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Optimal Excise Taxes in Exhaustible Resource Exploitation

and

The Intertemporal Externality in the Dynamic Common Property Renewable Resources Problem

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by

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Abstract

Optimal Excise Taxes in Exhaustible Resource Exploitation

Public authorities are charged with raising Z dollars, in present value, from a competitive industry exploiting an exhaustible resource. We derive Ramsey-type excise tax rules for such a problem. The L.C. Gray case of a single mine is then investigated and also the case of an industry experiencing declining quality of minerals.

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Optimal Excise Taxes in Exhaustible Resource Exploitation

1. Introduction

Suppose a government desires to tax each ton of output from an exhaustible mineral stock in order to raise Z dollars in total. What is the appropriate schedule of taxes, ton by ton? This is a Ramsey-type tax problem involving time and exhaustibility. We investigate it in variants of a Hotellingesque framework. First we solve for the appropriate tax schedules for the case of zero extraction costs (one could have constant unit costs without altering the analysis). We consider a Hotelling [1931] situation in which the industry price rises in the face of a stationary negatively sloped demand schedule and then we consider the L.C. Gray [1914] case of output price constant and stationary marginal costs positively sloped. We then take up the case of costs varying over time, say indexed by stock size, as in Levhari and Leviatan [1977].

Recent results on the taxation of exhaustible resources are reported in Dasgupta and Heal [1979, Chapter 12] and Dasgupta, Heal and Stiglitz [1980]. In these investigations, the issue focussed on was how various taxes change the time path of extraction from the competitive norm. Optimal taxation of a Ramsey sort was not taken up. For the case of a linear demand schedule, we observe that relative to the situation with a uniform excise tax, the period of exploitation of a stock is lengthened under an optimal excise tax scheme.

2. Optimal Excise Taxes: The Hotelling Model of the Industry

The framework for deriving optimal excise taxes is essentially of the Cournot-Nash sort. Agents are assumed to optimize in the face of taxes treated as parameters in their decision-making and a tax or revenue authority is assumed to optimize social welfare in the face of schedules of prices net of taxes, and quantities yielded by an agent's decision. For a Hotelling model, each agent's decision rule is the first order condition to the optimization problem: maximize by choice of q(t)

$$V = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^{t} \{B(q(t)) - \tau(t)q(t)\}$$

subject to

$$\sum_{t=0}^{T} q(t) \leqq S_0$$

where r is the discount or interest rate.

B(q(t)) is a benefit measure denominated in dollars (say the area under a demand schedule or gross consumer surplus) and $\frac{dB}{dq} = p$.

q(t) is the quantity of mineral mined at time t from the stock S_0 at t=0.

t is an index of the period in time.

$\tau(t)$ is the excise tax in \$ per unit of mineral.

T is the number of periods, endogenous to the problem, in the optimal program.

The first order condition for an interior solution is

$$p(t) - \tau(t) = \left(\frac{1}{1+r}\right)^{T-t} [p(T) - \tau(T)]$$

where p(T) is the cut-off price or price (equalling unit cost) of the backstop technology. Thus p(T) is exogenously given and price p(t) rises over time to p(T) at which time the stock is exhausted. (We gloss over the issue of "large" stocks and exploitation in the phase of constant prices, as did Hotelling. See for example Gilbert [1978].)

This condition indicates that rent, $p(t) - \tau(t)$, per ton rises to the terminal rent at a rate equal to the rate of interest (the so-called r per cent rule). Moreover, this condition represents the decision rule for each competitive mine operator. It indicates how much to extract in each period of the program. The tax authority takes this rule as given and uses the fact that $\left(\frac{dp(t)}{dq(t)}\right) dq(t) - d\tau(t) = 0$ or

$$\frac{dq(t)}{d\tau(t)} = \frac{dq(t)}{dp(t)} \qquad\qquad 0 < t < T.$$

where $\frac{dq(t)}{dp(t)}$ is the slope of the demand schedule for mineral facing the industry ($\frac{dq}{dp}$ is assumed to be negative).

The tax or revenue authority is charged with raising <u>in present value Z dollars of revenue</u> from this industry. Thus $Z = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t \tau(t)q(t)$. Its problem is to raise this revenue by choice of $\tau(t)$ while maximizing social welfare. That is, maximize by choice of $\tau(t)$,

$$W = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t \{B(q(t)) - \tau(t)q(t)\}$$

subject to

$$\sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t [\tau(t)q(t)] = Z$$

where $q(t)$ is now a function of $\tau(t)$ via the decision function of each agent. Assuming that an interior solution is optimal, the necessary condition for a maximum is

$$p(t)\,\frac{dq(t)}{dp(t)} - \tau(t)\,\frac{dq(t)}{dp(t)} - q(t) = \lambda\left\{\tau(t)\,\frac{dq(t)}{dp(t)} + q(t)\right\}$$

$$0 \leqq t < T$$

where λ is the Lagrangian multiplier on the "Z constraint" and is the shadow price of a unit of tax revenue. Since $B(q(t))-\tau(t)q(t)$ is consumer surplus ($\equiv C(t)$) at t and $\tau(t)q(t)(\equiv X(t))$ is total tax revenue at t, the first order condition can be expressed as

