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Utilitarianism, Voting and the Redistribution of Income

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Abstract: Utilitarianism can be misplaced or ambiguous. As a prescription for individual behaviour, the injunction to seek the *greatest good for the greatest number* is misplaced because there remains a domain of life where, within the bounds of law and custom, one is free to act as selfishly or as altruistically as one pleases. As a criterion for responsible government, it is ambiguous because there is no universally-recognized perception of the greatest good; people have different perceptions which can only be reconciled by compromise or by voting. The greatest number must be of citizens alive today, but governments may be vicariously concerned about people in other countries or yet to be born, in so far as citizens today have such concerns and are prepared to sacrifice for the benefit of others. The greatest good for the greatest number has no rival as a criterion for government, but it is vague nonetheless. Utilitarian ambiguity is inherited in any attempt to combine the ordinary measure of economic growth with changes in the distribution of income on a common scale.

Key Words: utilitarianism, voting, redistribution.

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"Ultimate ends are a matter of pure choice." R. F. Harrod[1]

Utilitarianism - the mandate to seek the greatest good for the greatest number - is almost meaningless without some indication of how the greatest good[2] is identified, who is commended to seek it and who the greatest number includes. This essay is a discussion of these matters with special reference to majority rule voting and the redistribution of income. It is argued that utilitarianism is is a prescription for governments rather than for citizens in the ordinary business of life, that there are innate difference among people in their understandings of the greatest good, that these differences can only be resolved by voting, that governments are responsible to citizens who are more concerned for their own welfare today than for the good of people in other countries or yet to be born. Discussion of these matters is followed by appendices on how to incorporate changes in the distribution of income into a unified measure of economic growth, how the greatest good is affected by the redistribution of income, the shape of the utility of income function and the consistency of public choice.

Citizen and Government

"The greatest good for the greatest number" is not just an ideal. It is a command, a command we make upon ourselves perhaps, but a command nonetheless. If so, then who is commanded and what exactly is the recipient of the command required to do? The injunction to seek the greatest good for the greatest number may be to the government or to the citizen. It could be a general principle for the choice of laws and customs, or it could be a direct command to people in their daily lives. It is understood here as "government-house utilitarianism"[3], a doctrine about how governments ought to behave.

Begin with Jeremy Bentham whose advocacy of utilitarianism was, so far as I can tell, not so much about individual morality but to keep the despot away. Defending utilitarianism against the accusation that it is dangerous, Bentham responded that "it unquestionably is, to every government which has for its *actual* end an object, the greatest happiness of a certain *one*, with or without the addition of some correspondingly small number of voters, is a matter of pleasure or accommodation to him to admit to a share of concern on the footing of so many junior partners." To an advocate any other rule than to seek the greatest good for the greatest number, "let him ask himself whether his

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[1]The quotation is from Harrod (1936, 145). Harrod adds that "I hold that when people use the terms good, right, virtue, etc., promotion of the common interest is what they mean. I have no reason for promoting the common interest. It is beyond the power of reason to prescribe ends." (page 144).

[2] By "the greatest good", I mean no more than what is usually called social welfare. The old fashioned language seems more appropriate when speaking about utilitarianism.

[3] Goodin, R.E., *Utilitarianism as a Public Philosophy*, Cambridge University Press, 1995

principle is not despotical and hostile to all the rest of the race."[4] It is at least arguable that utilitarianism was first and foremost a defence against despotism, a principle for government by the many rather than for the all-powerful leader. Concern about maximizing total happiness was to give content to this prescription.

But utilitarianism is not a directive to the benevolent dictator to do what is best for his subjects in each and every action he takes, for there is no benevolent dictator and would-be benevolent dictators have proved very different in practice. Though people may act on strict utilitarian principles when the law is vague, the doctrine is best seen as a broad rationale for institutions likely to supply the greatest good most of the time. It is best seen - and this appears to be Bentham's usage - as the moral foundation of the institutions of society: government by majority rule voting, the rule of law, an economy based upon private property, duties, not enforced by the threat of punishment, that the citizen is expected to respect, and enough redistribution of income to keep society on track, enabling less than perfect people to live together in peace and prosperity.

As will be discussed below, there is some question about how closely the outcome of voting corresponds to what most people think of as the greatest good, and there is considerable diversity of opinion among citizens about the meaning of the greatest good when conflicts of interest arise, but government by majority rule voting is the only alternative to autocracy, monarchy or dictatorship. Voting is compulsory in some countries, but in most country it is not, for fear that a person who only votes when compelled to do so will not vote thoughtfully or well. People may vote altruistically, selfishly or not at all, but government by majority rule voting is only workable as long as a significantly large proportion of the population recognize a duty to vote. [5]

Despite the inevitable gap between rich and poor, private property is the foundation of prosperity and a requirement for the preservation of democratic government. Any attempt to allocate the entire national income by voting or by government decree would sooner or later come under the hand of political or bureaucratic factions organized to appropriate the lion's share of the national income for their members, destroying the consensus to accept the outcome of the vote on which the continuation of democratic government depends. A degree of systematic redistribution of income - directly or by public provision of goods such as education and medical care - may be necessary to preserve the loyalty of the poor.

The injunction to seek the greatest good may bind citizens in one sense but not another. The

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[4]Quoted from Bentham's *Introduction to the Principles of Morals and Legislation* (1822) quoted in Jonathan Glover, *Utilitarianism and its Critics*, MacMillan Publishing Company, 1990, pages 13 and 14.

[5]It is remarkable how little studies of utilitarianism have to say about voting. *The Cambridge Companion to Utilitarianism* (2014) has just one reference to voting, a few lines on page 156 about John Harsanyi's argument that only rule utilitarianism supplies a duty to vote.

citizen is bound by law and custom which, one hopes, are designed to promote some notion of the greatest good, but, having done what law and custom require, the citizen is free to conduct his life as selfishly or as altruistically as he likes. Utilitarianism is no guide to individual behaviour except where laws and established customs give clear prescriptions of what one must or may not do. Nobody (or almost nobody) gives his entire fortune to the poor as, strictly speaking, the maximization of the greatest good for the greatest number would require. We spend the greater part of our incomes for our own benefit rather than for the benefit of others. We may feel compelled to be generous, but not to that extent. We admire Good King Wenceslas for his generosity, but would think him a fool were he to give away so much of his income that yonder peasant becomes as well off as he. Recently, a private donation financed a beautiful new concert hall in the City of Kingston. Was the donor immoral for not using his resources to alleviate poverty instead? Were the recipients in Kingston immoral for accepting the gift when money to finance the new concert hall might have been used to reduce starvation in Bangladesh? Donor and recipient may be immoral on some extreme utilitarian criterion, but not on the common understanding of right and wrong.

Not all rules of conduct are enforced by law. There is a moral obligation to be kind to one another, to help people in distress even though no punishment is imposed for one's failure to do so, but there is no clear specification of how far this obligation extends between the limits of pure selfishness and of unadulterated act utilitarianism devoted to attaining the greatest good for the greatest number in each and every thing you do. Few people go to the first extreme. Nobody but Mother Theresa goes to the second. We must each choose our stance in between.[6]

Citizens differ in how and to what extent duties are performed. Some go well out of their way to help people in distress. Others recognize no such obligation. Some give generously to charity. Others do not. Some vote thoughtfully. Others do not. What we see as a good society relies on citizens' sense of duty when enforcement of socially-beneficial behaviour is not possible or not desirable. A distinction may be drawn between duty when others can be expected to be dutiful too

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[6]In "Utilitarianism and Integrity" (reprinted in Jonathan Glover ed. *Utilitarianism and its Critics*, Macmillan, 1990), Bernard Williams presents an example of Jim who is captured by bandits and confronted with the choice of killing one innocent person to save the lives of twenty other innocent people. The utilitarian solution is to kill the one person to save twenty others, but Jim cannot do so without violating his deepest moral principles and seriously diminishing himself . This situation can be looked upon as exemplifying the inevitable choice one must make from time to time whether to help others at some cost to oneself. A government confronted with an angel who insists on killing either one person at random or twenty persons at random has no difficulty in its decision. Where there is no better remedy, we have no hesitation about licencing drugs or permitting operations that are occasionally lethal but can be expected to save a great many more people than they kill. Rules of personal morality block comparable behaviour by people acting independently because, if "justifiable" murder were allowed, too many people would manage to persuade themselves that murders they would like to commit are justifiable.

and duty when others cannot be relied upon to reciprocate.[7] For individual behaviour, our common understanding of right and wrong is what might be called the Wolf Cub morality[8]: to obey the law, to be helpful to others from time to time, but, in most things, to do what is best for our families and ourselves.

There are circumstances where personal and public considerations are intertwined. First, citizens govern vicariously. If government is to seek the greatest good for the greatest number, citizens must do what is necessary to procure government that does so. Hence the moral obligation to vote and to play some part in public life, giving to charity, participating on school boards and so on. Second, different rules kick in for dealing with bad law or in odd cases where ordinarily good law has bad consequences. Throwing boxes of tea overboard in Boston harbour was pressure by citizens to change a bad law. Rioting on the streets may be part of the process of democratic government. It is sometimes right to park in a no-parking zone outside a maternity hospital. The line between private and public action is blurred in such cases.

There may be some question as to whether this variant of government house utilitarianism is utilitarianism at all because it does not impose people to seek the greatest good, in their actions or in their rules of behaviour. Citizens delegate their governments to seek the greatest good, and

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[7] We may all feel a moral obligation to place ourselves in some danger in order to save the life of another person, but we differ in the amount of danger we are prepared to accept. A full-fledged utilitarian might act to minimize the expected number of lives lost, accepting up to a 50% chance of losing his own life to avoid a 50% chance that some other person's life is lost. Most people are far less noble. A person's behaviour might be represented as minimizing a weighted sun of the risks to his own life and to the lives of other people, where concern for the lives of other people may well depend on how one would expect others to behave if one's own life were in danger.

[8] When I was a Wolf Cub, we recited this pledge at each meeting:

> I promise to do my best
> to do my duty to Go and the King
> to keep the law of the wolf cub pack
> and to do a good turn for somebody every day.
> Dib, dib, dib, dib
> We'll dob, dob, dob, dob.

The pledge was to do one's duty, to obey the law and to be helpful to others, but we were free to choose whom to help and, beyond that, to play and enjoy ourselves as we pleased. We were certainly not obliged to do good deeds all day as a strict interpretation of utilitarianism would imply.

responsible government tries to do so.[9]

The Greatest Good

The "good" in the greatest good for the greatest number must include more than just money, but money is surely part of it and, if the greatest good means anything at all, it must be possible for a person to say whether the good as he sees it to be greater in one income distribution than in another. Imagine a society of just two people, 1 and 2, and suppose an angel is about to change their incomes in a way that will be equalizing but costly, reducing the higher of the two incomes by more than the lower income is increased. Suppose their initial incomes are $160,000 and $40,000, an average of $100,000, and suppose the angel equalizes incomes completely at, for example, $90,000 each. How might one judge whether the angel is increasing or decreasing the greatest good of person 1 and person 2 together?

