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Central Bank Screening, Moral Hazard, and the Lender of Last Resort Policy

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Abstract

This paper establishes a theoretical model to examine the LOLR policy when a central bank cannot distinguish between solvent and insolvent banks. We study two cases: a case where the central bank cannot screen insolvent banks and a case where the central bank can only imperfectly screen insolvent banks. The major results that our model produces are as follows: (1) It is impossible for any separating equilibrium to exist because insolvent banks always have an incentive to mimic solvent banks to gamble for resurrection. (2) The pooling equilibria in which, on one hand, all the banks borrow from the central bank and, on the other hand, all the banks do not borrow from the central bank, could exist given certain market beliefs off the equilibrium path. However, neither of the equilibria is socially efficient because insolvent banks will continue to hold their unproductive assets, rather than efficiently liquidating them. (3) When the central bank can screen banks imperfectly, the pooling equilibrium where all the banks borrow from the central bank becomes more likely, and the pooling equilibrium where all the banks do not borrow from the central bank becomes less likely. (4) Higher precision in central bank screening will improve social welfare not only by identifying insolvent banks and forcing them to efficiently liquidate their assets, but also by reducing moral hazard and deterring banks from choosing risky assets in the first place. (5) If a central bank can commit to a specific precision level before the banks choose their assets, rather than conducting a discretionary LOLR policy, it will choose a higher precision level to reduce moral hazard and will attain higher social welfare.

Keywords: Central Bank Screening, Moral Hazard, Lender of Last Resort

JEL Classifications: D82 G2

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1 Introduction

The 2007-2009 subprime mortgage crisis has revealed that the lender of last resort (LOLR) policy is a crucial tool for a central bank to tackle financial crises. During the crisis, three major central banks – the Federal Reserve, the European Central Bank, and the Bank of England – all employed the LOLR policy heavily to provide liquidity to banks. However, our understanding about how a central bank should conduct the LOLR policy still remains unclear.

The early discussion on the LOLR policy can be dated back to Thornton (1802) and Bagehot (1873). However, as time has gone by, our understanding about this issue has not become clearer. On the contrary, there have been many controversies around this issue (see, e.g., Goodhart (1999)). Many economists believe that with a more developed financial system, open market operations of central banks in a well-functioning interbank loan market are enough to maintain an efficient market. As a result, the LOLR policy becomes unnecessary (see, e.g., Goodfriend and King (1988)). Considering moral hazard associated with the LOLR policy, some economists even believe that we should stop using the LOLR policy.

The argument that open market operations of central banks make the LOLR unnecessary is based on a crucial assumption that the interbank loan market functions well without any information frictions. However, the LOLR policy can be justified during a financial crisis when all the financial markets suffer most heavily from information frictions. A large body of literature has suggested that when neither the central bank nor the market can distinguish between illiquidity and insolvency, the LOLR policy can improve social welfare by preventing contagion and alleviating market freezes (see, e.g., Goodhart and Huang (1999), Freixas, Parigi, and Rochet (2004), Rochet and Vives (2004), and Li, Milne and Qiu (2013) among many others).

Although it has become clear that the LOLR policy is needed when neither the central bank nor the market can distinguish between illiquidity and insolvency, our understanding about the optimal rules that a central bank should follow when conducting the LOLR policy in such a situation is far from clear. Our paper establishes a theoretical model to examine the LOLR policy when a central bank cannot distinguish between illiquidity and insolvency. In particular, we focus on how the LOLR policy will affect equilibrium outcomes and social welfare when the central bank can screen insolvent banks imperfectly. More specifically, in our model we assume that banks are divided into two types: solvent banks and insolvent ones. The market does not know which bank is which type, but knows the distribution of the two types. As an LOLR, the central bank offers central bank loans to these banks. We study two cases: a case where the central bank cannot screen insolvent banks and a case where the central bank can imperfectly screen insolvent banks. Then we examine the possible equilibria and the social welfare level associated with each equilibrium in these two cases. Finally, we extend our model to a case where banks can choose between a safe asset and a risky one and examine how the precision in central bank screening will affect banks' choice of assets in the first place.

Our model produces the following results: First, it is impossible for any separating equilibrium to exist, because insolvent banks always have an incentive to mimic solvent banks to gamble for resurrection. Second, the pooling equilibria in which, on one hand, both types of banks borrow from the central bank and, on the other hand, neither type of bank borrows from the central bank could exist given certain market beliefs off the equilibrium path. However, neither of the equilibria is socially efficient because insolvent banks will continue to hold their unproductive assets, rather than efficiently liquidating them. Third, when the central bank could screen banks imperfectly, the pooling equilibrium where all the banks borrow from the central bank becomes more likely, and the pooling equilibrium where all the banks do not borrow from the central bank becomes less likely. Fourth, higher precision in central bank screening will improve social welfare not only by identifying insolvent banks and forcing them to efficiently liquidate their assets, but also by reducing moral hazard and deterring banks from choosing risky assets in the first place. Finally, we find that if a central bank can commit to a specific precision level before the banks choose their assets, rather than conducting a discretionary LOLR policy, it will choose a higher precision level to reduce moral hazard and will attain higher social welfare.

The key insight in our paper is that central bank screening is crucial in the LOLR policy. A traditional criticism about the LOLR policy is that it induces moral hazard. Our paper reveals that the key cause of moral hazard is imperfect information, not the LOLR policy. When neither the central bank nor the market can distinguish between illiquidity and insolvency, moral hazard will exist even without the LOLR policy. Our model demonstrates that when certain conditions hold, the moral hazard problem is even *less* severe with the LOLR policy than without it. Moreover, if central bank screening can provide more precise information about each bank's type, the LOLR policy can further reduce moral hazard by deterring banks from choosing risky assets in the first place. This result coincides with Acharya and Backus (2009). They argue that the LOLR policy of the central banks during the subprime mortgage crisis was suboptimal and emphasize that the optimal LOLR policy has to be *conditional*. That is, when a central bank lends to banks as an LOLR, it must say no to the banks that cannot meet certain solvency conditions such as the maximum leverage ratio and minimum capital adequacy ratio. They further argue that the conditional LOLR policy can help reduce moral hazard induced by the LOLR policy. We build a rigorous model to show how precision in this conditional LOLR policy will affect equilibrium outcomes and social welfare. More importantly, our model reveals that if a central bank can commit to an LOLR policy with high precision in its screening, the LOLR policy will not induce moral hazard. On the contrary, it will reduce moral hazard.

Our paper is closely related to the literature on the LOLR policy with imperfect information. Rochet and Vives (2004) study the LOLR policy in a global game setup where depositors face both strategic complementarities and imperfect information. They find that the introduction of the LOLR policy in this case will improve social welfare by alleviating coordination failure. Goodhart and Huang (1999) build a model where the central bank employs the LOLR policy to prevent contagion, but has to suffer the loss caused by moral hazard when the central bank cannot perfectly distinguish between illiquid and insolvent banks. Freixas, Parigi, and Rochet (2004) also build a model to study the LOLR policy when the market cannot distinguish liquidity shocks from solvency shocks. Our model is most closely related to one particular case in their paper where insolvent banks have an incentive to gamble for resurrection and the central bank cannot distinguish insolvent banks from illiquid ones. They find that the LOLR policy is more useful in improving social welfare in this case. However, they do not further examine the optimal LOLR policy. Our paper complements this previous paper by examining the optimal LOLR policy in this case. In particular, we examine how precision in central bank screening when implementing the LOLR policy will affect social welfare and moral hazard.

The rest of the paper is organized as follows. Section 2 introduces a basic model where the central bank cannot screen insolvent banks in the LOLR policy and characterizes the equilibria. Section 3 introduces a model where the central bank can only imperfectly screen insolvent banks and characterizes the equilibria in this model. Section 4 extends the model in Section 3 where banks' assets are exogenously given to the case where banks can choose their assets at the beginning of the model. Section 5 concludes.

2 The basic model without central bank screening

2.1 The environment

The model is a two-period one with three dates of 0, 1 and 2. There is a continuum of banks with a central bank in the economy. The initial balance sheet of each bank at $t = 0$ is exogenously given as follows:

Table 1: A bank's balance sheet at $t = 0$

Bank Balance sheet at $t = 0$	
Long-term Assets: A	Short-term Debts : D
	Equity: e_0

From table 1, we can tell that at date 0 each bank has a long-term asset with a size of A. The asset is financed at date 0 by each bank's own equity e_0 and one-period short-term debts with a size of D. Thus, $A = D + e_0$. If the long-term asset is mature at date 2, its gross return rate will be $R_H > 1$. If the asset is liquidated prematurely at date 1, a liquidation cost will be incurred. We will specify the liquidation technology later.

The short-term debts' interest rate in period 1 (between date 0 and date 1) is exogenously given and assumed to be zero for simplicity. The roll-over rate of short-term debts is determined by short-term creditors' expectations. We assume that short-term creditors are risk neutral and aim for a riskless rate of zero.

We assume that at the beginning of date 1, before each bank rolls over its short-term debts, an unanticipated shock hits some banks' long-term assets. As a result, the banks are divided into two types. A proportion $0 < \lambda < 0$ of the banks is unaffected by the shock, which we call the high type (H-type) banks. However, for the remaining proportion

$1 - \lambda$ of the banks, with a probability of p, their long-term assets' return rate is R_H, and with a probability of $1 - p$, the rate is $R_L < 1$. We call these banks the low type (L-type) banks. The return rate of each L-type bank is independently and identically distributed.

Each bank knows its own type. Neither the central bank nor the short-term creditors know the type of each bank, but know the proportion of each type.

At date 1, after the shock, each bank needs to roll over its short-term debts of D through three options: borrowing from the central bank, borrowing from the market (short-term creditors), and liquidating its long-term assets. We assume that it happens in two stages:

In the first stage, the central bank offers to lend $L_{CB} < D$ to each bank at $r_{CB} \geq 0$. We focus on the case where r_{CB} is always lower than the prevailing market rate.[1] Each bank determines whether to borrow or not, which is publicly observed.

In the second stage, each bank determines how much to borrow on the market and how many long-term assets to liquidate. The market rate is determined as follows. For each bank, a short-term creditor decides whether to lend or not, and the interest rate if he does. The bank then decides whether to borrow or not. The creditor can not make his lending decision contingent on the quantity of debts that the bank will borrow. In addition, creditors cannot observe how many long-term assets are liquidated by a bank when determining the market rate.

The liquidation technology is as follows. For H-type and L-type banks, each unit of the assets liquidated at date 1 will yield γ_H and γ_L units of the proceeds, respectively, where $\gamma_L \leq \gamma_H < 1 < R_H$.

In addition, we assume that it is socially better off for an L-type bank to liquidate its asset at date 1 rather than continuing to hold the asset to date 2. More specifically, we assume that

$$\gamma_L > pR_H + (1 - p)R_L \tag{1}$$

Moreover, we assume that

$$\gamma_L A \leq \gamma_H A < D \tag{2}$$

[1] In our model, if r_{CB} is higher than the market rate, we will have an uninteresting case where banks never borrow from the central bank.

which means that the date 1 liquidation value of both H-type and L-type banks' assets is not enough to repay their debts. This condition will be satisfied when e_0 is sufficiently low such that the asset-debt ratio of $\frac{A}{D}$ is high and when γ_H and γ_L are sufficiently low. Note that by combining conditions (1) and (2), we derive

$$pR_H + (1-p)R_L < \gamma_L < \frac{D}{A}$$

Or

$$pAR_H + (1-p)AR_L < D \tag{3}$$

Since $AR_H > D$ (because $A > D$ and $R_H > 1$), it is straightforward to see that

$$AR_L < D \tag{4}$$

This implies that an L-type bank cannot repay its debts in the down state and will default.

For simplicity in our calculations, we assume that

$$p\frac{R_H}{\gamma_L} + (1-p)\frac{AR_L}{D} < 1 \tag{5}$$

This condition guarantees that a creditor will never roll over his debts if he knows that the bank is L-type. The proof of this condition is given in appendix A.

