial w_1} - \frac{\partial q}{\partial w_2}\right)\tau. \tag{12b}$$

The term on the left-hand side of (12a) can be interpreted as the benefit from lowering w_2 by one unit and simultaneously raising w_1 by $1-q$ units, holding turnover fixed. This procedure satisfies the budget constraint and takes \$q from each stayer and transfers \$(1-q) to each quitter. $(1-q)q$ is therefore the number of dollars transferred per worker. And since $\frac{EU_1-EU_2}{q}$ is the (average) marginal utility of income of a quitter while $\frac{EU_2}{1-q}$ is that of a stayer, $\frac{1}{\lambda}\left(\frac{EU_1-EU_2}{q} - \frac{EU_2}{1-q}\right)$ is the dollar benefit from each dollar transferred. Since the hypothesized transfers result in a unit increase in the turnover subsidy, (12b) indicates that the marginal benefit associated with an increase in the turnover subsidy equals the reduction in the deadweight loss associated with the disparity in the average marginal utility of income of stayers and quitters. The right-hand side of (12a) and (12b) is the marginal cost of an increase in the turnover subsidy, which equals the increase in the excess burden associated with excessive labor turnover.

This is a convenient point at which to note that the first-order conditions may characterize multiple local optima. This can readily be seen from plotting marginal benefit (MB) and marginal cost (MC), as defined below (12), against τ the turnover subsidy. $\frac{1}{\lambda}\left(\frac{EU_1-EU_2}{q} - \frac{EU_2}{1-q}\right)$ varies inversely with the size of the turnover subsidy. $(1-q)(q)$, however, rises from 0 (τ is so negative that $q=0$) to a maximum of ¼ and then falls to 0 again (τ is so large that $q=1$). Thus, the marginal benefit curve may fall or rise for $q < \frac{1}{2}$, but for $q > \frac{1}{2}$ it unambiguously falls. Since the sensitivity of the quit rate to the size of the turnover subsidy depends on the form of $F[j_1]$, the marginal cost curve can have almost any shape.[8] These results are shown in Figure 1.

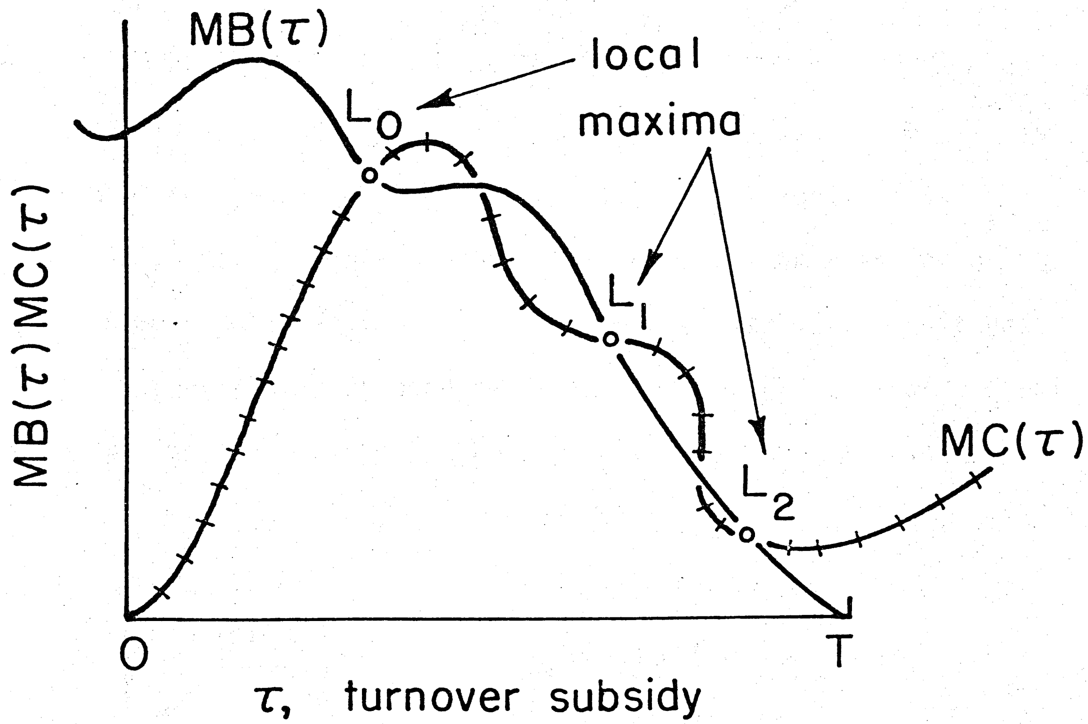

FIGURE 1: Multiple optimal turnover subsidies

The findings thus far in this subsection are summarized in Proposition 2.