The greatest good for the greatest number cannot be meaningful unless a sympathetic observer - call him Joe - can say for all such changes whether or not the greatest is good has increased. To supply consistency in Joe's judgments, it is common to suppose that Joe's understanding of the greatest good can be represented by the sum not of people's incomes, but of their utilities where utility is a function of income showing some concern for equality. A person's utility of income function, u(y), is assumed to be of the general form

$$u(y) = [1/(1 + \varepsilon)]y^{1+\varepsilon} \qquad (1)$$

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[9]As guide to people utilitarianism is customarily divided into act utilitarianism and rule utilitarianism. As defined by Eggleston ("Act Utilitarianism" in Eggleston and Miller, 2014, pages 125 and 130), act utilitarianism specifies that

"An act is right if and only if it results in at least as much overall well-being as any act the agent could have performed."

and rule utilitarianism specifies that

"An act is right if and only if it would be permitted by a system of rules whose general acceptance would result in at least as much overall well-being as would the general acceptance of any system of rules."

So defined, act utilitarianism would appear to leave no room at all for ordinary self-interested behaviour and rule utilitarianism seems to draw no distinction between the rules as they are and as the actor thinks they ought to be.

where the parameter ε - the elasticity of marginal utility of income with respect to income itself [10] - may differ from one person to the next. Concern not just about aggregate income, but about the degree of equality in the distribution of income requires the person's utility of income function to be concave. This in turn requires the parameter ε to be negative, though it may lie anywhere within the range from 0 to - ∞. The lower a person's value of ε, the greater the concavity of that person's utility of income function and the greater that person's concern about equality must be.

Interpreting ε as the elasticity of Joe's utility of income function, his assessment of the "good" of person 1 and person 2 together becomes

$$u^j(y_1) + u^j(y_2) = [1/(1 + \varepsilon)](y_1)^{1+\varepsilon} + [1/(1 + \varepsilon)](y_2)^{1+\varepsilon} \qquad (2)$$

where $u^j(y)$ is Joe's utility of income function. More generally, in a society of n people with net incomes $y_1, y_2, y_3, ...y_n$, Joe's sense of the greatest good can be represented as

$$W^j(y_1, y_2, y_3, ...y_n) = u^j(y_1) + u^j(y_2) +..... + u^j(y_n) \qquad (3)$$

where W is mnemonic for welfare. This measure of welfare may be converted to a dollar value, specifically the income - called the *uniform income equivalent* - such that welfare of the community would be the same if everybody had that income as it is with the existing distribution of income. Each person has his own perception of uniform income equivalents reflecting his utility of income function. Joe's *uniform income equivalent* is $\mathbf{I}_j$ defined implicitly as

$$u^j(\mathbf{I}_j) = (1/n)W^j(y_1, y_2, y_3, ...y_n) = (1/n)[u^j(y_1) + u^j(y_2) +..... + u^j(y_n)] \qquad (4)$$

Strictly speaking, the uniform income equivalent is only well-defined for some set of reference prices, but deflation by an appropriately-chosen price index will often prove sufficient. For the two-person society with incomes y_1 and y_2 , and with respect to the utility of income function in equation (1), one's uniform income equivalent becomes

$$\mathbf{I} = [½ (y_1)^{1+\varepsilon} + ½ (y_2)^{1+\varepsilon}]^{1/(1+\varepsilon)} \qquad (5)$$

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[10] For any given ε, the marginal utility of income is

$$u' \equiv \delta u/\delta y = y^{\varepsilon} > 0$$

and the second derivative is

$$u'' \equiv \delta u'/\delta y = \varepsilon y^{\varepsilon-1}$$

The elasticity of the marginal utility of income with respect to income itself becomes

$$(y/u')u'' = (y/y^{\varepsilon})(\varepsilon y^{\varepsilon+1}) = \varepsilon$$

When the angel replaces incomes of \$160,000 and \$40,000 with incomes of \$90,000 each, and when ε refers to Joe's utility of income function, Joe's sees combined welfare - as expressed by the uniform income equivalent - as changing

from $$I(\varepsilon; 160{,}000; 40{,}000) = [(½)\ 160{,}000^{(1+\varepsilon)} + (½)\ 40{,}000^{(1+\varepsilon)}]^{1/(1+\varepsilon)} \quad (6a)$$

to $$I(\varepsilon; 90{,}000; 90{,}000) = [(½)\ 90{,}000^{(1+\varepsilon)} + (½)\ 90{,}000^{(1+\varepsilon)}]^{1/(1+\varepsilon)} \quad (6b)$$

which may or may not be an increase in society's "good" depending on Joe's value of ε.

I(ε; 90,000; 90,000) is equal to 90,000 regardless of ε. If ε = 0, the uniform income equivalent becomes the average income which is *reduced* from \$100,000 to \$90,000. If ε = - ½, the uniform income equivalent is unchanged.[11] If ε = -2, the uniform income equivalent is *increased* [12]from \$64,000 to \$90,000. Joe sees the angel's action as helpful or harmful depending on whether the ε in his utility of income function is less than or greater than - ½.

Several features of this example should be noted. First, people's utility functions represent sympathy rather than benevolence[13]. Joe's perception of the greatest good for two other people is no indication of what he personally would feel obliged to donate to somebody who is less well off than he. Second, one's elasticity ε is assumed to apply uniformly to all comparisons of income. Ruled out by this assumption is the possibility that high weight is placed upon the income of the very poor but all dollars are valued equally among people with incomes above some limit. Third, the utility of income functions can be reinterpreted as representing people's attitude to risk. When ε = - ½, the \$90,000 in our example may be reinterpreted as the certainty equivalent of equal chances of acquiring incomes of \$160,000 and \$40,000. But aversion to inequality among other people is not the same as aversion to risk to oneself . A risk-taker may at the same time favour substantial redistribution of income.

Fourth, and most importantly, the value of ε in the utility of income function may differ radically from one person to the next; there is no God-given true value of ε that should govern everybody's moral preferences at once. One person may have a relatively high value of ε (close to 0) signifying little concern for inequality among his fellow citizens. Another person may have a relatively low value of ε signifying equality of income to be an important component of his notion of the common good. Ultimately, the greatest good for the greatest number is a matter of individual

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[11] Because $\mathbf{I}(-½\ ; 160{,}000; 40{,}000) = 10{,}000\ [(½)\ 16^{1/2} + (½)\ 4^{1/2}]^2 = 90{,}000$

[12] $\mathbf{I}(2; 160{,}000; 40{,}000) = [(½)(160{,}000)^{-1} + (½)(40{,}000)^{-1}]^{-1} = [1/320{,}000 + 1/80{,}000]^{-1}$
$= 320{,}000/5 = 64{,}000$

[13]On sympathy and benevolence, see Darwell, 1998.

preference. Differences among citizens in their values of ε can only be resolved politically, but that introduces new problems of its own.

Voting in One's Own Interest or for the Greatest Good

Utilitarianism supplies a broad sanction for majority rule voting, but the outcome of voting is not what utilitarianism would seem to require. An extension of the preceding example shows what may be at stake. Joe is now a citizen of a small country with one million other people, with two political parties, Liberal and Conservative, and with first-past-the-post voting in the country as a whole, so that the party with the most votes nation-wide wins the election. Suppose Joe believe that exactly half a million other people would becone $1,000 better off if the Liberals win the election, that the remaining half million other people would become $2,000 better off if the Conservatives win the election, and that Joe himself would become $2,000 better off if the Conservatives win the election. Suppose also that there is enough disagreement among other people about which party is best and enough uncertainty about the matter that, among the one million other people, the Liberal party is expected to have equal chances of winning anywhere between 45% and 55% of votes cast, that is between 450,001 and 550,000 votes, giving Liberals and Conservatives equal chances of winning the election. Suppose finally, that Joe's cost of voting, of getting himself to the ballot box and casting his vote, is $20.

Joe's vote has no effect upon the outcome of the election unless it is *pivotal,* turning what would be a win for the party Joe votes against into a win by the party Joe votes for. On th assumptions in the preceding paragraph, the chance of Joe's vote being pivotal is only 1-in-100,000.[14]

Joe must choose whether to vote or abstain and which party to vote for if he votes at all. If Joe is strictly selfish with no sense of duty to vote, he will surely abstain because his expected benefit from voting is only 2¢ (a 1 in 100,000 chance of a gain of $2,000 in the event that his vote is pivotal) which is nowhere near the $20 that it cost Joe to vote. Joe might vote from a sense of duty but not because it is in his own interest to do so.

On the other hand, if Joe votes Conservative, he confers a large expected benefit to his fellow

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[14] A person's vote is either pivotal or ineffectual. If Joe votes Liberal, the only possible impact of his vote is to turn what would otherwise be a win for the Conservatives into a win by the Liberals. With equal chances (1/100,000) of between 450,001 and 550,000 other people voting Liberal, Joe's vote can be pivotal in one of two ways. Either i) 499,999 other people vote Liberal, so that Joe's vote would creates a tie between Liberals and Conservatives and the coin-flip to break the tie would be in favour of the Liberals, or ii) 500,000 other people vote Liberal and, but for Joe's vote, there would be a tie which would have been broken in favour of the Conservatives, There is no other way for Joe's vote to have any impact on the outcome of the election. There is a 1 in 200,000 chance of each of these two outcomes [(1/100,000) x ½], creating a 1-in-100,000 chance that Joe's vote is pivotal.

citizens. By voting Conservative, Joe creates a 1 in 100,000 chance of a *gain* of $2,000 of each of 500,000 Conservative voters together with a $1,000 *loss* to each of the 500,000 Liberal voters. The expected gain to all Conservatives is $10,000 [(1/100,000 x $2,000 x 500,000]. The expected loss to all Liberals is $5,000 [(1/100,000 x $1,000 x 500,000]. The net gain to all other voters together is $5,000. At a cost of $20 in choosing to vote rather than abstain, Joe creates an expected net gain to his fellow citizens of $5,000. Very little else that one might do in life would create such a large benefit to others per dollar of cost to oneself. To be sure, the example is made up, but the gap between benefit and cost is so large that almost any plausible changes in the numbers would yield the same general result.[15]

So far, Joe is only concerned about income, his own and that of the nation a whole. Concern for other people would lead Joe to vote Conservative because average income is $500 higher if the Conservatives win the election. Alternatively, if concerned about equality as well as average income, Joe might be inclined to vote for what he sees as the greatest good which may be represented by the uniform income equivalent. To keep matters simple, suppose

- The electorate consists of equal numbers of rich and poor people, 500,000 of each.

