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An Alternative Method to Calculate Internal Rate of Return

Internal rate of return (IRR) is an important measure of the worth of investment project. It has been employed very widely in decision making relating to various types of investments. The IRR of a project is defined as the discount rate which makes net present value (NPV) of future stream of net cash flow of project equal to zero.

Despite the popularity of IRR in project appraisal/evaluation, the methods used to calculate IRR do not provide exact value of IRR as such. It is through hit and trial or through an iterative process that one tries to obtain the true or exact value of IRR. The method is very cumbersome and non-systematic. In most of the cases it becomes extremely difficult to get the exact value of IRR and a value close to real value of IRR is taken as true value of IRR. In this note an alternative method has been suggested which is systematic and provides exact value of IRR.

EXISTING METHOD

Mathematically, IRR is that value of r^* which satisfies the following equation:

$$\sum_{t=0}^{n} \frac{B_t - C_t}{(1+r)^t} = 0 \qquad \text{.... (1)}$$

where B_t = benefit from the project in period 't',
C_t = cost associated with the project in period 't',
n = total life of project and
r^* = internal rate of return

As mentioned earlier, there is no exact way to calculate value of r^* for the above equation. One starts with an arbitrary value of r and examines the value corresponding to r on the right-hand side expression of (1), *i.e.*, value of net present value. Depending upon whether the expression is positive, a higher or lower value of r is chosen and NPV is again calculated. This process is repeated till one strikes at a value of r^* at which NPV is zero.

ALTERNATIVE METHOD

To get unique (single) value of IRR the existing and alternative method must satisfy the assumption that there is only one change in sign of stream of cash flow (B_t - C_t). It should be noted that if there are more than one changes in sign of (B_t - C_t) in equation (1), then there exists multiple value of r^* for which the equation is satisfied. Under this assumption the alternative method is outlined below.

Expanding equation (1) and substituting (1+r) by x give

$$\frac{B_0 - C_0}{X^0} + \frac{B_1 - C_1}{X} + \frac{B_2 - C_2}{X^2} + \frac{B_3 - C_3}{X^3} + \dots + \frac{B_n - C_n}{X^n} = 0 \qquad \text{.... (2)}$$

or multiply throughout by x^n

$$(B_0 - C_0)\,x^n + (B_1 - C_1)\,x^{n-1} + (B_2 - C_2)\,x^{n-2} \ldots (B_n - C_n) = 0 \qquad \ldots (3)$$

$$\text{or } F(x) = 0 \qquad \ldots (4)$$

Under the assumption of one variation in sign of the right-hand side of equation (3), there is exactly one positive real root x of polynomial F (x) which satisfies (3). The problem addressed here is how to calculate the x and the procedure under alternative method is illustrated below.

Step I

Assume that the value of x for which (3) is satisfied lies between α and $\alpha + 1$. Substitute $x = 0$ in F (x) and note the sign of F (x). Increase the value of x by one and go on substituting x = 1,2,3...., so on, till the sign of F(x) changes (from negative to positive or positive to negative). The value of (x) for which F (x) = 0 lies between two positive integers at which sign of F (x) changes. Say, those positive integers are α and $\alpha + 1$ is integral part of the root. The next task is to find fractional part, which is IRR.

Step II

Diminish the roots of equation (3) by α and denote the transformed equation as

$$G(x) = 0 \qquad \ldots (5)$$

The procedure for diminishing the roots of an equation is given in Appendix 1.

Step III

Multiply the roots of the equation G(x) = 0 by 10 and denote the transformed equation as H(x) = 0

The procedure for multiplying the roots of an equation by ten is given in Appendix 2.

Step IV

Locate two positive integers for which sign of H(x) changes (from positive to negative or negative to positive) by substituting x = 0, 1, 2, ...9 in H(x). Say, those positive integers are B and B+1. B is the first decimal value of positive real root.

Step V

To evaluate the second decimal value of positive real root, the steps II to IV are repeated for equation H(x) = 0 and B as a root.

The procedure requires a number of iterations (four iterations are sufficient) to get IRR correct to two decimal places.