Proposition 2: When workers are risk-averse, when insurance markets against job-related contingencies are incomplete, and when there are costs to hiring a worker, multi-period employment contracts will typically provide implicit insurance. The amount of insurance provided will minimize the sum of the excess burdens associated with, first, inequality in workers' marginal utilities of income and, second, excessive (or inefficient) labor turnover.

A corollary of this proposition is that when the quit rate is unrelated to the wage structure, full (or, in the case of non-separable utility, as full as possible) insurance will be provided. In the next subsection, we shall discuss how the turnover subsidy changes in response to a rise in turnover costs.

I.3 Training

The treatment of specific and general training is a straightforward extension of the analysis of hiring costs. We measure the amount of training provided by the firm in terms of the increase it induces in a worker's marginal product.[9] The cost to the firm of providing a worker with g units of general training and s units of specific training is c(s,g), with c_s, c_g, c_{ss}, and $c_{gg} > 0$.[10] To simplify the analysis, the inconsequential assumption is made that the benefits from training accrue in the period subsequent to that in which the training occurs. An implication of this in the model is that type 2 firms do not train workers, while type 1 firms train them only in the first period.

We first treat general training. To simplify the discussion, we make two additional assumptions:

i) consumption is perfectly substitutable for job satisfaction; and

ii) job satisfaction of type 2 firms is certain and equals zero.

Thus,

$$U[w_1,\bar{w},j_1,j_2] = U[w_1+\bar{w}+j_1] \text{ and}$$

$$U[w_1,w_2,j_1,j_2] = U[w_1+w_2+2j_1].$$

A worker will quit if $p=w_2-\bar{w} < j_1$; i.e., if the gain in job satisfaction exceeds the turnover penalty. An important implication of this is that $\frac{\partial q}{\partial w_1} = 0$.

Consider a type 1 firm which is offering an equilibrium contract. It now increases general training by one unit, which reduces the worker's net marginal product by c_g in the first period, and increases his second-period marginal product by one unit. In what way should the firm alter the wage structure in response to this increase in general training? Suppose it lowers the first-period wage by c_g and raises the second-period wage by one. Under the stated assumptions, the quit rate is a function only of the turnover penalty, $w_2-\bar{w}$. Since the increase in general training, along with the hypothesized change in wage structure, raises w_2 and $\bar{w}$ by the same amount, it alters neither the quit rate nor the turnover penalty. The turnover subsidy is $\bar{w}-w_2$, which is also unaffected by the hypothesized change. Thus, from the interpretation of equation (12), the change in training cum wage structure does not affect the marginal cost associated with raising the turnover subsidy. The hypothesized change in wage structure therefore results in a turnover subsidy that is of the right size (too large, too small) when the change causes the marginal benefit in equation (12) to stay the same (fall, rise). Now, the hypothesized change increases all workers' income by $1-c_g$. It is easily demonstrated that an increase in all workers' incomes by the same amount causes marginal benefit to remain the same/fall/rise according to whether workers' tastes for risk exhibit constant/decreasing/increasing absolute risk aversion. Thus, in response to an increase in the amount of general training provided, the turnover subsidy should be increased if the amount of general training is suboptimal[11] ($c_g<1$) and workers' utility functions are characterized by increasing absolute risk aversion <u>or</u> if the amount of training is superoptimal and absolute risk aversion is decreasing. If either the amount of training is optimal or if there is constant absolute risk aversion, the turnover subsidy should remain the same. The other cases are left to the reader.

These results are intuitively reasonable. The turnover subsidy is the amount of insurance "purchased" by the worker. The extra unit of training increases all workers' incomes by $1-c_g$. And whether a worker purchases the same amount of/more/less insurance in response to a rise in income depends on whether his tastes exhibit constant/increasing/decreasing absolute risk aversion.[12,13] The above argument applies to any change that alters all workers' productivity by the same amount. This includes a technological improvement that raises workers' productivity by an equal amount in type 1 and type 2 firms.

These results are summarized in Propositions 3 and 4.

<u>Proposition 3</u>: When quitters are the unlucky ones and insurance markets against job-related contingencies are incomplete, the amount of insurance provided in a two-period employment contract is measured by the size of the turnover subsidy, which equals the difference in the workers' marginal product if he does and does not quit minus the corresponding difference in wages.

<u>Proposition 4</u>: When, additionally, capital markets are perfect and assumptions i) and ii) hold, in response to a rise in all workers' net marginal product by the same amount, the turnover subsidy in the employment contract will remain the same/rise/fall according to whether workers' tastes exhibit constant/increasing/decreasing absolute risk aversion.