- Incomes of all rich people, y_R , are the same.

- Incomes of all poor people, y_P , are the same.

- If the Liberals win the election, y_R = $200,000 and y_P = $40,000.

- If the Conservatives win the election, y_R = $202,000 and y_P = $39,000.

- There is a 1-in-100,000 chance that any person's vote will be pivotal.

- A person whose concern for equality is represented by ε in the utility function of equation (1) has a uniform income equivalent, **I**, as defined implicitly in the equation (5) above with y_1 replaced by y_R and y_2 replaced by y_L.[16]

On these assumptions, a person's perception of the greatest good conditional on the outcome of the election becomes **I**(ε, E) where E is either L or C depending on which party is elected. The gain or loss in the uniform income equivalent from a win by the Liberal party becomes **I**(ε, L) - **I**(ε, C)

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[15]On "voting for charity's sake", see Edlin et. al, (2007).

[16] The special assumption that incomes of all rich people are the same and incomes of all poor people are the same too, allows the uniform income equivalent to be computed with reference to only two persons, one rich and the other poor.

as shown for several values of ε in the final columns of table 1.[17]

Table 1: Uniform Income Equivalents ($) for Selected Values of ε and Depending on which Party Wins the Election.

ε	**I**(ε, L)	**I**(ε, C)	**I**(ε, L) - **I**(ε, C)
0	120,000	120,500	- 500
-1/4	112,505	112,718	- 115
-1/2	104,721	104,629	92
-1	89,493	88,758	665
-2	57,143	56,271	862
- ∞	40,000	39,000	1,000

When ε = 0, uniform income equivalents and average incomes are one and the same, $120, 000 in the event that the Liberals win the election and $120,500 in the event that the Conservatives win the election. At the other extreme where ε = - ∞, the uniform income equivalents become the maximin incomes, incomes of the worst off person in society, equal to $40,000 when the Liberals win the election and equal $39,000 when the Conservatives win the election.

If Joe's value of ε is -1/2, he sees a Liberal win as yielding a uniform income equivalent that is $92 higher than the uniform income equivalent as it would be if the Conservatives won instead, a total gain of $92,000,000 [the additional uniform income equivalent x total population] for society as a whole. When the chance of Joe's vote being pivotal is1-in-100,000, Joe's perception of the dollar value of the *expected* gain to society from his voting Liberal rather than Conservative becomes $920 which is still very much larger - about fifty times as large - as Joe's assumed cost ($20) of casting his vote.

In short, if Joe is strictly selfish he abstains; if he is sufficiently altruistic and if the absolute value of ε is small, he votes Conservative; if he is sufficiently altruistic and ε is large, he votes Liberal. The example is made-up, but the spreads between benefits and costs of voting are large enough to withstand almost any reasonable changes in the assumptions.

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[17]For example, the term **I**(- ½, C) shown in the third column of the fourth row in Table 1to equal $104,629, is the value of **I** for which

$$\mathbf{I} = [½\,(202{,}000)^{1-½} + ½\,(39{,}000)^{1-½}]^{1/(1-½)}$$

Computation of this and other values in the table is straightforward.

There are other possibilities. One may vote to promote the welfare of one's social class. Poor people may vote in the interest of other poor people. Rich people may vote in the interest of other rich people. By contrast, redistribution, public health care and the old age pension may be supported by people for whom the cost in taxation exceeds the benefits they can expect from these programs not out of sympathy for their beneficiaries, but in fear that the social order will be disrupted if poverty becomes too extreme. Joe may vote Liberal because Joe and most Liberals live in the same part of the country or because they share the same occupation or because they share the same religion or national origin. Joe may vote as he does because he believes that some party's policy is right - that abortion should, or should not, be permitted - regardless of what the rest of the electorate prefers. Joe may vote Liberal not just because he sees the Liberal party as striking the better balance between the size and distribution of income as discussed above, but because he sees the Liberal party as better for everybody. Joe may believe a Liberal government to be more likely than a Conservative government to find the right balance of military expenditure and diplomacy to keep the country out of war. Voters may divide between Liberal and Conservative not just from self-interest or concerns about the distribution of income but because of differences in beliefs about what the parties in office would achieve in the interest of all citizens at once. People may vote as they do out of pure self-interest, to advance what they sees as the good of the nation as a whole, out of sympathy for a subgroup of the population or from some complex combination of motives.

There is no guarantee that Joe will opt for the common good, but any inclination to do so may be reinforced by another consideration. If Joe were strictly self-interested, he would not bother vote at all. He would abstain because his expected gain from voting - 2¢ in this example - would be too small to justify the time and trouble of getting himself to the ballot box. The fact that Joe does vote suggests that Joe is public-spirited and that, being so, he might be inclined to vote for what he sees as the common good.

Voters may be doubly altruistic: in voting rather than abstaining, and in voting for what they see as the common good. If nobody (or almost nobody) abstains, it may not matter much that each person votes for what is best for himself alone. Just as an approximation to the common good (Pareto optimality) may be procured by universal self-interested behaviour in commercial markets, so too might an approximation to the common good be procured by self-interested voting in electoral markets, for an outcome preferred by a majority of the population may not too far from what most people see as the common good. Universal self-interested voting might even be preferable to a situation where some people vote self-interestedly and others vote altruistically, biassing outcomes toward the interests of purely selfish voters. With widespread abstention, outcomes tend to diverge from any reasonable notion of the common good, depending instead on which groups are best organized and which party has access to the most funds. Something more than mere self-interest - a duty to vote or a willingness to vote for what one sees as the common good - may be required to induce people to vote rather than abstain. Choice between voting and abstaining confronts the electorate with an extreme prisoners' dilemma: it is in nobody's interest to vote, but everybody is better off when everybody votes.

Focussing on the extremes of pure self-interest and unbiassed concern for the common good,

each person s (mnemonic for self) may be supposed to have a voting utility function

$$v_s = v^s(y_s, \mathbf{I}_s) \qquad (7)$$

where y_s is one's own income and where $\mathbf{I}_s$ is the uniform income equivalent, as seen by person s and as described in equation (4) , of the distribution of income in the nation as a whole. The voting utility function, v^s , would be concave in both y_s and $\mathbf{I}_s$. Person s votes for whichever party or policy supplies values of y_s and $\mathbf{I}_s$ leading to the largest value of v_s. People may well differ in their weighting of these two arguments. Relatively public-spirited people place more weight on $\mathbf{I}_s$; relatively selfish people place more weight on y_s . Someone for whom $v_s = y_s$ is an "energy voter". Someone for whom $v_s = \mathbf{I}_s$ is a saint. Personnel and platforms of competing political parties supply each person s with different values of y_s and of $\mathbf{I}_s$. Each person can be expected to vote for whichever party supplies the larger value of v_s. Recognizing how voters behave, political parties would adjust their platforms accordingly.

The two functions, u in equation (2) and v in equation (8), are both utility functions but they differ in content and scope. The function u depends upon one's own income and is a *component* of the good, W, as one sees it of society as a whole. The function v depends upon one's perception of the greatest good as well as upon one's own income, which together determine and how one votes.

Consider the redistribution of income by a negative income tax, a tax levied at a rate t with proceeds divided equally among all citizens so that, the higher t, the smaller the gap between the net incomes of rich and poor. Everybody's post-tax, post-transfer income, y, and their uniform income equivalents, I, become functions of t. The income of person s becomes $y_s(t)$, his uniform income equivalent becomes $\mathbf{I}_s(t)$ and his voting utility function becomes $v_s(t)$. Voting about the rate for a negative income tax becomes a single-peaked issue with an equilibrium outcome in the first preference of the median voter. Line up all voters according to their preferred values of t. The rate at the mid-point of the line beats any other rate in a pair-wise vote and can be expected to prevail in an election about the redistribution of income. The electoral equilibrium tax rate is a compound of ethical and selfish motives. The rate would be low if the rich are selfish and the poor ethical, and would be high if the rich are ethical and the poor selfish, with plenty of room for hypocrisy in between.

Politics is not always so accommodating, for there need be no unique electoral equilibrium when more than one issue is at stake. There need be no unique electoral equilibrium when, for example, voting is about redistribution and foreign affairs simultaneously or even when the tax structure is not confined to the negative income tax. In such cases, the functions $\mathbf{I}_s(t)$ and $v_s(t)$ in equation (8) must be replaced by $\mathbf{I}_s(P)$ and $v_s(P)$ where P is a vector of public policies on a variety of issues. Now voting is no longer self-sufficient and must be supplemented by bargaining in the formation of platforms of political parties or in the legislature's choice of public policy.

The ambiguity and imprecision in moral, as distinct from legal, obligation is especially pronounced in the duty to vote. Government by majority rule voting is an indispensable part of everybody's conception of the greatest good, and a duty to vote is an indispensable requirement of for government by majority rule voting. Self-interest alone will not do. Wide-spread abstention opens the system to manipulation of organized groups seeking to shift the outcome of elections from what the

majority of the population would prefer. The smaller the proportion of voters in the electorate, the more the outcome of the vote comes to depend on the organization of voters and the less on voter preference. Some countries have made voting compulsory, but that introduces problems of its own. A duty to vote includes an obligation to vote thoughtfully, and that cannot be enforced.

People differ in what they see as the duty to vote. Some see their own and other peoples' duty as nothing more than showing up at the ballot box and voting. Others see a duty to vote thoughtfully and to avoid supporting parties or candidates likely to be cruel to designated parts of the population. Some see a duty to vote for the greatest good for the greatest number. Others see nothing wrong in voting in one's interest exclusively or in the interest one's ethnic group or social class. There is no moral resolution of these points of view. People vote as they please and the outcome is what it is. A decent society requires majority rule voting regardless of why people vote as they do.

Ethics becomes subordinate to elections. When ethical prescriptions differ from one person to another, the best one can hope for is that there is a median voter whose preference can be expected to prevail. Otherwise, one can only hope that people's utility functions do not differ too much and that legislators can bargain their way to a mutually-acceptable public policy, Diversity of citizen's ethical preferences, as reflected in their different values of ε, are no greater threat to democracy than ordinary conflicts of interest. The same mixture of voting and compromise which resolves differences in one case is just as likely to do so in the other.

