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Limited Capacity in Project Selection: Competition Through Evidence Production

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LIMITED CAPACITY IN PROJECT SELECTION: COMPETITION THROUGH EVIDENCE PRODUCTION

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Abstract. An organization must decide which proposals to fund. In evaluating the proposals, the organization may rely on those applying for funding to produce evidence about the merits of their own proposals. We consider the role of a capacity constraint preventing the organization from funding all projects. Agents produce more (Blackwell) informative evidence about the merits of their proposals when there are capacity constraints. In a two agent model, we fully characterize the equilibrium under unlimited and limited capacity. Unless the prior strongly favors accepting both proposals, the funding organization is better off when its capacity is limited.

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1. Introduction

We model the allocation of funding by an organization: an institution funding research or community improvement grants, a legislature funding earmarks or government programs, or a firm deciding which R&D projects to pursue, which new products to bring to market, or which divisions to continue funding during downsizing. Each proposal is supported by a separate agent, who can produce evidence about the merits of his proposal. For example, grant applicants may conduct pilot studies; interest groups or lobbyists sponsor research and conduct constituent polls; and project managers conduct product testing and market studies for a prototype. The organization prefers to support only good projects, but agents want to receive funding regardless of their merits.

Within this environment, we consider the impact of organizational resource constraints. In a model of two proposals, we compare outcomes when the organization is unconstrained and can implement both proposals to outcomes when the organization faces a capacity constraint which restricts the organization to implement at most one of the two proposals. Straightforward intuition suggests that an unconstrained organization should be better off compared to an organization that lacks the resources to fund all proposals, because it can always choose not to fund a weak proposal if it believes the costs of implementing the proposal dominate the benefits. This reasoning suggests that an organization will never gain from capacity constraint on its ability to implement proposals it believes beneficial.

While this intuition holds when the quality of available evidence fixed, it does not hold when the quality of evidence is endogenous, determined by the agents' evidence production strategies. When the agents choose the process by which evidence is generated, their choice depends on the capacity constraints. We show how a capacity constraint induces competition between agents, and leads them to produce more informative evidence about the merits of their proposals as they compete for the limited resources. This increase in evidence quality allows the organization to more accurately identify worthwhile projects, but prevents the organization from supporting both projects, even if it believes both are deserving. The informational benefits can dominate the costs, and can leave the organization better off.

We model evidence production by the agents as a form of Bayesian persuasion (e.g. Kamenica and Gentzkow 2011). Consistent with the literature, there is full transparency: all information available to the agents, including prior information, is also available to the principal. Agents choose how much evidence to generate prior to knowing the outcome. Each agent's evidence production process is modeled as the design of a signal: a random variable jointly distributed with the true quality of a proposal. The evidence generated and observed by the funding organization corresponds to a public realization of this signal. The analysis puts little structure on the type of evidence generation process, or signal, that agents may design to produce information about their proposals. Our main theoretical contribution is to incorporate capacity constraints within such a persuasion framework, and to show how such constraints change evidence production strategies.

When resources are unlimited, agents must produce evidence that is just persuasive enough for the funding organization to believe their proposal has non negative expected benefits. In equilibrium, any evidence produced by the agents (if they choose to produce any evidence at all) either confirms the priors or leaves the organization indifferent between implementing and not implementing the agent's proposal. As a consequence, the organization does not benefit from evidence production in an environment where it faces no capacity constraint and the agents control the production of the evidence.

In contrast to the unlimited capacity case, when resources are limited, agents not only need to convince the organization that their proposal is worthwhile, they must also convince the organization that their proposal is more worthwhile than any alternative proposal. Capacity constraints induce competition between the agents, who compete through the provision of more informative evidence (in the sense of Blackwell) than they do in the absence of these competitive incentives.

This means that limited capacity introduces countervailing effects on the organization's payoffs. The cost of limited capacity comes from the organization only being able to implement one of the proposals, even in situations where it believes both proposals are worthwhile. The benefit of limited capacity comes from the fact that a capacity constraint effectively commits the organization to requiring stronger evidence in favor of a proposal before choosing to implement it. This leads agents to produce more informative evidence, which in turn leads the organization to be better informed and more likely to implement only beneficial proposals.

When there is a high ex ante probability that all proposals are beneficial, then the costs of limited capacity dominate the benefits, and the organization is better off when it can implement all proposals. In other cases, when there is greater uncertainty about the merits or any or all of the proposals, the information benefits of limited capacity dominate the costs. In these cases, the organization is better off when it has limited capacity.

The organization would never be better off under limited capacity if evidence generation were exogenous. This is because the benefits of limited capacity come from incentivizing agents to increase the quality of evidence they produce. In the absence of this effect, the organization prefers the flexibility allotted by having enough funding to implement all projects it believes are worthwhile.

These results have implications for a number of settings. First, a philanthropist may prefer to endow his foundation with limited funds in order to ensure the more-careful allocation of this funding across projects or grant applications. Even if the philanthropist can easily afford to endow the foundation with enough funding to pursue all beneficial projects, doing so may not be worthwhile. This has implications for the most effective allocation of charitable donations, highlighting a benefit of "underfunded" organizations. Second, voters may be better off when legislatures and bureaucracies are underfunded, as limited resources lead to more-persuasive lobbying, more-informed decision making, and the more-efficient allocation of resources. Our results highlight an unrecognized benefit of potentially underfunded budgets in governments and bureaucracies. Third, these results extend to the allocation of resource within firms. An executive who faces resource or budget constraints may become more informed and choose resource allocations that waste fewer resources than a similar executive who does not face such constraints. This suggests a potential benefit to corporate boards or central management of limiting resources available to decision makers within their organization, of restricting the number of new products they are willing to bring to market, or of otherwise limiting the number of projects which may be pursued within their firm. Our results highlight previously unrecognized benefits of capacity constraints and underfunding within organizations.

Section 2 surveys relevant literature. Section 3 presents the model and some preliminary analysis. Section 4 presents results for the unlimited capacity case. Section 5 presents results for the limited capacity case. Section 6 discusses the tradeoff inherent in limited capacity. Section 7 summarizes and presents some conclusions.

2. Literature

A handful of recent papers consider project review and selection in a principal-agent framework. Unlike our analysis, which is primarily concerned with the production of public information, this literature is primarily concerned with incentives for agents to truthfully reveal private information about project attributes. Che, Dessein and Kartik (2013), consider strategic communication of project attributes by an informed agent who shares the principal's preferences over potential projects but is biased with respect to the principal's outside option. Meanwhile Armstrong and Vickers (2010) and Dessein (2002) consider project selection in a delegation framework, whereby the principal allows a privately informed agent to choose a project, subject to certain restrictions or rules imposed by the principal. Bar and Gordon (2014) and Mylovanov and Zapechelnyuk (2013) consider the design of optimal mechanisms–employing either message contingent transfers or imposing costs on agents *ex post*—to elicit information about privately known project attributes. Lewis and Sappington (1997) consider a mechanism design problem in which the agent must be motivated both to acquire private information and to communicate it truthfully. Relative to these papers, we weaken the principal's commitment power: the organization (the principal in our framework) cannot commit to either a project selection rule or to transfers. Instead we consider how a limited capacity to approve projects can *substitute* for commitment power, improving the principal's payoff.

In its focus on the generation of public information, our paper is related to a recent strand of literature on evidence production and persuasion. Brocas and Carrillo (2007), consider a dynamic model of persuasion in which an agent can allow the realization of a public signal about an unknown state or can terminate the flow of information in each period, triggering the principal to select an action. The authors characterize situations in which the agent benefits from this type of information control. Where Brocas and Carrillo (2007) assume a single agent, Brocas, Carrillo and Palfrey (2012) and Gul and Pesendorfer (2012) focus on adversarial evidence production, considering multiple agents with opposing preferences over a single policy or action. In this strand of literature, agents produce evidence by drawing (public) realizations from a known signal structure, choosing when to terminate the information generation process; however, agents do not have direct control over the design of the signal.[1] This aspect is incorporated by Kamenica and Gentzkow (2011) who consider a general "Bayesian persuasion" framework with a single sender and receiver. The sender can design a signal whose realization

[1] Austen-Smith and Wright (1992) apply a related model to analyze informational lobbying. **?** consider a setting in which both interest groups and a policymaker may pay costs to produce public information before the policymaker chooses which policy to implement. They show that interest groups representing less efficient policies may choose to collect evidence, and that this evidence collection may decrease the incentives for information collection regarding the more-efficient policy.

is publicly observed. The receiver (whose preferences may differ from the sender's) observes the signal and its realization before selecting an action. With unlimited capacity, our framework reduces to two copies of the single agent game, and we highlight the conflict of interest identified by Kamenica and Gentzkow (2011), demonstrating that information generated by the agent is worthless to the principal.[2] Our main contribution, however, is to analyze the case of limited capacity, which induces competition between agents and incentivizes more informative evidence production.

There are a number of recent papers building on Kamenica and Gentzkow (2011). Only a handful of these papers consider competition between agents in a Bayesian persuasion environment. Boleslavsky and Cotton (2015) develop a Bayesian persuasion model of school competition in which schools simultaneously invest in developing graduate ability, and control how information about graduate ability is revealed to employers by designing grading policies. In work developed independently and concurrently with ours, Gentzkow and Kamenica (2012*a*) consider persuasion by multiple agents who can each produce evidence about the same underlying state in order to influence a decision by the principal. Their central insight is that no agent can design a signal that *reduces* the information available to the principal. Thus, an outcome is an equilibrium if no agent can benefit by supplying a more-informative signal to the principal. In our framework each agent produces information about an independent dimension of the state of the world (their own proposal's quality), while the assumptions of Gentzkow and Kamenica (2012*a*) require that agents produce evidence about all dimensions (the quality of all proposals). This assumption is significant both for the interpretation of the model and the results. For example, in Gentzkow and Kamenica (2012*a*), if one agent sends a signal that perfectly reveals the state of the world, all other agents are indifferent over all possible signals, and a fully-revealing signal is a best response. Hence, any profile of signals in which at least two agents send a signal that fully reveals the underlying state constitutes an equilibrium. In our analysis, agents reveal information about their own proposals, not all proposals under consideration. Thus, our analysis does not exhibit the strong incentives that arise in Gentzkow and Kamenica (2012*a*), and our results are less extreme. Gentzkow and Kamenica (2012*a*)'s assumption that all agents provide information about a single state is most appropriate for settings such as a criminal trial, in which the court is interested in determining the guilt or innocence of a defendant, and *both* prosecutor and defense attorney reveal public information about this state. In contrast our analysis is best suited to situations of project selection or review in which agents typically supply information only about their own proposals. Our framework also allows us to naturally consider capacity constraints limiting the number of proposals our organization is able to accept.

Our contribution is also related to the literature on disclosure of verifiable information. The bulk of the literature focuses on the incentives to truthfully disclose or withhold information that has already been uncovered (Milgrom 1981, Milgrom and Roberts 1986, Bull and Watson 2004, Cotton 2012). Cotton (2009) presents a model in which agents provide payments in competition for access to a time-constrained decision maker. Several recent papers consider the interaction of the incentive for disclosure and the incentive to acquire information. Henry (2009) considers the impact that mandatory research

[2]Kolotilin et al. (2015) considers a Bayesian persuasion model in which the principal is privately informed.

disclosure rules have on an agent's decision to acquire evidence, and Che and Kartik (2009) consider how differences of opinion between decision makers and agents affect the agents' incentives to acquire and transmit evidence. Because we focus on evidence that cannot be withheld, these considerations are absent from our analysis. [3] Carlin, Davies and Iannaccone (2012) develop a model of competitive disclosure in a financial market and show that competition between agents may *decrease* incentives for agents to *disclose* evidence. This contrasts with our analysis of evidence *production* (rather than evidence *disclosure*), in which competition increases evidence production and leads to more informed decisions by the principal.

3. Model

A funding organization chooses whether to provide funding for or not provide funding for each of two proposals, $i \in \{H, L\}$. Each proposal can be either "good" or "bad." Denote proposal i's type by $\tau_i \in \{g, b\}$. Let γ_i represent the common prior for both the organization and the agents about proposal i's quality: $\gamma_i = \Pr(\tau_i = g)$. With out loss of generality, assume $\gamma_H \geq \gamma_L$. Thus, proposal H has more promise ex ante.

The organization prefers to fund good proposals and to not fund bad proposals, although there is initial uncertainty about each proposal's type.[4] Its total payoff is $U_{org} = u_H + u_L$, where

$$u_i = \begin{cases} 1 - \theta & \text{if proposal } i \text{ is funded and } \tau_i = g \\ -\theta & \text{if proposal } i \text{ is funded and } \tau_i = b \\ 0 & \text{if proposal } i \text{ is not funded.} \end{cases}$$

Funding a good proposal increases the organization's payoff by $1 - \theta$ and funding a bad proposal reduces the organization's payoff by θ. Parameter $\theta \in (0, 1)$ represents the organization's tradeoff between Type I and Type II errors in its decision.[5] The organization's decision directly determines the payoffs of two agents, each of whom is affiliated with a separate proposal. Agent i receives payoff 1 whenever proposal i is funded and 0 whenever i is not funded, regardless of whether the organization accepts or reject the *other* agent's proposal. There is no private information; agents are unable to withhold any information, even ex ante information, from the organization, and therefore the organization and agents share an identical prior over the type of each proposal.

We abstract from collective decision making problems that may arise within the organization. We assume that the organization's decision making power is held by a single individual whose preferences are completely aligned with the organization as a whole. We also abstract from asymmetry in the size of the proposal, essentially assuming that

[3] Gentzkow and Kamenica (2012*b*) identify conditions under which a forced disclosure requirement does not change the set of equilibrium outcomes of a persuasion game with endogenous information.

[4] We also use "accept" and 'reject" as synonyms for "fund" and "not fund."

[5] If it rejects a good proposal, the organization's payoff is zero instead of $1 - \theta$ (Type I error), and if it accepts a bad proposal, the organization's payoff is $-\theta$ instead of zero (Type II error). Provided that the payoff of rejecting is zero, this payoff specification is equivalent to any other payoff structure in which the organization benefits from accepting good proposals and is hurt by accepting bad proposals. If the organization's payoff from accepting a good proposal is $v > 0$ while the payoff of accepting a bad proposal is $-c < 0$, then dividing both payoffs by $v + c$ gives the specification defined in the text.

each proposal requires the same commitment of resources from the funding organization.

In the first stage of the game, each agent simultaneously and independently generates evidence about the quality of his own proposal. The agent's evidence is a publicly observable realization of a random variable S_i that is jointly distributed with the quality of proposal i. Signal S_i represents the *process* by which evidence is generated, and this signal is designed by agent i. We represent signal S_i as a pair of conditional random variables (S_i^g, S_i^b). If $\tau_i = g$, then random variable S_i^g is realized; if $\tau_i = b$, then random variable S_i^b is realized. We focus on univariate random variables (S_i^g, S_i^b) which have a finite set of mass points; except at mass points, (S_i^g, S_i^b) admit continuous densities $(f_i^g(\cdot), f_i^b(\cdot))$ supported on sets (Σ_i^g, Σ_i^b), which can be represented as a countable union of intervals. We assume that the realizations in the support of S_i, $\Sigma_i \equiv \Sigma_i^g \cup \Sigma_i^b$, are ordered in such a way that the likelihood ratio $f_i^b(\cdot)/f_i^g(\cdot)$ is monotone decreasing; thus a higher signal realization is good news about proposal quality. We also assume that with the exception of mass points, the likelihood ratio is differentiable in the interior of Σ_i. Signals with these properties are "valid" for our analysis.