We now turn to specific training. As Becker [1962, pp. 19-21] rightly pointed out, the effects of specific training on the wage structure are more complex than those of general training. To simplify the discussion, we retain assumptions i) and ii). We shall perform a conceptual experiment analogous to that for general training. Suppose that the firm increases the amount of specific training provided by one unit, and simultaneously increases w_2 by one unit and lowers w_1 by c_s units. The turnover subsidy, $\overline{w}-w_2+s$, remains the same, but the turnover penalty, $w_2-\overline{w}$, increases and hence the quit rate decreases. From (12), the marginal cost associated with a rise in the turnover subsidy is $-\frac{\partial q}{\partial w_2}\tau$.[14] The hypothesized change increases the turnover penalty but has no effect on the turnover subsidy. Thus, $\left(\frac{\partial MC}{\partial p}\right)_{\overline{\tau}} = -\left(\frac{\partial^2 q}{\partial w_2^2}\tau\right)_{\overline{\tau}} = -f'[\theta]\overline{\tau}$; the change causes the marginal cost curve to rise if f is decreasing at θ, and to fall otherwise. The rise in the turnover penalty causes θ to fall, which, if f is decreasing at θ, increases the marginal deadweight loss associated with excessive labor turnover. The posited change has an <u>a priori</u> ambiguous effect on the marginal benefit curve as well. The posited change in the wage structure increases the income of stayers by $1-c_s$ but decreases that of quitters by c_s; thus, $\left(\frac{\partial\left(\frac{EU_1-EU_2}{q} - \frac{EU_2}{1-q}\right)}{\partial p}\right)_{\overline{\tau}} > 0$. However, since $\left(\frac{\partial((1-q)q)}{\partial p}\right)_{\overline{\tau}} = (1-2q)\frac{\partial q}{\partial p} = -(1-2q)f[\theta]$, then (unless $q>\frac{1}{2}$) the effect of the conjectured change in the wage structure on the marginal benefit curve is ambiguous. The above results imply that the effect of the posited change on the size of the turnover subsidy is also ambiguous. We can, however, say that in response to a unit increase in specific training, the turnover subsidy increases by more (decreases by less) the greater is worker risk aversion, the greater is $f'[\theta]$, and (with $q<\frac{1}{2}$) the greater is f. The responsiveness of the turnover subsidy to an increase in specific training equals the (marginal) share of the costs and benefits of training borne by the firm.[15] Contrary to the impression given in most previous studies, this share need not lie between zero and one; the turnover subsidy may fall in response to an increase in the value of specific training provided, or may rise by more than this increase in value.

<u>Proposition 5</u>: Under the conditions of Proposition 4, the share of the costs and benefits of specific training assumed by the worker is negatively related to his risk aversion and to the second derivative of the quit function with respect to the size of the turnover penalty.

The above arguments concerning how changes in the levels of general and specific training provided by a firm affects its wage structure can be employed to ascertain the effect on the size of the turnover subsidy of an increase in hiring or turnover costs. An increase in hiring costs of a type 1 firm decreases all workers' net marginal products by the same amount. Proposition 4 is therefore applicable to this increase. An increase in the hiring costs of a type 2 firm by one unit is equivalent to an increase in specific training of one unit combined with a unit decrease in general training, when the cost increase in specific training just offsets the cost decrease in general training. The increase in hiring costs can therefore be separated into two parts. The first comprises the increase in hiring costs of a type 1 firm along with the loss in surplus (benefits minus costs) from the decrease in the level of general training. Its effect on the wage structure will depend on whether workers' absolute risk aversion is constant, increasing or decreasing with income. The second is the gain in surplus from the increase in the level of specific training; its effects were just analyzed and depend on the worker's degree of risk aversion as well as the magnitude of $f'[\theta]$. It will usually be the case that, in response to a unit increase in hiring costs, the turnover penalty will rise by less than one unit. But with extreme values of $f'[\theta]$ or strongly increasing or decreasing absolute risk aversion, the turnover subsidy can either rise by more than the increase in hiring costs, or fall.

There has been a debate in the literature, which to our knowledge has never been resolved, concerning the disposition of the surplus from training. In our model, the entire surplus accrues to labor.[16] Since the demand curve is horizontal, the surplus does not accrue to consumers. By elimination, since there is only the single factor, it must go to labor. However, if there were capital in production which were not completely elastically supplied to the <u>industry</u>, if labor were elastically supplied to the industry at a given level of expected utility, and if the good produced were perfectly elastically demanded, then the entire surplus would accrue to the owners of capital. Alternatively, if both capital and labor were perfectly elastically supplied to the industry, but demand was not horizontal, the entire surplus would accrue to consumers. It should be evident from these three cases that the determination of the disposition of the surplus from training is a standard exercise in incidence analysis à la Harberger. The disposition between factors and consumers will depend on the elasticities of supply of the factors to the industry in question, the elasticity of demand facing the industry for its product, the partial elasticities of substitution between the various factors, and the relative shares of the factors.