Each bank aims to maximize its expected equity value at date 2. A bank does not care about the loss of its creditors. As a result, an L-type bank will borrow and continue its operation until date 2 as long as it has a higher expected equity value, even when it knows that it is not socially optimal to do so. In other words, it has an incentive to gamble for resurrection.

2.2 Equilibrium Characterization

Here we examine possible perfect Bayesian Nash equilibria in this model. We will first examine possible separating equilibria, and then examine possible pooling equilibria.

Before we characterize possible equilibria in this model, we first examine banks' optimal borrowing behavior in this model. Proposition 1 gives the results.

Proposition 1.
- *For an H-type bank,*
  - *if $1 + r_M < \frac{R_H}{\gamma_H}$, it will borrow on the market, and will never liquidate its asset.*

* *If it does not borrow from the central bank, then it will borrow an amount D on the market.*
  * *If it borrows from the central bank, then it will borrow an amount $D - L_{CB}$ on the market.*
- *If $1 + r_M \geq \frac{R_H}{\gamma_H}$, the bank will never borrow on the market, and will only liquidate its asset to repay its debts.*

- *An L-bank acts in a similar way to an H-type bank, except that $\frac{R_H}{\gamma_H}$ in the above conditions is changed into $\frac{R_H}{\gamma_L}$.*

Proof: see the appendix. ■

Note that $\frac{R_H}{\gamma_L} \geq \frac{R_H}{\gamma_H}$. So when $1 + r_M < \frac{R_H}{\gamma_H}$, both H-type and L-type banks will borrow on the market to meet all the liquidity need. When $1 + r_M \geq \frac{R_H}{\gamma_H}$, an H-type bank will stop borrowing on the market, while an L-type bank will still want to borrow if $1 + r_M \in [\frac{R_H}{\gamma_H}, \frac{R_H}{\gamma_L}]$. However, any value of $1 + r_M$ in the range of $[\frac{R_H}{\gamma_H}, \frac{R_H}{\gamma_L}]$ can not exist in equilibrium, because creditors know that at this level of $1 + r_M$, all the borrowers must be L-type. By assumption, creditors' expected rate of return from lending to an L-type is always below 1. As a result, creditors will never offer such a market rate in equilibrium.

In the proof of proposition 1, we also derive the following result:

Corollary 1. *An L-type bank's net asset value in the down state is negative in all situations. Thus its equity value in that state is always zero.*

Proof: see the proof of proposition 1. ■

This result implies that, to maximize the equity value, an L-type bank needs only to maximize its equity value in the up state.

2.2.1 Separating equilibria

Here we examine two separating equilibria: the equilibrium where only L-type banks borrow from the central bank and the equilibrium where only H-type banks borrow from the central bank. All the banks aim to maximize their expected equity value. In order to find out whether these equilibria exist or not, we need to find each type of banks' expected equity value when following the equilibrium strategy and when deviating. The no-deviation condition will be that the gap between the two values is positive. An equilibrium exists if and only if the no-deviation condition holds for both types of banks. Proposition 2 summarizes the results.

Proposition 2. *Both separating equilibria where only L-type banks borrow from the central bank and only H-type banks borrow from the central bank cannot exist.*

Proof: see the appendix. ■

The intuition of proposition 2 is quite straightforward. When we examine the no-deviation conditions for both types of banks, we find that, in both equilibria, L-type banks' no-deviation condition does not hold, and L-type banks always deviate. As a result, both equilibria cannot exist. The reason for L-type banks' deviation is as follows. A key feature of this basic model is that L-type banks can always imitate H-type banks without incurring any cost, because the market rate depends only on creditors' belief derived from a bank's action of whether to borrow from the central bank or not. Thus an L-type bank can always pretend to be H-type and get the most favorable rate (riskless rate) on the market without incurring any cost. Therefore, an L-type bank will always deviate.

2.2.2 Pooling equilibria

Here we examine two possible pooling equilibria in which both types of banks borrow and do not borrow from the central bank.

We first examine *the pooling equilibrium where both types of banks borrow from the central bank.* Similar to the separating equilibria case, we need to find each type of banks' expected equity value when following the equilibrium strategy and when deviating and the resultant no-deviation condition. In order to do so, we need to find the market rate when a bank does and does not deviate.

First, we find the equilibrium market rate when banks follow the equilibrium strategy. In this case, both types of banks borrow from the central bank, and banks' action of borrowing from the central bank will not reveal their type. Hence, creditors will maintain their prior belief that a bank could be H-type with a probability of λ and L-type with a probability of $1 - \lambda$. The equilibrium market rate is determined based on this belief. We focus on the case where an equilibrium market rate exists. In this case, creditors' expected return rate from an H-type bank will be $1 + r_M$. Similarly, their expected return rate from an L-type bank in the up state is also $1 + r_M$. Creditors' expected return rate from an L-type bank in the down state is $\frac{AR_L}{D}$. This is because, in the down state, an L-type bank's net asset value is $AR_L - (D - L_{CB})(1 + r_M) - L_{CB}(1 + r_{CB}) < AR_L - D < 0$.

As a result, its asset is allocated between the central bank and creditors proportional to their principals.[2]

Therefore, the equilibrium r_M is determined as follows:

$$\lambda(1 + r_M) + (1 - \lambda)[p(1 + r_M) + (1 - p)\frac{AR_L}{D}] = 1 \tag{6}$$

Or

$$r_M^* = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{\lambda + (1 - \lambda)p} - 1\right) \tag{7}$$

In order for an equilibrium market rate to exist, $1+r_M^* \leq min\{\frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}, \frac{R_H}{\gamma_H}\}$.[3] We focus on this case and assume that this condition always holds. Note that the quantity that an H-type or L-type bank will borrow is the same so that it will not reveal any information about the type of each bank.

Next, we find the market rate if a bank deviates to not borrowing from the central bank. Then we need to specify creditors' belief off the equilibrium path first. Let $0 \leq \hat{\lambda} \leq 1$ be the probability that a creditor assigns to a bank being H-type when observing it not borrow from the central bank. Given that $\hat{\lambda} \geq \lambda$, the market rate off the equilibrium path, denoted by $\hat{r}_M$, is always lower than r_M and is determined as follows:

$$\hat{r}_M = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{\hat{\lambda} + (1 - \hat{\lambda})p} - 1\right) \tag{8}$$

Proposition 3 summarizes the results.

Proposition 3.

- *There exists a pooling equilibrium where both types of banks borrow from the central bank as long as creditors' belief off the equilibrium path, $\hat{\lambda}$, is no greater than the prior belief, λ.*
- *When $\hat{\lambda} > \lambda$, this pooling equilibrium exists if and only if*

$$L_{CB}(r_M - r_{CB}) - D(r_M - \hat{r}_M(\hat{\lambda})) > 0 \tag{9}$$

Both types of banks will not deviate as long as the above condition is satisfied.

[2] We assume that when a bank's asset value is below the principals of its debts, its asset will be allocated among creditors proportional to their principals.

[3] The maximum market rate that an H-type bank can pay is $\frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}$. Thus any $1 + r_M^* > \frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}$ could not be an equilibrium market rate. $1 + r_M^* > \frac{R_H}{\gamma_H}$ could not be an equilibrium rate, as we explained previously.

- *There exists a threshold level of* $\hat{\lambda}$, $\hat{\lambda}_{th} \in (\lambda, 1)$, *above which such an equilibrium cannot exist.* $\hat{\lambda}_{th}$ *is determined by*

$$L_{CB}(r_M - r_{CB}) - D(r_M - \hat{r}_M(\hat{\lambda}_{th})) = 0 \tag{10}$$

Proof: see the appendix. ■

The intuition behind proposition 3 is as follows. Since in our model L-type banks can mimic H-type banks without any cost, the no-deviation condition is the same for both of them, which is given by condition (9). The first term $L_{CB}(r_M - r_{CB})$ of this condition can be thought of as a bank's interest cost reduction because the bank is borrowing a loan of L_{CB} from the central bank instead of from the market. The second term $D(r_M - \hat{r}_M)$ is the bank's interest cost reduction when it deviates and borrows D at a rate of $\hat{r}_M$ rather than r_M. Therefore, a bank's no-deviation condition is that the benefit of borrowing from the central bank dominates over the benefit of deviating.

It is straightforward to see that $\hat{r}_M$ is strictly decreasing in $\hat{\lambda}$. That is, if creditors have a higher belief that a bank is H-type when observing it deviate, they will charge a lower market rate. As a result, a bank will have a stronger incentive to deviate when $\hat{\lambda}$ is higher. When $\hat{\lambda} \leq \lambda$, $\hat{r}_M \geq r_M$ and condition (9) always holds.[4] Thus, no bank will deviate. Intuitively, when $\hat{\lambda} \leq \lambda$, a deviating bank will borrow from the market at a higher rate. Meanwhile, it will suffer a loss by not being able to borrow from the central bank at a low interest rate. Thus it will be definitely worse off by deviating. However, when $\hat{\lambda} > \hat{\lambda}_{th}$, $\hat{r}_M$ is so low that the no-deviation condition does not hold any more.

We can tell that the no-deviation condition is more likely to hold when L_{CB} is higher and r_{CB} is lower. Intuitively, more central bank loans at a lower interest rate will lower a bank's interest costs and, consequently, make the equilibrium strategy of borrowing from the central bank more attractive. As a result, this equilibrium is more likely to exist.

Now we consider *the pooling equilibrium where neither type of bank borrows from the central bank.*

First, we find the equilibrium market rate when banks follow the equilibrium strategy. It turns out the same as in the previous pooling equilibrium, which is given by equation (7). Again, we focus on the case of $1 + r_M^* < min\{\frac{R_H}{\gamma_H}, \frac{AR_H}{D}\}$ such that an equilibrium market rate exists.[5]

[4] Recall that we assume r_{CB} is always lower than the prevailing market rate.

[5] In this case without central bank loans, the maximum market rate that an H-type bank can pay is

Second, we find the market rate when a bank deviates. Again, we need to specify creditors' belief off the equilibrium path. Let $0 \leq \tilde{\lambda} \leq 1$ denote the probability a creditor assigns to a bank being H-type when observing a bank borrow from the central bank. We first consider the case where $\tilde{\lambda}$ is high enough such that a market rate exists. Similar to the previous case (equation (6)), we have

$$\tilde{r}_M = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{\tilde{\lambda} + (1-\tilde{\lambda})p} - 1\right) \tag{11}$$

Now consider the case where $\tilde{\lambda}$ is so low that the equilibrium market rate does not exist. It occurs when $1 + \tilde{r}^*_M > min\{\frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}, \frac{R_H}{\gamma_H}\}$. $\frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}$ is the maximum interest rate that an H-type bank can pay to the creditors when it deviates to borrowing from the central bank. Thus an equilibrium market rate cannot exceed it. We also proved previously that any $1 + \tilde{r}^*_M > \frac{R_H}{\gamma_H}$ could not be an equilibrium market rate. When the equilibrium market rate does not exist, a market freeze occurs. Banks will liquidate their assets to repay their debts of $D - L_{CB}$ at date 1. Since $\tilde{r}_M$ is strictly decreasing in $\tilde{\lambda}$, there exists a level of $\tilde{\lambda}$, $\tilde{\lambda}_{freeze}$, below which $1 + \tilde{r}^*_M > min\{\frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}, \frac{R_H}{\gamma_H}\}$, and a market freeze occurs. Let $r_M^{max,H} = \frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}} - 1$. Then $\tilde{\lambda}_{freeze}$ is determined as follows:

$$min\{\frac{R_H}{\gamma_H} - 1, r_M^{max,H}\} = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{\tilde{\lambda} + (1-\tilde{\lambda})p} - 1\right)$$

Or

$$\tilde{\lambda}_{freeze} = \frac{1}{1-p}\left[\frac{1}{\frac{min\{\frac{R_H}{\gamma_H} - 1, r_M^{max,H}\}}{1 - \frac{AR_L}{D}} + 1} - p\right] \tag{12}$$

Proposition 4 summarizes the results.