$$\frac{dC(t)}{dX(t)} = \lambda \qquad\qquad 0 \leqq t < T$$

or an extra \$ of taxes at time t "costs", in terms of consumer surplus foregone, the same at each t. In terms of elasticities of demand, the first order condition or optimal tax formula is

$$\frac{\frac{\tau(t)}{p(t)}\,\varepsilon(t)+1}{\varepsilon(t)} = \frac{1}{1+\lambda} \qquad\qquad 0 \leqq t < T$$

where $\varepsilon(t) \equiv \frac{dq(t)}{dp(t)} \cdot \frac{p(t)}{q(t)}$ is the elasticity of demand.

Example: Linear Demand

For the case of demand linear in price or $q(t) = \beta-\alpha p(t)$, the optimal tax formula becomes

$$\tau(t) = \frac{[1+\lambda]\frac{\beta}{\alpha} - \lambda Z(t)}{[1+2\lambda]}$$

where $Z(t) = [p(T)-\tau(T)]\left(\frac{1}{1+r}\right)^{T-t}$. Observe that $p(T) = \beta/\alpha$ and $q(T) = 0$. For the case of $\beta=10$, $\alpha=1$, $r=.1$, $\tau(T)=5$, and $\lambda=1$, we solved in a backward recursion. The results are reported in Table 1.

Table 1*

t	τ	p	q	$\left(\frac{1}{1+r}\right)^t \tau q$	$\left(\frac{1}{1+r}\right)^t \{B(q)-\tau q\}$
10	5.000	10.000	0.000	0.000	0.000
9	5.1548485	9.700303	0.299697	0.65518527	0.59677673
8	5.2925895	9.4248209	0.5751791	1.4201357	1.1859497
7	5.4178087	9.1743827	0.8256173	2.295375	1.7664516
6	5.5316442	8.9467115	1.0532885	3.2888606	2.34356
5	5.6351311	8.7397377	1.2602623	4.4096238	2.922521
4	5.7292101	8.5515798	1.4484202	5.667853	3.5085977
3	5.8147365	8.3805271	1.6194729	7.0749874	4.1071186
2	5.8924877	8.2250246	1.7749754	8.643819	4.723524
1	5.9631706	8.0836587	1.9163413	10.388609	5.363419
0	6.0274279	7.9551443	2.0448557	12.32522	6.0326196
			12.81811	56.169669	32.550538

* $q=\beta-\alpha p$; $\beta=10$, $\alpha=1.0$, $r=.1$, $\lambda=1.0$, $B(q) = \frac{1}{\alpha}\{\beta q-.5q^2\}$.

The interesting qualitative feature of the optimal tax schedule is the decline over time of tax per unit output from \$6.027 to \$5.000. As one moves along a linear demand schedule toward q=0, the elasticity eventually starts rising in absolute value, and one assumes, given earlier results on optimal excise taxation, that taxes should fall. Indeed they do in our example but our framework has the effective demand schedule shifting over time because of discounting and this should also result in τ declining over time. In summary, $\tau(t)$ does decline over time but we do not have a pithy explanation for this result.

For purposes of comparison, we solve for a program which, given the same stock, yields the same revenue in present value but with a uniform tax. Discounted welfare should be lower since the tax is not set optimally. Because time is treated discretely, it is not possible to construct a perfectly comparable program. It turns out that we need a program of about 8.5 periods. However we report on the appropriate comparison program in Table 2 and note that our comparison is imprecise though qualitatively correct and of interest. The same parameters are used in generating the results in Table 1.

Table 2*

t	τ	p	q	$\left(\frac{1}{1+r}\right)^t \tau q$	$\left(\frac{1}{1+r}\right)^t \{B(q)-\tau q\}$
9	5.7	10.000	0.000	0.000	0.000
8	5.7	9.609091	0.390909	1.039463	0.7485129
7	5.7	9.2537191	0.74628091	2.1828726	1.5038306
6	5.7	8.9306537	1.0693463	3.4406232	2.2728195
5	5.7	8.6369579	1.3630421	4.8241489	3.0624703
4	5.7	8.3699617	1.6300383	6.3460271	3.8799619
3	5.7	8.127238	1.872762	8.0200924	4.7327256
2	5.7	7.90658	2.09342	9.8615653	5.6285147
1	5.7	7.7059818	2.2940182	11.887185	6.575472
0	5.7	7.5236198	2.4763802	14.115367	7.582206
			13.936197	61.717345	35.986514

* See Table 1 for parameters.

As we noted above, the examples are not perfectly comparable because the one in Table 2 should hold for roughly 8.5 periods. However the qualitative features are that relative to a regime of uniform taxation, under optimal excise taxation, the length of the program is greater, price starts higher and welfare is slightly higher. (To obtain this last result, we halve the discounted consumer surplus at period 0 in Table 2 in order to shorten the program appropriately, and observe then that total discounted welfare becomes 32.195411 which is less than the 35.550538 for the optimal program.) Some experimentation with different uniform taxes and different horizons led to the conclusion that the above example reported in Table 2 is the correct comparison program but that a horizon with roughly 8.5 periods is required.