AN EXAMPLE

The application of the method is demonstrated below by taking a simple case of investment. Suppose the net cash flow from a project is given by

Year	0	1	2	3	4
Cash flow	-6000	-1000	4000	5000	6000

The IRR for this example is obtained by solving the following equation for r^*. Let

$$\frac{-6000}{(1+r^*)^0}-\frac{1000}{(1+r^*)^1}+\frac{4000}{(1+r^*)^2}+\frac{5000}{(1+r^*)^3}+\frac{6000}{(1+r^*)^4}=0$$

Multiplying throughout by $(1+r^*)^4$

$$-6000(1+r)^4 - 1000(1+r)^3 + 4000(1+r)^2 + 5000(1+r) + 6000 = 0$$

Let $(1+r^*) = x$, we get

$$-6000x^4 - 1000x^3 + 4000x^2 + 5000x + 6000 = 0$$

Divide throughout by -1000 to simplify the calculations

$G_1(x) = 6x^4 + x^3 - 4x^2 - 5x - 6 = 0$
$G_1(x)$ = negative, when $x = 0$
= negative, when $x = 1$
= positive, when $x = 2$

Thus the value of x for which $G_1(x)$ changes its sign lies between 1 and 2 and hence the positive root of polynomial equation $G_1(x) = 0$ lies between 1 and 2.

Now diminish the roots of previous equation $G_1(x) = 0$ by 1

1/	6	1	-4	-5	-6
		6	7	3	-2
		7	3	-2	-8
		6	13	16	
		13	16	14	
		6	19		
		19	35		
		6			
		25			
	$\bar{6}$				

Transformed equation is

$G_2(x) \equiv 6x^4 + 25x^3 + 35x^2 + 14x - 8 = 0$

Multiplying the roots of equation $G_2(x) = 0$ by ten, we get polynomial $G_3(x)$ as

$G_3(x) \equiv 6x^4 + 250x^3 + 3500x^2 + 14000x - 80000 = 0$
$G_3(x)$ = negative, when x = 0
= negative, when x = 1
= negative, when x = 2
= positive, when x = 3

The positive root of equation $G_3(x) = 0$ lies between 2 and 3 and hence the positive root of equation $G_1(x) = 0$ correct to one decimal place is 1.2.

Diminish the roots of equation $G_3(x) = 0$ by 2

2/	6	250	3500	14000	-80000
		12	524	8048	44096
		262	4024	22048	-35904
		12	548	9144	
		274	4572	31192	
		12	572		
		286	5144		
		12			
		298			
	6				

Transformed equation is:

$G_4(x) \equiv 6x^4 + 298x^3 + 5144x^2 + 31192x - 35904 = 0$

Multiplying the roots of equation $G_4(x) = 0$ by ten, we get polynomial $G_5(x)$ as

$G_5(x) \equiv 6x^4 + 2980x^3 + 514400x^2 + 31192000x - 359040000 = 0$
$G_5(x)$ = negative, when x = 0
= negative, when x = 1
= negative, when x = 2
= negative, when x = 3
= negative, when x = 4
= negative, when x = 5
= negative, when x = 6
= negative, when x = 7
= negative, when x = 8
= negative, when x = 9
= positive, when x = 10

The positive root of equation $G_5(x) = 0$ lies between 9 and 10 and hence positive root of equation $G_1(x) = 0$ correct to two decimal place is 1.29.

Diminish the roots of equation $G_5(x)$ by 9.

9/	6	2980	514400	31192000	-359040000
		54	27306	4875354	324606190
		3034	541706	36067354	-34433810
		54	27792	5125482	
		3088	569498	41192836	
		54	28278		
		3142	597776		
		54			
		3196			
	6				

Transformed equation is

$$G_6(x) \equiv 6x^4 + 3196x^3 + 597776x^2 + 41192836x - 34433810 = 0$$

Multiply the roots of equation $G_6(x)$ by ten, we get polynomial $G_7(x)$ as

$$G_7(x) \equiv 6x^4 + 31960x^3 + 59777600x^2 + 41192836000x - 344338100000 = 0$$

$G_7(x)$ = negative, when $x = 0$
= negative, when $x = 1$
= negative, when $x = 2$
= negative, when $x = 3$
= negative, when $x = 4$
= negative, when $x = 5$
= negative, when $x = 6$
= negative, when $x = 7$
= negative, when $x = 8$
= positive, when $x = 9$

The positive root of equation $G_7(x) = 0$ lies between 8 and 9 and hence positive root of equation $G_1(x) = 0$ correct to three decimal places is 1.298.