A distinction needs to be drawn between "social welfare" and "public choice". Social welfare is somebody's assessment - which may be expressed as a uniform income equivalent - of the well-being of society as a whole. As some person's assessment, it inherits the rationality to be expected in individual judgment. A rational person who prefers x to y when z is unavailable, does not come to prefer y to x if z becomes available; in a choice among x, y and z, this person must prefer either x or z. That remains so regardless of whether the person is choosing for himself alone or expressing his view of what is best for the entire nation. Public choice is different. Public choice - amalgamation of people's diverse preferences into a single course of action, by majority rule voting or by some other means - may display precisely the inconsistency in the x, y, z example. The outcome of majority rule voting may depend on the presence or absence of a candidate who himself has no chance of winning the election. Such inconsistency, called the spoiler problem, may arise because no unique measure of social welfare is valid for everybody at once. Arrow's Impossibility Theorem is interpretable as meaning that no mechanism of public choice can avoid such inconsistencies altogether, that there are always constellations of preferences for which such inconsistencies arise.[18]

Note finally that values of ε may be the basis of the left-right distinction in the political sphere. It is at least arguable that right-wing people see the job of responsible government to do whatever is appropriate to maximize total national income regardless of how income is allocated between rich and poor, while left-wing people place much more weight on distribution and would willingly sacrifice a large loss to the rich in return for a small gain to the poor.

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[18]On this interpretation of Arrow's impossibility Theorem, see Usher (2017).

The moral of the story is that utilitarianism and voting are connected in two conflicting ways. On the one hand, utilitarianism upholds majority rule voting as essential if government is to serve the many rather than just the few, as the only alternative to despotism. On the other hand, majority rule voting need not - and typically does not - supply objectively-defined utilitarian outcomes. People may vote self-interestedly rather than to promote some conception of the greatest good. Worse still, people may differ in their conceptions of the greatest good, for nothing forces people's values of ε to be the same. In the example, people with very concave utility of income functions see the greatest good in a win by the Liberals and people with less concave utility of income functions see the greatest good in a win by the Conservatives, with no commonly-recognized criterion for deciding whose perception is right. At best, differences can be resolved by majority rule voting despite voters' different mixtures of selfishness and altruism and different perceptions of the greatest good. Utilitarianism may forbid evil voting, such as voting for the dictator expected to murder some segment of the population, but within broad limits people may vote as they please. Voting like much of ordinary commerce is mostly outside the bounds of what utilitarianism prescribes.

Concern for Others in Different Countries or Yet to be Born

Governments may be responsible but not sympathetic, except vicariously. People alone are sympathetic toward their fellow citizens today, toward people elsewhere and toward people yet to be born. There is also a difference in degrees of concern. Responsible government counts all citizens equally. Citizens play favourites, being more concerned about the plight of some people than of others. Refugees from one country may be welcome; equally-distressed refugees from another may not.

To account for citizen's concerns above and beyond the welfare of their fellow citizens today, the voting utility function, v^s, of person s in equation (7) above might be expanded to

$$v_s = v^s(y_s, \mathbf{I}_s, \mathbf{F}_s, \mathbf{O}_s) \qquad (8)$$

where, as before, y_s and $\mathbf{I}_s$ are one's own net income and one's uniform income equivalent of the entire distribution of income of one's fellow citizens alive today, where $\mathbf{F}_s$ is a measure of the welfare of future generations of citizens and where $\mathbf{O}_s$ is a measure of the welfare of people abroad. Platforms of political parties must take all four variables y_s, $\mathbf{I}_s$, $\mathbf{F}_s$ and $\mathbf{O}_s$ into account.[19] This is obviously a gross

[19]More generally, the voting utility function of person s (oneself) may be represented as $v^s(y_1, y_2, y_3, \ldots\ldots, y_w)$ where w is the number of people (perhaps everybody in the world) about whom person s might be concerned, where the function v^s weighs people unequally according to person s's degree of concern and where people may be thought of as spread out over time and throughout the world. If person s is strictly selfish, then $\delta v^s/\delta y_i = 0$ except when s and i are the same. Unless person s is a saint, he is more concerned about himself than about other people in the sense that $\delta v^s/\delta y_s > \delta v^s/\delta y_i$ whenever $y_s = y_i$. He would never sacrifice (dollar for dollar) any of his own income for the benefit of others who are as well off as he. However, the inequality may be reversed when $y_i < y_s$; an altruistic person may give up part of his income for the benefit of others who are worse off than he. Person s may be said to care more about person i than about

simplification of citizen's concerns, but the contrast with equation (7) may be helpful by indicating what may be at stake. The four variables may all be thought of as influenced by public policy. In a two-party election, each person s would vote for the party with leadership and policy yielding the higher value of v_s.

This description of responsible government is at odds with the view that "all lives matter" equally, lives of people in one's own country, lives of people abroad and lives of people yet to be born.[20] On this view, every person has his own unique life experience with as much right to existence and to protection by public policy as any other person, so that, the larger the population, the larger, other things being equal, the greatest good must be. When all lives matter equally, the greatest good is transformed from **I** in equation (4) into n**I** where n is population, so that a doubling of population would be deemed as desirable as a doubling of the uniform income equivalent. In a country with an initial population of one million and an initial uniform income equivalent of $50,000, a government acting on this interpretation of utilitarianism would be as content with a rise of population from one to two million as with a rise in the uniform income equivalent from $50,000 to $100,000.

There are several opposing considerations: As long as incomes are unaffected, the greatest good should be independent of how a given total population is allocated among countries. One would not want to say that social welfare in Canada has increased for no other reason than more people choose to live there. The greatest good is obviously **I** rather than n**I** in this case. It is less clear how to treat population growth that is not just migration from one country to another, but is the birth of new people in one's country, so that an extra million people Canada is at the same time an extra million people in the world.

One's perception of social welfare may be sensitive to both **I** and n, but more sensitive to **I** than to n. A distinction may be drawn here between full-blown utilitarianism and number-damped utilitarianism (see Ng, 1986), the former requiring the maximization of n**I**, the latter requiring the maximization of f(n)**I** where the function f(n) is concave and may no longer increase once population has grown beyond some limit. Number-damped utility has the advantage of valuing additional people without at the same time giving rise to Parfitt's (1984) famous "repugnant conclusion" that social welfare is lower in any society, no matter how prosperous, than in some other society, no matter how poor, as long as the latter has a sufficiently large population and as long as people's utilities in that society are all positive no matter how small.[21]

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person j if $\delta v^s / \delta y_i > \delta v^s / \delta y_j$ when y_i and y_j are the same.

[20]For a thorough discussion of population problems, see Blackorby, Bossert and Donaldson, 2005.

[21]If **I** were a decreasing function of n, there would be an ethically optimal population at which the elasticity of **I**(n) with respect f(n) is equal to 1. The "repugnant conclusion" is valid on its premises, but the premise that actual lives and lives of people who might be born are valued

Much depends on the nature of population growth. With social welfare measured as n**I** rather than just **I**, population growth might be thought of as a magical transformation from a smaller to a larger population, leaving average income and the distribution of income unchanged. More people may well be better than less in such circumstances. One may be uneasy about this expanded measure of social welfare because there is no such magical transformation. Population growth takes place gradually and is typically (though not invariably) accompanied by income per head that is less than it would otherwise be. A rise in population next year may well cause income per head or population itself to fall the year after. The world's resources are scarce and may only be replenished slowly. The world may be Easter Island. The comparison between **I** and n**I** as representatives of social welfare is static, implicitly abstracting out of sight the reasons for fearing population growth. The proposition that "all lives matter" may be true but irrelevant in the light of the problems that the world actually faces.

There is also a counting problem. What is the value of n when, looking far enough ahead, there are an infinite number of descendants of Canadians alive today? Even interpreted as the number of people in some future year, there remains a choice of when n is to be observed. The concept n**I** tends to dissolve on close inspection.

There is finally some question as to whether all lives really do matter. A person may feel morally required to help poor people alive today and to better the lives of people destined to be born, but may at the same time be unconcerned about whether or not extra people are born. An analogy may be drawn between actual families and the family of man. Parents love their children, know that they would love additional children too, but choose to have fewer children than they might. People are concerned about future generations and would sacrifice a great deal to ensure the continuance of mankind forever, but, beyond some limit, may not much care whether the population a hundred years hence is large or small. Especially in view of the limits on land and natural resources, we are far less concerned, if we are concerned at all, about a *permanent* decrease in world population from 7.4 billion to 6.4 billion than about a decrease some time in the future from 1 billion to 0.

Turning from future people to people in other countries, public provision of foreign aid is compatible with responsible government's exclusive focus on its own citizens because foreign aid is a public good. Each citizen can be expected to place a higher weight (at any given income) on the welfare of fellow citizens today than upon people in other countries, but all citizens may favour foreign aid when people in recipient countries are very much worse off than people in one's own country. Vicarious altruism is sufficient.

Two questions can now be distinguished: Are citizens today concerned about future generations and about people elsewhere in the world? and Can such concerns be reasonably be addressed by the injunction to seek the greatest good for the greatest number? I suggest that the answers to these questions are "yes" and "no". Yes, all lives matter to some extent. Yes, we are concerned about future generations. Yes, we favour foreign aid to nations where people are less well off than we. But we

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equally seems totally at variance with most people's moral preferences.

subordinate these concerns to our concern for our own incomes and for the incomes of our fellow citizens today. Future generations, residents of foreign countries, our fellow citizens and ourselves are not equally weighted in our voting utility functions as some extreme interpretations of the injunction to seek the greatest good for the greatest number would seem to require. The vague representation of concern for others in equation (8) above is about as far as we can reasonably go. The greatest good for the greatest number is an objective of responsible government in its dealing with its own citizens and cannot be reasonably generalized to concern for future generations or for the rest of the world.

A standard objection to strict utilitarianism - the injunction to seek the greatest good for the greatest number - is that it demands too much. The objection is that people's concern for one another is genuine but limited. It would seem to be a fact of life that, though people are concerned about other people today and about other people yet to be born, one's degree of concern is less than the strict utilitarian ideal would seem to require. We are prepared to sacrifice for other people but not to that extent.

Comments on the Assumptions

Mana from Heaven: In modelling the redistribution of income, people's incomes were looked upon as having fallen mysteriously in different amounts on different people but open to modification by public policy, typically narrowing the gap between rich and poor at the cost of some reduction in average income. No account was taken of people's responsibility for their circumstances. No distinction was drawn between income disparities arising a) because one person is born wealthy and another is born poor and b) because one person works harder and earns more than some other equally-endowed person. Ideally, we would not want the government to redistribute income from Joe to Charlie if both are equally skilled, Joe works long hours and makes lots of money while Charlie works as little as necessary to survive, spending as much time as possible in the library reading philosophy. But that is exactly government does. Ideally, income in the definition of social welfare would be replaced by *potential* income, by income as it would be if everybody's effort to acquire income were the same. In practice, the Ministry of Finance has enough trouble identifying people's incomes without trying to identify their efforts as well. Modifications in the tax system that would be appropriate in an ideal world may in practice cause more trouble than they are worth. Better to subsidize the deserving poor, even at the cost of subsidizing the undeserving poor as well.[22]

Failure to recognize personal responsibility is part of a larger problem. People's welfare depends on more than just income. Ideally, laws and policies should be designed to promote people's welfare in some larger sense of the term. Perhaps happiness is the appropriate criterion; perhaps something more. The simple model with incomes treated like mana from heaven takes no account of freedom (Friedman, 1960) or the exercise of people's capabilities (Sen, 1985). There is a larger utilitarian project to specify people's welfare and the welfare of society comprehensively, and to design

[22]On the expansion of the definition of income to account for differences in people's supply of leisure, see (Boadway et. al., 2002). Incorporation of responsibility into measures of social welfare is discussed in detail in Fleurbaey (2008)

ideal public policy accordingly, incorporating cardinality of utility functions and comparability among utilities of people with different tastes, skills and temperaments.[23] The claim here is that, though the simple model with innate differences in people's ethical prescriptions has useful implications for the measurement economic progress, the redistribution of income, the choice of laws and other aspects of public policy, there is no objectively-correct measure of the greatest good; voting or compromise among people with different perceptions is a requirement for what we see as a good society.