3. Optimal Excise Taxes: The L.C. Gray Model of the Mine

In the classic Gray model of the mine, price of output is fixed and the mine owner with a fixed stock adjusts his output in the face of a stationary extraction cost structure so that his rent rises at a rate equal to the rate of interest. The cut-off occurs at the point at which marginal cost of extraction currently in effect at time t equals the average cost. The mine owner's objective function is the maximization, by choice of q(t), of

$$\pi = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^{t} \{pq(t)-C(q(t))-\tau(t)q(t)\}$$

subject to

$$\sum_{t=0}^{T} q(t) \leqq S_0$$

and his decision function (the first order condition for the maximization problem) is

$$p-c(t)-\tau(t) = \{p-c(T)-\tau(T)\}\left(\frac{1}{1+r}\right)^{T-t}$$

where $c(t)$ is the marginal cost of extraction $(c(t) \equiv dC/dq)$.

$\tau(t)$ is the tax per unit output.

$c(T)$ is given by the condition $c(T) = C(q(T))/q(T)$. The mine owner's decision function yields the condition $-\frac{dc(t)}{dq(t)} dq(t) = d\tau(t)$ or

$$\frac{dq(t)}{d\tau(t)} = -\frac{dq(t)}{dc(t)}$$

where $$\frac{dc(t)}{dq(t)} \equiv \frac{d^2C(q(t))}{dq(t)^2} .$$

The tax-revenue authority selects excise taxes in order to maximize discounted social welfare[1] subject to raising in present value terms Z dollars of revenue. It maximizes, by choice of $\tau(t)$,

$$W = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t \{pq(t) - C(q(t)) - \tau(t)q(t)\}$$

subject to

$$Z = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t \tau(t)q(t).$$

Assuming an interior solution exists, the first order condition and rule for optimal excise taxes is

$$p\frac{dq}{dc} - c\frac{dq}{dc} - \tau\frac{dq}{dc} + q(t) = \lambda\{\tau\frac{dq}{dc} - q\}$$

$$0 \leqq t < T$$

which in terms of elasticities becomes

$$\left[\frac{1 - \frac{\tau(t)}{c(t)}\eta(t)}{\eta(t) - \frac{p}{c(t)}\eta(t)}\right] = \frac{1}{1+\lambda}\ ; \qquad 0 \leqq t < T$$

where $\eta(t) \equiv \frac{dq}{dc} \cdot \frac{c(t)}{q(t)}$ and $\eta(t) > 0$.

Example: Linear Marginal Cost

We assume $C(q) = \gamma q + \frac{\delta}{2} q^2$. Then $\frac{C}{q} = \gamma + \frac{\delta q}{2}$ and $c = \gamma + \delta q$ yielding $\frac{dq}{dc} = \frac{1}{\delta}$. For this specification the optimal taxes are

$$\tau(t) = \frac{[p-\gamma][1+\lambda]-\lambda Y(t)}{[1+2\lambda]} .$$

where $Y(t) \equiv \{p-c(T)-\tau(T)\} \left(\frac{1}{1+r}\right)^{T-t}$ and $c(T) = \gamma$ and $q(T) = 0$. Observe that the formula for the optimal taxes has the identical structure to that for the Hotelling model with the linear demand specification. One can then adapt the numerical example in Tables 1 and 2 to this L.C. Gray model with excise taxes simply by labelling variables differently.

4. Optimal Excise Taxes and Shifting Extraction Cost Schedules

The classic case of exhaustible resource use and quality variation is formalized by assuming that extraction costs shift up as the stock of mineral is depleted. For this situation there are two important subproblems. First the physical stock may be exhausted at the terminal date in the program and second the extraction costs may rise sufficiently rapidly so that physical exhaustion is not reached at the terminal date but marginal profitability becomes negative for exploitation beyond the terminal date. Formally, there are two distinct end-point conditions depending on the parameters of the problem in hand. It is helpful to label the first case as one of physical exhaustion and the second as one of economic exhaustion of the stock. Under a competitive industry, the small deposit owner's decision to produce mineral or not at time t is the first order condition to the following optimization problem. (This is a discrete time formulation of that in Levhari and Leviatan [1977] with excise taxes inserted.) Maximize by choice of q(t)

$$V = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^{t} \{B(q(t))-C(q(t);\ S(t))-\tau(t)q(t)\}$$

subject to

$$\sum_{t=0}^{T} q(t) \leqq S_0$$

where $S(t) = S_0 - \sum_{j=0}^{t-1} q(j)$ and it is understood that $\sum_{j=0}^{0-1} q(j) = 0$ and $\sum_{j=0}^{0} q(j) = q(0)$. $S(t) = S_0 - \sum_{j=0}^{t-1} q(j)$ is the stock remaining at the beginning of time t. It is assumed that $\partial C/\partial q(t) > 0$ and $\frac{\partial^2 C}{\partial q(t)^2} < 0$ or that current extraction costs are increasing and concave in current quantity extracted and that $\partial C/\partial S(t) < 0$ so that larger stocks remaining in the ground imply lower current extraction costs. This is the formalization of the idea that the quality of mineral declines as cumulative quantity extracted increases.