In the second stage of the game, the organization evaluates each proposal. It observes both the design of each agent's signal and the realization of each signal, updating its beliefs about the quality of each proposal according to Bayes' Rule. In this way, both the process by which evidence is generated (the signal) and the evidence uncovered (the signal realization) play a role in the organization's evaluation of each proposal. The organization then chooses which proposal(s), if any, to accept. Its ability to fund proposals may be either *limited* or *unlimited*. If capacity is *unlimited*, the organization can fund neither, either or both proposals as it sees fit. In this case no link exists between proposals; the decision to fund each proposal is made independently. Alternatively, when capacity is *limited*, the organization can accept at most one proposal. This limitation may arise for a variety of reasons: the organization may be constrained by limited budgets or limited time, it may also be constrained by procedural or bureaucratic hurdles that require considerable effort to overcome. When capacity is limited, a decision to accept one proposal eliminates the possibility of accepting the other proposal. In this case, acceptance decisions cannot be made in isolation; the signals and realizations for both proposals influence the organization's decisions. The organization's capacity constraint (if one exists) is common knowledge.

We solve for the Perfect Bayesian Equilibria of this game under the limited and unlimited capacity systems. In the first stage, agents simultaneously choose their evidence strategy: signals (S_H, S_L). Once both agents choose their signals, both the signals and their realizations (s_H, s_L) are observed by the organization. In the second stage, the organization updates its beliefs about the quality of each proposal and decides which proposals to fund, subject to any capacity constraint.

Preliminary analysis. Bayesian Persuasion Representation of Signals. We present a representation of signals that considerably simplifies the analysis; this representation follows the approach of Kamenica and Gentzkow (2011). The organization's acceptance decisions in stage two are determined by its posterior beliefs about the quality of each proposal. These beliefs are generated from Bayes' Rule and depend on both the signal, S_i, supplied by the agent, and its realization, s_i: $\hat{\gamma}_i(s_i) = \Pr(\tau_i = g | S_i = s_i)$. Once the signal has been designed (the investigative process has been determined) but before the signal is realized (the evidence has been generated) the organization's posterior belief is a random variable: $\Gamma_i \equiv \hat{\gamma}_i(S_i) = \Pr(\tau_i = g|S_i)$. This posterior belief random variable summarizes the informational content of signal S_i: any signals generating the same posterior belief random variable are payoff equivalent for all players.

A posterior belief random variable generated by a valid signal must have certain properties. First, because it's realization represents a probability, the support of Γ_i is a subset of the unit interval. Under the conditions we impose on signals, Γ_i is supported on a finite set of mass points and outside the set of mass points possesses a continuous density supported on a countable union of intervals. Second, the Law of Iterated Expectations implies that the expected value of the posterior belief random variable is equal to the prior: $E[\Gamma_i] = \gamma_i$. Following Kamenica and Gentzkow (2011) we refer to a random variable with these properties as *Bayes-Plausible*. The following lemma establishes that any Bayes-Plausible random variable is the posterior belief random variable generated by a valid signal.

Lemma 1. *If random variable Γ_i is Bayes-Plausible, then there exists a valid signal S_i for which the posterior belief random variable is Γ_i.*

Lemma 1 establishes a correspondence between valid signals and Bayes-Plausible posterior belief random variables. As described above, a valid signal generates a posterior belief random variable that is Bayes-Plausible. The lemma shows that the reverse relationship also holds: provided a random variable is Bayes-Plausible, a signal exists (in fact, many) for which it is the posterior belief random variable. Therefore, we focus our analysis on an agent's choice of Bayes-Plausible posterior belief random variable Γ_i, rather than signal (S_i^g, S_i^b).In the rest of the paper, we refer to choice of posterior belief random variable as an "evidence strategy" or "signal," although it could be generated by a large set of signals (all of which are payoff equivalent).

Two extreme evidence strategies are always available to an agent. A *fully revealing* evidence strategy resolves all uncertainty about proposal quality, generating a Bernoulli posterior belief random variable: $\Pr(\Gamma_i = 1) = \gamma_i$ and $\Pr(\Gamma_i = 0) = 1 - \gamma_i$. A fully revealing strategy by the agents is optimal for the organization, as it learns a proposal's true quality before making an acceptance decision. An *uninformative* evidence strategy conveys no information to the organization about a proposal's quality, generating a degenerate posterior belief random variable in which all mass is concentrated on the prior: $\Pr(\Gamma_i = \gamma_i) = 1$. An *informative* evidence strategy conveys some information to the organization about proposal quality; the posterior belief random variable is not degenerate.

When the realized value of the posterior belief is high, an agent's proposal "looks good" to the organization. However, in order to be consistent with the Law of Iterated Expectations (and Bayesian Rationality), any probability mass on high realizations of the posterior must be offset by probability mass on low realizations of the posterior, imposing a tradeoff on the agent.

Organization's Funding Decision. Suppose that in stage two the organization believes that proposal i is good with probability $\widehat{\gamma}_i$. If it funds proposal i then its expected payoff is equal to $\widehat{\gamma}_i(1-\theta) - (1-\widehat{\gamma}_i)\theta = \widehat{\gamma}_i - \theta$. With unlimited capacity, the organization compares the expected payoff of funding with its payoff of rejecting and receiving zero. Therefore with unlimited capacity, the organization strictly prefers to accept whenever

$\widehat{\gamma}_i > \theta$ and is indifferent when $\widehat{\gamma}_i = \theta$. It is simple to show that in equilibrium the organization accepts when it is indifferent between accepting and rejecting.[6] Next, consider limited capacity. The organization's payoff of accepting proposal i is increasing in $\widehat{\gamma}_i$; thus it will prefer to accept the proposal that it believes is more likely to be high quality, provided that its belief that the proposal is high quality is no less than θ. If its belief about the quality of both proposals is the same and no less than θ we assume that it randomizes fairly between them. In equilibrium this can happen only when the organization is sure that both proposals are high quality, $\widehat{\gamma}_L = \widehat{\gamma}_H = 1$.

4. Unlimited Capacity

With unlimited capacity the organization funds a proposal whenever the posterior belief that the proposal is good is no less than θ. Anticipating the organization's funding strategy, each agent chooses random variable Γ_i to maximize $\Pr(\Gamma_i \geq \theta)$ subject to $E[\Gamma_i] = \gamma_i$ and $\Pr(0 \leq \Gamma_i \leq 1) = 1$.

Consider first the case in which the organization is predisposed in favor of accepting policy i; that is, it would accept based on the prior alone, $\gamma_i \geq \theta$. In this case, agent i prefers to choose a posterior belief random variable whose support is strictly above θ. Doing so eliminates the possibility of a realization that could overturn the favorable prior, and therefore guarantees that the organization accepts proposal i. Although the agent is indifferent over all posterior belief distributions that place all mass on realizations greater than θ, one of these stand out as most reasonable: it would be the unique equilibrium if there were even a very small cost of evidence production. The *focal equilibrium* involves agents producing no information (i.e. Γ_i is completely uninformative).

Next, consider the alternative case in which the organization is predisposed against accepting proposal i given its prior belief, $\gamma_i < \theta$. In this case, agent i never prefers an uninformative signal, which guarantees that proposal i is rejected. Instead, agent i prefers a signal that can overturn the prior in the event of a favorable realization. In this case, signals with non-zero probability mass on posterior beliefs in $(\theta, 1]$ or $(0, \theta)$ are strictly dominated by a signal that concentrates probability mass on only two realizations: 0 and θ.[7] Thus, when $\gamma_i < \theta$ the optimal signal requires only two realizations. One realization reveals that the proposal is bad for certain, while the good realization leaves the organization just indifferent between accepting and rejecting the proposal.

Lemma 2. *In equilibrium under unlimited capacity*

- *If $\gamma_i \geq \theta$, then agent i chooses a signal (evidence strategy) such that always generates a posterior belief realization above θ. On the equilibrium path, the organization funds proposal i with probability* 1. *In the focal equilibrium, $\Gamma_i = \gamma_i$ with probability* 1.

[6] Assume that in equilibrium an agent sends a signal that leaves the organization indifferent with non-zero probability. If the organization accepts with probability less than one in this case, the agent could profitably deviate by shifting probability mass up to $\theta + \epsilon$. This would result in a marginal decrease in the probability of this realization, but it would cause a jump in the probability of a proposal being accepted.

[7] Because $\gamma_i < \theta$, some mass must be allocated below θ in order to satisfy the constraint on the mean, $E[\Gamma_i] = \gamma_i$. If non-zero mass is allocated in interval $(\theta, 1]$ it can be reallocated to a mass point on θ without reducing the agent's payoff. This reallocation reduces the mean, allowing some of the mass assigned below θ to be moved to the mass point on θ, increasing the agent's payoff. A similar argument rules out mass inside $(0, \theta)$.

- *If $\gamma_i < \theta$, then agent i chooses a signal (evidence strategy) such that*

$$\Pr(\Gamma_i = \theta) = \frac{\gamma_i}{\theta} \quad \textit{and} \quad \Pr(\Gamma_i = 0) = 1 - \frac{\gamma_i}{\theta}.$$

On the equilibrium path, the organization funds proposal i if and only if $\Gamma_i = \theta$. This equilibrium is unique.

This result illustrates a conflict of interest between the organization and the agents it relies on for the production of information. Although agents have the capacity to choose signals which fully reveal the quality of their proposals, in equilibrium they never do so. The organization would obtain the same expected payoff if it did not observe any signal realization and simply acted according to the priors. These results are consistent with those derived in Kamenica and Gentzkow (2011) who establish that this conflict of interest is a common feature of Bayesian persuasion games with a single agent.

5. Limited Capacity

In the first stage of the game with limited capacity, each agent simultaneously chooses a Bayes-Plausible posterior belief random variable Γ_i. Once both agents have made their choices, realizations of each random variable $(\widehat{\gamma}_H, \widehat{\gamma}_L)$ are publicly observed. The organization then funds the proposal with the highest realization (i.e. the highest posterior beliefs about quality), as long as $\max\{\hat{\gamma}_L, \hat{\gamma}_H\} \geq \theta$. It does not fund any proposal whose realized posterior is below θ, given that such a proposal has a negative expected payoff. It randomizes fairly if $\hat{\gamma}_L = \hat{\gamma}_H \geq \theta$. Given the other agent j's posterior belief random variable Γ_j, agent i's payoff of choosing a Bayes-Plausible random variable Γ_j is

$$EU_i(\Gamma_i, \Gamma_j) = \Pr(\Gamma_i > \Gamma_j \cap \Gamma_i \geq \theta) + \frac{1}{2}\Pr(\Gamma_i = \Gamma_j \cap \Gamma_i \geq \theta).$$

The normal form representation of this game is closely related to the normal form of a full-information all-pay auction; although important distinctions exist. In the standard full-information all-pay auction, each agent's action is a choice of non-negative bid. The agent who chooses the highest bid wins a prize whose value is common knowledge (but can differ across players). All participants must pay their bids. A mixed strategy in this game is a choice of a random variable representing a player's random bid. The player whose random variable realization (i.e. bid) is highest wins the prize. In our framework, agents also design random variables, and the agent whose random variable generates the highest realization (that meets the minimum threshold θ) has his proposal accepted. The key difference between our framework and an all pay auction involves the constraints imposed by Bayes-Plausibility. In our framework, the agent's strategy Γ_i represents the distribution of posterior beliefs generated by his signal. Bayesian rationality therefore requires that the expected value of an agent's strategy equal the prior belief (which can be different for each agent).[8] A full information auction with a constraint on the mean bid is analyzed by Conitzer and Wagman (2011); however, our framework considers a number of additional significant features absent from the analysis of the standard all-pay

[8] In the all-pay auction setting, this constraint forces each bidder to adhere to a (potentially different) budget constraint that holds *in expectation* only. While in the all pay auction agent i chooses best response B_i to maximize $vPr(B_i > B_j) + \frac{1}{2}vPr(B_i = B_j) - E[B_i]$, in our game player i's best response maximizes $Pr(\Gamma_i > \Gamma_j) + \frac{1}{2}Pr(\Gamma_i = \Gamma_j)$ *subject to* $E[\Gamma_i] = \gamma_i$.

auction. Because realizations represent probabilities, the maximum possible realization is $\widehat{\gamma}_i = 1$ (in an all-pay auction, this requirement is effectively a bid cap). The organization's decision problem also imposes a minimum realization that is required for a proposal to be accepted, equivalent to a reservation price. Finally, because priors about each proposal quality can be different, agents face different constraints on their posterior belief random variables. Despite these important distinctions, our analysis brings to light an interesting connection between Bayesian persuasion games and all-pay auctions.

In the Appendix, we derive the equilibrium of the signal design game for all possible (γ_H, γ_L) combinations. For generic parameters, the equilibrium is unique. Each possible equilibrium of the game shares a similar structure. The disadvantaged player, L, chooses a strategy that consists of some combination of the following: a mass point on zero, uniform mixing between θ and some value $\bar{\gamma} < 1$, and a mass point on one. The advantaged player, H, plays a strategy similar in structure to L, expect that H may additionally concentrate probability mass on realization $\hat{\gamma}_H = \theta$ (the realization that is just persuasive enough to cause the organization to fund its proposal whenever the other agent's signal results in $\Gamma_L = 0$).

Lemma 3. *In every equilibrium, agents' evidence strategies have the following structure:*

$$\Gamma_H = \begin{cases} 0 & \textit{with probability} \quad f_{H0} \\ \theta & \textit{with probability} \quad f_{H\theta} \\ U[\theta, \bar{\gamma}] & \textit{with probability} \quad f_{HU} \\ 1 & \textit{with probability} \quad f_{H1} \end{cases} \qquad \Gamma_L = \begin{cases} 0 & \textit{with probability} \quad f_{L0} \\ U[\theta, \bar{\gamma}] & \textit{with probability} \quad f_{LU} \\ 1 & \textit{with probability} \quad f_{L1} \end{cases}$$

where $\bar{\gamma} \in (\theta, 1)$ depends on parameters (γ_H, γ_L). Agents put no probability mass on realizations outside of $\{0, \theta, 1\} \cup [\theta, \bar{\gamma}]$.