<u>Proposition 6</u>: The determination of the disposition of the surplus from specific and general training is a standard exercise in incidence analysis.

II. <u>Comparative Statics</u>

In this section, we provide a fairly thorough cataloguing of the comparative statics properties of the wage structure of competitive employment contracts in our hypothetical economy. We formally derive those comparative static results which were obtained in the last section using (12) supplemented by verbal argument. We also obtain a number of other results not discussed in the previous section; in particular, we examine the effects of increases in uncertainty and worker risk aversion.

To keep the analysis manageable, we retain assumptions i) and ii). We then obtain

$$EU = \int_{-\infty}^{m+g-T-w_2} U[w_1+m-T+g+j]f[j]dj + \int_{m+g-T-w_2}^{\infty} U[w_1+w_2+2j]f[j]dj. \tag{13}$$

The budget constraint of a type 1 firm is

$$m(2-q)+(1-q)(g+s)-w_1-c(s,g)-T-(1-q)w_2 = 0, \tag{14}$$

while that for a type 2 firm is given by

$$m-T+g-\bar{w} = 0, \tag{15}$$

which determines $\bar{w}$. Also,

$$q = F[\theta] = F[\bar{w}-w_2] = F[m+g-T-w_2]. \tag{16}$$

$-(m+g+T-w_2)$ is the turnover penalty, while $m+g+s-w_2$ is the turnover subsidy. The first-order conditions of a type 1 firm's profit maximization problem are:

$$w_1: \quad EU_1-\lambda = 0, \text{ and} \tag{17a}$$

$$w_2: \quad EU_2-\lambda\left((1-q)+\frac{\partial q}{\partial w_2}\tau\right) = 0 \quad \text{or}$$

$$EU_2-\lambda((1-F[\theta])-f[\theta]\tau) = 0. \tag{17b}$$

Total differentiation of (17a), (17b), and (13), using $\theta = m+g-T-w_2$ gives

$$\begin{bmatrix} EU_{11} & EU_{12} & -1 \\ EU_{12} & EU_{22}-\lambda h & -H \\ -1 & -H & 0 \end{bmatrix} \begin{bmatrix} dw_1 \\ dw_2 \\ d\lambda \end{bmatrix} = \begin{bmatrix} EU_{11}-EU_{12} \\ -U'[\Omega]f[\theta]+\lambda(h-f[\theta]) \\ H+F[\theta] \end{bmatrix} dT$$

$$+ \begin{bmatrix} EU_{12}-EU_{11} \\ U'[\Omega]f[\theta]-\lambda h \\ -H+c_g \end{bmatrix} dg + \begin{bmatrix} 0 \\ -\lambda f[\theta] \\ c_s(1-F[\theta]) \end{bmatrix} ds + \begin{bmatrix} EU_{12}-EU_{11} \\ U'[\Omega]f[\theta]-\lambda h \\ -1-H \end{bmatrix} dm,$$

where $h = 2f[\theta]+f'[\theta]\tau$, $\Omega = w_1+w_2+2\theta$, and $H = 1-F[\theta]-f[\theta]\tau$. Hereafter we suppress the argument of F, f, and f'.

It will prove instructive to examine a couple of special cases. First, for risk-neutrality,

$$\frac{dw_1}{dT} = -1, \ \frac{dw_2}{dT} = 0 \qquad \frac{dw_1}{dg} = -c_g, \ \frac{dw_2}{dg} = 1$$
$$\frac{dw_1}{ds} = -c_s, \ \frac{dw_2}{ds} = 1 \qquad \frac{dw_1}{dm} = 1, \ \frac{dw_2}{dm} = 1 \tag{19}$$

Workers are paid the value of their net marginal products, and assume the full costs and benefits of training. Second, for constant absolute risk aversion of degree $A = -\frac{U''}{U'}$,

$$\frac{dw_1}{dT} = -q-(1-q)\frac{\lambda f}{\Delta}, \ \frac{dw_2}{dT} = -1 + \frac{\lambda f}{\Delta} \qquad \frac{dw_1}{dg} = -c_g, \ \frac{dw_2}{dg} = 1$$
$$\frac{dw_1}{ds} = -\frac{\lambda f c_s}{\Delta}, \ \frac{dw_2}{ds} = \frac{\lambda f}{\Delta} \qquad \frac{dw_1}{dm} = 1, \ \frac{dw_2}{dm} = 1, \tag{20}$$

where $\Delta = AH\lambda(1-H)-U'[\Omega]f+\lambda h$ (>0 for a maximum). We discussed the effect of a change in the amount of general training provided for the constant absolute risk aversion case at some length in the previous section. $\frac{dw_2}{ds}$ may be rewritten as:

$$\frac{dw_2}{ds} = \frac{\lambda f}{\lambda f\left(1+\left(1-\frac{U'[\Omega]}{\lambda}\right)+\frac{f'}{f}\tau+\frac{AH(1-H)}{\lambda}\right)} . \qquad (21)$$