The first order condition, assuming physical exhaustion occurs, is[2]

$$p(t)-\tau(t)-C_q(t) = \left(\frac{1}{1+r}\right)^{T-t} [p(T)-\tau(T)-C_q(T)] - \sum_{j=1}^{T} \left(\frac{1}{1+r}\right)^{(j+1-t)} C_S(j+1)$$

where $C_q(t) \equiv \frac{\partial C(q(t);S(t))}{\partial q(t)}$ and $C_S(t) \equiv \frac{\partial C(q(t);S(t))}{\partial S(t)}$ and $C_S(T+1) = 0$. The interpretation of this condition is that rent per ton at time t equals the discounted rent at the end of the program minus the sum of discounted "degregation charges" (a term for $C_S(t)$ due to Solow and Wan [1976]) relevant beyond t. For the case of economic exhaustion, rent at T is zero and the deposit owner's decision rule is

$$p(t)-\tau(t)-C_q(t) = -\sum_{j=t}^{T}\left(\frac{1}{1+r}\right)^{(j+1-t)} C_S(j+1); \quad 0 \leqq t < T. \qquad (1)$$

We assume that p(T) is given exogenously by the cost of the backstop technology so that for the case of economic exhaustion, $p(T)-\tau(T)-C_q(T)=0$, or terminal rent is zero.

We will complete the analysis under the assumption that economic exhaustion is the case in hand (the reader can readily work out the case for physical exhaustion). Thus equation (1) is the agent's relevant decision rule. Let us jump ahead for a moment to consider the solution of the problem assuming that the formulae for the $\tau(t)$'s are known and \$Z, in present value, must be raised. Since p(T) is given exogenously, q(T) is given exogenously also from the stationary demand schedule. Given S_0 exogenous, there will in general be a unique value of $\tau(T)$ which will make $p(T)-\tau(T)-C_q(q(T);S(T)) = 0$ and have \$Z raised. Alternatively, there are only T $\tau(t)$'s which can be chosen given S_0, q(T), p(T) and Z.

The tax-revenue authority takes the mineral extracting agents decision rule as given and perturbs it to determine the response to small changes in the T $\tau(t)$'s given $\tau(T)$ assumed optimally chosen. That is, totally differentiating the T equations in (1), yields

$$\frac{dp(t)}{dq(t)}\,dq(t) - C_{qq}(t)dq(t) + C_{Sq}(t)\sum_{i=0}^{t-1} dq(i) + \sum_{j=t}^{T-2}\left(\frac{1}{1+r}\right)^{(j+1-t)} C_{Sq}(j+1)dq(j+1)$$

$$-\sum_{j=t}^{T-1}\left(\frac{1}{1+r}\right)^{(j+1-t)} C_{SS}(j+1)\sum_{i=0}^{j} dq(i) = d\tau(t)$$

$$t = 0,\ldots,T-1 \qquad (2)$$

This is a system of T linear equations in dq(0),...,dq(T-1) and can be solved for $\frac{dq(k)}{d\tau(t)}$ (k=0,...,T-1; t=0,...,T).

The tax-revenue authority now takes these responses, $\frac{dq(k)}{d\tau(t)}$'s, and computes the T optimal $\tau(t)$'s as solutions to the constrained welfare optimization problem: maximize by choice of $\tau(t)$'s,

$$W = \sum_{t=0}^{T}\left(\frac{1}{1+r}\right)^{t}\{B(q(t))-\tau(t)q(t)-C(q(t),S(t))\}$$

subject to

$$\sum_{t=0}^{T}\left(\frac{1}{1+r}\right)^{t}\tau(t)q(t) = Z$$

where now q(t) is a function of $\tau(0),\ldots,\tau(T)$ for t=0,...,T-1 and the derivatives $\frac{dq(k)}{d\tau(t)}$ are given from the solution to (2). The first order condition yields the optimal excise tax formulae

$$\frac{\sum_{t=0}^{T-1}\left(\frac{1}{1+r}\right)^{t}\left\{p(t)\frac{dq(t)}{d\tau(k)} - \tau(t)\frac{dq(t)}{d\tau(k)} - C_q(t)\frac{dq(t)}{d\tau(k)} + C_S(t)\sum_{i=1}^{t-1}\frac{dq(i)}{d\tau(k)}\right\} - q(k)\left(\frac{1}{1+r}\right)^{k}}{\sum_{t=0}^{T-1}\left\{\left(\frac{1}{1+r}\right)^{t}\tau(t)\frac{dq(t)}{d\tau(k)}\right\} + \left(\frac{1}{1+r}\right)^{k}q(k)} = \lambda$$

$$(k=0,\ldots,T) \qquad (3)$$

where λ is the Lagrangian multiplier corresponding to the "Z constraint" and is the shadow price of a dollar of tax revenue. Clearly the formulae for the optimal $\tau(t)$'s is complicated since the problem does not decompose into parts such that the calculation of the tax for one period can be determined relatively independently of the calculation for another period. Now there is an essential simultaneity in the computation of optimal taxes in general since "stock effects" are present.[3]