8/	6	31960	59777600	4.119283×10^{10}	-3.443381×10^{11}
		48	256064	4.8026931×10^{8}	3.333848×10^{11}
		32008	60033664	4.1673105×10^{10}	$-1.0953258 \times 10^{10}$
		48	256448	4.823209×10^{8}	
		32056	60290112	4.2155426×10^{10}	
		48	256832		
		32104	60546944		
		48			
		32152			
	6				

Transformed equation is

$$G_8(x) \equiv 6x^4 + 32152x^3 + 6054944x^2 + 4.2155426 \times 10^{10} x - 1.0953258 \times 10^{10} = 0$$

Multiply the roots of the equation $G_8(x) = 0$ by ten, we get polynomial $G_9(x)$ as

$$G_9(x) \equiv 6x^4 + 321520x^3 + 6.0546944 \times 10^9 x^2 + 4.2155426 \times 10^{13} x - 1.0953258 \times 10^{14} = 0$$

$G_9(x)$ = negative, when $x = 0$
= negative, when $x = 1$
= negative, when $x = 2$
= positive, when $x = 3$

The positive root of equation $G_9(x) = 0$ lies between 2 and 3 and hence the positive root of equation $G_1(x) = 0$ correct to four decimal place is 1.2982.

To check the results, substitute $x = 1.2982$ in $G_1(x)$.

$G_1(x) = -0.0025$, when $x = 1.2982$. Hence 1.2982 is the positive real root of $G_1(x) = 0$ correct to four decimal places.

Since $1+r^* = x$, therefore $r^* = 0.2982$

IRR for the above example is 29.82 per cent.

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APPENDIX 1

PROCEDURE FOR DIMINISHING ROOT OF AN EQUATION

For simplicity and to make the procedure for diminishing root of an equation clear to the reader, we make use of polynomial of 4th degree though the same procedure is applicable to a polynomial of any other degree. A polynomial of 4th degree can be written as

$$a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0 \qquad ....(1)$$

To diminish the root of this equation by say 'h', where h is a positive integer, four steps are required (the number of steps being equal to degree of polynomial).

Step I. Arrange the coefficients of the polynomial equation in descending order of power of x. Multiply the first coefficient by the root 'h' and add it to the next, *i.e.*, second coefficient and get the sum. Multiply the sum by 'h' and add to the next coefficient to get the next sum. Again multiply this sum by h and add to the next coefficient. Repeat the procedure till we get the sum under the last coefficient. Substitute the sum below the last coefficient in place of the coefficient in equation (1) to get the corresponding coefficient in modified equation.

Step II. Multiply the first coefficient by h and add to the first sum of step I to get the first sum of step II. Multiply the first sum of second step by h and add to the second sum of step I. Repeat the process till the last but one coefficient. This way the last sum of step II replace the corresponding coefficient of equation (1) to get modified equation.

Step III. Repeat step II till the last but two coefficients.

Step IV. Repeat step II till the last but three coefficients. Use last sum of each step in place of the corresponding coefficients of equation (1) to get the modified equation. The first coefficient is retained as such.

For further exposition the procedure required to transform equation (1) by diminishing its root by 'h' is illustrated below:

Step No.	a_4	a_3	a_2	a_1	a_0
I	Add	a_4h	$a_3h+a_4h^2$	$a_2h+a_3h^2+a_4h^3$	$a_1h+a_2h^2+a_3h^3+a_4h^4$
		a_3+a_4h	$a_2+a_3h+a_4h^2$	$a_1+a_2h+a_3h^2+a_4h^3$	$a_0+a_1h+a_2h^2+a_3h^3+a_4h^4$
II	Add	a_4h	$a_3h+2a_4h^2$	$a_2h+2a_3h^2+3a_4h^3$	
		a_3+2a_4h	$a_2+2a_3h+3a_4h^2$	$a_1+2a_2h+3a_3h^2+4a_4h^3$	
III	Add	a_4h	$a_3h+3a_4h^2$		
		a_3+3a_4h	$a_2+3a_3h+6a_4h^2$		
IV	Add	a_4h			
		a_3+4a_4h			
	a_4				

$a_4x^4 + (a_3+4a_4h)\,x^3 + (a_2+3a_3h+6a_4h^2)x^2 + (a_1+2a_2h+3a_3h^2+4a_4h^3)\,x +$ $(a_0+a_1h+a_2h^2+a_3h^3+a_4h^4) = 0$ is the transformed equation obtained from equation (1) by diminishing its roots by h.

APPENDIX 2

MULTIPLICATION OF THE ROOTS OF AN EQUATION BY TEN

Assume $f(x) \equiv a_0x^n + a_1x^{n-1} + \ldots + a_n = 0$ ....(1)

be a polynomial equation of degree n.

Substitute $x \equiv \frac{Y}{10}$ in equation (1), we get g (y)

$$g(y) \equiv a_0\left(\frac{Y}{10}\right)^n + a_1\left(\frac{Y}{10}\right)^{n-1} + \cdots + a_n = 0$$

That is $g(y) \equiv a_0y^n + 10a_1\,y^{n-1} + \ldots + 10^n\,a_n = 0$

This is the transformed equation whose roots are ten times the roots of equation (1).

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