Figure 1 divides the parameter space into six regions, each corresponding to a particular equilibrium structure. Table 1 gives more details about the structure of the unique equilibrium in each region. The table refers to the following equations involving $\bar{\gamma}$.

$$T_L(\gamma) \equiv \gamma + \sqrt{\gamma^2 + \theta^2} \quad \text{and } T_R(\gamma) \equiv \gamma + \sqrt{\gamma^2 - \theta^2} \tag{1}$$

The equilibrium has a number of notable features. First, competition through information provision can incentivize agents to supply fully-informative signals in equilibrium, but this is not always the case. This equilibrium exists only when both proposals are sufficiently likely to be good *ex ante* (Region A in Figure 1). This result contrasts with competitive information provision in settings where agents design signals about the entire state (for example, guilt or innocence of a defendant as in Gentzkow and Kamenica (2012*a*)), rather than about one dimension of the state (the quality of an agent's proposal). With this type of information production, full information from both agents would always be an equilibrium. Next, consider regions D and F, in which γ_L is too low to generate a fully revealing equilibrium, and the *ex ante* asymmetry between the proposals is also small. Here, agent L chooses a signal structure that sometimes reveals the true state of the world, but otherwise produces a noisy realization that leaves the organization favorable, but still uncertain about his proposal. Compared with agent L, agent H shifts mass from posterior realization zero onto realization θ, otherwise generating the same posterior belief distribution. Thus, agent H takes advantage of a favorable prior by choosing a signal that is less likely to inform the organization when his proposal is

FIGURE 1. Equilibrium Regions

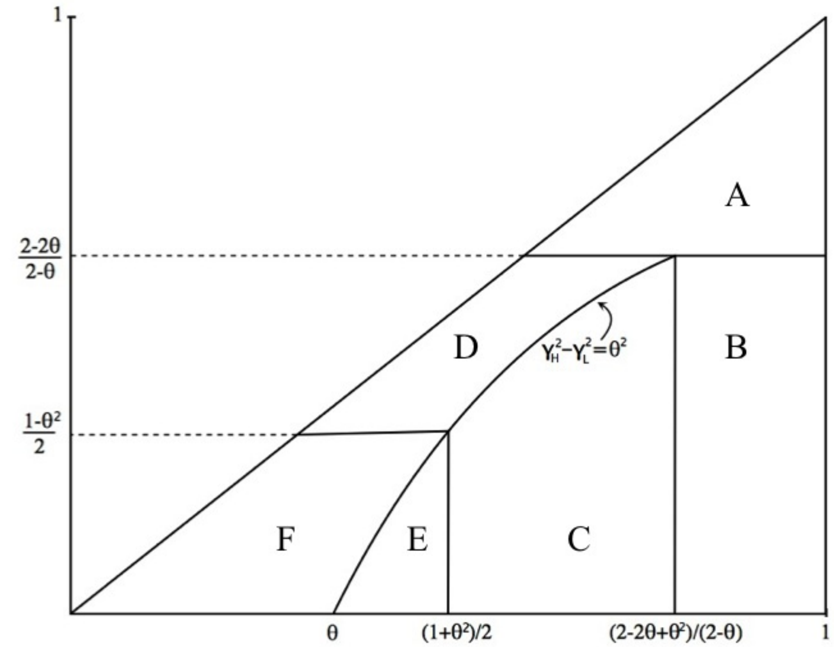


TABLE 1. Summary of equilibrium structure by region

Region	L Strategy	H Strategy	$\bar{\gamma}$
A	$f_{L0}, f_{L1} > 0$ **(FI)**	$f_{H0}, f_{H1} > 0$ **(FI)**	N/A
B	$f_{L0}, f_{L1} > 0$ **(FI)**	$f_{H\theta}, f_{H1} > 0$	N/A
C	$f_{L0}, f_{LU}, f_{L1} > 0$	$f_{H\theta}, f_{HU}, f_{H1} > 0$	$2 - T_R(\gamma_H)$
D	$f_{L0}, f_{LU}, f_{L1} > 0$	$f_{H0}, f_{H\theta}, f_{HU}, f_{H1} > 0$	$2 - T_L(\gamma_L)$
E	$f_{L0}, f_{LU} > 0$	$f_{H\theta}, f_{HU} > 0$	$T_R(\gamma_H)$
F	$f_{L0}, f_{LU} > 0$	$f_{H0}, f_{H\theta}, f_{HU} > 0$	$T_L(\gamma_L)$

All values of f not listed equal 0. **(FI)** denotes a fully-informative strategy. Along the diagonal in regions A, D and F where $\gamma_H = \gamma_L$, strategies are given above, except $f_{H\theta} = 0$, and $f_{Hi} = f_{Li}$ elsewhere. T_L and T_R are given by Eq. (1).

bad. In addition, agent L's equilibrium posterior belief random variable is the same as on the diagonal: agent L does not respond to the introduction of small asymmetries in the prior beliefs. When the asymmetry in priors becomes large enough, parameters cross into Regions E or C. Here, agent H is no longer able to satisfy the mean requirement by shifting mass from 0 to θ alone and begins to shift mass to higher realizations. In these regions agent L's equilibrium signal does respond to increases in γ_H (by becoming more informative, as we discuss below). In Region B, the asymmetry between γ_H and γ_L is sufficiently large that agent H shifts all probability mass from $(\theta, 1)$ to 1, generating a posterior belief random variable that puts mass only on realization θ and 1, and agent L adopts a fully revealing strategy .

To develop additional insight into competition through information provision, consider how an increase in the prior for proposal j affects agent i's posterior belief random variable. If γ_j increases, the constraint on the mean of Γ_j (arising from Bayes-Plausibility) is relaxed, allowing agent j to concentrate more probability mass on high realizations of his posterior belief random variable. When this increase is significant, competitive pressure motivates agent i to also reallocate mass onto high realizations. However, because γ_i is unchanged, to satisfy his mean constraint, agent i must simultaneously allocate mass to lower realizations of his posterior. Thus, in response to an increase in γ_j, player i shifts mass away from realizations near the prior onto realizations close to zero and one, while preserving the mean of Γ_i. In the appendix, we show that the reallocation of mass generates a posterior belief random variable that is second order stochastic dominated by the original. Results in Ganuza and Penalva (2010) establish that second order stochastic dominance of posterior belief random variables with the same mean is equivalent to Blackwell informativeness (in the context of binary states, as in our model).

Proposition 1. *In equilibrium, the Blackwell informativeness of agent i's signal (evidence strategy) is weakly increasing with γ_j. If $\gamma_j > \gamma_j'$, then in equilibrium either $\Gamma_i' \stackrel{d}{=} \Gamma_i$ or Γ_i' is more Blackwell informative than Γ_i.*

This proposition reveals a complementarity between an agent's signal informativeness and the competitive pressure that he faces: as his opponent's proposal looks more promising *ex ante*, the agent responds by supplying a more informative signal.

6. The capacity tradeoff

Under limited capacity, agents compete to generate the highest posterior beliefs about their proposals. Bayes-Plausibility requires that any probability mass on favorable belief realizations (above the prior) is offset (in a mean preserving way) by probability mass on unfavorable belief realizations (below the prior). Thus competition to generate higher beliefs realizations also forces agents to choose evidence strategies which generate lower belief realizations, spreading mass on posterior belief realizations inside the unit interval. Compared with the equilibrium posterior belief distributions under unlimited capacity, with limited capacity probability mass is shifted away from the mean, toward zero and one. Indeed, the posterior belief random variables arising in equilibrium under limited capacity are second order stochastic dominated by those arising under unlimited capacity, implying that under limited capacity, signals are more Blackwell informative.

Proposition 2. *In the unique equilibrium of the limited capacity game, each agent chooses a signal (evidence strategy) which is more Blackwell informative than in the unique (for an agent with $\gamma_i < \theta$) or focal (for an agent with $\gamma_i \geq \theta$) equilibrium under unlimited capacity.*

Limited capacity thus creates incentives for agents to supply more Blackwell informative signals, which benefits the organization. However, by restricting its ability to act *ex post*, limited capacity also imposes a cost on the organization. Indeed, if both proposals generate posterior belief realizations strictly greater than θ, then the organization would strictly benefit by accepting both proposals, but under limited capacity it is constrained to accept only one. Next, we argue that unless both proposals are sufficiently likely to be good *ex ante*, the benefits dominate the costs of limited capacity.

When the organization is predisposed against both proposals, the result is immediate. With unlimited capacity, the organization is, at best, indifferent between accepting and rejecting, giving its payoff zero. In this case, no *ex ante* cost is associated with limited capacity. At the same time, limited capacity motivates the agents to supply signals that are more informative, and a positive probability exists that the organization will accept one of the two proposals, leaving it with a positive expected payoff. Thus, when it is predisposed against both proposals, the organization prefers limited capacity.

If it is predisposed against L, but in favor of H, then under unlimited capacity, the organization always accepts proposal H and expects zero payoff from proposal L, resulting in expected payoff $\gamma_H - \theta$. In the limited capacity game, if the organization is forced to accept H and reject L, then its expected payoff will be $\gamma_H - \theta$, identical to its payoff in the unlimited capacity equilibrium. In the limited capacity equilibrium, however, the organization is not constrained to always accept H and reject L. In fact, the probability that the organization maximizes its payoff by accepting L or rejecting both proposals is always non-zero in the limited capacity equilibrium. Its expected payoff in the limited capacity equilibrium therefore exceeds its payoff when it is constrained (which is identical to its unlimited capacity payoff). Thus, if the organization is predisposed against either proposal it strictly prefers limited capacity.

We have therefore demonstrated that the organization strictly prefers the equilibrium with limited capacity whenever $\gamma_L \leq \theta$, independent of γ_H. Because the equilibrium changes in a continuous way as the parameters change, there also exists a region with $\gamma_L > \theta$ in which the organization prefers limited capacity.[9]

Proposition 3. *If $\gamma_L, \gamma_H \leq \theta$, then the organization's expected payoff always is strictly higher under limited capacity compared to unlimited capacity. If $\gamma_H > \theta$, then there exists a value $\tilde{\gamma}_L \in (\theta, \gamma_H)$ such that the organization's expected payoff is strictly higher under limited capacity for all $\gamma_L < \tilde{\gamma}_L$.*

In other words, the organization benefits from limited capacity, unless ex ante beliefs in favor of *both* proposals are sufficiently strong. In that case, limiting capacity forces the organization to forgo a proposals that has a high ex ante probability of being beneficial. In all other parameter cases, the organization is better off under limited capacity. In these situations, the benefits associated with better information dominate the costs of potentially forgoing a project that is worthwhile.

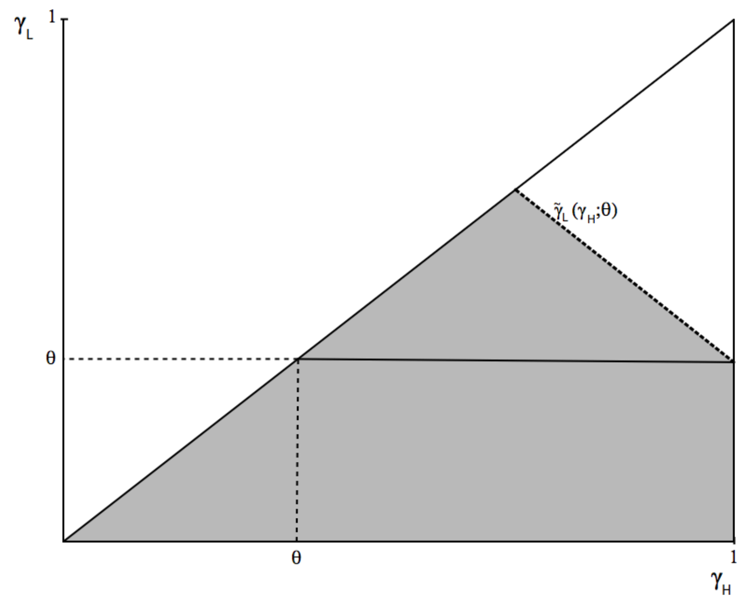
7. Conclusion

We model the allocation of limited funding across proposals as a persuasion game with limited capacity. Agents representing alternative proposals produce evidence regarding the merits of their proposals, in the hope of persuading a funding organization to allocate resources to their opportunity.

Not only do the agents choose whether to produce evidence, but they also choose how informative the evidence is that they collect. That is, they design the polls, research

[9]Technically, discontinuities occur when crossing into region A from parameters that are not inside region B. However, in the interior of all equilibrium regions, the equilibrium posterior belief distributions are continuous in the parameters, so this point is irrelevant for the argument.

FIGURE 2. Benefit of limited capacity



In the shaded region, the organization's payoff is higher under limited capacity than under unlimited capacity.

methodology, or search for evidence about the merits of their proposals. The funding organization prefers that the agents produce the most informative evidence possible. Agents, however, strategically choose to produce less informative evidence, a strategy which maximizes the probability that they receive funding. We show that the organization will be better informed and can be better off when the number of proposals that it can accept is limited. When the organization is unable to accept all proposals, the agents respond to the introduction of competitive pressure by producing more Blackwell informative evidence. Except when the priors strongly favor the implementation of both proposals, the organization is better off when its capacity to fund projects is limited.

Our analysis highlights a novel benefit of capacity constraints and underfunding in organizations. Limited capacity incentivizes the provision of more persuasive information by those vying for funding. This leads to better informed funding allocation decisions by organizations. An underfunded foundation or charity will be better informed and able to more efficiently allocate its resources across community projects or individuals, compared to an foundation or charity that can afford to give money to all community projects or individuals its sees as deserving. A legislature or government bureaucracy will be better informed and may more efficiently allocate resources across earmarks, internal projects, or policy reforms when those resources are limited. A firm manager will become better informed about the optimal allocation of funding within her division if she faces resource constraints than if she does not. These benefits come because capacity constraints incentivizes those competing for the limited resources to produce evidence that is more informative about the merits of their proposals. This can benefit the organization responsible for allocating the funding.

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Appendix A. Appendix

Proof. **Lemma 1** As Γ_i is Bayes-Plausible, its generalized density $p(x)$ can be written as

$$p(x) = f(x) + \sum_{k=1}^{M} \mu_k \delta(x - m_k), \tag{2}$$

where $f(x)$ is continuous in the interior of its support, m_k is a mass point of Γ_i, and μ_k is the mass on m_k. Define two new random variables, (S_i^b, S_i^g) by their generalized densities as follows:

$$p_g(x) = \left(\frac{x}{\gamma}\right) p(x) \quad \text{and} \quad p_b(x) = \left(\frac{1-x}{1-\gamma}\right) p(x).$$

Observe that the supports of (S_i^b, S_i^g) coincide exactly with the support of Γ_i, which must be inside the unit interval. If Γ_i is Bayes-Plausible, $E[\Gamma_i] = \gamma_i$ and these generalized densities integrate to one. Also observe also that the likelihood ratio for this signal structure is equal to

$$\frac{p_b(x)}{p_g(x)} = \frac{1-x}{x} \frac{\gamma}{1-\gamma},$$

which is differentiable for all x in the interior of the support of Γ_i. Consider the valid signal given by the pair (S_i^b, S_i^g). For this signal, the posterior belief associated with a draw of s is

$$\frac{\gamma p_g(s)}{\gamma p_g(s) + (1-\gamma) p_b(s)} = \frac{\gamma(\frac{s}{\gamma}) p(s)}{\gamma(\frac{s}{\gamma}) p(s) + (1-\gamma)(\frac{1-s}{1-\gamma}) p(s)} = s$$

Thus, for this signal, the posterior belief associated with a draw of s from this signal structure is simply s itself. Furthermore, the density of the posterior belief is therefore equal to the density of a draw from this signal structure:

$$\gamma p_g(x) + (1-\gamma) p_b(x) = p(x)$$

Thus, we have constructed a valid signal for which the ex ante posterior belief is Γ_i. Note that other constructions are possible. □

A.1. **Derivation of equilibria under limited capacity.** Lemma 4 shows that each player's equilibrium strategy always takes a certain structure.

Lemma 4. *In every equilibrium, Γ_i takes the following form:*

$$\Gamma_i = \begin{cases} 0 & \text{with probability} \quad f_{i0} \\ \theta & \text{with probability} \quad f_{i\theta} \\ U[\theta, \bar{\gamma}] & \text{with probability} \quad f_{iU} \\ 1 & \text{with probability} \quad f_{i1} \end{cases}$$

where $f_{i0}, f_{i\theta}, f_{iU}, f_{i1} \geq 0$ and $f_{i0} + f_{i\theta} + f_{iU} + f_{i1} = 1$.

Proof. A strategy for agent i, is a random variable Γ_i with support contained in the unit interval, and expectation γ_i. Because the underlying signal structure is valid, Γ_i has a finite number of mass points. Let M_i represent the set of all mass points in Γ_i, and m denotes an arbitrary mass point in M_i. Let $\mu_i(m) = Pr(\Gamma_i = m)$ for all $m \in M_i$. The cumulative distribution function of Γ_i, denoted P_i, is as follows:

$$P_i(x) = F_i(x) + \sum_{m \in M_i} H(x - m) \mu_i(m) \tag{3}$$

where $F_i(x)$ is a continuous and differentiable function, strictly increasing in the interior of set I_i, a countable union of closed intervals, and it is neither increasing nor decreasing outside of I_i; $H(\cdot)$ represents a right-continuous step function. The support of random variable Γ_i is $I_i \cup M_i \equiv S[\Gamma_i]$.