Proposition 4. • *When $\tilde{\lambda} > \tilde{\lambda}_{freeze}$, a deviating bank will not face a market freeze. In this case, the pooling equilibrium where neither type of bank borrows from the central bank exists if and only if*

$$D(\tilde{r}_M - r_M) - L_{CB}(\tilde{r}_M - r_{CB}) > 0 \tag{13}$$

$\frac{AR_H}{D}$. Thus $1 + r^*_M > \frac{AR_H}{D}$ could not be an equilibrium market rate. In addition, we proved previously that $1 + r^*_M > \frac{R_H}{\gamma_H}$ could not be an equilibrium rate either.

- *When $\tilde{\lambda} < \tilde{\lambda}_{freeze}$, a deviating bank will face a market freeze. In this case, the pooling equilibrium where neither type of bank borrows from the central bank exists if and only if*

$$D\left[\frac{R_H}{\gamma_H} - 1 - r_M\right] - L_{CB}\left(\frac{R_H}{\gamma_H} - 1 - r_{CB}\right) > 0 \tag{14}$$

This pooling equilibrium will never exist if condition (14) does not hold.

Proof: see the appendix. ■

The intuition behind proposition 4 is as follows. When $\tilde{\lambda} > \tilde{\lambda}_{freeze}$, the market does not freeze when a bank deviates. In this case, both types of banks have the identical no-deviation condition given by condition (13). Since $D > L_{CB}$, the RHS of condition (13) is strictly increasing in $\tilde{r}_M$, or strictly decreasing in $\tilde{\lambda}$. Intuitively, if a bank faces a higher market rate when it deviates, then it will be more likely to stick with the equilibrium strategy of not borrowing from the central bank.

Given that $\tilde{\lambda} < \tilde{\lambda}_{freeze}$, when a bank deviates, it faces a market freeze and will liquidate its asset to meet the liquidity demand of $D - L_{CB}$. The two types of banks' no-deviation conditions are slightly different in this case since their liquidation rates, γ, are different. It turns out that H-type banks' no-deviation condition, given by condition (14), is stricter because of its higher liquidation rate. That is, an L-type bank will never deviate as long as an H-type bank does not deviate. Thus the no-deviation condition for this equilibrium to exist is H-type banks' no-deviation condition, because neither of bank will deviate once it holds. Note that the worst case scenario for a deviating bank is the market freeze case. Thus if a bank deviates even when it will face a market freeze, then it will definitely deviate when not facing a market freeze. As a result, such an equilibrium can never exist.

Now we examine the range of the belief off the equilibrium path that supports this equilibrium. Given that the no-deviation condition (14) in the market freeze case holds, there are two possible cases. First, if there exists a $\tilde{\lambda}_{th} \geq \tilde{\lambda}_{freeze}$ such that when $\lambda \in [\tilde{\lambda}_{freeze}, \tilde{\lambda}_{th}]$, the no-deviation condition (13) in the no-market-freeze case holds, then in this range, the equilibrium will exist. Thus, the whole range of $\tilde{\lambda}$ in which the equilibrium exists is $[0, \tilde{\lambda}_{th})$. Second, if the no-deviation condition (13) does not hold for any $\tilde{\lambda} > \tilde{\lambda}_{freeze}$, then the valid range for the equilibrium to exist is simply $[0, \tilde{\lambda}_{freeze})$. This case is possible because when we compare the two no-deviation conditions, we find that when condition (14) holds, condition (13) does not necessarily hold.[6] That is, banks are more likely to deviate in the no market freeze case than in the market freeze case. Intuitively, this is because a market freeze imposes the highest cost on a deviating bank. Thus a bank is least likely to deviate in this case.

In addition, note that when λ is high enough, then banks will indeed deviate, and this equilibrium will not exist. For example, when $\tilde{\lambda} = \lambda$ and, consequently, $\tilde{r}_M = r_M$, condition (13) does not hold. Intuitively, if a bank can borrow at the same rate after it borrows from the central bank (where r_{CB} is lower than the market rate), then it will surely deviate and borrow from the central bank. Actually, as long as $\tilde{\lambda} \geq \lambda$ and $\tilde{r}_M \leq r_M$, condition (13) will not hold. Thus, $\tilde{\lambda}$ that supports the equilibrium must be smaller than λ.

From the no-deviation conditions, we can see that a bank is less likely to deviate when L_{CB} is smaller and r_{CB} is higher. Intuitively, less central bank loans at a higher cost will reduce the attractiveness of borrowing from the central bank and discourage banks from borrowing from the central bank. As a result, this equilibrium is less likely to exist.

3 A model with central bank screening

In our model, it is socially optimal for L-type banks to liquidate all of their assets at date 1. This implies that the first best allocation is the separating equilibrium where only H-type banks borrow from the central bank. However, our previous analysis demonstrates that this equilibrium can never exist. The two possible pooling equilibria are both socially inefficient. In both cases, L-type banks will survive to gamble for resurrection. In our basic model, we assume that the central bank plays a rather passive role. It simply offers a fixed amount of loans at a fixed interest rate. Here our question is, can the central bank improve social welfare by implementing a different LOLR policy?

The inefficiency in our model originates from imperfect information. In our basic model, the central bank has no information advantage over the market and does not provide additional information to the market. In this section, we introduce an assumption that the central bank can imperfectly screen insolvent banks from solvent banks and provide additional information to the market through the LOLR policy. We believe that

[6] Note that $\tilde{r}_M \leq \frac{R_H}{\gamma_H} - 1$ and $D > L_{CB}$

this assumption is realistic because, as a major superintendent of banks, the central bank should hold some private information about banks' quality.

The major results that we find are the following: with central bank screening, the pooling equilibrium where both types of banks apply for central bank loans becomes more likely, and the pooling equilibrium where neither type of bank applies for central bank loans becomes less likely. This is because central bank screening makes the strategy of applying for central bank loans more attractive to H-type banks: they will always receive central bank loans. More importantly, they will benefit from a lower market rate derived from a more optimistic market belief after receiving central bank loans, because they survive central bank screening. As a result, H-type banks have a weaker incentive to deviate in the pooling equilibrium where all the banks apply for central bank loans and have a stronger incentive to deviate in the pooling equilibrium where all the banks apply for central bank loans than in the case without central bank screening. Since in our model L-type banks always mimic H-type banks, the essential no-deviation condition is always determined by H-type banks' no-deviation condition. Thus we find that the former pooling equilibrium is more likely and the latter one is less likely.

3.1 The setup

We assume that each bank applying for central bank loans must agree to be inspected by the central bank. In addition, we assume that the central bank can perfectly identify an H-type bank, but can identify an L-type bank only imperfectly. More specifically, we assume that for each L-type bank, with a probability of $\phi < 1$, the central bank can identify it as L-type and will reject its application. With a probability of $1-\phi$, the central bank can not identify it as L-type and will lend to it. We believe that this assumption is realistic, because in reality a healthy bank may have safer assets, the quality of which is easier to verify, while an insolvent bank may have risker assets, the value of which could be more uncertain and, consequently, more difficult to judge. In addition, a healthy bank may be more cooperative, while an insolvent bank may try to hide information. In a more general case, the central bank can also mistake a healthy bank for an insolvent one. Here we consider this simpler case to reduce the complexity, and our qualitative results will not be affected by doing so.

Note that separating equilibria can not exist either in this model. Consider the equilibrium where only L-type banks apply for central bank loans. Then L-type banks will always deviate, because they can mimic H-type banks without incurring any cost by not applying central bank loans. By doing so, an L-type bank will be identified as H-type and get the lowest possible borrowing rate on the market. Consider the equilibrium where only H-type banks apply for central bank loans. If an L-type bank follows the equilibrium strategy and does not apply, it will be identified as an L-type bank by the market and will not be able to borrow on the market. As a result, it will be forced to liquidate all its asset and go bankrupt at date 1. If it mimics an H-type bank and borrows from the central bank, then at least with a probability of $1-\phi$, it will successfully get loans and gain positive equity in the up state. As a result, L-type banks will always deviate, and such an equilibrium can not exist either. Hence, we focus on the pooling equilibria.

3.1.1 Pooling equilibrium I: both types of banks apply for central bank loans

When a bank applies for central bank loans, it will be inspected by the central bank and will be rejected by the central bank if identified as L-type. We assume that whether a bank applies or not and whether it is rejected or not when it applies are all publicly observed.

The analysis is similar to the one in the basic model except that central bank screening now reveals additional information. First, we find the market rate if banks follow the equilibrium strategy. Now the banks rejected by the central bank are identified as L-type, face a market freeze, and are forced to liquidate all their assets at date 1. Creditors' belief about the remaining banks that successfully receive central bank loans is calculated as follows. For an H-type bank, the conditional probability that it will get the loan is 1, while for an L-type bank, the conditional probability that it will get the loan is $1-\phi$. Let g denote creditors' ex post belief that a bank is H-type. Thus

$$g = \frac{\lambda}{\lambda + (1-\lambda)(1-\phi)} > \lambda \tag{15}$$

It is obvious that g is higher than λ, the prior belief.

The equilibrium market rate, now denoted by $r_{M,g}$, is decided in a similar way as in equation (6) except that λ is replaced by g.

$$1 = g(1 + r_{M,g}) + (1-g)(p(1 + r_{M,g}) + (1-p)\frac{AR_L}{D})$$

Or

$$r_{M,g} = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{g + (1-g)p} - 1\right) \tag{16}$$

Because $g > \lambda$, $r_{M,g}$ is lower than the market rate without central bank screening, r_M.

Next, we examine the market rate when a bank deviates. Again let $\hat{\lambda}$ be creditors' belief off the equilibrium path that a bank is H-type when they observe a bank not apply for central bank loans. When $\hat{\lambda}$ is sufficiently low, the market freezes and a bank will gain the minimum payoff of zero equity. As a result, banks will have no incentive to deviate. We focus on the case where $\hat{\lambda}$ is high enough such that the market rate exists. The market rate, $\hat{r}_M$, is identical to the one in the case without central bank screening and is shown by equation (8).

Proposition 5 summarizes the results.

Proposition 5. *The pooling equilibrium where both types of banks apply for central bank loans exists if and only if*

$$L_{CB}(r_{M,g} - r_{CB}) - D(r_{M,g} - \hat{r}_M(\hat{\lambda})) > 0 \tag{17}$$

It implies that this equilibrium exists if and only if $\hat{\lambda}$ is lower than a threshold level, $\hat{\lambda}_{th,H}$, which is determined by

$$L_{CB}(r_{M,g} - r_{CB}) - D(r_{M,g} - \hat{r}_M(\hat{\lambda}_{th,H})) = 0 \tag{18}$$

Proof: see the appendix. ■

The intuition behind proposition 5 is as follows. Now H-type and L-type banks have different no-deviation conditions with central bank screening. H-type banks' no-deviation condition is less strict than L-type banks'. This is because, with central bank screening, H-type banks's payoff from following the equilibrium strategy of applying for central bank loans is higher than L-type banks: they will be certainly identified as H-type, while L-type banks may be identified as L-type with a probability of ϕ. Meanwhile, their payoffs from deviating to not applying for central bank loans are the same. However, L-type banks always have an incentive to mimic H-type banks in our model, because they will certainly be identified as L-type if they do not do so. Thus we can use the intuitive criterion to argue that the essential no-deviation condition is H-type banks' no-deviation condition (17). With a higher belief off the equilibrium path (a higher $\hat{\lambda}$), the creditors will charge a lower $r_{M,g}$ when a bank deviates, inducing a stronger incentive for banks to deviate. Thus we have a threshold level of $\hat{\lambda}$, $\hat{\lambda}_{th,H}$. Once $\hat{\lambda} > \hat{\lambda}_{th,H}$, $r_{M,g}$ is so low that condition (17) is violated.

