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Nonrenewable Resource Exploitation by a Dominant Seller and a Fringe Group with Rising Costs

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by

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Abstract

Nonrenewable Resource Exploitation by a Dominant Seller and a Fringe Group with Rising Costs

A dominant seller with a large finite stock of low cost nonrenewable resource sets a price path for a price-taking fringe group with deposits of rising extraction costs. The nature of profit-maximizing prices and quantities for the dominant seller is investigated and in particular the way in which the market is shared by the two selling agents period by period in a discrete time framework.

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February, 1980

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1. Introduction

We consider the nature of outcomes from a situation in which a monopolist controls a large stock of a low cost nonrenewable resource and is faced with a fringe group of rising cost, price-taking suppliers of the resource. Three strategies seem plausible for the dominant supplier: 1) Arrange prices so that the fringe group's holdings of low cost resource are exhausted as quickly as possible. The "cost" of this strategy is the discounted revenue foregone by the dominant seller while "waiting". 2) Arrange prices so that the fringe group is forced to supply their holdings relatively late in an intertemporal program. The "cost" to the dominant seller is in keeping prices relatively low in the early periods in order to "pre-empt" the fringe sellers. 3) Arrange prices so that the dominant seller and fringe sellers split or share the market in the early periods until the fringe sellers exhaust their low cost holdings. It is this last approach which we focus attention on here. With the dominant firm acting as a price-setter and the fringe group acting as price-takers, what sequence of prices is optimal for the dominant supplier to set? (We return to strategies 1) and 2) later but they are not central to our analysis.)

Gilbert [1978] has investigated this question of behavior by the dominant seller for a number of cases. With regard to the matter of optimally sharing the market at each instant of time, he worked with all suppliers with zero extraction costs and all agents facing a constant elasticity demand function. The analysis was reduced to finding the optimal initial price for the dominant seller. In subsequent periods, price and marginal revenue rose at the rate of interest. We deal with a fringe with rising extraction costs as stock is depleted in a discrete time framework. The analysis of how the dominant seller decides to split or share the market at each instant of time is central to our analysis. We are able to develop a precise Hotellingesque rule for the behavior of market price and quantities supplied over time by the dominant seller and the fringe group.

We feel that our specification of the market structure captures basic aspects of OPEC versus non-OPEC (ignoring the communist bloc countries) producers. Though OPEC's extraction costs may rise in the future, it seems reasonable to treat their extraction costs as negligible over the time span in which non-OPEC countries will likely exhaust their stocks of oil producible at costs below the backstop technology price. Conversely the threat of exhaustion of oil stocks in non-OPEC countries is one of low cost supplies being depleted and subsequent supplies becoming available only at costs approaching the backstop technology price - not a threat of a known stock at a known fixed extraction cost being run down to zero. It seems to be the run up of extraction costs which is crucial to the supply position of non-OPEC producers and this is the approach we have taken in the model developed below. For analyses of OPEC and cartels in mineral production, besides Gilbert, one should note the contributions of Salant [1976], Cremer and Weitzman [1976], Hnyilicza and Pindyck [1976], Pindyck [1978], Nichols and Zeckhauser [1977], Lewis and Schmalensee [1979], and Robson [1979].

The model is set out and developed in the next two sections. Numerical investigations are taken up next. Finally aspects of strategic behavior are analyzed.

2. A Model of a Price-Taking Fringe

We have a large group of owners of small deposits of a stock, no two strata of stock having the same cost of extraction per ton. The question is this: as price per ton moves exogenously upward over time period by period, how do the deposit owners react with regard to putting their mineral on the market or retaining it for future sale. If a ton with extraction cost c^* is sold this period at price $p(t)$ then the rate of return over the interval of a period is $\frac{(p(t)-c^*)r}{(p(t)-c^*)} = r$ where r is the rate of interest. If the deposit is left unsold, then the capital gain (or loss if $p(t+1)<p(t)$) is $p(t+1)-p(t)$, which as a rate of return is $\frac{p(t+1)-p(t)}{p(t)-c^*} = \gamma$. If $\gamma > r$ then obviously the maximizing strategy is to hold onto the deposit over the interval.

With a continuum of holdings corresponding to different values of c, then given $p(t+1)-p(t)$, there is a deposit holder with an extraction cost per ton $\hat{c}$ who is indifferent between extracting or retaining his holding. That is $\frac{p(t+1)-p(t)}{p(t)-\hat{c}} = r$.