Let $W_i(x)$ represent the probability proposal i is accepted when signal Γ_i generates posterior belief realization x.

$$W_i(x) = \begin{cases} 0 & \text{if} \quad x < \theta \\ \Pr(\Gamma_j < x) + \frac{1}{2}\Pr(\Gamma_j = x) & \text{if} \quad x \geq \theta. \end{cases}$$

Thus, for any point x this function is given by the following expression:

$$W_i(x) = \begin{cases} 0 & \text{if} \quad x < \theta \\ P_j(x) & \text{if} \quad x \geq \theta \text{ and } x \in I_j \text{ and } x \notin M_j \\ P_j(x) - \frac{1}{2}\mu_i(x) & \text{if} \quad x \geq \theta \text{ and } x \in M_j. \end{cases} \tag{4}$$

Function $P_j(x)$ neither increases nor decreases outside of set $S[\Gamma_j]$. Therefore, function $W_i(x)$ maintains a constant value in any interval that does not intersect $S[\Gamma_j]$.

Consider the best response of agent i to a choice of Γ_j by agent j. Agent i prefers is a choice of random variable Γ_i with generalized density p_i to solve the following maximization:

$$\max_{p_i(\cdot)} \int_0^1 p_i(x) W_i(x) dx$$

$$\text{s.t.} \int_0^1 p_i(x) x dx = \gamma_i \text{ and } \int_0^1 p_i(x) dx = 1 \text{ and } p_i(x) \geq 0 \; \forall x \in [0,1].$$

The first constraint comes from the law of iterated expectations given the ex ante distribution of proposal type. Consider the Lagrangian for agent i's maximization problem:

$$L = \int_0^1 p_i(x) W_i(x) dx - \lambda_{1i}\left(\int_0^1 p_i(x) x dx - \gamma_i\right) - \lambda_{2i}\left(\int_0^1 p_i(x) dx - 1\right) - \lambda_{3i}(x) p_i(x)$$

where a separate λ_{3i} may apply for each x. Simplifying the expression gives

$$L = \int_0^1 p_i(x)(W_i(x) - \lambda_{1i}(x - \gamma_i) - \lambda_{2i} - \lambda_{3i}(x)) dx - \lambda_{2i}$$

The stationarity condition with respect to $p_i(x)$ requires that for all $x \in [0,1]$,

$$W_i(x) - \lambda_{1i}(x - \gamma_i) - \lambda_{2i} - \lambda_{3i}(x) = 0$$

Consider any x for which $p_i(x) > 0$. Complementary slackness requires that $\lambda_{3i}(x) = 0$. Furthermore, if $p_i(x) = 0$, then $\lambda_{3i}(x) \geq 0$. Hence, defining

$$L_i(x) \equiv W_i(x) - \lambda_{1i}(x - \gamma_i) \tag{5}$$

we find the following conditions:

$$x \in S[\Gamma_i] \Rightarrow L_i(x) = \lambda_{2i} \tag{6}$$
$$x \notin S[\Gamma_i] \Rightarrow L_i(x) \leq \lambda_{2i},$$

for some $\lambda_{2i} \geq 0$. To summarize, all values of $x \in S[\Gamma_i]$ generate the same value of $L_i(x)$, which is at least as large as the value of $L_i(x)$ for any $x \notin S[\Gamma_i]$. Condition (6) implies several properties of best-responses.

I For any best response $\Pr(0 < \Gamma_i < \theta) = 0$.
Proof: Indeed inside $(0, \theta)$, probability of winning, $W_i(x) = 0$, but $\lambda_i(x - \gamma_i)$ is increasing. Thus, no set of realizations inside $(0, \theta)$ could generate the same value of $W_i(x) - \lambda_i(x - \gamma_i)$.

II If Γ_i is a best response to Γ_j, then I_i is a weak subset of I_j:
Proof: Let I_j^C represent the complement of set I_j. Because both I_i and I_j are unions of closed intervals, if $I_i \cap I_j^C$, is non-empty. If so, it contains an interval. Outside I_j, the only support of Γ_j is a finite set of mass points. Hence, exists a subinterval in $I_i \cap I_j^C$ that does not intersect G_j. On this interval, however, $W_i(x)$ is constant, while $\lambda_i(x - \gamma_i)$ is increasing.

III If Γ_i is a best response to γ_j, and if Γ_j has a mass point on $m_j \in M_j$, then there exists ϵ such that $(m_j - \epsilon, m_j)$ does not intersect I_i.
Proof: Because $W_i(x)$ jumps up at m_j but $\lambda_i x$ does not, $L_i(x)$ jumps up at m_j. Hence, any value of $x \geq m_j$ cannot give the same value of $L_i(.)$ as a value of x sufficiently close to m_j.

IV If Γ_j does not have a mass point on 1, then player i best response Γ_i does not have a mass point on 1.
Proof: If Γ_j does not have a mass point on 1, then $L_i(1-\epsilon) > L_i(1)$.

These properties have significant implications for the structure of possible equilibria.

(i) In no equilibrium does Γ_i or Γ_j put positive probability mass on realizations inside $(0, \theta)$.
Proof: Direct consequence of property I.

(ii) In equilibrium no mass point inside $[\theta, 1)$ can be common to both Γ_i and Γ_j.
Proof: Just above the mass point by ϵ, the probability of winning is discretely higher (by $\frac{\mu}{2}$), but the "cost" $\lambda_i(x - \gamma_i)$ is only marginally higher. Thus $L_i(x)$ is higher just above the common mass point. Only possible common mass point in equilibrium is 1.

(iii) In equilibrium $I_i = I_j \equiv \bar{I}$.
Proof: Direct consequence of property II.

(iv) In equilibrium $\bar{I}$ is a single interval.
Proof: Suppose $\bar{I}$ contains more than one interval. In this case there exist $\{x_L, x_R\}$ with $x_R > x_L$ such that $[x_L - \epsilon_L, x_L]$ and $[x_R, x_R + \epsilon_R]$ are inside $\bar{I}$ but (x_L, x_R) is not. Because $[x_R, x_R + \epsilon_R]$ is inside $[0, 1]$, $x_R < 1$. Because no mass point inside $(\theta, 1)$ can be common to Γ_i and Γ_j it cannot be that x_R is a mass point for both players. Therefore suppose that x_R is not a mass point for j. If x_R is not a mass point for j, then $W_i(x_L) = W_i(x_R)$, but $\lambda_i(x_R - \gamma_i) > \lambda_i(x_L - \gamma_i)$, contradicting the optimality condition.

(v) In equilibrium, the smallest element of the (identical) interval support $\bar{I}$, is θ.
Proof: In equilibrium, both random variables are supported on the same interval I. Suppose the smallest element of I, denoted $\underline{x}$ is strictly above θ. Property IV shows at most one of Γ_i and Γ_j can have a mass point on $\underline{x}$. If exactly one has a mass point on $\underline{x}$, let Γ_i be the random variable with no mass point on $\underline{x}$. Besides, I, Γ_i is supported on a set of mass points. This implies that for sufficiently small ϵ, no mass point exists between $\underline{x} - \epsilon$ and $\underline{x}$. No mass point exists on $\underline{x}$, and, because $\underline{x}$ is the smallest element of I, $F_j(\underline{x}) = 0$. Hence $W_i(\underline{x}) = W_i(\underline{x} - \epsilon)$. Thus $L(\underline{x} - \epsilon) > L(\underline{x})$.

(vi) In equilibrium, no mass point exists in $(\theta, 1)$ for either player.
Proof: In equilibrium each player's strategy has the same interval support $I = [\theta, r]$. By property III, no mass point can exist in interval $(\theta, r]$. If a player has mass point above r, then it must be shared with the other player. If it is not shared with the other player, then a point just below generates the same winning probability, but lower $\lambda_i x$ and hence a greater value of $L_i(.)$. However, property IV implies that only possible common mass point in equilibrium is 1, ruling out this possibility.

Equilibrium conditions (i) to (vi) allow us to characterize the structure of equilibrium strategies. An equilibrium strategy for player $i \in \{H, L\}$ must have the following structure:

$$\Gamma_i = \begin{cases} 0 & \text{with probability } \rho_i^0 \\ \theta & \text{with probability } \rho_i^\theta \\ \Phi_i & \text{with probability } \rho_i^\Phi \\ 1 & \text{with probability } \rho_i^1 \end{cases}$$

where $\rho_i^0 + \rho_i^\theta + \rho_i^\Phi + \rho_i^1 = 1$, and Φ_i is a random variable with support over an interval $I = [\theta, r]$ with no mass points, and where $r \in (\theta, 1]$. The CDF of Φ_i is given by $F_i(x)$, where $F_i(x)$ is continuous (and differentiable), $F_i(\theta) \geq 0$ and $F_i(r) = 1$. In this case the win-probability for player i has the following structure:

$$W_i(x) = \begin{cases} 0 & \text{if} \quad x \in [0, \theta) \\ \rho_j^0 + \rho_j^\theta \frac{1}{2} & \text{if} \quad x = \theta \\ \rho_j^0 + \rho_j^\theta + \rho_j^\Phi F_j(x) & \text{if} \quad x \in [\theta, r] \\ 1 - \rho_i^1 \frac{1}{2} & \text{if} \quad x = 1. \end{cases}$$

We make additional observations in terms of the ρ values:

(vi) If $\rho_j^\Phi > 0$, then $\rho_i^\Phi > 0$.

Proof: Suppose instead that $\rho_i^\Phi > 0$ and $\rho_j^\Phi = 0$. Since $\rho_j^\Phi = 0$, $W_i(x) = p_j^0 + p_j^\theta$ and $L_i(x) = p_j^0 + p_j^\theta - \lambda_{1i}(x - \gamma_i)$ for all $x \in I$. (6) establishes that $L_i(x) = v_i$ for all $x \in G_i$. However, $L_i(x)$ is strictly decreasing in x, contradicting the requirement that it takes on the same value for all $x \in [\theta, r]$. Therefore, if $\rho_j^\Phi = 0$, then $\rho_i^\Phi = 0$. This immediately implies that if $\rho_j^\Phi > 0$, then $\rho_i^\Phi > 0$.

(vii) If $\rho_j^\Phi > 0$, then F_j represents a uniform distribution on $[\theta, r]$.

Proof: If $\rho_j^\Phi > 0$, then by (vii) $\rho_i^\Phi > 0$. Given $\rho_i^\Phi > 0$, (6) implies that

$$\rho_j^0 + \rho_j^\theta + \rho_j^\Phi F_j(x) - \lambda_{1i}(x - \gamma_i) = v_i \text{ for all } x \in [\theta, r] \tag{7}$$

Rearranging the equality gives an expression for $F_j(x)$ which is linear in x:

$$F_j(x) = \frac{v_i - \rho_j^0 - \rho_j^\theta - \lambda_{1i}\gamma_i}{\rho_j^\Phi} + \frac{\lambda_{1i}}{\rho_j^\Phi} x.$$

Thus, if any mass exists on $(\theta, r]$ then it must be uniformly distributed with a possible mass point on θ. However, any mass point on θ is captured by ρ_j^θ rather than F_j. Therefore, when it contains mass, F_j must be a uniform distribution on $[\theta, r]$.

□

Next, we characterize the equilibria in the six regions of Figure 1.

Equilibrium in Region A. **Claim:** If and only if $\gamma_L \geq \frac{2-2\theta}{2-\theta}$ (i.e. iff we are in Region A of Figure 1) is there a Nash Equilibrium in which each agent plays *fully revealing strategy*:

$$\Gamma_i = \begin{cases} 0 & \text{with prob} \quad 1 - \gamma_i \\ 1 & \text{with prob} \quad \gamma_i. \end{cases}$$

Proof. In any fully revealing strategy, the constraint that $E[\Gamma_i] = \gamma_i$ implies the probabilities $f_{i0} = 1 - \gamma_i$ and $f_{i1} = \gamma_i$.

Suppose that agent j uses a fully revealing strategy. It must be a best response for agent i to also play a fully revealing strategy. The proof of Lemma 4 establishes that $L_i(x)$, defined by (5), achieves its maximum at each $x \in G_i$ (at each potential realization of Γ_i that occurs with positive probability in equilibrium).

Let B_i represent agent i's best response strategy. Claim I in the proof of Lemma 4 establishes that agent i's best response does not include any probability in $(0, \theta)$. To rule out B_i putting positive probability on $(\theta, 1)$, it is sufficient to show that $L_i(\theta) > L_i(x)$ for all $x \in (\theta, 1)$:

$$L_i(\theta) > L_i(x) \iff$$
$$(1 - \gamma) - \lambda_{1i}(\theta - \gamma_i) > (1 - \gamma_i) - \lambda_{1i}(x - \gamma_i) \iff$$
$$\theta < x,$$

which is satisfied given $x \in (\theta, 1)$. This means that the potential support of B_i is limited to mass points on realizations 0, θ and 1. Agent i chooses $f_{i\theta}$ and f_{i1} to maximizing the probability of having his proposal accepted: $(1-\gamma_j)f_{i\theta}+(1-\gamma_j/2)f_{i1}$. The constraint on the mean, $E[B_i]=\gamma_i$ implies $f_{i\theta}\theta+f_{i1}=\gamma_i$. Substituting $f_{i1}=\gamma_i-f_{i\theta}\theta$ into the probability of acceptance gives

$$EU_i=(1-\gamma_j)f_{i\theta}+(1-\gamma_j/2)(\gamma_i-f_{i\theta}\theta). \tag{8}$$

This expression is strictly decreasing in $f_{i\theta}$ when $\gamma_j>2(1-\theta)/(2-\theta)$, strictly decreasing in $f_{i\theta}$ when $\gamma_j<2(1-\theta)/(2-\theta)$, and independent of $f_{i\theta}$ when $\gamma_j=2(1-\theta)/(2-\theta)$. This means playing that $f_{i\theta}=0$ is a best response for player i if and only if

$$\gamma_j\geq\frac{2(1-\theta)}{2-\theta}.$$

In equilibrium, this inequality must hold for both L and H. Since, $\gamma_L\leq\gamma_H$, this implies the parameter condition $\gamma_L\geq 2(1-\theta)/(2-\theta)$. □

Equilibrium in Region B. **Claim:** If and only if $\gamma_L\leq\frac{2-2\theta}{2-\theta}$ and $\gamma_H\geq\frac{2-2\theta+\theta^2}{2-\theta}$ (i.e. iff we are in Region B of Figure 1) is there a Nash Equilibrium in which agent L plays a *fully revealing strategy*:

$$\Gamma_L=\begin{cases}0 & \text{with prob} \quad 1-\gamma_L\\ 1 & \text{with prob} \quad \gamma_L\end{cases}$$

and agent H plays a *deflationary strategy*:

$$\Gamma_H=\begin{cases}\theta & \text{with probability} \quad \frac{1-\gamma_H}{1-\theta}\\ 1 & \text{with probability} \quad \frac{\gamma_H-\theta}{1-\theta}\end{cases}$$

We refer to such a Γ_H as deflationary because it involves agent H choosing a signal that assigns only unfavorable realizations to bad proposals, and sometimes assigns favorable and sometimes unfavorable signals to good proposals. In this sense, the signal sometimes deflates the assessment of good proposals.

Proof. Notice that Γ_H is a well defined random variable when $\gamma_H\geq\theta$, a condition which is always satisfied given that here $\gamma_H>(2-2\theta+\theta^2)/(2-\theta)$. The random variale satisfies the mean constraint:

$$\theta(\frac{1-\gamma_H}{1-\theta})+\frac{\gamma_H-\theta}{1-\theta}=\gamma_H.$$

We first establish that Γ_H is a best reply to a fully revealing strategy for player L if and only if $\gamma_L\leq 2(1-\theta)/(2-\theta)$ and $\gamma_H>\theta$. The proof for Region A established that when $\gamma_L>2(1-\theta)/(2-\theta)$, player i prefers a fully informative strategy, ruling out the possibility that Γ_H is a best response for sufficiently large values of γ_L. We must now establish that γ_L is a best response for lower values of γ_L. Note that (8) in the proof for Region A is increasing in $f_{H\theta}$ when this inequality holds. This means that when the inequality holds, i chooses the highest value of $f_{H\theta}$ given the constraints that $f_{H\theta}\theta+f_{H1}=\gamma_i$ and $f_{H\theta},f_{H1}\in[0,1]$.