Now we compare creditors' belief off the equilibrium path that will support such a pooling equilibrium in two cases: one with central bank screening and one without. We find that the no-deviation condition with central bank screening (condition (17)) is more likely to hold than the one without central bank screening (condition (9)), because $r_{M,g} < r_M$ and $L_{CB} < D$. Intuitively, in the case with central bank screening, when an H-type bank borrows from the central bank, central bank inspection will boost the market belief. As a result, the H-type bank will be charged a lower market rate later when borrowing on the market, which will increase its payoff when following the equilibrium strategy. Thus, the threshold level of $\hat{\lambda}$, above which the equilibrium does not exist in the model without central bank screening, $\hat{\lambda}_{th}$, is lower than $\hat{\lambda}_{th,H}$, the threshold level with central bank screening. In other words, such a pooling equilibrium is more likely to exist with central bank screening than without central bank screening.

3.2 Pooling equilibrium II: neither type of bank applies for central bank loans

Now we consider the equilibrium where neither type of bank applies for central bank loans.

The market rate when a bank follows the equilibrium strategy, r_M, is the same as in the case without central bank screening and is given by equation (7).

When a bank deviates, the market rate depends on creditors' belief off the equilibrium path. Again, let $\tilde{\lambda}$ denote creditors' belief off the equilibrium path that a bank is H-type when observing it deviate to applying for central bank loans. If a bank's application is rejected, then creditors will know that the bank is L-type, and the bank will not be able to get loans from the market either. If a bank's application is accepted, then creditors' ex post belief that the bank is H-type will become

$$\tilde{g} = \frac{\tilde{\lambda}}{\tilde{\lambda} + (1 - \tilde{\lambda})(1 - \phi)} > \tilde{\lambda} \tag{19}$$

We first consider the case where $\tilde{\lambda}$ is high enough so that $\tilde{g}$ is high enough and an equilibrium market rate, now denoted by $\tilde{r}_{M,g}$, exists. The market rate is similar to the one in the basic model (equation (11)) except that $\tilde{\lambda}$ is replaced by $\tilde{g}$:

$$\tilde{r}_{M,g} = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{\tilde{g} + (1-\tilde{g})p} - 1\right) \tag{20}$$

For this equilibrium rate to exist, we have $1 + \tilde{r}^*_{M,g} \leq min\{\frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}, \frac{R_H}{\gamma_H}\}$.

Now consider the case where $\tilde{\lambda}$ is so low that a market freeze occurs. In this case, $\tilde{g}_{freeze} = \tilde{\lambda}_{freeze}$, which is given by equation (12). Since $\tilde{g}$ is always higher than $\tilde{\lambda}$, we have $\tilde{\lambda}^{CBS}_{freeze} < \tilde{\lambda}_{freeze}$. That is, a market freeze occurs at a lower level of $\tilde{\lambda}$ with central bank screening.

Proposition 6 summarizes the results.

Proposition 6.

- *When $\tilde{\lambda} > \tilde{\lambda}^{CBS}_{freeze}$, a deviating bank does not face a market freeze. In this case, a pooling equilibrium where neither type of bank applies for central bank loans exists if and only if*

$$D(\tilde{r}_{M,g} - r_M) - L_{CB}\left(\tilde{r}_{M,g} - r_{CB}\right) > 0 \tag{21}$$

- *When $\tilde{\lambda} < \tilde{\lambda}^{CBS}_{freeze}$, a deviating bank faces a market freeze. In this case, this pooling equilibrium exists if and only if*

$$D\left[\frac{R_H}{\gamma_H} - 1 - r_M\right] - L_{CB}(\frac{R_H}{\gamma_H} - 1 - r_{CB}) > 0 \tag{22}$$

  This pooling equilibrium will never exist if condition (22) does not hold.

Proof: see the appendix. ■

The intuition behind proposition 6 is as follows. When $\tilde{\lambda} > \tilde{\lambda}^{CBS}_{freeze}$, H-type banks' deviation condition is given by condition (21). L-type banks' deviation condition is different, and H-type banks' deviation condition is stricter to hold. That is, H-type banks are more likely to deviate. This result is quite intuitive because now with central bank screening, L-type banks will face a positive probability of ϕ of being identified as L-type when deviating to applying for central bank loans. As a result, they have a lower deviating payoff and are less likely to deviate. When $\tilde{\lambda} < \tilde{\lambda}^{CBS}_{freeze}$, H-type banks' deviation condition is given by condition (22). Again we find that H-type banks' deviation condition is stricter to hold, and H-type banks are more likely to deviate. In our model, L-type banks always mimic H-type banks such that they will not be identified as L-type. Thus we can use the intuitive criterion to argue that the no-deviation condition for H-type banks is the essential no-deviation condition in this equilibrium, and we can ignore L-type banks' no-deviation conditions because they will have to deviate if H-type banks do. Also note that a market freeze imposes the highest cost on a bank when it deviates. Thus if a bank chooses to deviate even when a market freeze occurs, then it will always deviate when a market freeze does not occur. As a result, condition (22) is a necessary condition for this equilibrium to exist.

Given the no-deviation conditions in proposition 6, we now consider creditors' belief off the equilibrium path that will support the equilibrium. Given that condition (22) is satisfied, there are two possible cases. First, if we find a threshold level of $\tilde{\lambda}$, say $\tilde{\lambda}_{th,H} > \tilde{\lambda}^{CBS}_{freeze}$, below which condition (21) holds, then the belief that supports this equilibrium is $\tilde{\lambda} \in [0, \tilde{\lambda}_{th,H}]$. Here $\tilde{\lambda}_{th,H}$ is determined by $\tilde{g}_{th,H}$ through equation (19), and $\tilde{g}_{th,H}$ is given by

$$D(\tilde{r}_{M,g}(\tilde{g}_{th,H}) - r_M) - L_{CB}\left(\tilde{r}_{M,g}(\tilde{g}_{th,H}) - r_{CB}\right) = 0 \tag{23}$$

It is straightforward to see that $\tilde{g}_{th,H} = \tilde{\lambda}_{th}$.

Second, if condition (21) does not hold for any $\tilde{\lambda} > \tilde{\lambda}^{CBS}_{freeze}$, then the belief that supports this equilibrium is $\tilde{\lambda} \in [0, \tilde{\lambda}^{CBS}_{freeze}]$.

Since $\tilde{\lambda}^{CBS}_{freeze} < \tilde{\lambda}_{freeze}$, with central bank screening we need more extreme values of $\tilde{\lambda}$ to induce a market freeze and prevent banks from deviating. Moreover, when there is no market freeze, the threshold level of $\tilde{\lambda}$ below which the equilibrium will exist is lower with central bank screening than without central bank screening ($\tilde{\lambda}_{th,H} < \tilde{\lambda}_{th}$). This is because $\tilde{\lambda}_{th,H} < \tilde{g}_{th,H} = \tilde{\lambda}_{th}$. As a result, we can conclude that with central bank screening, it is more likely for H-types bank to deviate to the alternative strategy of applying for central bank loans. Thus, this equilibrium of neither type of bank applying for central bank loans becomes less likely with central bank screening. Intuitively, central bank screening makes the strategy of applying for central bank loans more attractive to H-type banks, because they will face a more optimistic market belief about their asset quality. As a result, H-type banks have a stronger incentive to deviate to this alternative strategy, and the equilibrium is less likely to exist.

3.3 A numerical example

Here we provide a simple numerical example to illustrate the intuition of our analytical results.

The parameter values are given as follows: $\lambda = 0.7$, $A = 1$, $R_H = 1.2$, $R_L = 0.4$, $p = 0.25$, $D = 0.9$, $e_0 = 0.1$, $L_{CB} = 0.25$, $r_{CB} = 0$, $\gamma_H = 0.8$, $\gamma_L = 0.7$, and $\phi = 0.25$.

These values satisfy the assumptions in the model. First, the expected value of an L-type bank's asset is

$$pAR_H + (1-p)AR_L = 0.6 \tag{24}$$

which is lower than $\gamma_L A = 0.7$. So it is socially optimal for an L-type bank to liquidate its asset at date 1. Second, asset liquidation values at date 1 for an H-type and L-type banks are 0.8 and 0.7 respectively, both of which are smaller than $D = 0.9$. Third, creditors will not lend to an L-type bank if they know the bank is L-type. This is because $\frac{R_H}{\gamma_L} = 1.7143$, and we have

$$p\frac{R_H}{\gamma_L} + (1-p)A\frac{R_L}{D} = 0.7619 < D \tag{25}$$

In fact, the maximum rate that can be paid by the bank in the up state is $\frac{AR_H - L_{CB}}{D - L_{CB}} = 1.4615 < 1.7143$. So the bank will stop borrowing even before the rate reaches $\frac{R_H}{\gamma_L}$.

3.3.1 The case without central bank screening

We first show the results of the pooling equilibrium where both types of banks borrow from the central bank. Given the parameter values, we find that if a bank follows the equilibrium strategy of borrowing from the central bank, it will borrow $D - L_{CB}$ on the market at the market rate of $r_M = 0.1613$. Moreover, we find that $\hat{r}_M$ below which the banks will deviate is $\hat{r}_{M,devi} = 0.1165$, and the corresponding threshold of $\hat{\lambda}$ above which the banks will deviate is $\hat{\lambda}_{th} = 0.7689$. As a result, this equilibrium exists when $\hat{\lambda} \leq 0.7689$. That is, if the probability that creditors assign to a bank to be H-type when observing it deviate to not borrowing from the central bank is lower than 0.7689, this equilibrium will exist.

Next, we examine the case where neither type of bank borrows from the central bank. In this equilibrium, banks borrow only on the market at $r_M = 0.1613$. Without a market freeze, the threshold of $\tilde{r}_M$ below which banks will deviate is $\tilde{r}_{M,devi} = 0.2233$, and the corresponding $\tilde{\lambda}$ is $\tilde{\lambda}_{th} = 0.6177$. The level of $\tilde{\lambda}$ below which the market will freeze is $\tilde{\lambda}_{freeze} = 0.3950$, and in a market freeze, banks do not want to deviate. As a result, the overall threshold is $\tilde{\lambda}_{th} = 0.6177$. Thus the equilibrium exists when $\tilde{\lambda} < 0.6177$. That is, if the probability that creditors assign to a bank to be H-type when observing it deviate to borrowing from the central bank is lower than 0.6177, this equilibrium will exist.

Given the parameter values in our example, we find that both equilibria can exist and each equilibrium could be realized in equilibrium.

3.3.2 The case with central bank screening

We first show the results of pooling equilibrium where both types of banks borrow from the central bank. If a bank follows the equilibrium strategy of applying for central bank loans and successfully gets the loans, the market belief becomes $\tilde{g} = 0.7568$, and the equilibrium market rate is given by $r_{M,g} = 0.1240$, which is lower than $r_M = 0.1613$ in the case without central bank screening. We find that the threshold of $\hat{r}_{M,g}$ below which an H-type bank will deviate is 0.0895, and the corresponding $\hat{\lambda}_{th,H}$ below which the equilibrium exists is 0.8148, which is higher than $\hat{\lambda}_{th} = 0.7689$ in the case without central bank screening. Thus, this equilibrium is more likely to exist with central bank screening.

Next, we examine the case where neither type of bank borrows from the central bank. When banks follow the equilibrium strategy, they will borrow D on the market. $r_M = 0.1613$ is the same as in the case without central bank screening. The threshold level of $\tilde{r}_M$ below which an H-type bank will deviate is also the same as the $\tilde{r}_{M,devi}$ in the case without central bank screening. However, $\tilde{r}_M$ depends now on $\tilde{g}$ in the same way as it depends on $\tilde{\lambda}$ in the case without central bank screening. Thus $\tilde{g}_{th,H} = \tilde{\lambda}_{th} = 0.6177$, and the corresponding $\tilde{\lambda}_{th}$ is 0.5479. Similarly, $\tilde{g}_{freeze} = 0.3950$, and the corresponding $\tilde{\lambda}_{freeze}$ is 0.3287. As a result, the threshold of deviation is $\tilde{\lambda}_{th} = 0.5479$, which is lower than $\tilde{\lambda}_{th} = 0.6177$ in the case without central bank screening. Banks will not deviate when the market freezes. Thus this equilibrium exists when $\tilde{\lambda} < 0.5479$. Since without central bank screening this equilibrium exists when $\tilde{\lambda} < 0.6177$, we can conclude that this equilibrium is less likely to exist than in the case without central bank screening.