If the prevailing price in period 1 is $p(1)$ and the price is known to rise to $p(2)$ ($>p(1)$) next period and remain at that level permanently, we can solve for the amount of mineral that will be put on the market in period 1 and the amount in period 2. The marginal holder with a deposit with extraction cost $\hat{c}$ will be indifferent between selling his holding at $p(1)$ today or at $p(2)$ next period. For this holder $\frac{p(2)-p(1)}{p(1)-\hat{c}} = r$. Deposit holders with extraction cost per ton c less than $\hat{c}$ will observe that it is most profitable to sell in period 1, "reap" their rent $p(1)-\hat{c}^-$ where $\hat{c}^-$ is extraction cost per ton of a certain deposit with $\hat{c}^- < \hat{c}$. They will observe $p(2)-p(1)<r\cdot(p(1)-\hat{c}^-)$. The quantity brought to market will be $Q(1) = \int_{\bar{c}}^{\hat{c}} g(s)ds$ where $\bar{c}$ is the lowest cost of extracting a unit and g(s) is the quantity of mineral available at extraction cost s.

Those deposit holders with $c>\hat{c}$ will find it profitable to retain their holding until period 2 since $p(2)-p(1)>r\cdot(p(1)-\hat{c}^+)$ where $\hat{c}^+$ is the cost of extraction per ton of a deposit with extraction costs greater than $\hat{c}$ (but less than p(2)). The quantity brought to market in period 2 will be $Q(2) = \int_{\hat{c}}^{p(2)} g(s)ds$. Note rent per ton is negative if extraction costs exceed p(2). Hence p(2) defines the cut-off grade or quality of mineral.

We require Q(t) to be small relative to the amount of mineral in the market in period t so that the fringe sellers are indeed price takers of a distinctly fringe sort. Obviously our periods 1 and 2 could be an arbitrary adjacent pair indexed t and t+1. One can readily see that as long as p(t+1)>p(t) and deposits are arrayed with "regular" continuous extraction costs, one can determine a sequence of Q(t)'s corresponding to each period. Difficulties obviously emerge if the rate of price increase accelerates in distant periods. In such a case, the essentially myopic rule for determining whether a deposit holder should sell today or retain his holding until a later period breaks down because capital gains may be very large from awaiting a price in the distant future relative to those obtainable "early" in time. <u>Formally if $\hat{c}$ is the extraction cost of the marginal deposit</u>

holder between periods t and t+1 (i.e. $r\cdot p(t)-\hat{c})=p(t+1)-p(t)$) then $p(t+n)-p(t)<r^n\cdot(p(t)-\hat{c})$ for all $n>1$. This must be true for $\hat{c}$'s in all periods.[1]

Conversely if $p(t+1)-p(t)<r(p(t)-\hat{c})$ with $p(t)-\hat{c}>0$ and $p(t+n)-p(t)<r^n\cdot(p(t)-\hat{c})$ for all $n>1$, then deposit holders at time t will maximize their return by mining and selling all holdings with c's less than or equal to $\hat{c}$, reaping current income $[p(t)-c(\cdot)]g(s)$ for a holding with g(s) tons and extraction costs $c(\cdot)$. In a model with a dominant seller, the price path $\{p(t)\}$ will be set by the dominant seller and he has the possibility of "pre-empting" the fringe group by "choking off" the capital gains of the fringe group in the fashion set out immediately above. We return to this mode of strategic behavior by the dominant seller, below.

For the case of "normal" capital gains, we have then, assuming price-taking quantities available at different values of c, and a regular sequence of prices over the periods, a well-defined sequence of quantities delivered (i.e. Q(t)'s) by the fringe mineral deposit holders.

3. A Dominant Mineral Seller with a Price-Taking Fringe

The dominant seller (DS) is defined to be dominant in the sense of being a price-setter. Presumably the DS supplies a relatively large share of the mineral at each instant of time in which he is "competing" with the fringe sellers (FS). We consider briefly some other sorts of threat or contingent actions which might be associated with a dominant seller below. The DS has a large finite stock of mineral extractable at zero cost per unit and plans its actions in order to maximize its discounted profits which in this case are its discounted revenues derived from selling portions of its stock over time. The DS sets the price in each period under the assumption that the fringe sellers supply any amount they choose in each period. However the DS makes an intertemporal plan under the assumption that the FS behave in a price-taking optimal fashion i.e. the FS are assumed by the DS to behave in the fashion set out in the preceding section.