From this equation, it can be seen that the share of marginal benefits and costs of training borne by the firm is directly proportional to the degree of worker risk aversion and to f', the derivative of the quit rate with respect to the turnover penalty, as claimed in the previous section. Note also that $\frac{dw_2}{dT} = \frac{dw_2}{ds} - \frac{dw_2}{dg}$ and $\frac{dw_1}{dm} = \frac{dw_2}{dm} = 1$, which accord with arguments made in the last section. Since $\lim_{A\to 0} \Delta = \lambda f$, the results for the constant absolute risk aversion case, as the degree of risk aversion approaches zero, collapse to the risk neutrality case, as one would expect. One may rewrite (21) as $\frac{dw_2}{ds} = 1-D$, where

$$D = \frac{1-\frac{U'[\Omega]}{\lambda}+\frac{f'\tau}{f}+\frac{AH(1-H)}{\lambda}}{1+\left(1-\frac{U'[\Omega]}{\lambda}+\frac{f'\tau}{f}+\frac{AH(1-H)}{\lambda}\right)} \qquad (22)$$

is interpreted as a dampening coefficient, which equals zero when the worker is risk-neutral, and one when he is completely risk-averse. Note, however, that for intermediate values of A, D is not bounded between zero and one.

Intuitively, we expect that the more rapidly is the degree of absolute risk aversion increasing in income (or more precisely in income plus money-equivalent job satisfaction) the greater the increase in insurance purchased, i.e. the larger the turnover subsidy, in response to a change which increases workers' expected utility. Unfortunately, we have been unable to formalize this intuition.

We next examine the effects of increases in workers' risk aversion, and then of changes in the characteristics of F[j].

We may write a type 1 firm's maximization problem as

$$\max_{w_2} EU[w_1[w_2],w_2],$$

where $w_1[w_2]$ is solved for from a type 1 firm's budget constraint. Theorem 4 of Diamond and Stiglitz [1974] states:

> Let α^* be the level of control variable which maximizes $\int V(U(\phi,\alpha),\rho)dF(\phi))$. If increases in ρ represent increases in risk-aversion ---, then α^* increases (decreases) with ρ if there exists a ϕ^* such that $U_\alpha \geq (\leq 0)$ for $\phi \leq \phi^*$ and $U_\alpha \leq 0 (\geq 0)$ for $\phi \geq \phi^*$.

In our problem, j corresponds to ϕ, and w_2 to α. Thus, to obtain an unambiguous comparative static result, it is sufficient to show that there exists a j^* such that $U_{w_2} \leq 0$ for $j \leq j^*$ and $U_{w_2} \geq 0$ for $j \geq j^*$. We show below that with $j^*=\theta$, these conditions are satisfied.

From (13) and (14),

$$U[j,w_2] = \begin{cases} U[m(3-q)+g(2-q)+s(1-q)-c(s,g)-2T-(1-q)w_2+j] & \text{for } j \leq \theta \\ U[m(2-q)+(1-q)(g+s)-c(s,g)-T+qw_2+2j] & \text{for } j \geq \theta \end{cases} \tag{23}$$

Thus

$$U_{w_2}[j,w_2] = \begin{cases} -\left((1-q) + \dfrac{\partial q}{\partial w_2}\tau\right) & \text{for } j \leq \theta \\ q - \dfrac{\partial q}{\partial w_2}\tau & \text{for } j \geq \theta. \end{cases} \tag{24}$$

Since $EU_2>0$, then from (17b), $U_{w_2}<0$ for $j<\theta$. Also since $EU_1>EU_2$, then $U_{w_2}>0$ for $j>\theta$. Thus, an increase in worker risk aversion results in a decrease in w_2 and an increase in the turnover subsidy, the expected result.

The most natural form of increasing risk in the context of this problem preserves expected utility (Diamond and Stiglitz [1974]). Let e denote a shift parameter, with larger values of e indicating more expected-utility-preserving risk. Since changes in e do not affect expected utility, $EU_e=0$. We obtain

$$\begin{bmatrix} EU_{11} & EU_{12} & -1 \\ EU_{12} & EU_{22}-\lambda h & -H \\ -1 & -H & 0 \end{bmatrix} \begin{bmatrix} dw_1 \\ dw_2 \\ d\lambda \end{bmatrix} = \begin{bmatrix} -EU_{1e} \\ -EU_{2e}-\lambda(F_e+f_e\tau) \\ F_e\tau \end{bmatrix} de \tag{25}$$