4 Central bank screening and banks' ex ante choices

The LOLR policy is often thought to be controversial because it may induce moral hazard. In this section, we extend the model to allow banks to choose between a safe and risky assets at date 0. We will examine how the LOLR policy with central bank screening may affect banks' ex ante choices. Our model reveals that an LOLR policy with high precision in central bank screening can actually reduce moral hazard. Moreover, if a central bank can commit to a specific screening precision level before the banks choose their assets, instead of conducting a discretionary LOLR policy, the central bank will attain higher social welfare by choosing a higher screening precision level and reducing moral hazard.

4.1 A model where ϕ is exogenously given

We first study the case where ϕ is exogenously given. Later we will examine the case where the central bank optimally chooses ϕ.

We assume that there is a continuum of banks with mass 1. A typical bank can choose between a safe and a risky long-term asset at date 0. The safe asset will mature at date 2 with a return of $R_H > 1$. With a probability of π, the risky asset's return at date 2 is R_H, and with a probability of $1 - \pi$ its return is uncertain. With a probability of p, its return is R_H, and with a probability of $1 - p$, its return is $R_L < 1$. Here we can interpret $1 - \pi$ as the probability of a financial crisis, such as the recent subprime mortgage crisis, that is an aggregate shock to all the risky assets. For simplicity, we assume that the return of all the risky assets are perfectly correlated. Similar to the previous models, each asset has a fixed size of A, and each bank finances its asset by its equity of e_0 and its short-term debts of D. Other assumptions in the previous models about the two types of banks remain unchanged here.

We assume that banks can derive private benefits from investing in risky assets, but cannot derive any private benefit from investing in safe assets.[7] Banks are heterogenous in terms of private benefits that they derive. More specifically, a bank derives a private benefit of $PB_i = a_0 + a_1 h_i$ from investing in risky assets, where $a_0 \geq 0$ and $a_1 > 0$ are constant. Each bank has a different h_i that we assume is uniformly distributed between

[7]We introduce private benefits by following Holmstrom and Tirole (1997). The private benefit can be thought of as actual benefits that a manager can derive or as the costs reduced by adopting a less strict risk management procedure.

0 and 1 among all the banks. As a result, a bank with a higher h_i will have a stronger incentive to choose the risky asset. We denote a bank with a private benefit of PB_i by h_i.

4.1.1 Equilibrium Characterization

Consider the optimal choices of banks when the precision of central bank screening, ϕ, is exogenously given. In order to compute the expected payoff of the banks, we must first specify which equilibrium will happen at date 1. Recall that there may be multiple equilibria in this model. Here we assume that the pooling equilibrium where all the banks apply for central bank loans exists, and the central bank can always coordinate all the banks towards this equilibrium.[8] In addition, we focus on the the case with no market freeze in equilibrium.

Proposition 7 characterizes the symmetric trigger strategy equilibrium in this model.

Proposition 7. *There exists a symmetric trigger strategy Nash equilibrium at date 0 in this model. In this equilibrium, all the banks with h_i below a threshold level of h_1 will choose the safe asset. All the banks with h_i above the threshold level of h_1 will choose the risky asset. h_1 is determined by the following equation:*

$$(1-\pi)[1-(1-\phi)p][AR_H - D(1+r_{M,g_1}) + L_{CB}(r_{M,g_1} - r_{CB})] = a_0 + a_1 h_1 \tag{26}$$

Here

$$r_{M,g_1} = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{g_1 + (1-g_1)p} - 1\right) \tag{27}$$

where

$$g_1 = \frac{h_1}{h_1 + (1-h_1)(1-\phi)} \tag{28}$$

Proof: see the appendix. ■

The intuition behind proposition 7 is as follows. In this equilibrium, bank h_1 must be indifferent between the safe and risky assets. Equation (26) gives bank h_1's indifference condition. The LHS of equation (26) is the gap of bank h_1's expected equity values

[8] We believe that this assumption is realistic. The central bank has an incentive to coordinate all the banks towards this equilibrium, because its screening will improve social welfare only in this equilibrium. Since the no-deviation condition for this equilibrium depends crucially on the central bank's lending conditions of L_{CB} and r_{CB}, the central bank can ensure that this equilibrium exists by setting L_{CB} high and r_{CB} low. In addition, a higher ϕ can also make this equilibrium more likely.

between the safe and risky assets. The RHS is the private benefit that bank h_1 derives from choosing the risky asset. Thus bank h_1 is indifferent if and only if equation (26) holds. Any bank with $h_i > h_1$ has the LHS less than the RHS and will prefer the risky asset. Any bank with $h_i < h_1$ has the LHS greater than the RHS and will prefer the safe asset.

4.1.2 Numerical Examples

Here we give a numerical example to illustrate how the equilibrium threshold level of h_i, h_1, is determined. Note that the equilibrium proportion of H-type banks, λ, equals exactly h_1. Let $\pi = 0.9$, meaning that, with a probability of 0.9, no financial crisis occurs. Let $a_0 = -0.042$ and $a_1 = 0.08$. Parameter values are chosen such that solutions to h_1 exist when $h_1 = \lambda \in (\lambda_{freeze}, 1)$, given that $\phi \in [0, 1]$.

Figure 1 illustrates how h_1, or equivalently the equilibrium λ, is determined when $\phi = 0.5$. The private benefit curve intersects with the expected payoff difference at $h_1 = \lambda = 0.8219$. At this point, the bank is indifferent between the two assets. For the banks with $h < h_1$, their private benefit is lower than the payoff difference, and it will choose the safe asset. For banks with $h > h_1$, we have the opposite result. Figure 2

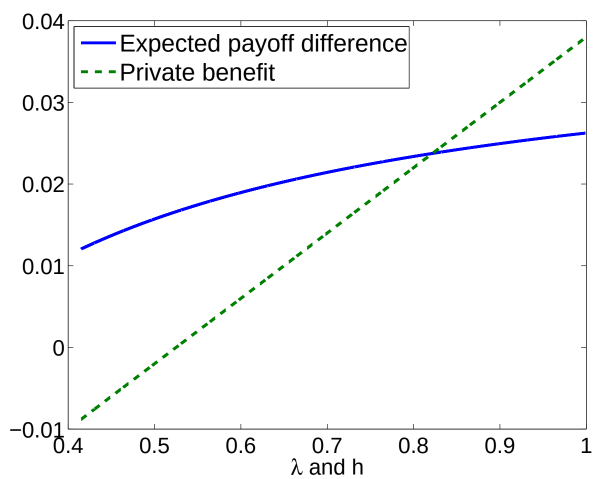

Figure 1: The determination of equilibrium λ given $\phi = 0.5$.

shows the values of equilibrium λ at different levels of ϕ. We can see that higher values of ϕ will induce more banks to choose the safe asset. This is mainly because with higher values of ϕ, insolvent banks will be more likely to be identified and fail to get central

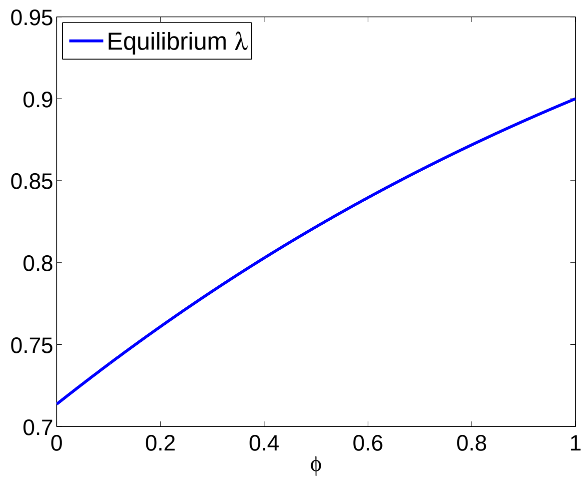

Figure 2: Equilibrium λ at different levels of ϕ.

bank loans, inducing a lower expected payoff for the risky asset. Thus we can see that the LOLR policy with central bank screening can effectively reduce moral hazard, especially when the precision in central bank screening is high.

4.1.3 A special case where $\phi = 0$: moral hazard with and without the LOLR policy

The LOLR policy is often criticized because it may induce moral hazard. This argument implicitly assumes that the market has perfect information and will refuse to lend to insolvent banks. As a result, only insolvent banks will approach the central bank for a loan, and the LOLR policy will save only the insolvent banks and induce moral hazard. However, as we argued in the introduction, during a financial crisis when the LOLR policy is needed, neither the central bank nor the market can distinguish between insolvent and solvent banks. Here we examine how the LOLR policy will affect moral hazard in this more realistic case. Our result is surprising: The LOLR policy will reduce moral hazard instead of inducing moral hazard. In other words, the moral hazard problem is actually *less* severe with the LOLR policy than without it, even when the central bank cannot screen insolvent banks in its LOLR policy (that is, $\phi = 0$). Figure 3 shows the result.

Note that when there is no central bank loan, the market will be in a freeze as long as $\lambda < \lambda_{freeze,NCB} = 0.5$. So a market freeze is more likely to happen after a shock without central bank loans than with central bank loans. Since we focus on the case without

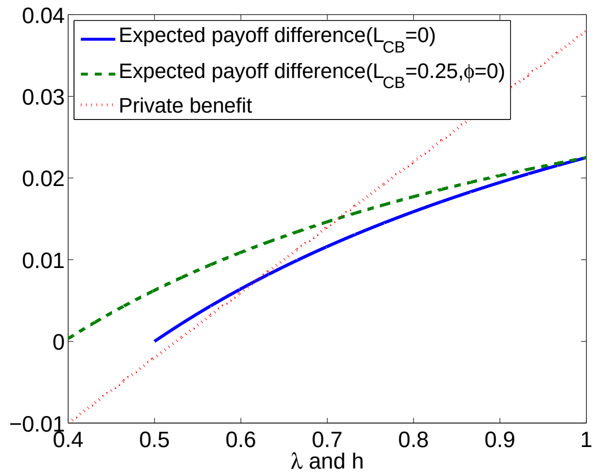

Figure 3: The determination of equilibrium λ when $L_{CB} = 0$ and when $L_{CB} = 0.25$ with $\phi = 0$.

market freezes, for the case of $L_{CB} = 0$, we show the expected payoff difference only for $\lambda > \lambda_{freeze,NCB}$. We can see that when $\lambda < 1$, the expected payoff difference in the case of $L_{CB} > 0$ and $\phi = 0$ is higher than that in the case of $L_{CB} = 0$. However, when $\lambda = 1$, the two values are the same because $r_M = r_{CB} = 0$ and central bank loans make no difference. The equilibrium λ with the LOLR policy is 0.7137, while the equilibrium λ without central bank loan is 0.6181. Therefore, the benefit provided by central bank loans actually encourages more banks to choose the safe asset, consequently reducing moral hazard instead of inducing it.

We can analytically prove that the expected payoff difference is higher with central bank loans at $\phi = 0$ than without central bank loans, inducing a higher equilibrium λ. The proof is given in appendix I. The intuition is as follows. With central bank loans at an interest rate of $r_{CB} < r_M$, banks' equity value is raised by $L_{CB}(r_M - r_{CB})$ as long as it is positive. However, this benefit disappears when a bank's equity value is zero. After the shock, H-type banks always have a positive equity value, while L-type banks have a positive equity value only in the up state with a probability of $p < 1$. As a result, central bank loans raise H-type banks' expected payoff more than L-type banks'. Therefore, the safe asset becomes more attractive and the equilibrium λ is higher.

Note that as our previous analysis showed, given the same level of L_{CB}, when ϕ is higher than zero, the expected payoff difference will be even higher with central bank loans, which will further increase λ and reduce moral hazard.