There is a demand function defined in each period. We will treat it as stationary, without loss of generality, and also assume that there is a relatively high price, $\bar{p}$, at which a backstop technology becomes available and places a price ceiling $\bar{p}$ and a quantity ceiling $\bar{Q}$ on quantities produced per unit of time. Thus we have

$$Q(p(t)) = Q^D(t) + \int_{c(t-1)}^{c(t)} g(s)ds \qquad (t=0,1,2,\ldots,n)$$

where $c(n) = p(n)$, and $Q(\bar{p}) = \bar{Q}$ at $\bar{p} = p(n)$.

The DS' optimization problem is, by choice of $p(0),\ldots,p(n-1)$, and T, to maximize

$$\pi = \sum_{t=0}^{n} \left(\frac{1}{1+r}\right)^t p(t)Q^D(t) + \left(\frac{1}{1+r}\right)^{n+1} \bar{p}\bar{Q}\int_0^T e^{-rv}dv$$

subject to $\sum_{i=0}^{n} Q^D(t) \leqq \bar{S} - \int_0^T \bar{Q}dv$

where π is discounted total profit for the monopolist

r is the interest rate (assumed, with no loss of generality, to be the same for the DS and FS)

p(t) is the price of the mineral in period t

$Q^D(t)$ is the quantity mined, delivered and sold by the DS in period t n is the number of periods or the horizon to which exploitation is planned before the price of the backstop technology is reached

$\bar{p}$ is the price at which the backstop technology is supplied

$\bar{Q}$ is the quantity demanded at price $\bar{p}$

T is the length of time over which mineral is sold at the backstop price $\bar{p}$

$\bar{S}$ is the total stock constraint facing the DS.

Let λ be the Lagrangian multiplier on the $\bar{S}$-constraint; let $R(t)=p(t)Q(t)$, and $MR(t)=dR(t)/dQ(t)$.

Consider T fixed exogenously for a moment. Assume a solution exists and is interior to the above constrained maximization problem.[3] We can derive the following necessary conditions for an optimum (treating the problem as one in n prices: $p(0),\dots,p(n-1)$ and an endogenous horizon n. Recall that $c(t) = \frac{p(t)(1+r)-p(t+1)}{r}$. See Appendix for detailed derivation.)

$$MR(0) - \{Q^F(0)+c(0)g(c(0))\} \frac{dp_0}{dQ_0} = \lambda$$

$$MR(t) - \{Q^F(t)+c(t-1)g(c(t-1))+c(t)g(c(t))\} \frac{dp(t)}{dQ(t)} = (1+r)^t\lambda \quad (1)$$

$$(t=1,\dots,n-1)$$

$$\sum_{i=0}^{n} Q^D(t) = \bar{S}-\int_0^T \bar{Q}dv$$

Given a feasible value for λ, we can solve for the n prices: $p(0),\dots,p(n-1)$ and the horizon n, using the fact that $c(n) = p(n) = \bar{p}$.

We require an endpoint condition in order for λ to be endogenous. An obvious endpoint condition is: in period n, the marginal profitability (in present value terms) of an additional unit of mineral sold equals that marginal profitability in any previous period. Since marginal profitability in period t is $MR(t)-[Q^F(t)-c(t-1)g(c(t-1))+c(t)g(c(t))]\frac{dp(t)}{dQ(t)} = (1+r)^t\lambda$ we have for period n, given $p(n) = \bar{p} = c(n)$

$$MR(n)-[Q^F(n)-c(n-1)g(c(n-1))+c(n)g(c(n))]\frac{dp(n)}{dQ(n)} = (1+r)^n\lambda$$

Thus for a fixed value of T, we have n+2 equations in n prices, λ and the horizon, n, the number of periods.

To now make T endogenous (the new horizon is nt+T), we observe that at the optimum[4]

$$\lambda(1+r)^n = \partial\pi^T/\partial Q^T$$

where $\pi^T = \bar{p}\bar{Q}\int_0^T e^{-rv}dv$ and $\partial\pi^T/\partial Q^T = (d\pi^T/dT)/(dQ^T/dT)$ where $Q^T = \int_0^T\bar{Q}dv$. We have then, using $c(n)=\bar{p}$, a system of n+2 equations in n prices, λ and T (or n).