Extreme Egalitarianism: An implication of the postulated forms of the social welfare function and the utility of income function in equations (2) and (4) is that greatest possible egalitarianism (when ε is equal to $-\infty$) requires a maximin solution: the correct public policy in choosing among sets of incomes would be to select the set in which the income of the worst-off person is largest. Actually, egalitarianism could go further than that. Consider a public choice between sets of income A and B for a population of 7 people. Set A is (4.9, 10,10,10,10,10,10). Set B is (5, 6,6,6,14,14,14). It is entirely possible that a person's sense of the greatest good for the greatest number places set A ahead of set B despite the fact that both the average income and the income of the very worst off person are lower in set A than in set B. Equality in the rest of the population might take precedence over the slight loss of income for the worst off person. The example is of a case where the representation of greatest good in equations (2) and (4) is inadequate.

Utility, Income and Happiness: In the core model, responsible government was assumed to maximize something called utility, but without explaining exactly what utility means. Utility was a device for describing preferences, not a good in its own right. Concavity of the utility of income function was justified by an aversion to inequality or by an analogy with risk aversion. Public choice was compared with private choice in risky situations. Alternatively, a person's choice behind the veil of ignorance is sometimes looked upon as the maximization of total nation-wide happiness where more income makes one happier but by an ever-decreasing amount. Suppose for the sake of the argument that there is a well-defined measure of happiness and that a person's happiness is a concave function, h(y) of income, so that total happiness is always increased by a dollar-for-dollar transfer from rich to poor. If it can be assumed that responsible government seeks to maximize total happiness, then the happiness function, h(y), replaces the utility function, u(y), in the core model above, but the story about redistribution of income remains essentially the same. Each person's prescription for the behaviour of responsible government depends on that person's elasticity of the marginal happiness, rather than utility, with respect to income.

But equality of happiness, rather than just total happiness, may be important. People's

[23] Adler (2014) replaces the simple weighing of incomes in equation (2) with an extensive comparison of utilities, asking such questions as whether the impartial observer would rather be Shakespeare or Cleopatra and how to account for "the wrong kind of preferences", questions that would seem more relevant to a psychiatrist than to the benevolent dictator in responsible government. Be that as it may, the focus in the present article is upon incomes on the working assumption that even responsible government maintains some distance from lives of its citizens. Adler's concerns may be more relevant to the choice of laws than to the redistribution of income.

happiness is typically measured by asking people to say how happy they are on a scale from 1 to 10 as described in Layard, Mayraz and Nickell (2008). Suppose for the sake of the argument that the measure is a perfect indicator of true happiness, so that, for example, a measure of 6 means exactly the same thing in Bangladesh as in New York City.[24] Even so, a person may prefer a sure happiness of 6 to equal chances of a happiness of 4 and a happiness of 8, and a person choosing between two societies on the understanding that he has equal chances of occupying the place of each and every person in whichever society he chooses, may prefer a society where everybody's happiness is 6 to a society where half the population has a happiness of 4 and the other half has a happiness of 8.

A person's assessment of the greatest good, W, remains the sum of utilities, u, but utility becomes a concave function of h. The function u(y) in the core model is transformed, ignoring subscripts and superscripts, into u(h(y) and the elasticity, ε, of the marginal utility of income with respect to income itself becomes a weighted average of two elasticities, the elasticity of marginal happiness with respect to income and the elasticity of marginal utility with respect to happiness. Both elasticities may differ substantially from one person to the next.

There is also some question about when and to what extent responsible government should take account of differences among people in their happiness at any given income. In the core model, each person j valued everybody else's income y_i in accordance with person j's utility of income function, $u^j(y_i)$, taking no account of differences among people's income-to-happiness schedules. Person j's assessment of person i's portion of social welfare depended on person i's income regardless of how happy person i turns out to be. That may change when utility and income are linked by happiness. Now, at least in principle, responsible government might feel obliged to take account of differences among people in their income-to-happiness functions. A person with a great capacity for turning money into happiness might be awarded more money than a person without that capacity. The bookish fellow who spends his days in the public library would be awarded less than the fellow who loves to take expensive vacations. In practice, the government does no such thing, if only in recognition of the likely harm from empowering civil servants to decide who is whom. On the other hand, some account is taken of "needs' informally in, for example, the design of programs to subsidize child care, as described in Atkinson (2015, chapter 8), and in public expenditure on care for people with physical or mental disabilities.

As perceived by person j, person i's portion of social welfare, $u^j(y_i)$, could be interpreted as either $u^j(h^j(y_i)$ or as $u^j(h^i(y_i)$; person i's income could be evaluated according to the happiness that person j would acquire from having person i's income, or according to person i's own happiness-to-income function. The former requires person j to know no more about person i than his income. The latter requires genuine empathy, a real ability on the part of person j to place himself in person i's shoes and to know how person i feels. That may be asking too much.

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[24] What exactly does a person mean when he asserts that his happiness is 6 rather than 7? Does everybody have the same definition of happiness? Might some people report happiness when all they mean is prosperity, creating an artificial correlation between the two?

Diversity of Ethical Preference: Some authors[25] feel strongly that a unique conception of the greatest good for the greatest number, a common value of ε, is hardwired into our brains. On this view, a common humanity gives rise to a common sense of right, wrong and the greatest good for the greatest number. The question is how similar people might reasonably be supposed to be. We have different hair colour, different IQs, different tastes, different talents, different trade-offs between income and leisure, and different propensities to commit crime. Might we not have different values of ε as well, or, more generally, different utility of income functions and different uniform income equivalents? The vehemence of Rawls' advocacy of the maximin principle (identifying justice with the maximization of the income of the poorest person and requiring that $\varepsilon = -\infty$) and of the advocacy of efficiency as the sole criterion for cost-benefit analysis and for the choice of laws in the literature of law and economics (tantamount to setting $\varepsilon = 0$) is evidence enough that ethical principles differ. Any value of ε between these limits would seem to be possible. No ethical counterpart of the price mechanism keeps people's valuations of ε the same.[26]

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Governments should act in accordance with the will of the electorate as expressed by majority rule voting. Governments should seek the greatest good for the greatest number. These propositions connect in several ways:

Government must accept the outcome of elections regardless of whether the expressed will of the electorate is in accordance with what the government sees as the common good, for there is no higher power to define and enforce the utilitarian prescription. Government by majority rule voting is a complex institution not just because voters differ in their perceptions of the greatest good, but because of the dynamics among political parties seeking office and of the unavoidable role of bargaining in the formation of platforms of political parties, among parties in formation of the government in office and among legislatures in the passage of laws. The utilitarian ideal may be seen as the under-pinning of government by majority rule voting because the alternative is dictatorship.

Voters may be utilitarians. In view of the potentially enormous benefit to society per dollar of cost to the voter himself in the event that one`s vote is pivotal, it is reasonable to suppose that citizens who vote rather than abstain would cast ballots to promote what they see as the greatest good for the greatest number or, at least, for a mix of personal and societal concerns.

Which brings us back to the main story in this article that there is no single ethical prescription which all voters recognize as correct. When you and I disagree about facts, there is usually a truth out there which may, at least in principle, be discovered. When you and I disagree about the trade-off between average income and equality in the distribution of income, there is no objective standard to

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[25]On the interpersonal comparability of happiness as a measure of utility, see Ng(1997)

[26] There would seem to be little support for what Harsanyi (1982, page 51) calls the similarity postulate which, in our simple model, would be that everybody's value of ε is the same.

which we may appeal. Each person has his own true measure of economic welfare reflecting his elasticity of marginal utility of income with respect to income itself . People differ their in their interests and in their views of what is best for society. Such differences can only be reconciled by voting or by some other political means that we agree to respect because the alternative is chaos.

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Appendix A: A Utilitarian Measure of Economic Growth

We rely on statistics of economic growth - the growth rate of income per head or per household as the case may be - to tell us how much better off we are becoming over time, but we are inclined to think of economic growth as more desirable when its benefits are widely shared than when benefits are concentrated upon the privileged few. A 2% rate of growth is seen as more socially desirable when everybody's income rises by 2% then when the entire increase in income accrues to the top one percent, while the income of the remaining ninety-nine percent does not rise at all. Recognition of this aspect of most people's sense of social welfare suggests that the ordinary measure of economic growth might be supplemented - supplemented, not replaced - by a *utilitarian measure of economic growth* that takes changes in the distribution of income into account. We would like to measure the growth rate not just of income per person, but of the uniform income equivalent of the entire distribution of income. We can construct such a measure but it is not unique because people's sense of economic progress vary with the elasticities of their utility of income functions.

Consider the growth of household income. Table A1 shows quintiles of mean household income in the United States in constant (2014) dollars for the years 1973 and 2014, together with the percentage increases over this period. On average, as shown in the last column, real household income rose by 29.4% over the entire period, but, as shown in the bottom row, percentage increases were lower for the poor than for the rich, varying steadily from a decline of 3.4% for the poorest fifth to a rise of 51.1% for the richest fifth. If the overall increase in average household income had been spread out evenly among quintiles, mean household income in the poorest quintile would have risen from $12,097 to $15,653 instead of falling to $11,676. Of growth in total income, 76.3% accrued to the richest fifth of the population. Looking at such data, we are ordinarily inclined to say that average income has increased but the distribution has deteriorated, without trying to place these changes on a common scale, but, just as consumption of apples and oranges is weighted on a common scale in the measurement of ordinary income, it may be useful to ask how changes in average income and in the distribution of income might be combined into a single measure of economic progress.