4.2 The optimal ϕ under pre-commitment policy

"Constructive ambiguity" is often discussed to prevent moral hazard associated with the LOLR policy. Here we argue that if the central bank can commit to a specific screening precision level at date 0, then it will choose a higher precision level than if it cannot commit at date 0. As a result, fewer banks will invest in the risky asset at date 0, and moral hazard will be reduced. Thus, our model reveals that when the central bank can screen insolvent banks as an LOLR, a clearly specified pre-committed LOLR policy in the first place can actually reduce moral hazard.

Now suppose the central bank can commit to a specific level of ϕ at date 0. In order to find the optimal pre-committed level for the central bank, we need to define the central bank's objective function. So far we have assumed that the precision of central bank screening is exogenously given. Now we relax this assumption by assuming that the central bank has a screening technology as follows: to attain a precision of ϕ, the central bank will incur a cost of $aA\phi^2$, where $a > 0$ is a constant. Note that the cost is convex, meaning that the marginal cost for additional precision is higher when the precision increases. Here we assume that $r_{CB} = 0$ for simplicity. Again, we assume that the pooling equilibrium where both types of banks apply for central bank loans exist, and the central bank can always coordinate all the banks towards this equilibrium. Next, we define the central bank's ex ante expected loss function as:

$$\begin{aligned} EL^{ex\ ante} &= (1-\lambda)A[R_H - \phi\gamma_L - (1-\phi)(pR_H + (1-p)R_L)] + aA\phi^2 + \\ &\quad b(1-\phi)(1-\lambda)(1-p)L_{CB}(1 - \frac{AR_L}{D}) \end{aligned} \tag{29}$$

where b is the weight the central bank assigns to the loss from lending to L-type banks. A higher b means that a central bank is more reluctant to use taxpayers' money to finance insolvent banks.

Thus the central bank's ex ante expected loss function consists of three components. The first component is the expected output losses caused by the proportion $1-\lambda$ of banks choosing the risky asset instead of the safe one. $A[R_H - \phi\gamma_L - (1-\phi)(pR_H + (1-p)R_L)]$ gives the expected output gap between the safe and risky assets. The second component is the costs incurred by employing the screening technology. The third component is the expected losses caused by lending to L-type banks, because they may fail to repay their full debts. It is obvious that the second component is decreasing in ϕ, and the first and

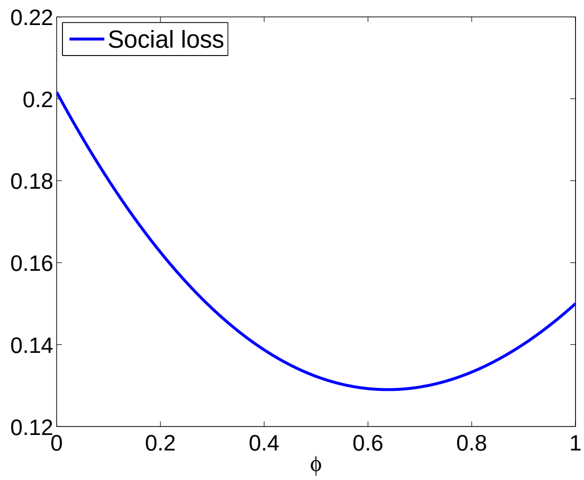

Figure 4: Ex ante social losses given different levels of ϕ.

third components are increasing in ϕ.

Note that, strictly speaking, all the above three losses will occur only with a probability of $1 - \pi$ when a financial crisis occurs. We do not multiply each component by $1 - \pi$ because it will not affect our results.

The central bank minimizes its expected social loss by choosing an optimal level of ϕ. Note that since the central bank can commit to a specific level of ϕ, λ is now a function of ϕ that is given by equation (26). The closed-form solution to the central bank's loss minimization problem is not available. Here we give a numerical example with $a = 0.1$ and $b = 1$ to illustrate the intuition. Figure 4 shows the result. Figure 4 shows the result. The social loss reaches its minimum value of 0.1290 when $\phi^*_{commit} = 0.6380$. The resultant equilibrium λ is $\lambda^*_{commit} = 0.8461$. That is, with this pre-committed LOLR policy, 84.61% of banks will choose the safe asset at date 0.

4.3 The optimal ϕ under discretionary policy

Now we consider the case where the central bank cannot commit to a specific level of ϕ at date 0, that is, the central bank conducts discretionary policy. Backward induction will be used to find the equilibrium in this case. First, at date 1, the central bank will minimize its loss function, taking λ as given. Note that the central bank's ex post loss function is now given by

$$EL^{ex\ post} = (1-\lambda)(1-\phi)A[\gamma_L - pR_H - (1-p)R_L] + aA\phi^2 + b(1-\phi)(1-\lambda)(1-p)L_{CB}(1-\frac{AR_L}{D}) \quad (30)$$

Its first component differs from the one in the commitment case. This is because now the central bank takes λ as given, and the first best allocation occurs when all the L-type banks liquidate at date 1. Thus the central bank's expected output gap between the actual and first best allocation for each L-type bank is $\gamma_L A - \phi\gamma_L A - (1-\phi)(pR_H + (1-p)R_L)A = (1-\phi)A[\gamma_L - pR_H - (1-p)R_L]$.

The first order condition gives us $\phi^* = min\{\frac{1-\lambda}{aA}[\gamma_L A - pAR_H - (1-p)AR_L + b(1-p)L_{CB}(1-\frac{AR_L}{D})], 1\}$.[9] It is straightforward to see that the central bank will choose a higher ϕ if the proportion of L-type banks is large ($1-\lambda$ is high), the screening technology cost is cheap (a is low), the social cost of an unliquidated asset of L-type banks is high ($\gamma_L A - pAR_H - (1-p)AR_L$ is high), and the expected loss from lending to L-type banks is high ($(1-p)L_{CB}(1-\frac{AR_L}{D})$ is high).

Now we move back to date 0. The rational banks will take into account the optimal choice of the central bank at each level of λ, and choose between the safe and risky assets. Therefore there exists an equilibrium level for both λ and ϕ, $\lambda^*_{discretion}$ and $\phi^*_{discretion}$ such that given that $\phi = \phi^*_{discretion}$, the trigger strategy $h_1 = \lambda^*_{discretion}$ is optimal for all the banks. On the other hand, given that $\lambda = \lambda^*_{discretion}$, the central bank will optimally choose $\phi = \phi^*_{discretion}$ to minimize its ex post loss function.

The closed-form solutions for $\lambda^*_{discretion}$ and $\phi^*_{discretion}$ are not available. Here we give a numerical example with the same parameter values as in the commitment case to illustrate the result. We find the solutions numerically as follows. First, we know that at each given level of ϕ, we will have a corresponding equilibrium trigger strategy h_1 among banks at date 0. Thus h_1 is a function of ϕ that we denote by $h_1 = \Gamma(\phi)$. Second, as we find above, at each given level of $\lambda = h_1$, the central bank will choose a corresponding optimal ϕ. Thus ϕ is a function of h_1. The equilibrium occurs at the intersection of the two functions.

Figure 5 shows the equilibrium. It turns out that under the discretionary policy, $\lambda^*_{discretion} = 0.8032$, and the corresponding $\phi^*_{discretion} = 0.4019$. Recall that under the pre-commitment policy, $\phi^*_{commit} = 0.6380$ and $\lambda^*_{commit} = 0.8461$. Then we can tell that

[9] The second order condition is satisfied to guarantee a minimum solution.

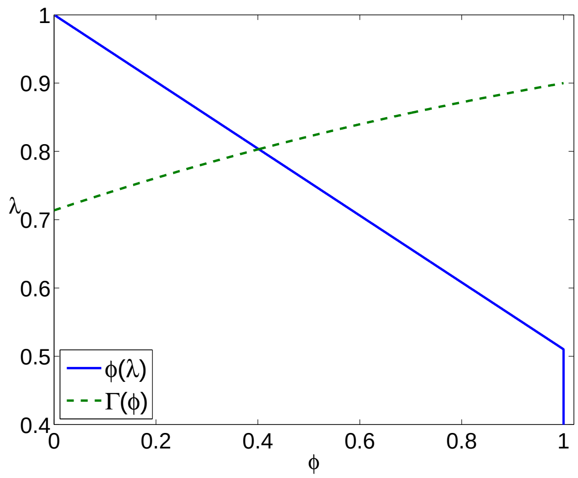

Figure 5: Equilibrium λ and ϕ under the discretionary policy

both equilibrium ϕ and λ are higher under the pre-commitment policy than under the discretionary policy. Moreover, we find that the ex ante social welfare loss defined by equation (29) under the discretionary policy is 0.1385, which is higher than the one of 0.1290 under the pre-commitment policy. Thus, we find that if the central bank can commit to an LOLR policy with an optimal screening precision level, it will attain higher social welfare and reduce moral hazard. The reason that the central bank will choose a higher ϕ under the pre-commitment policy is that the central bank has an incentive to use the LOLR policy to affect banks' choices at date 0 if it can commit. If it cannot commit, this incentive disappears. Thus the central bank will choose a higher ϕ to deter more banks from choosing the risky asset at date 0 if it can commit.

5 Conclusions

This paper studies the LOLR policy when insolvent banks have an incentive to gamble for resurrection, and the central bank cannot distinguish between illiquidity and insolvency. We find that when the central bank cannot screen insolvent banks in its LOLR policy, both the pooling equilibria in which, on one hand, all the banks borrow from the central bank and, on the other hand, all the banks do not borrow from the central bank, could exist, conditional on creditors' beliefs off the equilibrium path. However, neither of the equilibria is socially efficient because insolvent banks will inefficiently continue to operate. When the central bank can screen insolvent banks imperfectly in its LOLR policy, we find that both the pooling equilibria could exist too. However, central bank screening makes the pooling equilibrium where all the banks apply for central bank loans more likely and the equilibrium where all the banks do not apply for central bank loans less likely. Finally, we find that if we allow banks to choose their assets at the beginning of the model, the precision in central bank screening will greatly affect banks' decisions. An LOLR policy with a high precision in central bank screening can greatly improve social welfare not only by singling out insolvent banks and forcing them to liquidate early, but also by deterring banks from choosing risky assets in the first place. In other words, an LOLR policy with a high precision in central bank screening can reduce moral hazard instead of inducing moral hazard. Moreover, we find that if a central bank can commit to a specific precision level before the banks choose their assets, rather than conducting a discretionary LOLR policy, it will choose a higher precision level, reduce moral hazard, and attain higher social welfare.

A Proof of condition 5

We know that a bank will never borrow on the market if $1 + r_M > \frac{R_H}{\gamma}$, where $\gamma = \gamma_H$ for H-type banks and $\gamma = \gamma_L$ for L-type banks. This is because with such a high market rate, the bank is always better off by liquidating its own assets (please see appendix B for a rigorous proof). As a result, the highest possible return rate that a creditor can gain is $1 + r_M = \frac{R_H}{\gamma_L}$ (because $\gamma_L < \gamma_H$), and the highest possible expected return that a creditor can gain is $p\frac{R_H}{\gamma_L} + (1-p)\frac{AR_L}{D}$. By assuming $p\frac{R_H}{\gamma_L} + (1-p)\frac{AR_L}{D} < 1$, creditors' expected return rate from lending to an L-type bank can never exceed 1. Thus they will never lend to an L-type bank. ■

B Proof of proposition 1

We first prove the optimal choices for H-type banks. More specifically, we prove that when $1 + r_M < \frac{R_H}{\gamma_H}$, an H-type bank will always roll over all of its debts by borrowing on the market and will never liquidate its asset. On the other hand, when $1 + r_M > \frac{R_H}{\gamma_H}$, an

H-type bank will never borrow on the market and will always liquidate its asset until its debts are repaid.