The striking qualitative property of the solution is that the marginal revenue in the market at time t (this is not the DS' marginal revenue nor the FS' marginal revenue) plus an adjustment factor of positive magnitude for each t is required to grow at a rate equal to the rate of interest. Let $\Delta q(t)=\{Q^F(t)+c(t-1)g(c(t-1))+c(t)g(c(t))\}$ (t=1,...,n-1). (1) can be written

$$MR(t)-\Delta q(t)\,\frac{dp(t)}{dq(t)} = (1+r)^t\lambda \qquad (t=1,\ldots,n-1)$$

This compares with the rule for the competitive case with no production from a competitive and higher cost fringe: $MR(t)-q(t)\,\frac{dp(t)}{dq(t)} = (1+r)^t\gamma$ where γ is a shadow price on the stock $\bar{S}$ and with the rule for the monopoly case with no production from a competitive fringe: $MR(t)=(1+r)^t\delta$. One should not make snap inferences about the nature of the relative speeds of depletion of $\bar{S}$ under the different regimes since there are the boundary conditions to be satisfied - namely the sum of quantities extracted over the life of the program must not exceed the size of the initial stock. However, a superficial inquiry based on the above rules suggests that the case with the price taking fringe "lies between" in terms of speed of price increase that under competition and that under monopoly.

4. The Supply Response of the Price-Taking Fringe

There is the familiar formulation of resource availability from successively poorer quality deposits (Solow and Wan [1976], Heal [1976], Levhari and Leviatan [1977], Hartwick [1980]). In this formulation there is a function $C(q,S)$ which indicates the cost of extracting q tons given that a total of S tons have already been extracted. It is evident that $q=\Delta S$ or $\dot{S}$ in a formulation of the problem in dynamic terms. Now $\frac{\partial C}{\partial q}$ indicates the cost of extracting "an extra ton" of mineral given S fixed and $\frac{\partial C}{\partial S}$ indicates the cost of extracting q tons of mineral if the program were delayed or shifted forward in time by changing S by one ton.

Consider the same concepts in our formulation of the supply response from the price-taking fringe. First

$$S(c(1)) = \int_{c(0)}^{c(1)} g(s)ds$$

and $\partial S/\partial c(1)=g(c(1))$. Then

$$Q(c(2);\ S(c(1)) = \int_{c(1)}^{c(2)} g(s)ds$$

and $\partial Q/\partial c(2)=g(c(2))$ and $\partial Q/\partial c(1)=-g(c(1))$. Thus we have precise definitions of $S(t)$ and $Q(t)$ analogous to those in the Solow-Wan et. al. formulation. We have then

$$C(Q;\ S(c(1)) = \int_{c(1)}^{c(2)} sg(s)ds$$

For $dQ=0$, we obtain $g(c(2))dc(2)-g(c(1))dc(1)=0$, and

$$\left.\frac{dC}{dc(1)}\right|_{dQ=0} = c(2)g(c(2))\,\frac{dc(2)}{dc(1)} - c(1)g(c(1))$$

$$= [c(2)-c(1)]g(c(1))$$

whence

$$\left.\frac{dC}{dS_1}\right|_{dQ=0} = [c(2)-c(1)]$$

One also obtains

$$\left.\frac{dC}{dQ}\right|_{dS_1=0} = \frac{\dfrac{\partial C}{\partial c(2)}}{\dfrac{\partial Q}{\partial c(2)}} = c(2)$$

There is then a precise translation of concepts in our formulation of the supply response of the price-taking fringe to those in the Solow-Wan et. al. formulation.

A special case we shall appeal to in a numerical investigation has $g(s)=\alpha$. In this specification, any amount of mineral can be produced if the price of the mineral is sufficiently large. There is an equal amount of mineral of each grade for this specification - grade being indicated by the cost of extraction of a ton.

If there were no DS, then the fringe group would have the market to themselves and would act as competitors. The "portfolio equilibrium" rule is

$$\frac{[p(t+1)-c(t)]-[p(t)-c(t)]}{[p(t)-c(t)]} = r$$

which can be expressed as

$$p(t)-c(t) = \left(\frac{1}{1+r}\right) [c(t+1)-c(t)] + \left(\frac{1}{1+r}\right) [p(t+1)-c(t+1)]$$

or

$$p(t)-c(t) = \sum_{i=(t+1)}^{n} \left(\frac{1}{1+r}\right)^{i-t} [c(i)-c(i-1)] \qquad (2)$$

$$(t=0,...,(n-1))$$

Note that p(n)-c(n)=0 since costs rise to the exogenously given cut-off price p(n).

If, in turn, the fringe group organized itself as a monopoly and there were no DS, then the "portfolio equilibrium" rule followed would be

$$\frac{[MR(t+1)-c(t)]-[MR(t)-c(t)]}{[MR(t)-c(t)]} = r$$

which can be expressed as

$$MR(t)-c(t) = \sum_{i=(t+1)}^{n} \left(\frac{1}{1+r}\right)^{i-t} [c(i)-c(i-1)] \qquad (3)$$

$$(t=0,...,(n-1))$$

where MR(t) is the marginal revenue in the market.