Table A1: United States Mean Household Income Received by Each Fifth of the Population (in 2014 dollars) US Census, Table H3

	lowest quintile	second quintile	third quintile	fourth quintile	highest quintile	average
1973	$12,097	$30,416	$49,889	$71,768	$128,413	$58,517
2014	$11,676	$31,087	$54,041	$87,834	$194,053	$75,738
percentage increase	- 3.4%	2.1%	8.3%	22.2%	51.1%	29.4%

Let m be the population and $(y_1^{1973}, y_2^{1973}, \ldots, y_m^{1973})$ be the distribution of income in 1973. Let n be the population and $(y_1^{2014}, y_2^{2014}, \ldots, y_n^{2014})$ be the distribution of income in 2014. Mean incomes are $\mathbf{Y}^{1973}$ and $\mathbf{Y}^{2014}$ where

$$\mathbf{Y}^{1973} = (1/m)(y_1^{1973} + y_2^{1973} + \ldots\ldots + y_m^{1973}) = \$58{,}517 \qquad \text{(A1)}$$

$$\mathbf{Y}^{2014} = (1/n)\,(y_1^{2014} + y_2^{2014} + \ldots\ldots + y_n^{2014}) = \$75{,}738 \qquad \text{(A2)}$$

As ordinarily measured, the rate of economic growth, of average rather than total income, between year 1973 and year 2014 is

$$g(\mathbf{Y}) = (1/[2014 - 1973])\ln\,(\mathbf{Y}^{2014}/\mathbf{Y}^{1973}) = 0.63\% \qquad \text{(A3)}$$

But the rate of improvement as seen by a person with a given utility of income function, u(y), may be represented by the growth rate not of income alone, but of that person's uniform income equivalents, $\mathbf{I}^{1973}$ and $\mathbf{I}^{2014}$ defined implicitly by the equations

$$u(\mathbf{I}^{1973}) = (1/m)[u(y_1^{1973}) + u(y_2^{1973}) + \ldots\ldots + u(y_m^{1973})] \qquad \text{(A4)}$$

and

$$u(\mathbf{I}^{2014}) = (1/n)[u(y_1^{2014}) + u(y_2^{2014}) + \ldots\ldots + u(y_n^{2014})] \qquad \text{(A5)}$$

With u(y) as specified in equation (4), a person's values of $\mathbf{I}^{1973}$ and $\mathbf{I}^{2014}$ are dependent on ε and may be computed from equations (A4) and (A5) above. For any given ε, the utilitarian measure of economic growth becomes

$$g(\mathbf{I}) = (1/[2014 - 1973])\ln\,(\mathbf{I}^{2014}/\mathbf{I}^{1973}) \qquad \text{(A6)}$$

With changes in quintiles of household income in the United States between 1973 and 2014 as shown in table 1 above, the uniform income equivalents, the trade-offs in social welfare between dollars to the rich and dollars to the poor and the implied utilitarian rates of economic growth are shown for selected values of ε in Table A2.

Table A2: Uniform Income Equivalents,
Rates of Trade-off Between Incomes in the Highest and Lowest Quintiles
and Utilitarian Rates of Economic Growth at Alternative Values of ε

postulated ε	0	- ½	- 1	- 1½	- 2	- 6	- ∞
$\mathbf{I}^{1973}$	$58,517	$51,437	$44,225	$37,481	$31,785	$16,654	$12,097
$\mathbf{I}^{2014}$	$75,738	$62,873	$50,680	$40,412	$32,710	$16,084	$11,676
$-\delta y_T/\delta y_B$ 1973	1	3.26	10.62	34.59	112.68	1,430,829.50	∞
$-\delta y_T/\delta y_B$ 2014	1	4.08	16.62	67.75	276.22	21,074,518.00	∞
growth rate, g(**I**)	0.63%	0.49%	0.33%	0.18%	0.07%	- 0.085%	- 0.086%

For each value of ε, dollar values of uniform income equivalents in 1973 and 2014 are shown in the second and third rows. The next two rows show the amounts, $-\delta y_T/\delta y_B$ in 1973 and 2014, by

which income in the top fifth of the distribution can be reduced in compensation for an increase of $1 in the income in the bottom fifth without altering social welfare in the nation as a whole. The last row shows utilitarian rates of economic growth computed for each value of ε from equation (A6).

The story in the table is that, though the ordinary measure of the growth rate of real income per year is 0.63%, the growth rate of the uniform income equivalent diminishes steadily as the utility of income function becomes increasingly concave until, eventually, the rate of growth turns negative because all weight is placed upon the lowest quintile of the distribution.[27]

If most people's values of ε were about -1 - implying a willingness to see income in the highest quintile reduced by about $16 in return for a $1 increase in income in the lowest quintile - the utilitarian rate of economic growth would be about half the rate of economic growth as ordinarily computed, 0.33% per year rather than 0.63%.

The numbers in table 1 differ somewhat from the ideal numbers for illustrating the principles in this essay. Ideally, the measures should take account of the entire distribution of income rather than the quintile means. Utilities might be compared over people's entire lives rather than at a moment of time. Changes in the age distribution of the population may be important because people who consume the same amount each year of their lives would have higher observed incomes in their working years than in retirement. Allowance should be made for changes over time in tax rates, provision of public services and transfers in kind, such as food stamps, care and public education, together with some accounting for shares of corporate profits and capital gains and for changes over time in family size. For any given distribution of income as commonly measured, a society with substantial public services is likely to have more real equality than a society where each person is entirely on his own. No such corrections are included here. The numbers in the table remain useful, though they must be taken with a grain of salt.[28]

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[27] A computation for the United States (Beckerman, 1978, table 4.1) between 1952 and 1972 yielded much less spectacular results. As conventionally measured, gross national product per head grew at a rate of 4.07%. The comparable growth rate of the uniform income equivalent income (Beckerman called it "equally distributed income") for a postulated ε of -2 was 4.04%. The switch from the conventional measure to the uniform income equivalent mattered less in those years because they were the final years of what Gordon (2016) called "the golden age of economic growth" (1870-1970) after which the growth of average income declined substantially and what growth did occur accrued for the most part to the top decile of the income distribution. A similar estimate was made for Canada in Usher (1980).

[28] Atkinson (1970) uses a model of utility of income like that used here to supply a measure of inequality of income rather than a utilitarian measure of economic growth. Atkinson classifies measures of inequality as what might be called (my terminology) technical and utilitarian. Technical measures include variance, the ratio of variance to mean, the Gini coefficient, the share of the top 1%, and so on. Utilitarian measures are of the form

For many years, beginning perhaps with "Is Growth Obsolete?" (Nordhaus and Tobin, 1971) and represented today by *The Oxford Handbook of Well-being and Public Policy* (Adler and Fleurbaey, 2016), there has been what might be called a new utilitarian project to supplement or replace gross national product (GNP) as the basis for measuring economic progress with a wider and more representative measure of economic welfare (MEW), removing aspects of government best seen as intermediate products and incorporating leisure, health, life-expectancy, benefits from improvements in goods and services and benefits from the introduction of new products.

One would expect a measure of economic welfare to account for the distribution of income. A narrowing of the distribution of income would increase economic welfare, a widening of the distribution of income would decrease economic welfare, and the measure of economic welfare would serve as the criterion for redistribution. The difficulty with this procedure is that there is no standard that everybody is ethically-obliged to recognize as correct. Prices of apples and oranges are the same for everybody because we can all buy apples and oranges at the grocery store. The price of equality - the appropriate number of dollars from the rich that should be sacrificed to procure an extra dollar for the poor - differs from one person to the next, with no over-arching criterion of right and wrong.

A distinction may be drawn between *market goods* like apples and oranges with prices that are the same for everybody at any given time and *personal goods*, notably leisure and life expectancy, with prices that differ from one person to the next because people cannot sell such goods to one another. A person's "price" of leisure is his alternative cost, i.e. his wage. A person's "price" of life is a reflection of what he would pay to avoid a small probability of losing his life in disease or by accident.[29] Like leisure and life expectancy, and unlike apples and oranges, equality in the distribution of income is a

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$$Q = 1 - \mathbf{I}/Y$$

where Q is inequality, Y is average income as ordinarily defined and **I** is the dollar value of expected utility (what Atkinson calls "equally distributed equivalent"). Once estimates of **I** have been computed, they can be used equally well for Atkinson's measure of inequality or for the utilitarian measure of economic growth. For example, with ε set equal to -1, the value of Q for India in 1950 is computed to be 0.297 implying a value of **I** equal to 70.3% of income, Y, in that year (table 1, page 259). It is arguable, however, that Q is a measure of the cost of inequality rather than of inequality itself; it is the percentage loss of potential uniform income equivalent, comparable to the cost of oranges as distinct from the number of oranges. Better perhaps to think of inequality as a family of objectively-defined measures - variance, ratio of variance to mean, and so on - and to think of **I** as the net effect of size and distribution combined. The combination depends on taste, as represented by ε. Measures of the ingredients do not.

[29] One's price of life may be thought of as 1,000 times what one would pay to avoid a 1-in-1,000 chance of losing one's life. A variant of this price is used in cost-benefit analysis when lives lost or saved are among the costs and benefits that must be compared. Some such measure would also be required to impute for changes in life expectancy in an expanded measure of economic growth.

personal good. It is a public rather than private good, but a personal good all the same, for the marginal utility of income with respect to income itself varies from person to person with no "correct" market-determined value for interpersonal comparison.

To incorporate changes in quantities of personal goods into a more comprehensive measure of economic growth, quantities of personal goods must be evaluated, together with quantities of market goods, on a common scale. There are two main procedures. Nation-wide changes in quantities of personal goods might be evaluated at arbitrarily-chosen prices. Extra leisure might be evaluated at average wages. Increases in life expectancy might be evaluated at some average value of life. Alternatively, changes in each person's quantities of personal good might be evaluated at that person's own price, so that, for example, an extra hour of leisure for a person with a high wage would contribute more to the expanded measure of economic growth than an extra hour of leisure for a person with a low wage, and an increase in life expectancy of the rich would count more than an equal increase in life expectancy of the poor. The latter procedure is implicit in cost-benefit analysis and in the economics of law[30] whenever projects or laws are evaluated on an efficiency criterion in which "A dollar is a dollar is a dollar to whomsoever it may accrue". By the same token, there is no universally-recognized right way to evaluate changes in the distribution of income when people's own valuations are not the same.

In *Beyond GDP*,(2013, page 249), Fleurbaey and Blanchet argue that we should "accept the idea of having more than one number to tell us whether the current state of affairs is likely to last or not". One cannot help feeling that they are pushing against an open door because that is surely what we do now. To enlarge GDP by imputations does not stop anybody from considering the imputations separately. On the other hand, the government may prefer to avoid constructing an imputation for changes in the distribution of income because people differ in their moral judgments. Looking at the statistics of quintiles of real household income in table 1, left-wing people for whom the absolute value of ε is large may see no progress at all, while right-wing people for whom the absolute value of ε is small may see considerable progress. Both are morally correct. The government may be wary of placing its stamp on either view, leaving to private organizations the task of combining changes in size and distribution of income into a single measure of progress.