First, we examine the case where an H-type bank does not borrow from the central bank. In this case, suppose that the bank chooses to liquidate l_H of its asset, where $0 \leq l_H \leq A$. Thus its net asset value is given by

$$NV_H = (A - l_H)R_H - (D - \gamma_H l_H)(1 + r_M) \tag{31}$$

The first-order derivative of NV_H with respect to l_H is given by $\frac{\partial NV_H}{\partial l_H} = -R_H + \gamma_H(1+r_M)$. It is straightforward to see that given that $1+r_M < \frac{R_H}{\gamma_H}$, $\frac{\partial NV_H}{\partial l_H} < 0$. Thus $l_H^* = 0$. That is, an H-type bank will never liquidate its asset when $1+r_M < \frac{R_H}{\gamma_H}$. Given that $1+r_M > \frac{R_H}{\gamma_H}$, $\frac{\partial NV_H}{\partial l_H} > 0$. Thus $l_H^* = A$. That is, an H-type bank will liquidate all of its asset when $1 + r_M > \frac{R_H}{\gamma_H}$. Note that the bank will have to liquidate all of its asset and go bankrupt in this case because we assume $\gamma_H A < D$.

Next, we examine the case where an H-type bank borrows from the central bank. In this case, suppose that the bank chooses to liquidate l_H of its asset, where $0 \leq l_H \leq A$. Thus its net asset value is given by

$$NV_H = (A - l_H)R_H - (D - \gamma_H l_H - L_{CB})(1 + r_M) - L_{CB}(1 + r_{CB}) \tag{32}$$

The first-order derivative of NV_H with respect to l_H is given by $\frac{\partial NV_H}{\partial l_H} = -R_H + \gamma_H(1+r_M)$, which is identical to the one in the previous case. Thus when $1+r_M < \frac{R_H}{\gamma_H}$, $l_H^* = 0$. When $1 + r_M > \frac{R_H}{\gamma_H}$, the bank will never borrow on the market and will liquidate its asset until its debts of $D - L_{CB}$ are repaid.

Now we prove the optimal choices for L-type banks. More specifically, we prove that when $1 + r_M < \frac{R_H}{\gamma_L}$, an L-type bank will always roll over all of its debts by borrowing on the market and will never liquidate its asset. On the other hand, when $1 + r_M > \frac{R_H}{\gamma_L}$, an L-type bank will never borrow on the market and will liquidate its asset until its debts are repaid.

In order to prove the above results, we first prove that an L-type bank's net asset value is always negative such that its equity value is always zero in the down state. The proof is as follows. No matter whether an L-type bank borrows from the central bank or not, the maximum payoff that an L-type bank can gain from its asset in the down state is the one when it liquidates all the asset at date 1, $A\gamma_L$. This is because $\gamma_L > R_L$ by assumption.

No matter whether an L-type bank borrows from the central bank or not, the minimum repayment for an L-type bank's debt is D when it is charged a zero interest rate. Thus, the maximum net asset value for an L-type bank is $A\gamma_L - D$. However, by assumption, $A\gamma_L < D$. Thus, an L-type bank's net asset value in the down state is always negative. As a result, an L-type bank's equity value in the down state is always zero. This result implies that an L-type bank aims only at maximizing its equity value in the up state.

An L-type bank's net asset value in the up state is the same as that of an H-type bank except that now γ_L replaces γ_H. Thus we prove the results. ■

C Proof of proposition 2

We first examine a separating equilibrium in which only L-type banks borrow from the central bank and H-type banks do not borrow from the central bank.

In order to find out whether this equilibrium exists or not, we compare the payoffs of each type of banks when they follow the equilibrium strategy and when they deviate. For an H-type bank, if it follows the equilibrium strategy of not borrowing from the central bank, then the market will believe that it is H-type. Therefore, it can borrow on the market at a zero interest rate. In this case, its equity at date 2 will be

$$e_H = AR_H - D \tag{33}$$

If it borrows from the central bank, the market will believe that it is L-type. Due to condition (5) that we impose, short-term creditors will never roll over their debts to it. As a result, the bank has to liquidate its asset to meet the liquidity need of $D - L_{CB}$. The liquidated asset is decided according to

$$\gamma_H l_H = D - L_{CB} \Rightarrow l_H = \frac{D - L_{CB}}{\gamma_H} \tag{34}$$

and its equity will be

$$e_{H,CB} = (A - l_H)R_H - L_{CB}(1 + r_{CB}) = AR_H - L_{CB}(1 + r_{CB}) - \frac{D - L_{CB}}{\gamma_H}R_H \tag{35}$$

where CB denotes central bank loans. Thus we have

$$e_H - e_{H,CB} = L_{CB}(1 + r_{CB}) + \frac{D - L_{CB}}{\gamma_H}R_H - D \tag{36}$$

Because $1 + r_{CB} \geq 1$, $\frac{R_H}{\gamma_H} > 1$,

$$L_{CB}(1 + r_{CB}) + \frac{D - L_{CB}}{\gamma_H} R_H > L_{CB} + D - L_{CB} = D \tag{37}$$

Therefore, $e_H - e_{H,CB} > 0$, and an H-type bank has no incentive to deviate. Intuitively, an H-type bank can borrow at the lowest possible rate on the market to meet all its liquidity need if it does not borrow from the central bank. However, if it borrows from the central bank, it will not be able to raise the required liquidity on the market and will have to liquidate assets, which is costly. Thus, it has no incentive to borrow from the central bank.

For an L-type bank, if it follows the equilibrium strategy of borrowing L_{CB} from the central bank, it will need to liquidate assets to meet the liquidity need of $D - L_{CB}$. When $\gamma_L A < D - L_{CB}$, the bank liquidates all the assets and goes bankrupt at date 1. If $L_{CB} + \gamma_L A > D$, then the bank can survive date 1. In this case, l_L is decided according to

$$\gamma_L l_L = D - L_{CB} \Rightarrow l_L = \frac{D - L_{CB}}{\gamma_L} \tag{38}$$

At date 2, its equity values in the up and down states are

$$e_L^u = (A - l_L)R_H - L_{CB}(1 + r_{CB}) = (A - \frac{D - L_{CB}}{\gamma_L})R_H - L_{CB}(1 + r_{CB}) \tag{39}$$

$$e_L^d = 0 \tag{40}$$

If an L-type bank pretends to be H-type and does not borrow from the central bank, then the market will believe that it is H-type. Thus it can roll over D at a zero interest rate. Its equity values in the up and downs states are

$$e_{L,NCB}^u = AR_H - D \tag{41}$$

$$e_{L,NCB}^d = 0 \tag{42}$$

Since in both cases the bank gains zero equity in the down state, we need only to compare $e_{L,NCB}^h$ and e_L^h to see in which case its expected equity is higher.

Since $\frac{R_H}{\gamma_L} > 1$

$$\begin{aligned} e_L^u &= (A - \frac{D - L_{CB}}{\gamma_L})R_H - L_{CB}(1 + r_{CB}) \\ &< AR_H - (D - L_{CB}) - L_{CB}(1 + r_{CB}) < AR_H - D = e_{L,NCB}^u \end{aligned} \tag{43}$$

As a result, an L-type bank will deviate, and this separating equilibrium does not exist.

Similarly, when we examine the separating equilibrium where only H-type banks borrow from the central bank, we will find that an L-type bank will always have an incentive to deviate and mimic H-type banks. Again an L-type bank has zero equity in the down state and cares only about its equity value in the up state. If it follows the equilibrium strategy,

$$e_L^u = 0 \tag{44}$$

because it is identified as L-type, and creditors will refuse to roll over their debts. If it deviates by borrowing from the central bank,

$$e_{L,CB}^u = AR_H - (D - L_{CB}) - L_{CB}(1 + r_{CB}) > 0 \tag{45}$$

because we assume that r_{CB} is no higher than the market rate, which is zero in this case. Thus an L-type bank will always deviate in such an equilibrium, and this separating equilibrium cannot exist. ■

D Proof of Proposition 3

For an H-type bank, if it borrows from the central bank, then it can borrow the remaining debts of $D - L_{CB}$ on the market at the market rate r_M. Its equity at date 2 is

$$\begin{aligned} e_H &= AR_H - L_{CB}(1 + r_{CB}) - (D - L_{CB})(1 + r_M) \\ &= AR_H - D(1 + r_M) + L_{CB}(r_M - r_{CB}) \end{aligned} \tag{46}$$

Similarly, for an L-type bank, if it borrows from the central bank, it can borrow the remaining debts of $D - L_{CB}$ on the market at the rate of r_M because the market can not identify the bank's type. Then its equity values in the up and down states are

$$e_L^u = AR_H - L_{CB}(1 + r_{CB}) - (D - L_{CB})(1 + r_M) = e_H \tag{47}$$

$$e_L^d = 0 \tag{48}$$

Note that its equity value is zero in the down state as we proved previously.

The equity value of an individual bank deviating to not borrowing from the central bank is determined as follows. We first look at the case of $\hat{\lambda} \geq \lambda$. In this case, $\hat{r}_M$ is given by equation (8). If an H-type bank deviates, it will borrow D on the market. Its date 2 equity will be

$$\hat{e}_{H,NCB} = AR_H - D(1 + \hat{r}_M) \tag{49}$$

Similarly, if an L-type bank deviates, it will borrow D on the market, and its date 2 equity will be

$$\hat{e}^u_{L,NCB} = AR_H - D(1 + \hat{r}_M) = \hat{e}_{H,NCB} \tag{50}$$

$$\hat{e}^d_{L,NCB} = 0 \tag{51}$$

For an H-type bank not to deviate, we need $e_H \geq \hat{e}_{H,NCB}$, or

$$\begin{aligned} e_H - \hat{e}_{H,NCB} &= [AR_H - D(1 + r_M) + L_{CB}(r_M - r_{CB})] - [AR_H - D(1 + \hat{r}_M)] \\ &= L_{CB}(r_M - r_{CB}) - D(r_M - \hat{r}_M) > 0 \end{aligned}$$

which is condition (9). It is straightforward to see that an L-type bank cares only about its up state equity value, and its no-deviation condition of $e^u_L > \hat{e}^u_{L,NCB}$ is identical to the one for an H-type bank. As a result, as long as condition (9) is satisfied, both types of banks have no incentive to deviate.

Now consider the case where $\hat{\lambda} < \lambda$. When $\hat{\lambda} < \lambda$, $\hat{r}_M > r_M$ as long as an equilibrium rate exists. When it does not exist, a market freeze occurs and the banks will be forced to liquidate their assets and suffer a higher loss than in the case without a market freeze. The no-deviation condition (9) shows that as long as $\hat{r}_M > r_M$, this condition always holds and no banks will deviate.

$\hat{r}_M$ is an decreasing function of $\hat{\lambda}$. As a result, banks are more likely to deviate with a higher $\hat{\lambda}$. In the extreme case of $\hat{\lambda} = 1$, $\hat{r}_M = 0$, and condition (9) becomes $L_{CB}(r_M - r_{CB}) - Dr_M \geq 0$, which cannot be satisfied because $L_{CB} < D$. Thus there exists a threshold level of $\hat{\lambda}$, $\hat{\lambda}_{th} \in (\lambda, 1)$ above which the pooling equilibrium can not exist, where $\hat{\lambda}_{th}$ is determined by

$$L_{CB}(r_M - r_{CB}) - D(r_M - \hat{r}_M(\hat{\lambda}_{th})) = 0$$

which is equation (10). ■

E Proof of proposition 4

First, let us examine the case where $\tilde{\lambda} > \tilde{\lambda}_{freeze}$. In this case, a deviating bank will not face a market freeze. If an H-type bank follows the equilibrium strategy and borrows only from the market, its equity at date 2 will be

$$e_{H,NCB} = AR_H - D(1 + r_M) \tag{52}$$

Similarly, if an L-type bank follows the equilibrium strategy and borrows only from the market, its equity at date 2 in the up and down states will be

$$e^u_{L,NCB} = AR_H - D(1 + r_M) \tag{53}$$
$$e^d_{L,NCB} = 0 \tag{54}$$

If a bank deviates to borrowing from the central bank, it will need to borrow $D - L_{CB}$ on the market at the rate $\tilde{r}_M$ that is given by equation (11). If an H-type bank deviates, its date 2 equity will be

$$\begin{aligned} \tilde{e}_H &= AR_H - L_{CB}(1 + r_{CB}) - (D - L_{CB})(1 + \tilde{r}_M) \\ &= AR_H - D(1 + \tilde{r}_M) + L_{CB}(\tilde{r}_M - r_{CB}) \end{aligned} \tag{55}$$

If an L-type bank deviates, its date 2 equity values in the up and down states are

$$\tilde{e}^u_L = AR_H - L_{CB}(1 + r_{CB}) - (D - L_{CB})(1 + \tilde{r}_M) \tag{56}$$
$$\tilde{e}^d_L = 0 \tag{57}$$

The no-deviation condition for an H-type bank is $e_{H,NCB} \geq \tilde{e}_H$, or

$$\begin{aligned} &[AR_H - D(1 + r_M)] - [AR_H - D(1 + \tilde{r}_M) + L_{CB}(\tilde{r}_M - r_{CB})] \\ &= D(\tilde{r}_M - r_M) - L_{CB}(\tilde{r}_M - r_{CB}) > 0 \end{aligned}$$

which is condition (13). The no-deviation condition for an L-type bank is $e^h_{L,NCB} \geq \tilde{e}^u_L$, which is identical to the one for an H-type bank.