Two closing remarks. One can redevelop the L.C. Gray "theory of the mine" problem with "stock effects" or indices of mineral quality variation in a way analogous to our redevelopment of the Hotelling analysis. Clearly the optimal tax formulae will be very complex since again an essential simultaneity of the tax equations will be present. The procedure for deriving such tax formulae is the same as the one we set out for deriving formulae for the Hotelling problem with mineral quality variation. Secondly, we note that though as Dasgupta, Heal and Stiglitz observe, any time path of depletion can be obtained with appropriate taxes, we have isolated a second best socially optimal time path - one that maximizes social welfare subject to the distortion of raising Z dollars of revenue by excise taxes.

FOOTNOTES

1. The L.C. Gray "theory of the mine" deals with a profit maximizing, price-taking firm. The industry is left aside. Thus when we speak of the tax-revenue authority maximizing social welfare subject to a revenue constraint, we are dealing with a restrictive notion of social welfare. Consumers' surplus is neglected.
2. This is equation 7 in Levhari and Leviatan with a term inserted for the excise tax.
3. Dasgupta, Heal and Stiglitz elected not to take up mineral taxation for cases involving "stock effects". Clearly elegant expressions are difficult to come by.

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Abstract

The Intertemporal Externality in the Dynamic Common Property Renewable Resource Problem

We develop a model in which agent i's "catch" in time t affects his and all other agents' "catches" in time t+1. There is no static externality in this dynamic model of the common property, only an intertemporal externality. We demonstrate that common property regimes correspond to lower steady state stocks and lower welfare as compared to the competitive solution with no externalities.

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The Intertemporal Externality in the Dynamic Common Property Renewable Resource Problem

Dasgupta and Heal [1979; pp. 55-73] have recently provided some micro-foundations for the notion of common property developed by Gordon [1954], Weitzman [1974] and others. Their analysis like the earlier ones treats common property as market failure in a static context.[1] Essentially, firm i realizes that its current "catch" is influenced by the quantities taken by all other agents exploiting a common property renewable resource like the fishery. It is the explicit cost interdependencies which represent the externality caused by the institution of common property ownership in the traditional view. But this approach seems to miss the basic intuition of the externality in fishing under a common property regime. That intuition is: each agent acts under the assumption that there is no use foregoing some "catch" today in the interests of having a reasonably sized stock tomorrow because what he does not catch today will be caught by another agent - entry is "free". This intuition can be embodied readily in a formal model of the fishery under free access or common property - an explicitly dynamic model - and the traditional steady state free access equilibrium with zero rent can be obtained. We will observe that free access leads to over-exploitation of the stock relative to a private ownership regime.[2] In this formulation, market failure occurs with free entry because agent i in foregoing some "catch" today (practising saving with respect to the stock of fish) can expect to reap in the future the discounted average payoff of such "investment" rather than his discounted marginal payoff (average being defined over the number of agents in a steady state). The course to overcome the market failure is to have the appropriate tax on fish caught. The formula for the correct tax will involve time and the discount rate explicitly.

We proceed to make formal our intuitions concerning common property with two agents. We will compare the free access or common property outcome with two agents to the planning solution for the problem. (The N>2 agent problem is discussed below.) Each agent under the common property regime maximizes discounted profits from fishing in

$$\pi^i = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \{p-c(S_t,q_t^i)\}q_t^i \qquad (i=1,2)$$

where π^i is the present value of profit for agent i

r is the interest or discount rate

p is the price of a unit of fish caught

S_t is the stock of fish exploited at time t

q_t^i is the quantity of fish caught by agent i at time t

$c(\cdot,\cdot)$ is the cost per unit of fish caught. It depends on the stock (a proxy for the "density" of fish in the fishing ground) and on the current quantity caught. We explicitly omit the familiar interdependency of current costs of i on current catches of i <u>and</u> j emphasized by for example Dasgupta and Heal. We are attempting to focus attention on the intertemporal interdependencies. It is assumed that $\frac{\partial c}{\partial S_t} \equiv c_{S_t} < 0$, $\frac{\partial c}{\partial q_t^i} \equiv c_{q_t^i} \leqq 0$.

The exploitation-biological growth dynamics are represented by

$$S_{t+1}-S_t = \phi(S_t)-q_t^1-q_t^2 \qquad (t=0,\ldots,\infty)$$

where $\phi(S_t)$ is the natural growth of the stock indicating net births over deaths of fish. We will appeal to the familiar logistic form below: $\phi(S_t) \equiv aS_t(1-(S_t/K))$ where a>0 and K>0 is the "carrying capacity" of the fishing growth. Note that $\phi(0)=\phi(K)=0$ and $\frac{d\phi}{dS_t} \equiv \phi_{S_t} \gtrless 0$ as $S_t \lessgtr (K/2)$.