[30] See, for example, Kaplow and Shavell (1994). Admittedly, advocates of a pure efficiency criterion for cost-benefit analysis and the choice of laws might see their prescriptions as conditional on a presumed division of labour among disciplines in which the redistribution of income is attended to elsewhere, but, typically, not much is said about what to do when that is not so.

Appendix B: Ethically-warranted Redistribution of Income

Imagine a society with a given distribution of gross incomes, with responsible government that acts in the interest of citizens and with public policy confined to the redistribution of income by negative income tax. All incomes taxed at a uniform rate t and total revenue reallocated equally among all citizens. Redistribution is costly; the net gain to the poor falls short of the full cost to the rich because, one way or another, taxpayers devote resources to concealing income from the tax collector. Society consists of n citizens whose gross (pre-tax, pre-transfer) incomes, Y_1, Y_2, Y_2, ...,Y_n , have fallen upon them like mana from heaven. The negative income tax converts gross incomes into net (post-tax, post-transfer) incomes y_1, y_2, y_2 , ...,y_n. Public choice of a rate for the negative income becomes a trade-off between equality and quantity: narrowing the gap between rich and poor at the cost of reducing average disposable income as taxpayers divert resources from producing goods to tax evasion. Full equality of net incomes can sometimes be obtained by setting a tax rate of 100%, but few would favour such a high rate because the corresponding deadweight loss from tax evasion could leave everybody worse off than at some lower rate. Each person's preferred tax rate depends on a mixture of self-interest and concern for the welfare of society as a whole.

Public policy can be analysed in stages: the taxpayer's response to taxation, the identification of social welfare in accordance with the preferences of a person with a given elasticity ε of the marginal utility of income and the political resolution of differences among people with different values of ε.

Tax avoidance can occur in many ways, through the labour-leisure choice, by relocating abroad, by outright concealment of income, and so on. For our purposes, it is sufficient to focus on concealment. Assume that a portion of one's income can be concealed from the tax collector without risk of detection but at a cost that increases in proportion to the to the share of income concealed and that punishment for tax evasion is severe enough to deter evasion beyond what can be securely concealed. If so, the taxpayer's response to taxation is to conceal income from the tax collector up to the point where the additional cost of concealment would exceed the additional tax that would otherwise be paid.[31] On these assumptions, a person's gross and net incomes are connected as follows: For each person,

net income = gross income + average tax paid - tax paid - cost of tax avoidance (B1)

Define r as the proportion of income concealed and suppose everybody's marginal cost of concealment is βrY where Y is a person's gross income and where β is a parameter reflecting the government's efficiency at tax collection; the larger the amount rY of income concealed and the higher β, the greater the marginal cost of evasion must be. For any given β, r and Y, the full cost of tax

[31]This model of tax evasion is set out in greater detail in Usher (2016)

evasion[32] is $(\beta r^2/2)Y$. The cost of concealing income from the tax collector is a pure deadweight loss because the gain to the tax evader is entirely at the expense of the rest of the population whose transfer is reduced by the amount of tax concealed.

At a tax rate of t, the total tax paid is $(1 - r)tY$ and the full cost of taxation, T, to the taxpayer (the last two expressions in equation (18)) is the sum of tax paid and the cost of tax avoidance

$$T = (1 - r)tY + (\beta r^2/2)Y \qquad \text{(B2)}$$

Taxpayers choose r to minimize the full cost of taxation by setting

$$r = t/\beta \qquad \text{(B3)}$$

so that the full cost of taxation to the taxpayer becomes

$$T = tY + (t^2/\beta)Y \qquad \text{(B4)}$$

Now, to keep things simple, suppose the population consists of only two people, one rich and one poor, or, equivalently, of equal numbers of rich and poor where incomes within each group are the same. Gross incomes of rich and poor are Y_R and Y_P. Their net incomes are y_R and y_P. Redistribution of income converts gross to net incomes by a negative income tax at a uniform rate, t.

With only two people, one rich and the other poor, and with uniform taxation at a rate t, the total tax revenue, R, becomes

$$R = t(1-r)(Y_P + Y_R) = t(1-t/\beta)(Y_P + Y_R) \qquad \text{(B5)}$$

The effect of a tax increase on revenue is

$$\delta R/\delta t = (1 - 2t/\beta)(Y_P + Y_R) \qquad \text{(B6)}$$

so that revenue increases with the tax rate as long as $t < \beta/2$ creating two possibilities depending on the efficiency of tax collection: When $\beta \geq 2$, total revenue increases together with the tax rate all the way from $t = 0$ to $t = 1$ (the maximum attainable tax rate). When $\beta < 2$, revenue is a humped function of t (a classic Laffer curve) with a maximum revenue at $t = \beta/2$.

With tax revenue reallocated equally and taking account of deadweight loss, the net incomes of rich and poor in accordance with equation (B1) above become

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[32]The full cost of concealing a portion r of one's income from the tax collector is

$$\int_0^r \beta r Y dr = \beta r^2 Y / 2$$

$$y_R = Y_R + t(1-t/\beta)(Y_P + Y_R)/2 - t(1 - t/\beta)Y_R - (t^2/2\beta)Y_R$$

$$= Y_R - t(1-t/\beta)(Y_R - Y_P)/2 - (t^2/2\beta)Y_R \qquad \text{(B7)}$$

and

$$y_P = Y_P + t(1-t/\beta)(Y_P + Y_R)/2 - t(1 - t/\beta)Y_p - (t^2/2\beta)Y_P$$

$$= Y_P + t(1-t/\beta)(Y_R - Y_P)/2 - (t^2/2\beta)Y_P \qquad \text{(B8)}$$

The negative income tax transfers $t(1-t/\beta)(Y_R - Y_P)/2$ from rich to poor, but at the same time generates deadweight loss to both parties, $(t^2/2\beta)Y_R$ to the rich and $(t^2/2\beta)Y_P$ to the poor.

Clearly, y_R is always greater than or, in one limiting case, equal to y_P because the second terms in the expressions for y_R and y_P in equations (B7) and (B8) are the same and the other three terms are all multiples of Y_R and Y_P respectively. The special case is where $\beta = \infty$ at which any tax evasion would detected and punished severely enough to deter evasion completely. In that case, equations (B7) and (B8) are reduced to

$$y_R = Y_R - t(Y_R - Y_P)/2 = (Y_R + Y_P)/2 + (1 - t)(Y_R - Y_P)/2 \qquad \text{(B9)}$$

and

$$y_P = Y_P + t(Y_R - Y_P)/2 = (Y_R + Y_P)/2 - (1 - t)(Y_R - Y_P)/2 \qquad \text{(B10)}$$

which are downward-sloping and upward-sloping straight lines beginning at Y_R and Y_P respectively when $t = 0$ and meeting at $(Y_R + Y_P)/2$ when $t = 1$.

Regardless of β, y_R is a decreasing function of t. The higher the tax rate, the smaller the post-tax, post-transfer income of the rich must be. By contrast, y_P may increase with t all the way up to $t = 1$, but, depending on β, may instead increase up to some rate less than 1 beyond which y_P declines. From equation (B7), it follows that

$$\delta y_P/\delta t = (1-2t/\beta)(Y_R - Y_P)/2 - (t/\beta)Y_P$$

$$= (Y_R - Y_P)/2 - (t/\beta)Y_R \qquad \text{(B11)}$$

The income of the poor is maximized when $\delta y_P/\delta t$ is set equal to 0 subject to the qualification that the tax rate may not exceed 1. Define t* as the rate for which y_P is as large as possible.

$$t^* = (\beta/2)\,(Y_R - Y_P)/Y_R \qquad \text{(B12)}$$

as long as the right hand side of this equation is less than 1, and otherwise $t^* = 1$. Provided that t* is less than 1, $\beta < 2\,Y_R/(Y_R - Y_P)$ which is less than the tax rate, $(\beta/2)$, at which tax revenue is maximized except in the special case where $Y_P = 0$ at which the poor, having no income, pay no tax and suffer no

deadweight loss.[33]

How in these circumstances is the tax rate chosen? A convenient introductory assumption is that the tax rate is chosen by benevolent dictator, by someone with a given value of ε in equation (4) who is empowered to choose the tax rate and who does so to maximize social welfare as he sees it regardless of his own personal interest in the matter. The benevolent dictator sees **I** as a function of ε, t, Y_R and Y_P

$$\mathbf{I}^{\varepsilon+1} = (½)\,[y_P^{\varepsilon+1} + y_R^{\varepsilon+1}] \qquad \text{(B13)}$$

$$= (½)\,[\{Y_R - t(1-t/\beta)(Y_R - Y_P)/2 - (t^2/2\beta)Y_R\}^{\varepsilon+1} + \{Y_P + t(1-t/\beta)(Y_R - Y_P)/2 - (t^2/2\beta)Y_P\}^{\varepsilon+1}$$

which reduces to

$$\mathbf{I} = \mathbf{I}(\varepsilon, t, Y_R, Y_P) \qquad \text{(B14)}$$

The benevolent dictator chooses t to maximize **I**, allowing t (for any given pair of gross incomes, Y_R and Y_P) to be expressed as a function of ε alone.

$$t = t(\varepsilon, Y_R, Y_P) \qquad \text{(B15)}$$

It follows immediately from equation (B13) that, when ε = 0, **I** is maximized at t = 0 meaning that there is no redistribution at all.[34] At the other extreme where ε = - ∞, the income of the poor is maximized and the tax rate is as shown in equation (B12) above.[35] The tax rate increases steadily between these limits.

There are two large problems with this story, that there is no benevolent dictator and that

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[33]That a tax rate of as high as 100% might be in the interest of the poor seems more plausible in a model where gross income falls on people like mana from heaven than in a model where gross income is acquired by work and investment. The case where t* < 1 is far more realistic.

[34] In that case, $\mathbf{I} = (½)\,[y_P + y_R] = (½)\,[\{Y_P + Y_R\} - (t^2/2\beta)(Y_R + Y_P\}$ which is obviously maximized at t = 0.

[35]When ε = - ∞, equation (B13) can be rewritten as

$$\mathbf{I} = y_P\ (½)^{1/(1-\infty)}[(y_R/y_P)^{1-\infty} + (y_P/y_P)^{1-\infty}]^{1/(1-\infty)} = y_{min}$$

because each expression $(y_R/y_P)^{1-\infty}$ can be rewritten as $(y_P/y_R)^{\infty-1}$ which is equal to 0 whenever $y_R > y_P$ and which is equal to 1 when y_P and y_R are the same. The expression $(½)^{1/(1-\infty)}$ $= 2^{1/(\infty-1)} = 1$.

citizens are not saints. The first problem is that redistribution must be determined politically in circumstances where differences in people's perceptions of social welfare are somehow reconciled by majority rule voting or by some comparable means. The second is that, just as people look to their own interests primarily in the ordinary business of life, so too can they be expected to vote self-interestedly at least to some extent. Admittedly, citizens are likely to give more weight to what they see as social welfare in voting than in everyday life, but, as mentioned above, devotion of a person's entire life to the greatest good for the greatest number would be almost suicidal and may even be counter-productive when too many people act that way. Self-interest does not stop at the ballot box, and most voters can be expected to strike a balance between what they see as best for themselves and what they see as best for the nation as a whole.