Again, we focus on the case of constant absolute risk aversion. Now,

$$EU_e = \int_{-\infty}^{\theta} Uf_e dj + \int_{\theta}^{\infty} Uf_e dj, \tag{26}$$

where the arguments of U (see (13)) are suppressed to simplify notation. Integrating (26) by parts gives

$$\begin{aligned} EU_e &= UF_e\Big|_{-\infty}^{\theta} - \int_{-\infty}^{\theta} U'F_e dj + UF_e\Big|_{\theta}^{\infty} - 2\int_{\theta}^{\infty} U'F_e dj \\ &= -\int_{-\infty}^{\theta} U'F_e dj - 2\int_{\theta}^{\infty} U'F_e dj \end{aligned} \tag{27}$$

Proceeding similarly, one obtains

$$EU_{1e} = -\int_{-\infty}^{\theta} U''F_e dj = 2\int_{\theta}^{\infty} U''F_e dj, \text{ and} \tag{28a}$$

$$EU_{2e} = -U'[\Omega]F_e[\theta] - 2\int_{\theta}^{\infty} U''F_e dj. \tag{28b}$$

With constant absolute risk aversion, $EU_e=0$, (27), and (28a) together imply, $EU_{1e}=0$. Thus,

$$\frac{dw_2}{de} = \underbrace{(\lambda - U'[\Omega])F_e[\theta] + \lambda f_e[\theta]\tau)}_{(1)} - \underbrace{2\int_\theta^\infty U''F_e dj}_{(2)} \div \Delta \tag{29}$$

It follows from (29) that an expected-utility-preserving increase in risk has two effects. The quit rate effect, which results from changes in F[θ] and f[θ], is captured by term (1) and is of ambiguous sign. The pure uncertainty effect, reflected in term (2), is negative since $\Delta>0$ and

$$-\int_\theta^\infty U''F_e dj = A\int_\theta^\infty U'F_e dj < 0,$$

which follows from the definition of an expected-utility-preserving increase in risk. With more uncertainty, <u>c.p.</u>, the employment contract contains more implicit insurance.

(25) can also be used to determine the effect of small changes in F and f in the neighborhood of θ, by interpreting e as the size of a perturbation in F[θ], holding f[θ] constant, and of a perturbation in f[θ] holding F[θ] constant, respectively. For such changes $EU_{1e} \approx EU_{2e} \approx 0$. For an increase in F[θ], $\frac{dw_2}{de} = (\lambda - U'[\Omega])F_e[\theta] \div \Delta$, the sign of which is ambiguous. For an increase in f[θ], $\frac{dw_2}{de} = \lambda f_e[\theta]\tau \div \Delta > 0$. Such a change increases the responsiveness of the quit rate to the turnover subsidy, thereby increasing the marginal cost associated with an increase in the turnover subsidy and lowering the turnover subsidy.

The principal results concerning the effects of increased risk and risk aversion on the magnitude of the turnover subsidy are summarized in Proposition 7.

<u>Proposition 7</u>: In the economy of the paper, when assumptions i) and ii) apply:

a) An increase in workers' degree of risk aversion causes the turnover subsidy to increase.

b) An expected-utility-preserving increase in risk which causes no change in F in the neighborhood of θ causes the turnover subsidy to increase.

Both these results are what one would have expected. Less obvious is that an expected-utility-preserving increase in risk generally has an ambiguous effect on the size of the turnover subsidy, since it has an ambiguous effect on the quit rate and its sensitivity to the turnover penalty, both of which influence the magnitude of the turnover subsidy.

III. Concluding Comments

The object of this paper was to investigate the characteristics of multi-period employment contracts in economies in which, as a result of asymmetric information, markets for insurance against job-related contingencies are incomplete. We constructed just about the simplest possible model in which to examine the structure of such contracts. The only factor of production was labor. Laborers were ex ante identical and worked two periods. The only source of uncertainty was the quality of a worker-firm match or, equivalently, job satisfaction. Despite the simplicity of the model, the qualitative results obtained clearly generalize:

> When insurance markets against job-related contingencies are incomplete, multi-period competitive employment contracts will contain implicit insurance. Since moral hazard is present, only partial insurance is provided. The amount provided minimizes the sum of the deadweight losses associated, first, with not equalizing ex post marginal utilities of income and, second, with the distortions which arise from the provision of insurance.

In our model, insurers were able to observe only whether or not a worker quit, not his job satisfaction. As a result, insurance was provided against quitting, which resulted in moral hazard. Quitters were the unlucky workers. The insurance compensated them by paying them in excess of their net marginal product. Whenever quitters are the unlucky ones and are unable to purchase full insurance against the outcome of the random variable(s) that determines the quit decision, one expects the same result. However, if quitters are the lucky ones, which would be the case if, for instance, workers quit exclusively to take higher-paying jobs, then insurance would instead result in the firm charging a turnover tax rather than providing a turnover subsidy.