When $\gamma_H<\theta$, the constraint on the mean implies that $f_{i\theta}$ is maximizes by a distribution in which $f_{i0}=1-\gamma_H/\theta$, $f_{i\theta}=\gamma_H\theta$ and $f_{i1}=0$. When $\gamma_H>\theta$, the constraint on the mean implies that $f_{i\theta}$ is maximized by a distribution in which $f_{i0}=0$, $f_{i\theta}=(1-\gamma_H)/(1-\theta)$ and $f_{i1}=(\gamma_H-\theta)/(1-\theta)$. When $\gamma_H=\theta$, the deflationary strategy Γ_H becomes an uninformative strategy, and $f_{i\theta}=1$. Therefore, Γ_H requires $\gamma_H>\theta$, a condition guaranteed when $\gamma_H\geq(2-2\theta+\theta^2)/(2-\theta)$.

Next, we establish that Γ_L is a best reply to Γ_H if and only if $\gamma_H\geq(2-2\theta+\theta^2)/(2-\theta)$. Given the deflationary strategy by agent H, player L's best response may put mass on realizations 0, 1 or "just above" θ. Let $f_{L\epsilon}$ denote the probability mass L concentrates on realizations just above θ.

Agent L chooses f_{L0}, $f_{L\epsilon}$ and f_{L1} to maximize the probability of having his proposal accepted, $EU_i = f_{H\theta}f_{L\epsilon} + (f_{H\theta} + f_{H1}/2)f_{L1}$, subject to the constraint on the mean which requires $f_{L\epsilon}\theta + f_{L1} = \gamma_L$ given the mass point just above θ is arbitrarily close to θ. Substituting into EU_i for f_{L1} and substituting in for $f_{H\theta}$ and f_{H1} from Γ_H gives

$$EU_i = \frac{1-\gamma_H}{1-\theta}f_{L\epsilon} + (\frac{1-\gamma_H}{1-\theta} + \frac{\gamma_H-\theta}{2(1-\theta)})(\gamma_L - f_{L\epsilon}\theta).$$

Agent L must have no incentive to choose $f_{L\epsilon} > 0$. Therefore, this expression for EU_i must be decreasing in $f_{L\epsilon}$. This is the case when,

$$\frac{1-\gamma_H}{1-\theta} - (\frac{1-\gamma_H}{1-\theta} + \frac{\gamma_H-\theta}{2(1-\theta)})\theta \le 0 \iff$$

$$2(1-\gamma_H)(1-\theta) - (\gamma_H-\theta)\theta \le 0 \iff$$

$$\frac{2-2\theta+\theta^2}{2-\theta} \le \gamma_H.$$

If and only if this condition holds, it is a best response for agent L to play a fully revealing strategy when H plays the deflationary strategy. ☐

Equilibrium in Region C. **Claim:** If and only if $\theta \le \sqrt{\gamma_H^2 - \gamma_L^2}$ and $\frac{1}{2}(1+\theta^2) \le \gamma_H \le \frac{2-2\theta+\theta^2}{2-\theta}$ (i.e. iff we are in Region C of Figure 1) is there a Nash Equilibrium in which agent L plays

$$\Gamma_L = \begin{cases} 0 & \text{with probability} \quad 1 - f_{LU} - f_{L1} \\ U[\theta,\bar{\gamma}] & \text{with probability} \quad f_{LU} = \frac{2\gamma_L(\bar{\gamma}-\theta)}{(2-\bar{\gamma})^2-\theta^2} \\ 1 & \text{with probability} \quad f_{L1} = \frac{4\gamma_L(1-\bar{\gamma})}{(2-\bar{\gamma})^2-\theta^2} \end{cases}$$

and agent H plays

$$\Gamma_H = \begin{cases} \theta & \text{with probability} \quad \frac{\theta}{2-\bar{\gamma}} \\ U[\theta,\bar{\gamma}] & \text{with probability} \quad \frac{\bar{\gamma}-\theta}{2-\bar{\gamma}} \\ 1 & \text{with probability} \quad \frac{2-2\bar{\gamma}}{2-\bar{\gamma}} \end{cases}$$

where

$$\bar{\gamma} = 2 - \gamma_H - \sqrt{\gamma_H^2 - \theta^2}.$$

Proof. We first demonstrate that the strategies of both players are feasible. As $\theta \le \sqrt{\gamma_H^2 - \gamma_L^2} \le \gamma_H$, $\bar{\gamma}$ is a real number. Observe that $\frac{1}{2}(1+\theta^2) \le \gamma_H \le \frac{2-2\theta+\theta^2}{2-\theta} \Leftrightarrow \theta \le \bar{\gamma} \le 1$. Under these conditions, clearly both $f_{H\theta}, f_{HU}$ are positive. Furthermore, $f_{H\theta} + f_{HU} - 1 = -\frac{\theta}{2-\bar{\gamma}} < 0$. Observe next that,

$$\theta \le \sqrt{\gamma_H^2 - \gamma_L^2} \Leftrightarrow \bar{\gamma} \le 2 - \gamma_L - \sqrt{\gamma_L^2 + \theta^2}$$

This inequality will be both necessary and sufficient later in the proof. For now, note that

$$2 - \gamma_L - \sqrt{\gamma_L^2+\theta^2} - (\theta + 2(1-\gamma_L)) = \gamma_L - \theta - \sqrt{\gamma_L^2+\theta^2} < 0$$

The right hand side equals zero when $\theta = 0$ and has a negative derivative in θ. Thus,

$$\bar{\gamma} \le 2 - \gamma_L - \sqrt{\gamma_L^2+\theta^2} \rightarrow \bar{\gamma} \le \theta + 2(1-\gamma_L)$$

Observe that $\theta \leq \gamma_H \to 2-\bar{\gamma} = \gamma_H + \sqrt{\gamma_H^2 - \theta^2} \geq \theta$. Hence $f_{LU}, f_{L1} \geq 0$. Next, observe that $f_{LU} + f_{L1} - 1 = \frac{\bar{\gamma}-(\theta+2(1-\gamma_L))}{2-\bar{\gamma}-\theta} \leq 0$ as described above. Therefore Γ_H, Γ_L are random variables. They are feasible if they satisfy the constraints on the expected values.

$$E[\Gamma_H] = (1 - f_{H\theta} + f_{HU})\theta + f_{H\theta}\frac{\theta+\bar{\gamma}}{2} + f_{HU} = \frac{\theta^2 + (2-\bar{\gamma})^2}{2(2-\bar{\gamma})} = \gamma_H$$

$$E[\Gamma_L] = f_{LU}\frac{\theta+\bar{\gamma}}{2} + f_{L1} = \gamma_L$$

Note also that $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2}$ and $\frac{1}{2}(1+\theta^2) \leq \gamma_H \leq \frac{2-2\theta+\theta^2}{2-\theta}$ means that $\gamma_L \leq \frac{2-2\theta}{2-\theta}$.

Next, we establish that the proposed strategies are mutual best responses. According to Lemma 1, any possible best response to Γ_H, denoted $\hat{\Gamma}_L$ must have the following structure:

$$\hat{\Gamma}_L = \begin{cases} 0 & \text{with probability} \quad 1-\phi_M-\phi_H-\phi_{H\theta} \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \\ 1 & \text{with probability} \quad \phi_{H\theta} \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M, and if a mass point exists at 1, it is not part of G_H (no mass point exists at endpoints of G_H). Furthermore, no mass point at θ exists in G_M. Such mass point leads to ties with positive probability; using a mass point of $\theta+\epsilon$ leads to all ties at θ breaking in favor of player L. In order for this strategy to be feasible, it must be that

$$\phi_M\bar{g}_M + \phi_H\bar{g}_H + \phi_{H\theta} = \gamma_L \iff \bar{g}_M = \frac{\gamma_L - \phi_H\bar{g}_H - \phi_{H\theta}}{\phi_M}.$$

The expected payoff of using such a strategy against Γ_H is given by:

$$\begin{aligned}
& \phi_M(1 - f_{H\theta} - f_{HU} + f_{H\theta}\int_\theta^{\bar{\gamma}} \frac{x-\theta}{\bar{\gamma}-\theta}\, g_M(x)\, dx) + \phi_H(1-f_{HU}) + \phi_{H\theta}(1-\frac{f_{HU}}{2}) \\
= \;& \phi_M(1 - f_{H\theta} - f_{HU} + f_{H\theta}\frac{\bar{g}_M-\theta}{\bar{\gamma}-\theta}) + \phi_H(1-f_{HU}) + \phi_{H\theta}(1-\frac{f_{HU}}{2}) \\
= \;& \phi_M(1 - f_{H\theta} - f_{HU} + f_{H\theta}\frac{\frac{\gamma_L-\phi_H\bar{g}_H-\phi_{H\theta}}{\phi_M}-\theta}{\bar{\gamma}-\theta}) + \phi_H(1-f_{HU}) + \phi_{H\theta}(1-\frac{f_{HU}}{2}) \\
= \;& \phi_M\frac{(\bar{\gamma}-\theta)(1-f_{HU}) - \bar{\gamma}f_{H\theta}}{(\bar{\gamma}-\theta)} + \phi_{H\theta}\frac{(2-f_{HU})(\bar{\gamma}-\theta) - 2f_{H\theta}}{2(\bar{\gamma}-\theta)} \\
& + \phi_H(\frac{(2-f_{HU})(\bar{\gamma}-\theta)-2f_{H\theta}}{2(\bar{\gamma}-\theta)} - \frac{f_{H\theta}(\bar{g}_H-\bar{\gamma})}{\bar{\gamma}-\theta}) + \frac{\gamma_L f_{H\theta}}{\bar{\gamma}-\theta}.
\end{aligned}$$

Clearly, the coefficient on ϕ_H is less than the coefficient on $\phi_{H\theta}$. Thus, in any best response, $\phi_H = 0$. It is also easy to check that the coefficient on $\phi_M, \phi_{H\theta}$ are equal to zero. Thus the payoff to using a strategy of type $\hat{\Gamma}_L$ is independent of $\phi_M, \phi_{H\theta}, G_M$. Thus any random variable in class $\hat{\Gamma}_L$ is a best response, provided $\phi_H = 0$. As Γ_L satisfies these criteria, it is a best response.

Next, we show that Γ_H is a best response to Γ_L. According to Lemma 1, any admissible best response to Γ_L, denoted $\hat{\Gamma}_H$ must have the following structure:

$$\hat{\Gamma}_H = \begin{cases} 0 & \text{with probability} \quad 1-\phi_M-\phi_H-\phi_{H\theta} \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \\ 1 & \text{with probability} \quad \phi_{H\theta} \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M, and if a mass point exists at 1, it is not part of G_H (no mass point exists in G_H). In order for this strategy to be admissible, it must be that

$$\phi_M \bar{g}_M + \phi_H \bar{g}_H + \phi_{H\theta} = \gamma_H \iff \bar{g}_M = \frac{\gamma_H - \phi_H \bar{g}_H - \phi_{H\theta}}{\phi_M}$$

The expected payoff of using such a strategy against Γ_L is given by:

$$\phi_M(1 - f_{LU} - f_{L1} + f_{LU} \int_\theta^{\bar{\gamma}} \frac{x-\theta}{\bar{\gamma}-\theta}\, g_M(x)\, dx) + \phi_H(1 - f_{L1}) + \phi_{H\theta}(1 - \frac{f_{L1}}{2})$$

By symmetry with the previous calculations this simplifies to:

$$\phi_M \frac{(\bar{\gamma}-\theta)(1-f_{L1}) - \bar{\gamma} f_{LU}}{(\bar{\gamma}-\theta)} + \phi_{H\theta} \frac{(2-f_{L1})(\bar{\gamma}-\theta) - 2f_{LU}}{2(\bar{\gamma}-\theta)}$$
$$+\phi_H\left(\frac{(2-f_{L1})(\bar{\gamma}-\theta) - 2f_{LU}}{2(\bar{\gamma}-\theta)} - \frac{f_{LU}(\bar{g}_H - \bar{\gamma})}{\bar{\gamma}-\theta}\right) + \frac{\gamma_H f_{LU}}{\bar{\gamma}-\theta}$$

As in the previous calculation, $\phi_{H\theta} = 0$ for any best response. Next observe that

$$2((\bar{\gamma}-\theta)(1-f_{L1}) - \bar{\gamma} f_{LU}) = (2-f_{L1})(\bar{\gamma}-\theta) - 2f_{LU} = \frac{2(\bar{\gamma}-\theta)}{(2-\bar{\gamma})^2 - \theta^2}$$

$$= \frac{2(\bar{\gamma}-\theta)}{(2-\bar{\gamma})^2 - \theta^2}(-\bar{\gamma}^2 + (4-2\gamma_L) + \theta^2 + 4\gamma_L - 4)$$

This is larger than zero, provided $\bar{\gamma} \leq 2 - \gamma_L - \sqrt{\gamma_L^2 + \theta^2}$, which was demonstrated previously and is equivalent to If $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2}$. Hence, any random variable in class $\hat{\Gamma}_H$ is a best response, provided $\phi_H = 0$. As the strategy Γ_H satisfies these criteria, it is a best response. □

Equilibrium in Region D. **Claim:** If and only if $\sqrt{\gamma_H^2 - \gamma_L^2} \leq \theta$ and $\frac{1}{2}(1-\theta^2) \leq \gamma_L \leq \frac{2-2\theta}{2-\theta}$ (i.e. iff we are in Region D of Figure 1) is there a Nash Equilibrium in which agent L plays

$$\Gamma_L = \begin{cases} 0 & \text{with probability } \frac{\theta}{2-\bar{\gamma}} \\ U[\theta, \bar{\gamma}] & \text{with probability } \frac{\bar{\gamma}-\theta}{2-\bar{\gamma}} \\ 1 & \text{with probability } \frac{2-2\bar{\gamma}}{2-\bar{\gamma}} \end{cases}$$

and agent H plays

$$\Gamma_H = \begin{cases} 0 & \text{with probability } \frac{\theta}{2-\bar{\gamma}} - \frac{\gamma_H - \gamma_L}{\theta} \\ \theta & \text{with probability } \frac{\gamma_H - \gamma_L}{\theta} \\ U[\theta, \bar{\gamma}] & \text{with probability } \frac{\bar{\gamma}-\theta}{2-\bar{\gamma}} \\ 1 & \text{with probability } \frac{2-2\bar{\gamma}}{2-\bar{\gamma}} \end{cases}$$

where

$$\bar{\gamma} = 2 - \gamma_L - \sqrt{\gamma_L^2 + \theta^2}.$$

Proof. We first demonstrate that the strategies of both players have support inside the unit interval and satisfy the mean constraint. First, observe that $\frac{1}{2}(1-\theta^2) \leq \gamma_L \leq \frac{2-2\theta}{2-\theta} \Leftrightarrow \theta \leq \bar{\gamma} \leq 1$, that is, the second group of inequalities is necessary for either strategy to be feasible. These inequalities also imply that $f_{HU} = f_{LU} \geq 0$ and $f_{H1} = f_{L1} \geq 0$. It is also obvious that $f_{H\theta} \geq 0$. To prove that all probabilities are less than 1, we establish that $f_{H\theta} + f_{HU} + f_{H1} \leq 1$. This inequality implies that $f_{LU} + f_{L1} \leq 1$.

$$f_{H\theta} + f_{HU} + f_{H1} - 1 = \frac{\gamma_H - \sqrt{\gamma_L^2 + \theta^2}}{\theta} \leq 0 \Leftrightarrow \theta \geq \sqrt{\gamma_H^2 - \gamma_L^2}$$

Thus the second inequality is necessary for H's strategy to be a random variable. The inequalities defined in the proposition are therefore necessary and sufficient for Γ_H, Γ_L to be well defined. Finally, we demonstrate that both random variables have the correct expected values.