Below, we compare the solution under a regime of a dominant seller and price taking fringe group with a purely competitive regime and a purely monopolistic regime. We will have occasion to use the formulas above in solving examples.

5. A Numerical Example of a Dominant Seller and a Fringe Group

We make the demand schedule linear and stationary

$Q = 50 - p$

We assume a cut-off price of $p(n)=40$ leaving $Q(n)=10$.

The endowment of the fringe group is "rectangular" with $\alpha=1$ unit available at any cost c for $0 \leqq c \leqq 40$. Thus $Q^F(t) = \int_{c(t-1)}^{c(t)} ds = c(t)-c(t-1)$. Using the endpoint condition $MR(n)-[Q^F(n)+c(n-1)g(c(n-1))+c(n)g(c(n))] \frac{dp(n)}{dQ(n)} = \lambda(1+r)^n$ we obtain $\lambda(1+r)^n=110$. Stock available is $\bar{S}=126.53$ for the DS. $r=.10$.

If one returns to the general relation in (1) and substitutes with parameters above, one obtains the general recursive relationship

$$p(t-1)=2.083 + .0417\ \lambda(1+r)^{t-1} + .83p(t)$$

$$(t=1,\dots,n)$$

and

$$p(0)=2.083 + .0417\ \lambda + .83p(1) + \bar{c}$$

where $\bar{c}=0$ under our assumptions. $n=10$ exhausts the DS' $\bar{S}$. (The striking aspect of our particular specification of the problem is that the problem becomes, given λ, perfectly recursive. In general, given λ and β, the problem would involve n simultaneous equations in n prices. Formally it would be "nearly recursive".)

The numerical results are reported in Table 1. The c's are computed from the relation $c(t) = \frac{(1+r)p(t)-p(t+1)}{r}$.

TABLE 1

n	p	c	Q^F	Q	Q^D	$\pi^D\left(\frac{1}{1+r}\right)^t$
10	40	40	6.017	10	3.983	61.424757
9	39.453	33.983	3.694111	10.547	6.852889	114.66201
8	38.619899	30.288889	3.066112	11.380101	8.313989	149.78872
7	37.583797	27.222777	2.544873	12.416203	9.87133	190.38272
6	36.410534	24.677904	2.112234	13.589466	11.477232	235.88922
5	35.15191	22.56567	1.753173	14.84809	13.094917	285.81713
4	33.848327	20.812497	1.455119	16.151673	14.696554	339.76761
3	32.530968	19.357378	1.207766	17.469032	16.261266	397.44157
2	31.223572	18.149612	1.002421	18.776428	17.774007	458.65123
1	29.943901	17.147191	.832026	20.056099	19.224073	523.31249
0	28.704925	16.315165	16.315165	21.295075	4.97991	142.94794
					126.52917	2900.0854

We observe in Table 1 prices decline from the fixed terminal price of 40 to 28.7. At this initial price, the fringe puts up 16.3 units for sale. This leaves the fringe a large proportion of current quantity sold only in the initial period (about 16.3/21.3 is the proportion at t=0). The proportion falls to about .8/20 in t=1 and rises slowly to 6/10 at t=10.

We now turn to a comparison of the speed of extraction under this regime (dominant seller and fringe) relative to a pure monopoly regime and a pure competitive regime.

6. A Numerical Example with A Monopolist Controlling All Supplies

We take up this case in order to compare the nature of a path under a monopoly regime with that under the dominant seller and fringe regime in the previous section. The monopolist in this section is presumed to control all supplies. The optimal solution obviously involves using the higher cost deposits after the lower cost ones.[5] We solve for the rising price interval by a backward recursion using the expression (3) and the fact that MR(n) = c(n) given p(n) = 40. In the last period, MR is constant as $\bar{p}$ and the monopolist sells the residual holdings with costs below $\bar{p}$ at price $\bar{p}$. At the price, say (p(n-t*), at which the deposits with positive extraction costs are used up, we continue the backward recursion using the relation $MR(t) = \left(\frac{1}{1+r}\right)^{n-t^*-t} \times MR(n-t^*)$ to determine price and output in any period t < n-t*. The initial interval t=0 (or value of n) is determined, using the stock of the zero-cost deposit, namely 126.53 from the previous section.

We use the same parameterization as in the previous section; r=.10, Q=50-p, p(n)=40, g(s)=α=1. The numerical results for the pure monopoly case are reported in Table 2.