The extension of the analysis of the paper to n periods or to continuous time is conceptually, if not notationally, straightforward. Two interesting complications arise. First, the quit rate from a particular firm for workers of a given seniority depends not only on that firm's wage structure, but on those of other firms as well. With this interdependence, employment contracts still provide implicit insurance, but not the optimal amount. We return to this point later. Second, unless job satisfaction and consumption are strongly separable in utility, a worker's attitude towards risk will typically depend on his <u>job satisfaction</u> history. Thus, different employment contracts will typically be chosen by workers with the same <u>employment</u> history, some more risky, others less.

With the source of uncertainty we considered, all terminations were quits. With other sources of uncertainty, terminations might occur as a result of firing. For example, suppose the quality of a job match instead reflected the productivity of a worker <u>in a particular job</u>, which was observable only by the firm employing the worker. If implicit insurance were provided against a worker's productivity, the firm would have an incentive to state that each worker's productivity was the lowest possible consistent with the firm's actions. As a result, the firm would have to pay all its workers of a given seniority the same amount. In the absence of insurance, it would pay each group of workers (differing in seniority) the average net marginal product of that group. And its firing policy would maximize the average net marginal product of all its workers. Insurance would take the form of paying less senior workers more than their average net marginal product and reducing the firing rate.

When there is more than one source of uncertainty, one expects the effective turnover subsidy provided to workers of a given seniority to differ depending on the reason for the worker leaving the firm, to the extent this is observable. Early retirement provisions in the firm's pension, disability payments, and the conditions under which severance pay is granted all reflect this phenomenon. Relatedly, the complexity of actual employment contracts is explainable to a large degree as resulting from the contract's providing many forms of implicit insurance.

The employment contract is but one of many social institutions that involves an (implicit) multi-period contract. Friendship and family are the two most obvious other such social institutions. Just as with the employment contract, one expects these institutions to provide partial insurance against contingencies. The theory we have developed can therefore be applied to a wide variety of problems. In its current form, however, it is unrealistic in an important respect - we assumed that all individuals were <u>ex ante</u> identical. This was a useful simplifying assumption since it allowed us to abstract from adverse selection problems. However, one expects these problems to significantly influence the form of multi-period contracts, and in particular to result in less insurance being provided than would be the case if the insurer could ascertain to which risk-class each insured belonged.[17] Thus, extending the model of the paper to treat adverse selection would be most useful.

The goal of this paper was to describe competitive multi-period employment contracts. The next paper (Arnott and Stiglitz [1980]) in the series examines the efficiency properties of such contracts. It is argued there that these contracts are generally inefficient. In deciding how much insurance to provide, a firm's manager neglects to consider that the contract he offers affects the profitability of the contracts offered by firms of other types.[18] The third paper in the series modifies the structure of the model so that some workers who quit are unable to find another job immediately and consequently become unemployed, and investigates whether the amount of unemployment is efficient.

This paper also touched on a couple of issues in the traditional labor economics literature. First, Becker [1962] argued that the share of the costs and benefits of specific training borne by workers depends on workers' risk aversion, among other things. The subsequent literature on the sharing of specific training has ignored this by assuming risk-neutral workers. We formalized Becker's insight. Second, the disposition of the surplus from training appears to be an unresolved issue. We argued that determination of the disposition of this surplus is a standard exercise in incidence analysis.

FOOTNOTES

* This paper is a considerably revised version of "The Control of Labor Turnover with Incomplete Insurance Markets: The One-Group Case", Institute for Economic Research, Queen's University, discussion paper 292. It is an outgrowth of joint research with, separately, Mark Gersovitz and Joseph Stiglitz. David Card provided a most useful set of comments on an earlier draft. None of the aforementioned is responsible for remaining errors.

1. Extending the analysis of this paper to treat adverse selection would be useful. Rothschild and Stiglitz [1976] provides an excellent discussion of some of the problems which arise from adverse selection.

2. Arrow [1965], Spence and Zeckhauser [1971], and Pauly [1974] together provide a good discussion of moral hazard.

3. Ordinarily, workers with lower job satisfaction will quit and others stay. However, if a worker quits, his future job satisfaction is uncertain, while if he does not quit it is certain. It is possible that a reduction in period one job satisfaction makes a worker so much more risk-averse that he is more reluctant to quit. Thus, it may occur that the subset of levels of job satisfaction corresponding to which workers quit is not connected.

4. By continual experimentation, perhaps.

5. This expression and the results of the paper can be modified to take account of the point made in footnote 3.

6. Whether or not capital markets are perfect should be derived rather than assumed. Capital markets will be perfect if there is no risk of default. If the penalty for default on a loan made during the first period is confiscation of a worker's second-period wage plus his savings, his second period consumption is zero. If we assume that $(\mathcal{U}_{c_2}[c_1,c_2,j_1,j_2])_{c_2=0} = \infty$, where c_i, $i=1,2$ is consumption during period i, and $\mathcal{U}[\cdot]$ is the worker's direct utility function, the worker will choose not to default. Thus, the assumption of perfect capital markets is consistent with our model for at least a certain subset of utility functions.