$$E[\Gamma_L] = f_{LU}\frac{\theta + \bar{\gamma}}{2} + f_{L1} = \frac{\bar{\gamma} - \theta}{2 - \bar{\gamma}}(\frac{\theta + \bar{\gamma}}{2}) + \frac{2 - 2\bar{\gamma}}{2 - \bar{\gamma}} = \gamma_L$$

$$E[\Gamma_H] = f_{H\theta}\theta + E[\Gamma_L] = \gamma_H$$

Next, we establish that the proposed strategies are mutual best responses. According to Lemma 1, any admissible best response to Γ_H, denoted $\hat{\Gamma}_L$ must have the following structure:

$$\hat{\Gamma}_L = \begin{cases} 0 & \text{with probability} \quad 1 - \phi_M - \phi_H - \phi_{H\theta} \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \\ 1 & \text{with probability} \quad \phi_{H\theta} \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M, and if a mass point exists at 1, it is not part of G_H (no mass point exists at endpoints of G_H). Furthermore, no mass point at θ exists in G_M. Such mass point leads to ties with positive probability; using a mass point of $\theta + \epsilon$ leads to all ties at θ breaking in favor of player L. In order for this strategy to be admissible, it must be that

$$\phi_M \bar{g}_M + \phi_H \bar{g}_H + \phi_{H\theta} = \gamma_L \iff \bar{g}_M = \frac{\gamma_L - \phi_H \bar{g}_H - \phi_{H\theta}}{\phi_M}$$

The expected payoff of using such a strategy against Γ_H is given by:

$$\begin{aligned} & \phi_M(1 - f_{HU} - f_{H1} + f_{HU}\int_\theta^{\bar{\gamma}} \frac{x - \theta}{\bar{\gamma} - \theta}\, g_M(x)\, dx) + \phi_H(1 - f_{H1}) + \phi_{H\theta}(1 - \frac{f_{H1}}{2}) \\ = \; & \phi_M(1 - f_{HU} - f_{H1} + f_{HU}\frac{\bar{g}_M - \theta}{\bar{\gamma} - \theta}) + \phi_H(1 - f_{H1}) + \phi_{H\theta}(1 - \frac{f_{H1}}{2}) \\ = \; & \phi_M(1 - f_{HU} - f_{H1} + f_{HU}\frac{\frac{\gamma_L - \phi_H\bar{g}_H - \phi_{H\theta}}{\phi_M} - \theta}{\bar{\gamma} - \theta}) + \phi_H(1 - f_{H1}) + \phi_{H\theta}(1 - \frac{f_{H1}}{2}) \\ = \; & \phi_M\frac{2(\bar{\gamma} - \theta)(1 - f_{H1}) - 2\bar{\gamma}f_{HU}}{2(\bar{\gamma} - \theta)} + \phi_H\frac{2(\bar{\gamma} - \theta)(1 - f_{H1}) - 2\bar{\gamma}f_{HU} - 2f_{HU}(\bar{g}_H - \bar{\gamma})}{2(\bar{\gamma} - \theta)} \\ & + \phi_{H\theta}\frac{2(\bar{\gamma} - \theta)(1 - f_{H1}) - 2\bar{\gamma}f_{HU}}{2(\bar{\gamma} - \theta)} + \frac{2\gamma_L f_{HU}}{2(\bar{\gamma} - \theta)}. \end{aligned}$$

Observe first that the coefficient on ϕ_H is less than the coefficient on either ϕ_M or ϕ_H, hence, for any best response, $\phi_H = 0$. Furthermore,

$$2(\bar{\gamma} - \theta)(1 - f_{H1}) - 2\bar{\gamma}f_{HU} = 2(\bar{\gamma} - \theta)(1 - \frac{2 - 2\bar{\gamma}}{2 - \bar{\gamma}}) - 2\bar{\gamma}\frac{\bar{\gamma} - \theta}{2 - \bar{\gamma}} = 0.$$

Thus, the payoff of any admissible random variable does not depends on $\phi_{H\theta}, \phi_M, G_M$. Thus any random variable with the structure $\hat{\Gamma}_L$ and $\phi_H = 0$ is a best response to Γ_H. In particular, Γ_L is a best response to Γ_H.

Next, we show that Γ_H is a best response to Γ_L. According to Lemma 1, any admissible best response to Γ_L, denoted $\hat{\Gamma}_H$ must have the following structure:

$$\hat{\Gamma}_H = \begin{cases} 0 & \text{with probability} \quad 1-\phi_M-\phi_H-\phi_{H\theta} \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \\ 1 & \text{with probability} \quad \phi_{H\theta} \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M, and if a mass point exists at 1, it is not part of G_H (no mass point exists in G_H). In order for this strategy to be admissible, it must be that

$$\phi_M\bar{g}_M + \phi_H\bar{g}_H + \phi_{H\theta} = \gamma_H \iff \bar{g}_M = \frac{\gamma_H - \phi_H\bar{g}_H - \phi_{H\theta}}{\phi_M}.$$

The expected payoff of using such a strategy against Γ_L is given by:

$$\phi_M(1 - f_{LU} - f_{L1} + f_{LU}\int_\theta^{\bar{\gamma}} \frac{x-\theta}{\bar{\gamma}-\theta}\, g_M(x)\, dx) + \phi_H(1-f_{L1}) + \phi_{H\theta}(1-\frac{f_{L1}}{2})$$

Because of the equalities $f_{LU} = f_{HU}, f_{L1} = f_{H1}$ this equation becomes

$$\begin{aligned}
&\phi_M(1 - f_{HU} - f_{H1} + f_{HU}\frac{\bar{g}_M-\theta}{\bar{\gamma}-\theta}) + \phi_H(1-f_{H1}) + \phi_{H\theta}(1-\frac{f_{H1}}{2}) \\
= \ &\phi_M(1 - f_{HU} - f_{H1} + f_{HU}\frac{\frac{\gamma_L-\phi_H\bar{g}_H-\phi_{H\theta}}{\phi_M}-\theta}{\bar{\gamma}-\theta}) + \phi_H(1-f_{H1}) + \phi_{H\theta}(1-\frac{f_{H1}}{2}) \\
= \ &\phi_M\frac{2(\bar{\gamma}-\theta)(1-f_{H1}) - 2\bar{\gamma}f_{HU}}{2(\bar{\gamma}-\theta)} + \phi_H\frac{2(\bar{\gamma}-\theta)(1-f_{H1}) - 2\bar{\gamma}f_{HU} - 2f_{HU}(\bar{g}_H-\bar{\gamma})}{2(\bar{\gamma}-\theta)} \\
&+ \phi_{H\theta}\frac{2(\bar{\gamma}-\theta)(1-f_{H1}) - 2\bar{\gamma}f_{HU}}{2(\bar{\gamma}-\theta)} + \frac{2\gamma_H f_{HU}}{2(\bar{\gamma}-\theta)}.
\end{aligned}$$

Thus, from the previous equation, it follows that in any best response $\phi_H = 0$. Furthermore, the payoff of any admissible strategy is independent of $\phi_M, \phi_{H\theta}, G_M$, thus any admissible strategy with $\phi_H = 0$ is a best response. As Γ_H satisfies these criteria it is a best response. □

Equilibrium in Region E. **Claim:** If and only if $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2}$ and $\gamma_H \leq \frac{1}{2}(1+\theta^2)$ (i.e. iff we are in Region E of Figure 1) is there a Nash Equilibrium in which agent L plays

$$\Gamma_L = \begin{cases} 0 & \text{with probability} \quad 1 - \frac{2\gamma_L}{\bar{\gamma}+\theta} = 1 - f_L \\ U[\theta, \bar{\gamma}] & \text{with probability} \quad \frac{2\gamma_L}{\bar{\gamma}+\theta} = f_L \end{cases}$$

and agent H plays

$$\Gamma_H = \begin{cases} \theta & \text{with probability} \quad 1 - \frac{2(\gamma_H-\theta)}{\bar{\gamma}-\theta} = 1 - f_H \\ U[\theta, \bar{\gamma}] & \text{with probability} \quad \frac{2(\gamma_H-\theta)}{\bar{\gamma}-\theta} = f_H \end{cases}$$

where

$$\bar{\gamma} = \gamma_H + \sqrt{\gamma_H^2 - \theta^2}.$$

Proof. First we establish that the proposed strategies are admissible.

By assumption $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2} \leq \gamma_H$, therefore $\bar{\gamma}$ is a real number. These same conditions imply that $\bar{\gamma} \geq \theta$. Furthermore, $\theta \leq \sqrt{\gamma_H^2 - \gamma_L^2} \Leftrightarrow f_H \geq f_L$ (this condition is not necessary for the random variables to be well defined but it will be both necessary and sufficient later in the proof). As $\theta \leq \gamma_H, f_H \geq 0$, f_L is obviously positive. Finally, substituting and simplifying gives $f_H - 1 = -\frac{\gamma_H - \sqrt{\gamma_H^2 - \theta^2}}{\theta} < 0$; $f_L \leq f_H \leq 1$. If $\gamma_H \leq \frac{1}{2}(1+\theta^2)$ then $\bar{\gamma} \leq 1$. Finally, we check that both random variables have the required expectations.

$$E[\Gamma_L] = \frac{2\gamma_L}{\bar{\gamma}+\theta}\frac{\bar{\gamma}+\theta}{2} = \gamma_L$$

$$E[\Gamma_H] = (1 - \frac{2(\gamma_H - \theta)}{\bar{\gamma}-\theta})\theta + \frac{2(\gamma_H - \theta)}{\bar{\gamma}-\theta}\frac{\bar{\gamma}+\theta}{2} = \gamma_H$$

We now show that the proposed strategies are mutual best responses. According to Lemma 1, any best reply to Γ_H has the following structure:

$$\hat{\Gamma}_L = \begin{cases} 0 & \text{with probability} \quad 1 - \phi_M - \phi_H \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M (no mass point at left endpoint exists in G_H). Furthermore, no mass point at θ exists in G_M. Such mass point leads to ties with positive probability; using a mass point of $\theta + \epsilon$ leads to all ties at θ breaking in favor of player L.

In order for $\hat{\Gamma}_L$ to be admissible, it must be that $\phi_M \bar{g}_M + \phi_H \bar{g}_H = \gamma_L$, which implies

$$\bar{g}_M = \frac{\gamma_L - \phi_H \bar{g}_H}{\phi_M}$$

Consider the expected payoff of playing $\hat{\Gamma}_L$ against Γ_H:

$$\begin{aligned} & \phi_M(1 - f_H + f_H \int_\theta^{\bar{\gamma}} \frac{x-\theta}{\bar{\gamma}-\theta} g_M(x)\, dx) + \phi_H \\ = \ & \phi_M(1 - f_H + f_H \frac{\bar{g}_M - \theta}{\bar{\gamma}-\theta}) + \phi_H \\ = \ & \phi_M(1 - f_H + f_H \frac{\frac{\gamma_L - \phi_H \bar{g}_H}{\phi_M} - \theta}{\bar{\gamma}-\theta}) + \phi_H \\ = \ & \phi_H \frac{\bar{\gamma} - \theta - f_H \bar{g}_H}{\bar{\gamma}-\theta} + \phi_M \frac{\bar{\gamma} - \theta - f_H \bar{\gamma}}{\bar{\gamma}-\theta} + \frac{\gamma_L f_H}{\bar{\gamma}-\theta}. \end{aligned}$$

Observe that the coefficient on ϕ_M is equal to 0:

$$\begin{aligned} \bar{\gamma} - \theta - f_H \bar{\gamma} \ &= \ \bar{\gamma} - \theta - \frac{2(\gamma_H - \theta)}{\bar{\gamma}-\theta}\bar{\gamma} \\ = \ \frac{\gamma_H^2 - \theta^2 - (\bar{\gamma} - \gamma_H)^2}{\bar{\gamma}-\theta} \ &= \ \frac{\gamma_H^2 - \theta^2 - (\sqrt{\gamma_H^2 - \theta^2})^2}{\bar{\gamma}-\theta} \ = \ 0. \end{aligned}$$

Thus, the payoff of any admissible best response $\hat{\Gamma}_L$ does not depend on the value of ϕ_M or on the random variable G_M. Moreover, as no mass point exists in G_H at the left endpoint, $\bar{g}_H > \bar{\gamma}$. Therefore $\bar{\gamma} - \theta - f_H \bar{\gamma} = 0 \rightarrow \bar{\gamma} - \theta - f_H \bar{g}_H < 0$. Hence, in any best response, it must be that $\phi_H = 0$. Therefore, a random variable is a best response to Γ_H if and only if it has the structure of Γ_L, with $\phi_H = 0$. As the strategy Γ_L proposed in the proposition, satisfies these criteria, Γ_L is a best reply to Γ_H.

As we have already shown, any best reply to Γ_L has the following structure:

$$\hat{\Gamma}_H = \begin{cases} 0 & \text{with probability} \quad 1-\phi_L-\phi_M-\phi_H \\ \theta & \text{with probability} \quad \phi_L \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M (no mass point at left endpoint exists in G_H). Also, G_M has no mass point at θ.

In order for $\hat{\Gamma}_H$ to be admissible, it must be that $\phi_L\theta + \phi_M\bar{g}_M + \phi_H\bar{g}_H = \gamma_H$, which implies

$$\bar{g}_M = \frac{\gamma_H - \phi_H\bar{g}_H - \phi_L\theta}{\phi_M}.$$

Consider the expected payoff of playing $\hat{\Gamma}_H$ against Γ_L:

$$\begin{aligned} & \phi_L(1-f_L) + \phi_M(1 - f_L + f_L\int_\theta^{\bar{\gamma}} \frac{x-\theta}{\bar{\gamma}-\theta}\, g_M(x)\, dx) + \phi_H \\ = \; & \phi_L(1-f_L) + \phi_M(1 - f_L + f_L\frac{\bar{g}_M - \theta}{\bar{\gamma}-\theta}) + \phi_H \\ = \; & \phi_L(1-f_L) + \phi_M(1 - f_L + f_L\frac{\frac{\gamma_L - \phi_H\bar{g}_H - \phi_L\theta}{\phi_M} - \theta}{\bar{\gamma}-\theta}) + \phi_H \\ = \; & \phi_H\frac{\bar{\gamma}-\theta-f_L\bar{g}_H}{\bar{\gamma}-\theta} + (\phi_L+\phi_M)\frac{\bar{\gamma}-\theta-f_L\bar{\gamma}}{\bar{\gamma}-\theta} + \frac{\gamma_H f_L}{\bar{\gamma}-\theta}. \end{aligned}$$

Observe that the coefficient on $(\phi_L + \phi_M)$ is positive, if and only if $f_H \geq f_L \Leftrightarrow \theta \leq \sqrt{\gamma_H^2 - \gamma_L^2}$. Moreover, because $\bar{g}_H > \bar{\gamma}$ the coefficient on ϕ_H is strictly less than the one on $(\phi_L + \phi_M)$. Therefore, in the best response, $\phi_H = 0$ and $\phi_L + \phi_M = 1$. As the strategy in the proposition satisfies these criteria, it is a best response. □