Appendix C: Concavity of the Utility of Income Function when Utility is Represented by the Uniform Income Equivalent

It has been assumed above that people's utility of income function are of the general form $u = [(1/(1 + \varepsilon)]y^{1+\varepsilon}$ in equation (1) where ε may be anywhere between 0 and $-\infty$. One would like the measure of utility to be positive, but that is not necessary and not always so. Utility is only positive as long as $\varepsilon > -1$. The important requirement is that utility be an increasing function of income. For example, if $\varepsilon = -3$, then $u(5) = -1/50$ and $u(10) = -1/200$, but it is still the case that $u(10) > u(5)$. Fortunately regardless of ε, everything works well when the relation between u and y is replaced by the relation between **I** and y_1 where **I** is the dollar value of expected utility in equation (4) - which reduces in a society with just two people to $(\mathbf{I})^{1+\varepsilon} = (½)[(y_1)^{1+\varepsilon} + (y_2)^{1+\varepsilon}]$ - and where y_2 is assumed to be invariant.[36] Specifically, as long as $0 > \varepsilon > -\infty$,

$$\mathbf{I} > 0 \qquad \text{(C1)}$$

$$\delta\mathbf{I}/\delta y_1 > 0 \qquad \text{(C2)}$$

and

$$\delta^2\mathbf{I}/\delta y_1^2 < 0 \qquad \text{(C3)}$$

The proof is simple. From equation (4), it follows that

$$\mathbf{I}^{\varepsilon}\, d\mathbf{I} = (½)\, y_1^{\varepsilon} dy_1 \qquad \text{(C4)}$$

so that

$$\delta\mathbf{I}/\delta y_1 = (½)\,(y_1/\mathbf{I})^{\varepsilon} = > 0 \qquad \text{(C5)}$$

as long as y_2 is held constant. From this, it follows immediately that the elasticity of **I** with respect to y_1 is

$$(y_1/\mathbf{I})(\delta\mathbf{I}/\delta y_1) = (½)\,(y_1/\mathbf{I})^{1+\varepsilon} = (½)(y_1^{1+\varepsilon})/(y_1^{1+\varepsilon} + y_2^{1+\varepsilon}) < 1 \qquad \text{(C7)}$$

That **I** is a concave function of y_1 now follows from the fact that

$$\delta^2\mathbf{I}/\delta y_1^2 = \delta\{(½)\,(y_1/\mathbf{I})^{\varepsilon}\}/\delta y_1 = \delta\{(½)\,(\mathbf{I}/y_1)^{-\varepsilon}\}/\delta y_1$$

$$= (½)\,(\varepsilon)\,(y_1/\mathbf{I})^{\varepsilon-1}\{\mathbf{I} - y_1 d\mathbf{I}/dy_1\}/\mathbf{I}^2$$

$$= (½)\,(\varepsilon)\,(y_1/\mathbf{I})^{\varepsilon-1}\{1 - (y_1/\mathbf{I})(\delta\mathbf{I}/\delta y_1)\}\mathbf{I} < 0 \qquad \text{(C8)}$$

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[36]Essentially the same formula holds when **I** varies with y_2 and when y_1 is invariant. The analysis is easily generalized from a society with just two people to a society with any number of people.

because $\varepsilon < 0$ and every other component of this expression is positive.

The concavity of the utility of income function - **I** as a function of y_1 - is illustrated in table A and figure A for two person society where the income of person 1 may vary from 0 to 10 and the income of person 2 remains constant at 5. The table shows values of **I** for integral values of y_1 from 0 to 10 and for ε equal to 0, -1, -2, -3 and $-\infty$. Changes in Δ**I** are also shown. Regardless of ε, **I** is equal to 5 when $y_1 = 5$ because, in that case, **I** becomes the common value of y_1 and y_2. As indicated in the table but is especially evident in the figure, the concavity of the function connecting **I** to y_1 increases steadily as ε declines; The function shifts steadily from an upward straight line when $\varepsilon = 0$ to a kinked line when $\varepsilon = -\infty$. When $\varepsilon = 1$, **I** becomes the average of y_1 and y_2. When $\varepsilon = -\infty$, **I** becomes the smaller of y_1 and y_2, equal to y_1 as long as $y_1 < 5$ but remaining constant at 5 when y_1 is higher.

Table C1: Certainty-equivalent Income, **I**, as a Function of the Income of Person #1, y_1, for Selected Values of y_1 and ε, when $(\mathbf{I})^{1+\varepsilon} = (½)[(y_1)^{1+\varepsilon} + (y_2)^{1+\varepsilon}]$ and the Income of Person #2, y_2, is equal to 5

	$\varepsilon = 0$	$\varepsilon = 0$	$\varepsilon = -1$	$\varepsilon = -1$	$\varepsilon = -2$	$\varepsilon = -2$	$\varepsilon = -3$	$\varepsilon = -3$	$\varepsilon = -\infty$	$\varepsilon = -\infty$
y_1	**I**	Δ**I**	**I**	Δ**I**	**I**	Δ**I**	**I**	Δ**I**	**I**	Δ**I**
0	2.5	-	0	-	0	-	0	-	0	-
1	3	.5	2.236	2.236	1.667	1.667	1.387	1.387	1	1
2	3.5	.5	3.162	.926	2.857	1.190	2.626	1.239	2	1
3	4	.5	3.873	.711	3.750	.893	3.638	1.012	3	1
4	4.5	.5	4.472	.599	4.444	.694	4.417	.779	4	1
5	5	.5	5	.528	5	. 556	5	.583	5	1
6	5.5	.5	5.477	.477	5.455	.455	5.432	.432	5	0
7	6	.5	5.916	.439	5.833	.378	5.754	.322	5	0
8	6.5	.5	6.325	.409	6.154	. 321	5.996	.242	5	0
9	7	.5	6.708	.383	6.429	.275	6.181	.185	5	0
10	7.5	.5	7.071	.363	6.667	.238	6.325	.144	5	0

When $\varepsilon = -1$, equation (4) reduces to $\mathbf{I} = (y_1)^{1/2}(y_2)^{½}$.
When $\varepsilon = -\infty$, equation (4) reduces to $\mathbf{I} = \min(y_1, y_2)$.
If $y_1 = 0$ and $\varepsilon \leq 0$, then $\mathbf{I} = 0$.
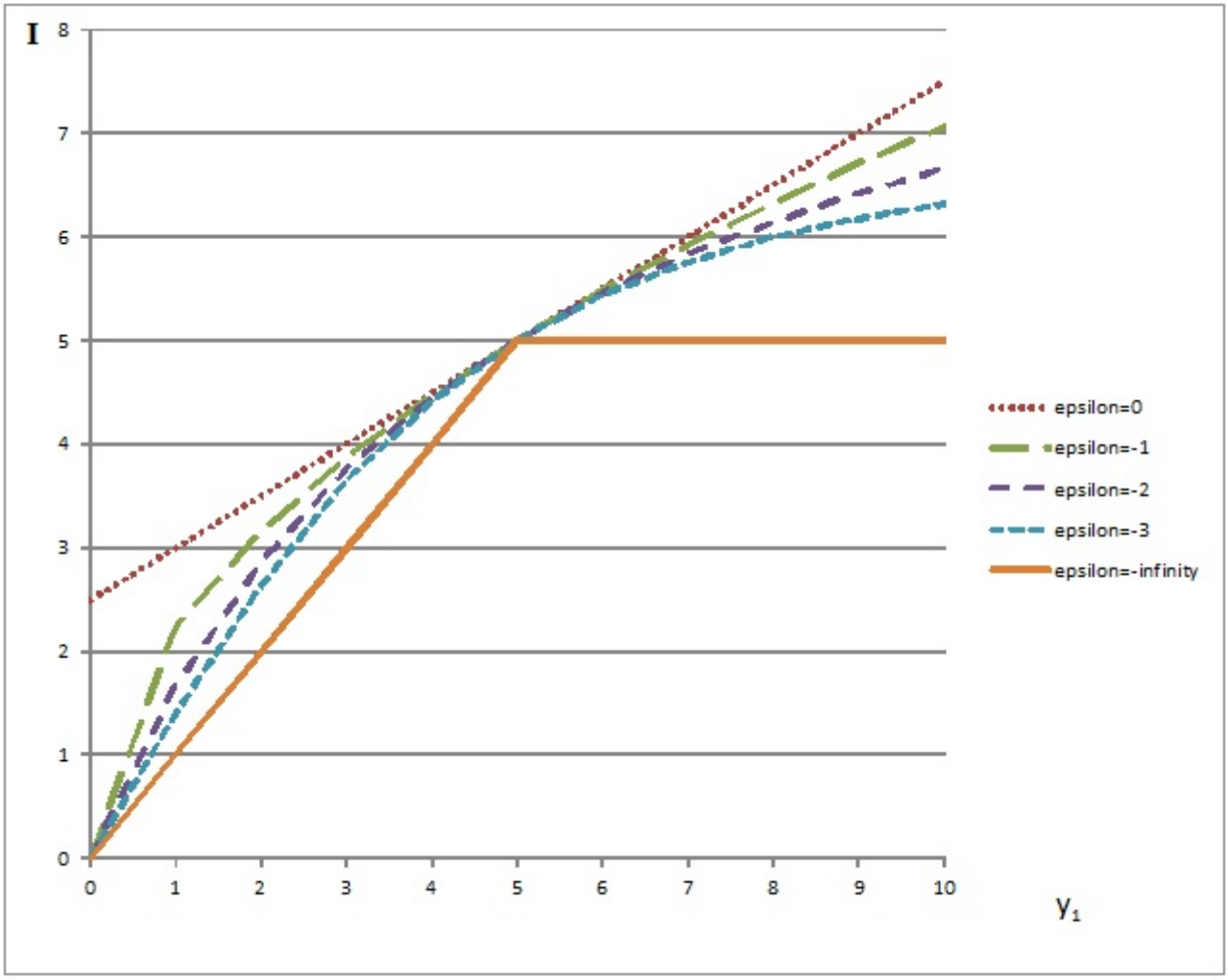
Figure C1: The Uniform Income Equivalent, **I**, as a Function of the Income, y_1, of Person #1 when the Income, y_2, of Person #2is Held Constant