We observe in Table 2, that in the penultimate period, 10 units are supplied at p=40 and c=30 since MR(n)=30=c. Thus in all, only 30 units are supplied from the "costly" endowment during the rising price interval. The last 10 units are sold at price $\bar{p}$=MR. Price rises more slowly in the pure monopoly case (an initial price of \$31.4 vs. \$28.7 for the same time period of exploitation) and exploitation to exhaustion takes 12 rather than 11 periods. Note that over the complete 12 periods, we have the monopolist providing 129 units from the zero-cost endowment rather than the 126.5 units he "should". The

TABLE 2

t	c	MR	p	Q	$\pi^D(1+r)^{-t}$
11	40		40	10	
10	30		40	10	
9	20		39.545455	10.454545	
8	9.5454545		38.657025	11.342975 (Q^F=9.5454545)	
8		27.31405	38.657025	11.342975 (Q^D=1.7975209)*	32.41611
7		24.830955	37.415478	12.584523	241.62355
6		22.573595	36.286798	13.713203	280.88687
5		20.52145	34.260725	14.739275	322.70369
4		18.655864	34.327932	15.672068	367.45419
3		16.959876	33.479938	16.520062	415.54519
2		15.418069	32.709035	17.290966	467.41389
1		14.016426	32.008213	17.991787	523.53177
0		12.742206	31.371103	18.628897	584.40905**
					3028.0192

* In period 8, 1.7975 units are supplied from the zero extraction cost endowment and 9.5454 units are supplied from the positive extraction cost endowment.

discrepancy occurs because time is treated in discrete units in the problem and we cannot consider a fraction of a period. Thus the illustrations, monopoly vs. dominant seller are not precisely comparable. We note also that our qualitative results are based on a particular parameterization of the general problem. In particular, we have a stationary linear demand schedule. Recall that Hotelling discovered: with a stationary linear demand schedule, constant extraction costs and a terminal quantity demanded of zero, the monopolist takes longer to exploit a fixed stock than does a competitive group. Our monopoly case is not perfectly comparable since part of our monopolist's supply has rising extraction costs. However, our monopolist still has price rising more slowly than is the case with a dominant seller and a competitive fringe.

Total profits to the holder of the first 126.5 units extracted are \$3028.0 which is a larger sum than that obtained under the "dominant firm - price taking fringe" regime. Not surprisingly perhaps profits are higher for the zero cost, resource stock holders if he can behave as a monopolist in the first phase of exploitation.

7. A Numerical Example with A Competitive Group Supplying the Resource

This case has a similar structure to the one with the monopoly supplier. Now it assumed that all quantities brought to market are supplied by a competitive group of resource stock owners. Again it is clear that the zero cost deposit will be exploited first, followed by exploitation of the rising-cost holdings. We solve by backward recursion, using the formula in (2) for the final or high cost interval of exploitation. We note that $p(n) = c(n)$. For the interval of exploitation of the zero cost deposit, we will have price rising at a rate equal to the rate of interest.

The same parameterization is assumed to hold. The results are reported in Table 3.

In Table 3, we observe that the optimal program under the competitive regime takes less time than either the monopoly or dominant seller regimes

TABLE 3

t	c	p	Q	$\pi(1+r)^{-t}$
8	40	40	10	
7	30	39.090909	10.909091	
6	19.090909	37.272727	12.727273	
5	6.3636362	34.46281	15.53719 (Q^F=6.3636362)*	
5		34.46281	15.53719 (Q^D=9.1735539)*	196.30207
4		31.329827	18.670173	399.51731
3		28.481661	21.518339	460.46434
2		25.892419	24.107581	515.87073
1		23.538563	26.461437	566.24018
0		21.398694	28.601307	612.03062
				2848.8313

* In period 5, 9.1735539 units are supplied from the zero extraction cost endowment and 6.3636362 units are supplied from the positive extraction cost endowment.

(9 vs. 11 and 12 periods for the latter two) and that the competitive regime has a program with a lower initial price ($21.4 vs. approximately $28.7 and $31.4 for the other regime). Thus the rate of price rise is faster for the competitive program than for the other two programs. This result on relative speeds of exploitation of a fixed stock under monopoly vs. competition is the same as Hotelling's original finding but of course his model did not have a partial endowment with rising extraction costs. We remark that in the program reported in Table 3, 128.53239 units are extracted from the zero extraction cost deposit. This exceeds the so-called initial stock of 126.5 units because the problem involves discrete time periods. We have not admitted fractions of a period. Thus the three alternate programs are not precisely com-

parable. Nevertheless discrepancies are small. We note that discounted profits or rents accruing to the zero cost, resource stock holder are smaller in the competitive regime than in the other two regimes.