Equilibrium in Region F. **Claim:** If and only if $\sqrt{\gamma_H^2 - \gamma_L^2} \leq \theta$ and $\gamma_L \leq \frac{1}{2}(1-\theta^2)$ (i.e. iff we are in Region F of Figure 1) is there a Nash Equilibrium in which agent L plays

$$\Gamma_L = \begin{cases} 0 & \text{with probability} \quad \frac{\bar{\gamma}-2\gamma_L}{\theta} \\ U[\theta, \bar{\gamma}] & \text{with probability} \quad 1-\frac{\bar{\gamma}-2\gamma_L}{\theta} \end{cases}$$

and agent H plays

$$\Gamma_H = \begin{cases} 0 & \text{with probability} \quad \frac{\bar{\gamma}-\gamma_H-\gamma_L}{\theta} \\ \theta & \text{with probability} \quad \frac{\gamma_H-\gamma_L}{\theta} \\ U[\theta, \bar{\gamma}] & \text{with probability} \quad 1-\frac{\bar{\gamma}-2\gamma_L}{\theta} \end{cases}$$

where

$$\bar{\gamma} = \gamma_L + \sqrt{\gamma_L^2 + \theta^2}.$$

Proof. First, we establish that the proposed strategies are random variables with support in the unit interval that satisfy the mean constraint. Obviously, $\bar{\gamma} \geq \theta$, $f_{H\theta} \geq 0$. A simple calculation shows that $\sqrt{\gamma_L^2 + \theta^2} - \gamma_L \leq \theta$, and therefore, $f_{HU} = f_L \geq 0$. Furthermore, $f_{H\theta} + f_{HU} - 1 = \frac{\gamma_H - \sqrt{\gamma_L^2+\theta^2}}{\theta}$. This difference is negative if and only if $\theta \geq \sqrt{\gamma_H^2 - \gamma_L^2}$. Thus, all probabilities are in the unit interval if and only if the first required inequality is satisfied. Observe that

$\gamma_L \leq \frac{1}{2}(1-\theta^2) \Leftrightarrow \bar{\gamma} \leq 1$, thus the second required inequality ensures that $\bar{\gamma}$ is less than 1. Next we demonstrate that the expected values are correct:

$$E[\Gamma_L] = f_L(\frac{\theta+\bar{\gamma}}{2}) = (1 - \frac{\sqrt{\gamma_L^2+\theta^2}-\gamma_L}{\theta})(\frac{\theta+\gamma_L+\sqrt{\gamma_L^2+\theta^2}}{2}) = \gamma_L$$

$$E[\Gamma_H] = f_{H\theta}\theta + f_{HU}(\frac{\theta+\bar{\gamma}}{2}) = \theta\frac{\gamma_H-\gamma_L}{\theta} + \gamma_L = \gamma_H$$

Next, we demonstrate that the strategies are mutual best replies. According to Lemma 1, any admissible best reply to Γ_H, denoted $\hat{\Gamma}_L$ must have the following structure:

$$\hat{\Gamma}_L = \begin{cases} 0 & \text{with probability} \quad 1-\phi_M-\phi_H \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M (no mass point at left endpoint exists in G_H). Furthermore, no mass point at θ exists in G_M. Such mass point leads to ties with positive probability; using a mass point of $\theta+\epsilon$ leads to all ties at θ breaking in favor of player L. In order for this strategy to be admissible, it must be that

$$\phi_M\bar{g}_M + \phi_H\bar{g}_H = \gamma_L \iff \bar{g}_M = \frac{\gamma_L - \phi_H\bar{g}_H}{\phi_M}$$

The expected payoff of using such a strategy against Γ_H is given by:

$$\begin{aligned} & \phi_M(1-f_{HU}+f_{HU}\int_\theta^{\bar{\gamma}}\frac{x-\theta}{\bar{\gamma}-\theta}\,g_M(x)\,dx) + \phi_H \\ = \;& \phi_M(1-f_{HU}+f_{HU}\frac{\bar{g}_M-\theta}{\bar{\gamma}-\theta}) + \phi_H \\ = \;& \phi_M(1-f_{HU}+f_{HU}\frac{\frac{\gamma_L-\phi_H\bar{g}_H}{\phi_M}-\theta}{\bar{\gamma}-\theta}) + \phi_H \\ = \;& \phi_H\frac{\bar{\gamma}-\theta-f_{HU}\bar{g}_H}{\bar{\gamma}-\theta} + \phi_M\frac{\bar{\gamma}-\theta-f_{HU}\bar{\gamma}}{\bar{\gamma}-\theta} + \frac{\gamma_L f_{HU}}{\bar{\gamma}-\theta}. \end{aligned}$$

Observe that the coefficient on ϕ_H is always less than the coefficient on ϕ_M, hence, for a best response it must be that $\phi_H = 0$. Furthermore, observe that the coefficient on $\phi_M = 0$. To see this, note

$$\bar{\gamma}(1-f_{HU}) = (\gamma_L+\sqrt{\gamma_L^2+\theta^2})\frac{(-\gamma_L+\sqrt{\gamma_L^2+\theta^2}}{\theta} = \frac{\gamma_L^2-\gamma_L^2+\theta^2}{\theta} = \theta.$$

Thus, the payoff of any strategy of the type $\hat{\Gamma}_L$ is independent of ϕ_M and G_M. Therefore, any admissible random variable of structure $\hat{\Gamma}_L$is a best response, provided $\phi_H = 0$. As the strategy Γ_L is consistent with these requirements, it is a best response.

We now consider the best response to Γ_L. According to Lemma 1, any best reply to Γ_L has the following structure:

$$\hat{\Gamma}_H = \begin{cases} 0 & \text{with probability} \quad 1-\phi_L-\phi_M-\phi_H \\ \theta & \text{with probability} \quad \phi_L \\ G_M & \text{with probability} \quad \phi_M \\ G_H & \text{with probability} \quad \phi_H \end{cases}$$

Where G_M is a random variable with support contained in $[\theta, \bar{\gamma}]$, $E[G_M] = \bar{g}_M$, and density $g_M(x)$, while G_H is a random variable with support contained in $[\bar{\gamma}, 1]$ and $E[G_H] = \bar{g}_H$ and density $g_H(x)$. If a mass point exists at $\bar{\gamma}$ then it is part of G_M (no mass point at left endpoint exists in G_H). Also, G_M has no mass point at θ.

In order for $\hat{\Gamma}_H$ to be admissible, it must be that $\phi_L \theta + \phi_M \bar{g}_M + \phi_H \bar{g}_H = \gamma_H$, which implies

$$\bar{g}_M = \frac{\gamma_H - \phi_H \bar{g}_H - \phi_L \theta}{\phi_M}.$$

Consider the expected payoff of playing $\hat{\Gamma}_H$ against Γ_L:

$$\begin{aligned}
& \phi_L(1 - f_L) + \phi_M(1 - f_L + f_L \int_\theta^{\bar{\gamma}} \frac{x - \theta}{\bar{\gamma} - \theta} g_M(x)\, dx) + \phi_H \\
= \; & \phi_L(1 - f_L) + \phi_M(1 - f_L + f_L \frac{\bar{g}_M - \theta}{\bar{\gamma} - \theta}) + \phi_H \\
= \; & \phi_L(1 - f_L) + \phi_M(1 - f_L + f_L \frac{\frac{\gamma_H - \phi_H \bar{g}_H - \phi_L \theta}{\phi_M} - \theta}{\bar{\gamma} - \theta}) + \phi_H \\
= \; & \phi_H \frac{\bar{\gamma} - \theta - f_L \bar{g}_H}{\bar{\gamma} - \theta} + (\phi_L + \phi_M) \frac{\bar{\gamma} - \theta - f_L \bar{\gamma}}{\bar{\gamma} - \theta} + \frac{\gamma_H f_L}{\bar{\gamma} - \theta}.
\end{aligned}$$

Observe that the coefficient on $(\phi_L + \phi_M) = 0$ because, $f_L = f_{HU}$ and, as demonstrated previously $\bar{\gamma}(1 - f_{HU}) - \theta = 0$. Thus, the payoff to any admissible $\hat{\Gamma}_L$ is independent of ϕ_L, ϕ_M, G_M. However, because $\bar{g}_H > \bar{\gamma}$ the coefficient on ϕ_H is negative. Thus, in a best response, it must be that $\phi_H = 0$. Hence, any random variable of the structure $\hat{\Gamma}_H$ is a best response, provided $\phi_H = 0$. As the strategy in the proposition satisfies these criteria, it is a best response. □

A.2. **Establishing uniqueness of equilibria.** We continue to narrow down the characteristics of potential equilibrium strategies which we began in the proof to Lemma 4.

(viii) If $\rho_j^\theta > 0$, then $\rho_i^\theta = 0$.

Proof: Follows from condition (ii) and the structure of equilibrium established in the restatement of Γ_i above.

(ix) If either $\rho_i^1 > 0$ or $\rho_j^1 > 0$ or both, then $r < 1$ when $\rho_i^\Phi, \rho_j^\Phi > 0$.

Proof: Alternatively, suppose that $\rho_j^1 > 0$, and $\rho_i^\Phi, \rho_j^\Phi > 0$ with $I = [\theta, 1]$ (i.e. $r = 1$). For each $x \in I$, (6) requires that $L_i(x) = L_i(1)$ if $\rho_i^1 > 0$ and $L_i(x) \geq L_i(1)$ if $\rho_i^1 = 0$. We show that these requirements fail to hold for x close enough to 1. Consider the requirement that for all $x \in I$, $L_i(x) \geq L_i(1)$, and substitute in for the expressions:

$$\rho_j^0 + \rho_j^\theta + \rho_j^\Phi F_j(x) - \lambda_i(x - \gamma_i) \geq \rho_j^0 + \rho_j^\theta + \rho_j^\Phi + \frac{\rho_j^1}{2} - \lambda_i(1 - \gamma_i).$$

Simplifying this expression gives the inequality

$$\lambda_i(1 - x) \geq \rho_j^\Phi(1 - F_j(x)) + \frac{\rho_j^1}{2}.$$

Further, considering the case of $x = r$ gives

$$\lambda_i(1 - r) \geq \frac{\rho_j^1}{2}.$$

For r approaching 1, this inequality clearly fails, contradicting the initial assumption that $r = 1$.

(x) If $\rho_j^1 > 0$, then $\rho_i^1 > 0$.

Proof: If i has a mass point on 1, then there exists an interval $(1-\epsilon, 1)$ outside of the support of Γ_j (this follows from equilibrium condition (viii)). If Γ_j does not include a mass point on 1, then $L_i(1-\epsilon) > L_i(1)$ because the probability of i winning is constant outside of G_j.

(xi) No equilibrium exists in which $\rho_i^0 = \rho_j^0 = 0$.

Proof: Suppose that neither player's equilibrium strategy has a mass point on 0. It cannot be the case that both Γ_i and Γ_j have mass points on θ since equilibrium property (ii) established that there could be no common mass points in $[\theta, 1)$. Suppose therefore that Γ_i does not have a mass point on θ. The probability that player j wins with realization θ therefore equals 0, i.e. $W_j(\theta) = \rho_i^0 + \rho_i^\theta = 0$. However, $W_j(0) = 0$. Thus, $L_j(\theta) < L_j(0)$, and 0 is outside of the support of Γ_j, which contradicts (6).

We proceed by considering two possible cases. First, we consider the possibility that $\rho_i^\Phi = \rho_j^\Phi = 0$. Second, we consider the possibility that $\rho_i^\Phi, \rho_j^\Phi > 0$. Equilibrium condition (vii) above rules out the possibility that $\rho_i^\Phi > 0$ and $\rho_j^\Phi = 0$.

A.2.1. *Possibility 1:* $\mathbf{æ_i^\Phi = æ_j^\Phi = 0}$. In this case, the only possible elements of the support of each random variable are $\{0, \theta, 1\}$. However, equilibrium condition vi. shows that at most one player's strategy can have a mass point on θ. Without loss of generality, let j be the player that potentially puts mass on θ. Therefore, Γ_i is confined to support $\{0, 1\}$. Hence,

$$\Gamma_i = \begin{cases} 0 & \text{with probability} \quad \rho_i^0 \\ 1 & \text{with probability} \quad \rho_i^1 \end{cases}$$

and[10]

$$\Gamma_j = \begin{cases} 0 & \text{with probability} \quad \rho_j^0 \\ \theta & \text{with probability} \quad \rho_j^\theta \\ 1 & \text{with probability} \quad \rho_j^1. \end{cases}$$

Given that the law of iterated expectations requires that the expected realization of Γ_i and Γ_j equal γ_i and γ_j, respectively, it follows that

$$\rho_i^1 = \gamma_i \quad \text{and} \quad \theta\rho_j^\theta + \rho_j^1 = \gamma_j.$$

Given that $\rho_i^1 = \gamma_i > 0$, equilibrium condition (vi) guarantees that $\rho_i^1 > 0$. Therefore, three possibilities exist: (1) $\rho_j^0 > 0$ and $\rho_j^\theta = 0$, (2) $\rho_j^0 = 0$ and $\rho_j^\theta > 0$, and (3) $\rho_j^0 > 0$ and $\rho_j^\theta > 0$. In each of these cases, $\rho_j^1 > 0$, $\rho_i^0 = 1 - \gamma_i$ and $\rho_i^1 = \gamma_i$.

Equation (6) requires that $L_i(x) = v_i$ for each $x \in G_i = \{0, 1\}$, and that$L_j(x) = v_j$ for each $x \in G_j = \{0, \theta, 1\}$. Applying (6) to Γ_i gives the following conditions:

$$L_i(0) = v_i \iff \lambda_i\gamma_i = v_i \tag{9}$$

$$L_i(1) = v_i \iff 1 - \frac{\rho_j^1}{2} - \lambda_i(1-\gamma_i) = v_i$$

Applying (6) to Γ_j gives the following conditions:

$$L_j(0) \geq v_j \iff \lambda_j\gamma_j \geq v_j \quad \text{[with equality if } \rho_j^0 > 0]$$

$$L_j(\theta) \geq v_j \iff 1 - \gamma_i - \lambda_j(\theta - \gamma_j) \geq v_j \quad \text{[with equality if } \rho_j^\theta > 0]$$

[10] Because all realizations inside $(\theta, 1)$ generate the same win-probability for j, no best response Γ_j can have positive mass inside this interval.

$$L_j(1) = v_j \iff 1 - \frac{\gamma_i}{2} - \lambda_j(1-\gamma_j) = v_j$$

Case I: $z_j > 0$ and $m_j = 0$ and $n_j > 0$
The mean constraint therefore implies that $n_j = \gamma_j$ and the constraint on probabilities implies that $z_j = 1 - \gamma_j$. Thus, the fully-revealing equilibrium is the only equilibrium with this structure. In the analysis of Region A in the appendix we show that the combination of fully-revealing signals is an equilibrium if and only if $\gamma_L \geq \frac{2-2\theta}{2-\theta}$. This analysis shows that no other equilibrium with this structure exists.

Case II: $z_j = 0$ and $m_j > 0$ and $n_j > 0$
Here the equilibrium must satisfy the following system of equations:

$$\lambda_i\gamma_i = v_i \tag{10}$$

$$1 - \frac{n_j}{2} - \lambda_i(1-\gamma_i) = v_i$$

$$1 - \gamma_i - \lambda_j(\theta - \gamma_j) = v_j$$

$$1 - \frac{\gamma_i}{2} - \lambda_j(1-\gamma_j) = v_j$$

$$m_j + n_j = 1$$

$$\theta m_j + n_j = \gamma_i$$

$$\lambda_j\gamma_j \leq v_j$$

$$m_j - \lambda_i(\theta - \gamma_i) \leq v_i$$

The solution of the system of equation has $m_j = \frac{1-\gamma_j}{1-\theta}$. In order for the inequalities to be satisfied, it must be that $\gamma_i \leq \frac{2-2\theta}{2-\theta}$ and $\gamma_j \geq \frac{2-2\theta+\theta^2}{2-\theta}$, and hence, $\gamma_i < \gamma_j$. Therefore in this case, L must be labelled i and H must be labelled j. Thus, the quasi-revealing equilibrium identified in the appendix is the only equilibrium with this structure, and exists if and only if these two inequalities are satisfied.

Case III: $z_j > 0$ and $m_j > 0$ and $n_j > 0$
In this case the equilibrium would need to satisfy the following three conditions

$$\lambda_j\gamma_j = v_j$$

$$1 - \gamma_i - \lambda_j(\theta - \gamma_j) = v_j$$

$$1 - \frac{\gamma_i}{2} - \lambda_j(1-\gamma_j) = v_j$$

This system is inconsistent, except for the knife-edge case in which $\gamma_i = \frac{2-2\theta}{2-\theta}$.