7. Becker implicitly assumes in his argument that when there is no cost to hiring a worker and no training, workers will be paid the value of their marginal product at each point in time.

8. However, unless $F[j_1]$ is discontinuous at the θ corresponding to $m=w_2$, the marginal cost curve must start at the origin.

9. This involves no ambiguity since we have assumed that the marginal product of labor is constant. However, if there were diminishing marginal

productivity of labor, the way in which the amount of training is measured would have to be modified.

10. Presumably $c_{sg} < 0$. This is not of consequence in our analysis, however.

11. The object of the firm is to

$$\max_{w_1, w_2, s, g} EU = \int_{-\infty}^{\theta} U[w_1+\bar{w}+j]dF[j] + \int_{\theta}^{\infty} U[w_1+w_2+2j]dF[j]$$

s.t. i) $\theta = m+g-T-w_2$

ii) $m(2-q)+(1-q)(g+s)-w_1-c(s,g)-T-(1-q)w_2 = 0$

iii) $q = F[\theta]$

iv) $\bar{w} = m+g$

The first-order conditions are:

$$s: \quad \lambda((1-q)-c_s) = 0$$

$$g: \quad (EU_1-EU_2)+\lambda((1-q)-c_g-\frac{\partial q}{\partial g}(m+g+s-w_2)) = 0$$

$$w_1: \quad EU_1-\lambda = 0$$

$$w_2: \quad EU_2-\lambda((1-q)+\frac{\partial q}{\partial w_2}(m+g+s-w_2)) = 0$$

Substituting the F.O.C.'s with respect to w_1 and w_2 into that for g gives $\lambda(1-c_g) = 0$. Thus, at least with assumptions i) and ii), the formula characterizing the optimal amounts of specific and general training are the same as in the absence of uncertainty.

12. Results simplify with an additive random variable when there is constant absolute risk aversion, and with a multiplicative r.v. when there is constant relative risk aversion. Here the assumption of perfect substitutability between consumption and job satisfaction (the random variable) has the effect of making the r.v. additive. This is why the results of the paper are so neat when there is constant absolute risk aversion.

13. Becker incorrectly argued that in response to a unit increase in general training, workers' wages will <u>always</u> fall by the cost of the general training in the training period, and rise by the increase in marginal product in subsequent periods. Becker's arguments are however correct for the

special case of perfect capital markets, perfect substitutability between consumption and job satisfaction, and constant absolute risk aversion.

14. Recall that under the stated assumptions, $\frac{\partial q}{\partial w_1} = 0$ and $\theta = -p$.

15. Becker [1962, pp. 19-21] discussed the sharing of the costs and benefits of specific training in the context of an economy in which a job separation could be due either to a worker being fired or to his quitting.

> If a firm had paid for the specific training of a worker who quit to take another job, its capital expenditure would be partly wasted, for no further return could be collected. Likewise, a worker fired after he had paid for specific training would be unable to collect any further return and would suffer a capital loss. The willingness of workers or firms to pay for specific training should, therefore, closely depend on the likelihood of labor turnover.--
> -- The shares [of specific training costs paid] depend on the relation between quit rates and wages, layoffs and profits, and on factors not discussed here, such as the cost of funds, attitudes toward risk, and desires for liquidity.

If our model were elaborated to include firing, it would substantiate Becker's argument.

Hashimoto [1978] solves for the optimal sharing rule in an economy with risk-neutral firms and workers. The quality of a job match has two dimensions - worker productivity and job satisfaction. The proportion of costs and benefits from specific training assumed by workers is determined so as to maximize the sum of output and output-equivalent job satisfaction. This will occur when the average gain in job satisfaction from marginal workers (marginal quitters or firees) leaving the firm just offsets the average loss in output from these workers quitting.

Donaldson and Eaton [1976] argue that the firm will impose a very high turnover penalty to protect their investment in specific human capital. Hashimoto points out that their argument is erroneous, since it ignores that the higher is the turnover penalty, the lower is (the analog to) average worker job satisfaction (in his model), and the higher will wages have to be to attract workers to the firm.

16. Becker assumes that the entire surplus accrues to labor.

17. Adverse selection also leads to the possibility of non-existence of equilibrium in the Rothschild-Stiglitz [1976] sense.

18. This argument applies in an economy in which workers are employed for more than two periods. However, in a two-period economy, since the type 2 firm wage, $\overline{w}$, is fully determined by its budget constraint, firms of neither type can influence the profitability of firms of the other type.

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