8. The Possibility of Strategic Behaviour by the Dominant Seller

In the normal case of a moderate sized stock held by the DS, profits maximum maximorum would be attained by the DS by setting prices so that his (the DS') marginal revenue, equal to the market marginal revenue, grew at the rate of interest up to exhaustion of the stock, at which time price just reached $\bar{p}$. For the case of a linear demand schedule $Q=A-Bp$, the FS would face a price path $p(t)(1+r)-p(t+1) = \frac{rA}{2B}$. This constant difference in prices over time implies that the fringe sellers would be receiving no capital gains and would thus consider alternative approaches to gain revenues for their holdings. The profit maximizing response by a price-taking fringe would be to attempt to sell all quantities with extraction costs less than or equal to the prevailing price, period by period. But this action would require that the DS accommodate such supplies by revising its original price schedule (the one based on its and the market MR growing at the rate of interest). Thus if the DS behaves as the Stackelberg price-setter we have assumed, it cannot behave so as to achieve the profits maximum maximorum attainable if it were operating in the absence of the fringe. The fringe group can "upset" a price path set by the DS so as to achieve its maximum maximorum profits.

We can label a price path along which $p(t)(1+r)-p(t+1)$ as constant over many consecutive periods as <u>pre-emptive</u>. The DS is preventing capital gains from rising over time for the fringe group with such a price path. The fringe group has its capital gains pre-empted and must respond by "dumping" all quantities with extraction costs lower than current price, period by period. Observe that a price path in which prices rise at the rate of interest (a competitive path) is also pre-emptive since $p(t)(1+r)-p(t+1)=0$ along such a path. Of significance is the fact that a competitive-pre-emptive path set by the DS over which it exhausts its stock and then a terminal competitive period over which the fringe exhausts its low cost holdings yields lower profits to the DS than does our optimal sharing-path set by the DS (Tables 1 and 3). In other words our optimal solution set out in section 3 dominates the particular pre-emptive course immediately above adoptable by the DS. That is for the DS to behave as a competitor is effectively pre-emptive in our model but it yields lower profits to the DS than does the quantity sharing solution set out in section 3.

A passively pre-emptive strategy by the DS would be to let the FS exhaust its low cost holdings by behaving as competitors alone in the market. When the FS had exhausted their low cost holdings, the DS would sell off its stock, period by period at the cost for the backstop technology, $\bar{p}$. Profits for the DS at the time the DS starts selling would be

$$40 \times 10 \int_0^{(126.5/10)} e^{-.1t}\, dt = 2871.0428$$

This value is lower than the 2900.0854 that the DS receives in the "shared solution" reported in Table 1. Moreover 2871.0428 should be discounted three periods while the FS are depleting their low cost holdings. Thus a "passive pre-emptive" strategy of waiting on the part of the DS is suboptimal.

A co-operative solution clearly dominates a solution in which the market is shared period by period by the DS and the FS. The DS could buy out the FS and operate as a pure monopolist and make higher profits after paying the FS for their holdings at the value implicit in the "shared" solution than by acting as a price setting DS facing the competitive fringe. However, we are assuming that such institutional arrangements are not available. Non-OPEC oil holders would presumably be legally barred from selling themselves to OPEC!

The relative size of the holdings clearly matters. For example if the lowest cost deposits of the fringe group have extraction costs close to $\bar{p}$ (that is $\bar{c}$ is slightly less than $\bar{p}$), then the DS can act as a monopolist in setting prices less than $\bar{c}$. His profit-maximizing strategy is presumably to take $\bar{c}$ as the effective cut-off or backstop price and act as a monopolist.

Appendix

The problem for the DS is to maximize

$$\pi = \sum_{t=0}^{n} \left(\frac{1}{1+r}\right)^{t} p(Q(t)) \cdot [Q(t)-Q^{F}(t)]$$

subject to

$$\sum_{t=0}^{n} [Q(t)-Q^{F}(t)] = \overline{S}$$

where p(Q(t)) is the inverse demand function

$$Q^{F}(t) \triangleq \int_{c(t-1)}^{c(t)} g(s)ds.$$

We ignore the possibility of resources being sold while the backstop technology with price $\overline{p}$ is in place, in order to avoid clutter.