The interdependency leading to an intertemporal externality in the above model is that agent i's costs in period t+1 are affected adversely by agent j's catch in period t. The effects are reciprocal and we assume here that the size of the agents and their costs are the same.

The first order conditions for profit maximization yield the simultaneous recursive equations

$$\left[\frac{1+\phi_{S_{t+1}}}{1+r}\right]\left[p-c(S_{t+1},q^i_{t+1})-q^i_{t+1}c_{q^i_{t+1}}\right] = \left[p-c(S_t,q^i_t)-q^i_t c_{q^i_t}\right]+\left(\frac{1}{1+r}\right)q^i_{t+1}c_{S_{t+1}} \qquad (i=1,2) \qquad (1)$$

We move to consider steady state solutions, now. Let $\hat{q}$ be the steady output <u>for one agent</u> and $\hat{S}$ be the steady state stock. Now $\hat{q} = \frac{\phi(\hat{S})}{2}$. The basic equation characterizing our common property steady state is then, from (1),

$$\left[\frac{1+\phi_{\hat{S}}}{1+r}\right]\left[p-c(\hat{S},\frac{\phi}{2})-\hat{q}c_{\hat{q}}\right] = \left[p-c(\hat{S},\frac{\phi}{2})-\hat{q}c_{\hat{q}}\right] + \left(\frac{1}{1+r}\right)\hat{q}c_{\hat{S}} \qquad (2)$$

One can by the same procedure above obtain the basic equation for the social optimum steady state (one treats the problem as if there is just one firm maximizing profits). It is[3]

$$\left[\frac{1+\phi_S}{1+r}\right]\left[p-c(S,\phi)-qc_q\right] = \left[p-c(S,\phi)-qc_q\right] + \left(\frac{1}{1+r}\right)qc_S \qquad (3)$$

Existence of interior solutions requires additional assumptions on the properties of $\phi(S)$ and $c(S,q)$. We proceed to specialize without searching for generality at this point. We let $\phi(S)$ be the logistic set out above and $c(S,q)\equiv q/S$ or ϕ/S in a steady under optimal planning. This form for costs is used extensively in Clark [1976]. Our steady state equations (2) and (3) become

$$\left[\frac{1+\phi_{\hat{S}}}{1+r}\right]\left[p - \frac{\phi}{\hat{S}}\right] = \left[p - \frac{\phi}{\hat{S}}\right] - \left(\frac{1}{1+r}\right)\left(\frac{\phi}{2\hat{S}}\right)^2 \qquad (4)$$

and

$$\left[\frac{1+\phi_S}{1+r}\right]\left[p - \frac{2\phi}{S}\right] = \left[p - \frac{2\phi}{S}\right] - \left(\frac{1}{1+r}\right)\left(\frac{\phi}{S}\right)^2 \qquad (5)$$

respectively, where $\phi_S=a[1-(2S/K)]$ and $\phi/S=a[1-(S/K)]$.

Note that $(\phi/S)=0$ for $S=K$ and $(\phi/S)=\phi_S=a$ for $S=0$. We manipulate (4) and (5) to obtain the same RHS, namely $\left[p - \left(\frac{2\phi}{S}\right)\right] - \left(\frac{1}{1+r}\right)\left(\frac{\phi}{S}\right)^2$, plotted as schedule hb in Figure 1. It increases in S. The new LHS of (4) is $\left(\frac{1+\phi_S}{1+r}\right)\left[p - \frac{\phi}{S}\right] - \left(\frac{\phi}{S}\right) - \left[\frac{3}{4}\left(\frac{1}{1+r}\right)\left(\frac{\phi}{S}\right)^2\right]$ plotted as schedule ef in Figure 1. The LHS of (5) is $\left(\frac{1+\phi_S}{1+r}\right)\left[p - \frac{2\phi}{S}\right]$, plotted as schedule gf in Figure 1. Routine calculation reveals each schedule to be concave in S.[4] hb reaches a value of p as S=K. gf and ef reach a value of $\left(\frac{1-a}{1+r}\right)p$ for S=K at f and clearly $\left(\frac{1-a}{1+r}\right)p > 0$ for $0<a<1$ and $p>0$, which we assume. If $p-1+a<0$, then gf and ef will be increasing in S provided g and e are negative at S=0. Now g corresponds to a negative value provided p-2a<0 which we assume. Point e corresponds to a value less than g if $r > \frac{1}{4}a$. Finally point e lies above point h if $\frac{r}{a} > \frac{2a-p}{3a-p}$. (These conditions are satisfied, for example if r=.7, a=.8, p=.1). We have in light of Figure 1,

Proposition 1 (Over exploitation of the stock under common property): $S^{cp} < S^{so}$ where S^{cp} and S^{so} are the stocks in steady state under common property and social optimality respectively.