Thus, the only possible equilibria in which Φ_i and Φ_j are degenerate are the equilibria identified in the appendix.

A.2.2. Φ_k *is non-degenerate.* Now consider the case in which $\rho_i^\Phi, \rho_j^\Phi > 0$. Given $\rho_i^\Phi > 0$, (6) implies that

$$\rho_j^0 + \rho_j^\theta + \rho_j^\Phi F_j(x) - \lambda_i(x - \gamma_i) = v_i \text{ for all } x \in [\theta, r] \tag{11}$$

Rearranging the equality gives

$$F_j(x) = \frac{v_i - \rho_j^0 - \rho_j^\theta - \lambda_i\gamma_i}{\rho_j^\Phi} + \frac{\lambda_i}{\rho_j^\Phi}x$$

Thus, if any mass exists inside $(\theta, 1)$ then it must be uniformly distributed with a possible mass point on θ. However, because it can not be that both i, j have a mass point on θ (No common mass points in $[\theta, 1)$) then at most one player has a mass point on θ. Let this be player j.

$$z_i > 0 \Rightarrow \lambda_i\gamma_i = v_i$$
$$n_i > 0 \Rightarrow 1 - \frac{n_j}{2} - \lambda_i(1 - \gamma_i) = v_i$$

$$F_i(\theta) = 0 \text{ and } F_j(\theta) \geq 0 \Leftrightarrow$$
$$\frac{v_j - \lambda_j\gamma_j}{m_i} + \frac{\lambda_j}{m_i}\theta = 0 \text{ and } \frac{v_i - \lambda_i\gamma_i}{m_j} + \frac{\lambda_i}{m_j}\theta \geq 0$$

If $r > \theta$, then neither player can have a mass point at r. Therefore,

$$F_i(r) = F_j(r) = 1 \Leftrightarrow$$
$$\frac{v_j - \lambda_j\gamma_j}{m_i} + \frac{\lambda_j}{m_i}r = 1 \text{ and } \frac{v_i - \lambda_i\gamma_i}{m_j} + \frac{\lambda_i}{m_j}r = 1$$

In addition each strategy must satisfy the appropriate mean constraint. Given the above conditions,

$$E[\Phi_i] = \frac{\lambda_j}{m_i}(\frac{r^2 - \theta^2}{2}) \text{ and } E[\Phi_j] = (\frac{v_i - \lambda_i\gamma_i}{m_j} + \frac{\lambda_i}{m_j}\theta)\theta + \frac{\lambda_i}{m_j}(\frac{r^2 - \theta^2}{2})$$

Therefore the mean constraints are:

$$m_iE[\Phi_i] + n_i = \gamma_i \Leftrightarrow \lambda_j(\frac{r^2 - \theta^2}{2}) + n_i = \gamma_i$$
$$m_jE[\Phi_j] + n_j = \gamma_j \Leftrightarrow (\frac{v_i - \lambda_i\gamma_i}{m_j} + \frac{\lambda_i}{m_j}\theta)\theta + \lambda_i(\frac{r^2 - \theta^2}{2}) + n_j = \gamma_j$$

Thus every equilibrium must satisfy the following conditions (SC) (collected from above):

$$\frac{v_j - \lambda_j\gamma_j}{m_i} + \frac{\lambda_j}{m_i}\theta = 0 \text{ and } \frac{v_i - \lambda_i\gamma_i}{m_j} + \frac{\lambda_i}{m_j}\theta \geq 0$$
$$\frac{v_j - \lambda_j\gamma_j}{m_i} + \frac{\lambda_j}{m_i}r = 1 \text{ and } \frac{v_i - \lambda_i\gamma_i}{m_j} + \frac{\lambda_i}{m_j}r = 1$$
$$\lambda_j(\frac{r^2 - \theta^2}{2}) + n_i = \gamma_i$$
$$(\frac{v_i - \lambda_i\gamma_i}{m_j} + \frac{\lambda_i}{m_j}\theta)\theta + \lambda_i(\frac{r^2 - \theta^2}{2}) + n_j = \gamma_j$$
$$z_i + m_i + n_i = 1$$
$$z_j + m_j + n_j = 1$$

Given a particular equilibrium structure, additional conditions also arise from the remaining indifference conditions.

Case I: $z_i > 0$, $z_j > 0$, $n_i = n_j = 0$.
In this case, both equilibrium random variables have mass points on 0, but no mass points on 1. In addition to conditions (SC), the indifference conditions in (6,11) require that

$$\lambda_i \gamma_i = v_i \text{ and } \lambda_j \gamma_j = v_j$$

$$1 - \lambda_i(1-\gamma_i) \leq v_i \text{ and } 1 - \lambda_j(1-\gamma_j) \leq v_j$$

Solving the equations gives two solutions. In one of these solutions, the value of r is negative, ruling it out. Furthermore, in order for $F_j(\theta) \geq 0$ it must be that $\gamma_j \geq \gamma_i$; thus it must be that player H is labelled j, and L is labelled i. With this identification, the equilibrium is identical to the one presented in the analysis of Region F in the appendix. This solutions has admissible probabilities and $r \in (\theta, 1)$ if and only if $\sqrt{\gamma_H^2 - \gamma_L^2} \leq \theta$ and $\gamma_L \leq \frac{1}{2}(1-\theta^2)$. Under the same conditions the required inequalities $1 - \lambda_i(1-\gamma_i) \leq v_i$ and $1 - \lambda_j(1-\gamma_j) \leq v_j$ are satisfied. Thus, the equilibrium presented in the analysis of Region F is the only equilibrium for which $z_i > 0$, $z_j > 0$, $n_i = n_j = 0$, and it exists if and only if $\sqrt{\gamma_H^2 - \gamma_L^2} \leq \theta$ and $\gamma_L \leq \frac{1}{2}(1-\theta^2)$.

Case II: $z_i > 0$, $z_j > 0$, $n_i > 0$, $n_j > 0$.
In this case, both equilibrium random variables have mass points on 0 and1. In addition to conditions (SC), the indifference conditions in (6,11) require that

$$\lambda_i \gamma_i = v_i \text{ and } \lambda_j \gamma_j = v_j$$

$$1 - \lambda_i(1-\gamma_i) = v_i \text{ and } 1 - \lambda_j(1-\gamma_j) = v_j$$

Solving the equations gives two solutions. In one of these solutions, the value of r is greater than 2, ruling it out. Furthermore, in order for $F_j(\theta) \geq 0$ it must be that $\gamma_j \geq \gamma_i$; thus it must be that player H is labelled j, and L is labelled i. With this identification, the equilibrium is identical to the one presented in the analysis of Region D in the appendix. This solutions has admissible probabilities and $r \in (\theta, 1)$ if and only if $\sqrt{\gamma_H^2 - \gamma_L^2} \leq \theta$ and $\frac{1}{2}(1-\theta^2) \leq \gamma_L \leq \frac{2-2\theta}{2-\theta}$. Thus, the equilibrium presented in the analysis of Region D is the only equilibrium for which $z_i > 0$, $z_j > 0$, $n_i > 0$ and $n_j > 0$, and it exists if and only if $\sqrt{\gamma_H^2 - \gamma_L^2} \leq \theta$ and $\frac{1}{2}(1-\theta^2) \leq \gamma_L \leq \frac{2-2\theta}{2-\theta}$.

Case III: $z_i > 0$, $z_j = 0$, $n_i = 0$, $n_j = 0$
In this case, Γ_i has a mass point on 0, but neither equilibrium random variable has a mass points on 1. This immediately implies that $m_j = 1$. In addition to conditions (SC), the indifference conditions in (6,11) require that

$$\lambda_i \gamma_i = v_i \text{ and } \lambda_j \gamma_j \leq v_j$$

$$1 - \lambda_i(1-\gamma_i) \leq v_i \text{ and } 1 - \lambda_j(1-\gamma_j) \leq v_j$$

Solving the equations gives two solutions. In either solution, $\gamma_j \geq r$ in order for r to be real. In this case, however, one solution has a value of $r \leq \theta$, ruling it out. The remaining solution satisfies $\lambda_j \gamma_j \leq v_j$ if and only if $\sqrt{\gamma_j^2 - \gamma_i^2} \geq \theta$, which implies that $\gamma_j \geq \gamma_i$. Hence, player i is labelled L and j is labelled H. With these identifications, the solution reduces to the one identified for Region E in the appendix, which satisfies constraints on probabilities and $r \in (\theta, 1)$ if and only if $\sqrt{\gamma_H^2 - \gamma_L^2} \geq \theta$ and $\gamma_H \leq \frac{1}{2}(1+\theta^2)$. The equilibrium identified for Region E exists if and only if these conditions are satisfied and is the only equilibrium for which $z_i > 0$, $z_j = 0$, $n_i = 0$, $n_j = 0$.

Case IV: $z_i > 0$, $z_j = 0$, $n_i = 1$, $n_j = 1$

In this case, player Γ_i has a mass point on 0, and both equilibrium random variables have a mass point on 1. In addition to conditions (SC), the indifference conditions in (6,11) require that

$$\lambda_i\gamma_i = v_i \text{ and } \lambda_j\gamma_j \leq v_j$$

$$1-\lambda_i(1-\gamma_i) = v_i \text{ and } 1-\lambda_j(1-\gamma_j) = v_j$$

Solving the equations gives two solutions. In either solution, $\gamma_j \geq r$ in order for r to be real. In this case, however, one solution has a value of $r \geq 1$, ruling it out. The remaining solution satisfy $\lambda_j\gamma_j \leq v_j$ if and only if $\sqrt{\gamma_j^2-\gamma_i^2} \geq \theta$, which implies that $\gamma_j \geq \gamma_i$. Hence, player i is identified with L and j is identified with H. With these identifications, the solution reduces to the one identified for Region C in the appendix, which satisfies constraints on probabilities and $r \in (\theta, 1)$ if and only if $\sqrt{\gamma_H^2-\gamma_L^2} \geq \theta$ and $\frac{1}{2}(1+\theta^2) \leq \gamma_H \leq \frac{2-2\theta+\theta^2}{2-\theta}$. The equilibrium identified for Region C exists if and only if these conditions are satisfied and is the only equilibrium for which $z_i > 0$, $z_j = 0$, $n_i = 1$, $n_j = 1$.

The above analysis shows that each equilibrium identified in the appendix is the only equilibrium that exists with the structure of the support. In addition, the regions in which each equilibrium exists are non-overlapping, and their union (along with the regions A and B) exhaust the parameter space. By ruling out equilibria with the remaining possible equilibrium structures, we establish uniqueness of our characterization. We do this below.

Case III-E $z_i = 0$, $z_j > 0$, $n_i = 0$, $n_j = 0$
Recall that Γ_i has no mass point on zero. In case III we consider Γ_i with a mass point on zero. Here we consider the case in which j has a mass point on zero, but i does not. Because $z_i = 0$ and $n_i = 0$, it must be that $m_i = 1$. In addition, because $z_j > 0$ it must be that $v_j = \lambda_j\gamma_j$. Consider then two equations that must hold simultaneously, $F_i(\theta) = 0$ and $F_i(r) = 1$.

$$\frac{v_j-\lambda_j\gamma_j}{m_i} + \frac{\lambda_j}{m_i}\theta = 0 \text{ and } \frac{v_j-\lambda_j\gamma_j}{m_i} + \frac{\lambda_j}{m_i}r = 1$$

Using $m_i = 1$ and $v_j = \lambda_j\gamma_j$ in these equations gives

$$\lambda_j\theta = 0 \text{ and } \lambda_j r = 1$$

Which gives an immediate contradiction because the Lagrange multiplier is non-zero: $\lambda_i \neq 0$.

Case IV-E $z_i = 0$, $z_j > 0$, $n_i > 0$, $n_j > 0$
Recall that Γ_i has no mass point on zero. In case IV we consider Γ_i with a mass point on zero and one. Here we consider the case in which j has a mass point on zero and one, but i has a mass point on one, but not on zero. . As above, because $z_j > 0$ it must be that $v_j = \lambda_j\gamma_j$. Consider, as above, two equations that must hold simultaneously, $F_i(\theta) = 0$ and $F_i(r) = 1$.

$$\frac{v_j-\lambda_j\gamma_j}{m_i} + \frac{\lambda_j}{m_i}\theta = 0 \text{ and } \frac{v_j-\lambda_j\gamma_j}{m_i} + \frac{\lambda_j}{m_i}r = 1$$

Using $v_j = \lambda_j\gamma_j$ in these equations gives

$$\frac{\lambda_j}{m_i}\theta = 0 \text{ and } \frac{\lambda_j}{m_i}r = 1$$

Again, an immediate contradiction arises because the Lagrange multiplier is non-zero: $\lambda_i \neq 0$.

A.3. **Proofs for Blackwell Informativeness.** Both proofs follow results that establish that for the case of binary states, if posterior belief random variables Γ and Γ' have the same mean and Γ second order stochastic dominates Γ', then Γ' is more informative, see Ganuza and Penalva (2010) Theorem 2, and the accompanying discussion.

Proof of Proposition 1: Establishing the second order stochastic relationship between player i's posterior belief random variable before and after an increase in proposal j's prior γ_j, requires an integration of the CDFs. We omit the details in the interest of brevity; these are available upon request.

Proof of Proposition 2: First consider the case where $\gamma_i \geq \theta$. Here, agent i provides an uninformative signal in the case of unlimited capacity. In the case of limited capacity, each agent chooses a signal in accordance with Lemma 3. Both the unlimited and limited capacity signals have expected value equal to the prior, γ_i, for each agent. The signal in the unlimited capacity environment returns posterior belief γ_i with probability 1, where as the signal in the limited capacity environment returns a range of posterior beliefs, but in expectation returns a posterior belief equal to γ_i. Therefore, the signal in the game with unlimited capacity represents a garbling of the signal in the game with limited capacity, and as such, the signal in the limited capacity game is more Blackwell informative.

Next consider the case where $\gamma_i < \theta$. In the case of unlimited capacity, agent i provides a signal that returns posterior belief realization 0 with probability $(\theta - \gamma_i)/\theta$ and 1 with probability γ_i/θ. Call this posterior belief distribution Γ_i; let its CDF be $F(\cdot)$. In the case of limited capacity, depending on parameter values, agent i chooses a posterior belief random variable with support confined to $\{0, \theta, 1\} \cup [\theta, \bar{\gamma}]$. Call the limited capacity posterior belief random variable Γ_i'; let its CDF be $F'(\cdot)$. Both Γ_i and Γ_i' have expected value γ_i. We will argue that $x \in [\theta, 1] \Rightarrow F(x) \geq F'(x)$ and that $x \in [0, \theta) \Rightarrow F(x) < F'(x)$, with equality only at $x = 1$. This single crossing condition of the CDFs implies that Γ_i second order stochastic dominates Γ_i'.

To argue that $x \in [\theta, 1] \Rightarrow F(x) \geq F'(x)$, observe that $F(x) = 1$ in this interval, as the support of Γ_i is the set $\{0, \theta\}$. Furthermore, in every possible equilibrium configuration under limited capacity, $\Pr \Gamma_i > \theta > 0$, and therefore, for some set of $x \in [\theta, 1]$, the inequality is strict.

Next we argue that $x \in [0, \theta) \Rightarrow F(x) < F'(x)$. Observe first that both $F(x)$ and $F'(x)$ are constant over this interval, $F(x) = F(0)$ and $F'(x) = F'(0)$, as the only realization in this interval in the support of either random variable is 0. Suppose that $F(0) \geq F'(0)$. Combined with the previous point, this supposition implies that for all $x \in [0, 1]$, $F(x) \geq F'(x)$, and for some values of x (see previous point) the inequality is strict. Thus Γ_i' first order stochastic dominates Γ_i, which immediately implies $E[\Gamma_i'] > E[\gamma_i]$, a contradiction. Therefore, we must have $F(0) < F'(0)$.