$$\text{Since } c(t) = \frac{(1+r)p(t)-p(t+1)}{r}$$

we can express the problem in terms of n prices, p(0),...,p(n-1). That is

$$Q^{F}(t) = \int_{\left[\frac{(1+r)p(t-1)-p(t)}{r}\right]}^{\left[\frac{(1+r)p(t)-p(t+1)}{r}\right]} g(s)ds$$

and Q(t) is a single valued function of p(t). We set up the lagrangian with multiplier λ

$$L = \sum_{t=0}^{n} \left(\frac{1}{1+r}\right)^{t} [p(t)\cdot Q(p(t))-p(t)Q^{F}(t)] + \lambda[\overline{S}- \sum_{t=0}^{n} [Q(p(t))-Q^{E}(t)]].$$

Then $\frac{\partial L}{\partial p(t)} = 0$ yields

$$\left(\frac{1}{1+r}\right)^t \left\{\frac{d(p(t)\cdot Q(p(t)))}{dp(t)} - Q^F(t)-p(t)[g(c(t))\ \frac{1+r}{r}\ +g(c(t-1))\left(\frac{1}{r}\right)]\right\}$$

$$+\left(\frac{1}{1+r}\right)^{t+1} p(t+1)[g(c(t))\left(\frac{1+r}{r}\right)] + \left(\frac{1}{1+r}\right)^{t-1} p(t-1)[g(c(t-1))\left(\frac{1}{r}\right)]$$

$$-\ \lambda\left[-\ \frac{dQ(p(t))}{dp(t)} + g(c(t))\left(\frac{1+r}{r}\right) + g(c(t-1))\left(\frac{1}{r}\right) - g(c(t))\left(\frac{1+r}{r}\right) - g(c(t-1))\left(\frac{1}{r}\right)\right] = 0$$

$$(0 < t < n)$$

which can be written as

$$\frac{d(p(t)\cdot Qp(t))}{dp(t)} - Q^F(t) - p(t)[g(c(t)\frac{(1+r)}{r} + g(c(t-1)\left(\frac{1}{r}\right)] + p(t+1)g(c(t))\left(\frac{1}{r}\right)$$

$$+ p(t-1)g(c(t-1))\left(\frac{1+r}{r}\right)$$

$$= (1+r)^t\lambda\ \frac{dQ(p(t))}{dp(t)}$$

which in turn yields (1) in the text

$$MR(t)-[Q^F(t)+c(t-1)g(c(t-1))+c(t)g(c(t))]\ \frac{dp(t)}{dQ(p(t))} = (1+r)^t\lambda. \qquad ||$$

FOOTNOTES

* An earlier draft of this paper was presented at a Journée d'etude sur les ressources naturelles, Université Laval, February, 1980. My thanks to the participants. This paper is also to be presented at the Fourth World Congress of the Econometric Society, Aix-en-Provence, August, 1980.

1. We are treating the interest rate r as constant over time. Appropriate changes have to be made for the interest varying over time.

2. We shall consider plausible forms for g(c) below.

3. Existence of solutions is not of great concern to us since numerical examples are easy to construct. One follows below. Interiority of solutions is however of consequence. We must be careful that there is not a solution in which the DS can act as a pure monopolist and leave the fringe group "waiting for an entry" into a program of exploitation and observe the DS at a maximum maximorum profit level. We take up this possibility in some detail in the final section of the paper. With regard to existence and interiority, see also footnote 4.

4. If it is optimal for the DS to sell some stock at price $\bar{p}$, beyond the transition point from rising prices to constant prices, in our model, this equation is only satisfied by inequality. The implication is that positive quantity should be provided in the phase before the transition point so that λ is pushed to approximately zero. We might consider this situation, with λ approximately zero, as the "abundant stock regime". Contrarily, no stock will be sold by the monopolist beyond the transition point and λ will be strictly positive in a "scarce stock regime". We deal with the "scarce stock regime" in this paper. In the "abundant stock regime" the DS would set aside an $\bar{S}$ for the rising price phase such that λ was approximately zero. In the subsequent phase, $\bar{p}$ prevailing in all per-

iods, the monopolist would simply supply the demanders at $\bar{p}$ until his complete stock was exhausted. He would pre-empt the market of the owner of the backstop technology until his (the DS) stock was exhausted. Note that we are assuming that the low cost stocks held by the fringe group are small relative to the quantity demandable below $\bar{p}$: that is $\int_{\underline{c}}^{\bar{p}} g(s)ds < \int_{\underline{c}}^{\bar{p}} q(p)dp$. The DS must be able to set price by providing some fraction of quantity demanded at any $p<\bar{p}$.

5. See Hartwick [1978].

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