Figure 1

Figure 2

In a free access/common property equilibrium (steady state), agents, assumed identical, will enter until $\left[p-c\left(S, \frac{\phi}{N}\right)\right] = 0$ where N is the number of agents. One can work out the equilibrium for any N>2 by replacing $\phi(S)/2$ above by $\phi(S)/N$. The welfare result is of course that consumer and producer surplus is higher under a socially optimal plan than under free access/common property. We can demonstrate this result for N=2 and $\left[p-\left(\frac{\phi}{2S}\right)\right] = 0$ in Figure 2. Existence of 0<S<K for $\left[p-\left(\frac{\phi}{2S}\right)\right] = 0$, given our form for $\phi(S)$ above, requires $0 < \frac{a-2p}{a} < 1$;

then $S = \left(\frac{a-2p}{a}\right) K$. We insert the linear schedule kb in Figure 2 corresponding to $\left[p-\left(\frac{\phi}{2S}\right)\right]$. In Figure 2 we assume that the steady state under common property has entry "complete" at 2 agents so that S^{cp} satisfies $\left[p-\left(\frac{\phi}{2S}\right)\right] = 0$. Linear schedule mb corresponds to $\left[p-\frac{\phi}{S}\right]$. Scrutiny of Figure 2 yields

Proposition 2 (Welfare Cost of Common Property Equilibrium): If S^{cp} corresponds to a situation zero profit for an agent (i.e. $p-\left(\frac{\phi}{2S}\right) = 0$) then S^{so} corresponds to a situation of positive producer and consumer surplus (i.e. $\left(p-\frac{\phi}{S}\right) > 0$).

We have then reworked the notion of a common property renewable resource equilibrium under exploitation and derived notions of over-exploitation and welfare costs in an explicitly dynamic model with no externalities of the traditional static kind. One could obviously rework the analysis with the familiar static common property externalities inserted. One could have current catch of agent i affected by the current catch of agents other than i and/or the current catch of agent i affected by the number of agents currently exploiting the resource. Then there would be two sources of market failure under common property - a static externality and our intertemporal externality.

We note, as a post script, that foregoing fishing is a form of saving and of investment in the stock for future use. However our model differs essentially from a general many person consumption-savings analysis involving the pooling of savings. In our model the stock enters directly into each element in the sum in the objective function and the pooling of savings affects the magnitude of the stock. There are no such direct "stock effects" in a many person Ramsey-type model of accumulation and over-consumption does not occur in these models.

FOOTNOTES

1. Exceptions are those studies which view an interior solution as non-existent or dynamically unstable. See for example Smith [1975], Clark [1976], and Hartwick [1979].

2. Khalatbari [1977] has a dynamic common property model dealing with two exploiters of an exhaustible pool of oil. The model and the context are quite different from ours for the fishery but Khalatbari also observes that the presence of a common property setting leads to over-exploitation of the stock - in this latter case, overly rapid exploitation relative to that under a socially optimal plan. See Dasgupta and Heal [1970, pp. 372-375]. We note also that Clark [1976; p. 43] points out that for some models the common property equilibrium can be viewed as the socially optimal outcome in which the discount rate tends to infinity. Our model is an attempt to provide micro-foundations for why common property regimes display a "bias" to the present (high discount rates) relative to the future.

3. This of course can be made into the familiar steady state intertemporal arbitrage rule if r is placed on the RHS.

4. The slopes of schedule hb is $\frac{2a}{(1+r)K}\left[1+r+a-\frac{aS}{K}\right]$; of schedule ef is $\frac{2a}{(1+r)K}\left[\frac{9a}{4}+1-p+\frac{r}{2}-\left(\frac{11}{4}\right)\frac{aS}{K}\right]$; and of schedule gf is $\frac{2a}{(1+r)K}\left[3a+1-p-\frac{4aS}{K}\right]$; all <u>linear in S</u>. Thus, given our other assumptions on parameter values, no two schedules can intersect more than once. Also as $S \to K$, the positive slope of ef must exceed the positive slope of gf. Given point g above point e by assumption, schedules gf and ef cannot intersect.

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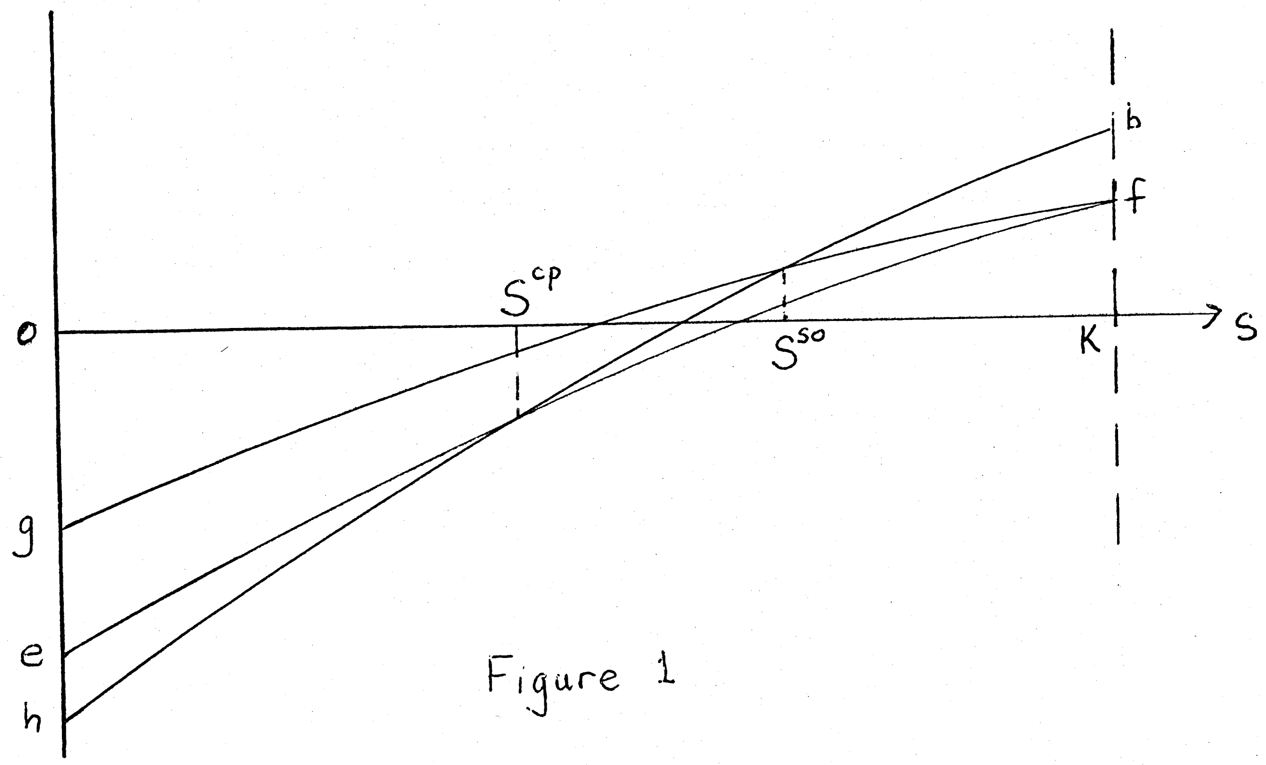

Figure 1

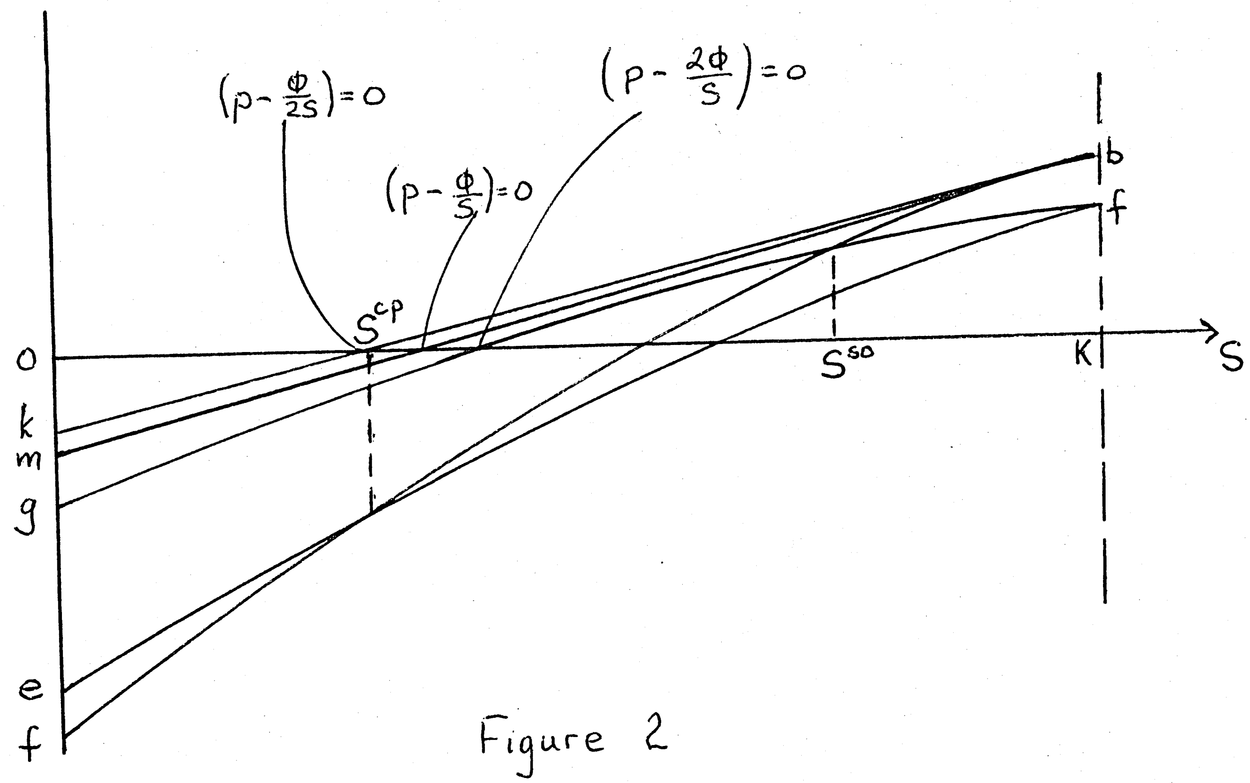

Figure 2