Next, we examine the case where $\tilde{\lambda} < \tilde{\lambda}_{freeze}$. In this case, a deviating bank faces a market freeze and has to liquidate its assets. For an H-type bank,

$$\gamma_H l_H = D - L_{CB} \Rightarrow l_H = \frac{D - L_{CB}}{\gamma_H} \tag{58}$$

and

$$\begin{aligned}\tilde{e}_{H,freeze} &= (A - l_H)R_H - L_{CB}(1 + r_{CB}) \\ &= AR_H - \frac{D}{\gamma_H}R_H + L_{CB}(\frac{R_H}{\gamma_H} - 1 - r_{CB}) \end{aligned} \quad (59)$$

Similarly, for an L-type bank

$$\gamma_L l_L = D - L_{CB} \Rightarrow l_L = \frac{D - L_{CB}}{\gamma_L} \quad (60)$$

and

$$\tilde{e}^u_{L,freeze} = (A - l_L)R_H - L_{CB}(1 + r_{CB}) \quad (61)$$

$$\tilde{e}^d_{L,freeze} = 0 \quad (62)$$

The no-deviation condition for an H-type bank is $e_{H,NCB} \geq \tilde{e}_{H,freeze}$. Using equations (52) and (59), we get

$$\begin{aligned} & e_{H,NCB} - \tilde{e}_{H,freeze} \\ = \; & [AR_H - D(1 + r_M)] - \left[AR_H - \frac{D}{\gamma_H}R_H + L_{CB}(\frac{R_H}{\gamma_H} - 1 - r_{CB})\right] \\ = \; & D\left[\frac{R_H}{\gamma_H} - 1 - r_M\right] - L_{CB}(\frac{R_H}{\gamma_H} - 1 - r_{CB}) > 0 \end{aligned}$$

which is condition (14). An L-type bank's no-deviation condition is $e^u_{L,NCB} \geq \tilde{e}^u_{L,freeze}$, or

$$e^u_{L,NCB} - \tilde{e}^u_{L,freeze} = D\left[\frac{R_H}{\gamma_L} - 1 - r_M\right] - L_{CB}(\frac{R_H}{\gamma_L} - 1 - r_{CB}) > 0 \quad (63)$$

Since $e^u_{L,NCB} = e_{H,NCB}$ and $\tilde{e}^u_{L,freeze} \leq \tilde{e}_{H,freeze}$ (because $l_H \leq l_L$), we can see that the no-deviation conditions for both types of banks are now different. H-type banks have a stronger incentive to deviate than L-type banks. In an equilibrium, both types of banks should have no incentive to deviate. As a result, the no-deviation condition is condition (14).

Note that if the no-deviation condition in the no-market-freeze case (condition (13)) holds, the no-deviation condition in the market freeze case will always hold (condition (13)). This is because the equilibrium rate $\tilde{r}_M$ is always lower than $\frac{R_H}{\gamma_H} - 1$. The intuition behind this result is that a market freeze imposes higher costs on a deviating bank. Thus if a bank does not deviate when the equilibrium market rate, $\tilde{r}_M$, exists, then it will not deviate when the market freezes. As a result, if the no-deviation condition (14) in the market freeze case does not hold, this pooling equilibrium will never exist. This is because a market freeze imposes a higher deviation cost. If a bank deviates when the market freezes, it will certainly deviate when the market does not freeze. ■

F Proof of proposition 5

When banks follow the equilibrium strategy, an H-type bank's equity value is the same as in the basic model (equation (46) except that the market rate is $r_{M,g}$ now. Thus

$$\begin{aligned} e_H &= AR_H - L_{CB}(1 + r_{CB}) - (D - L_{CB})(1 + r_{M,g}) \\ &= AR_H - D(1 + r_{M,g}) + L_{CB}(r_{M,g} - r_{CB}) \end{aligned} \tag{64}$$

If an L-type bank applies for central bank loans, it will be rejected with a probability of ϕ and will be accepted with a probability of $1 - \phi$. Using subscripts Acc and Rej to denote the cases where the loan application is accepted and rejected, respectively, we have

$$e^u_{L,Acc} = AR_H - L_{CB}(1 + r_{CB}) - (D - L_{CB})(1 + r_{M,g}) \tag{65}$$

$$e^d_{L,Acc} = 0 \tag{66}$$

$$e_{L,Rej} = 0 \tag{67}$$

Note that an L-type bank's equity value is higher when the up state is realized than in the basic model without central bank screening, because $r_{M,g} < r_M$. Intuitively, once an L-type bank successfully passes the central bank's screening, it faces a more optimistic creditor belief and a lower market rate that will lower its interest costs.

When banks deviate to not borrowing from the central bank, their payoffs are identical to those in the pooling equilibrium without central bank screening, which are given by $\hat{e}_{H,NCB}$ (equation (49)), $\hat{e}^u_{L,NCB}$ (equation (50)) and $\hat{e}^d_{L,NCB}$ (equation (51)).

An H-type bank's no-deviation condition is $e_H \geq \hat{e}_{H,NCB}$. We have

$$\begin{aligned} & e_H - \hat{e}_{H,NCB} \\ = \ & [AR_H - D(1 + r_{M,g}) + L_{CB}(r_{M,g} - r_{CB})] - [AR_H - D(1 + \hat{r}_M)] \\ = \ & L_{CB}(r_{M,g} - r_{CB}) - D(r_{M,g} - \hat{r}_M) > 0 \end{aligned}$$

which is condition (17).

An L-type bank's no-deviation condition is $(1-\phi)pe^u_{L,Acc} \geq p\hat{e}^u_{L,NCB}$, or $(1-\phi)e^u_{L,Acc} \geq \hat{e}^u_{L,NCB}$. We have

$$\begin{aligned} & (1-\phi)e^u_{L,Acc} - \hat{e}^u_{L,NCB} \\ = \; & (1-\phi)\left[AR_H - D(1+r_{M,g}) + L_{CB}(r_{M,g} - r_{CB})\right] - \left[AR_H - D(1+\hat{r}_M)\right] > 0 \end{aligned} \quad (68)$$

Comparing these two conditions, we can see that $e_H - \hat{e}_{H,NCB} > (1-\phi)e^u_{L,Acc} - \hat{e}^u_{L,NCB}$ such that an H-type bank's no-deviation condition is easier to hold. The intuition behind this result is that an L-type bank cares only about its equity value in the up state. An H-type bank has a higher equity value from applying for central bank loans than an L-type bank in the up state because it will always be identified as H-type. On the other hand, the equity values of H-type banks and L-type banks in the up state are the same when they deviate.

This result differs from the one in the basic model without central bank screening where the no-deviation conditions of both types of banks are the same. This is because now the central bank can distinguish between the two types of banks to some degree, inducing different payoffs for both types.

Note that the RHS of both types of banks' no-deviation condition is strictly decreasing in $r_{M,g}$. Since $r_{M,g}$ is strictly decreasing in $\hat{\lambda}$, there exists a threshold level of $\hat{\lambda}$, say $\hat{\lambda}_{th,H}$, above which an H-type bank will always deviate. Meanwhile, there exists a threshold level of $\hat{\lambda}$, say $\hat{\lambda}_{th,L}$, above which an L-type bank will always deviate. Since $e_H - \hat{e}_{H,NCB} > (1-\phi)e^u_{L,Acc} - \hat{e}^u_{L,NCB}$, we have $\hat{\lambda}_{th,H} > \hat{\lambda}_{th,L}$.

Can we argue that the equilibrium will not exist whenever $\lambda > \hat{\lambda}_{th,L}$ because the L-type bank will deviate? The answer is no if we use the intuitive criterion to refine the belief off the equilibrium path. The reason is as follows. Suppose creditors have a belief off the equilibrium path, $\hat{\lambda} \in (\hat{\lambda}_{th,L}, \hat{\lambda}_{th,H})$. Then given $\hat{\lambda}$ and the corresponding $\hat{r}_M$, creditors know that an L-type bank will indeed deviate, while an H-type bank will not deviate. Then creditors should conclude that at this particular level of $\hat{r}_M$, the bank that deviates can only be L-type. As a result, the market freezes, and the bank cannot borrow on the market. Thus, L-type banks will not deviate when $\hat{\lambda} \in (\hat{\lambda}_{th,L}, \hat{\lambda}_{th,H})$. This implies that in this range of the belief off the equilibrium path, the equilibrium will exist. Only when $\hat{\lambda} > \hat{\lambda}_{th,H}$ where both types of banks will deviate, then can we say that the pooling equilibrium will not exist. Note that $\hat{\lambda}_{th,H}$ is determined by

$$L_{CB}(r_{M,g} - r_{CB}) - D(r_{M,g} - \hat{r}_M(\hat{\lambda}_{th,H})) = 0$$

which is equation (18). ■

G Proof of proposition 6

When banks follow the equilibrium strategy and borrow only on the market, both types of banks' payoffs are the same as in the basic model without central bank screening (equations (52), (53), and (54)) that we replicate here:

$$\begin{aligned} e_{H,NCB} &= AR_H - D(1 + r_M) \\ e^u_{L,NCB} &= AR_H - D(1 + r_M) \\ e^d_{L,NCB} &= 0 \end{aligned}$$

Now consider banks' payoffs when they deviate. We first consider the case without market freeze ($\tilde{\lambda} > \tilde{\lambda}^{CBS}_{freeze}$). The market rate $\tilde{r}_{M,g}$ is given by equation (20). If an H-type bank deviates, its date 2 equity will be

$$\begin{aligned} \tilde{e}_H &= AR_H - L_{CB}(1 + r_{CB}) - (D - L_{CB})(1 + \tilde{r}_{M,g}) \\ &= AR_H - D(1 + \tilde{r}_{M,g}) + L_{CB}(\tilde{r}_{M,g} - r_{CB}) \end{aligned} \tag{69}$$

If an L-type bank deviates, it will be identified as L-type and gain zero equity with a probability of ϕ. With a probability of $1 - \phi$, it can successfully get central bank loans and then borrow on the market at the rate of $\tilde{r}_{M,g}$. Its equity values in different cases are specified as follows.

$$\tilde{e}_{L,Rej} = 0 \tag{70}$$

$$\tilde{e}^u_{L,Acc} = AR_H - L_{CB}(1 + r_{CB}) - (D - L_{CB})(1 + \tilde{r}_{M,g}) = \tilde{e}_H \tag{71}$$

$$\tilde{e}^d_{L,Acc} = 0 \tag{72}$$

An H-type bank's no-deviation condition is $e_{H,NCB} \geq \tilde{e}_H$, or

$$\begin{aligned} & [AR_H - D(1 + r_M)] - [AR_H - D(1 + \tilde{r}_{M,g}) + L_{CB}(\tilde{r}_{M,g} - r_{CB})] \\ = \; & D(\tilde{r}_{M,g} - r_M) - L_{CB}\,(\tilde{r}_{M,g} - r_{CB}) > 0 \end{aligned}$$

which is condition (21).

An L-type bank's no-deviation condition is $pe^u_{L,NCB} > (1-\phi)p\tilde{e}^u_{L,Acc}$, or

$$[AR_H - D(1+r_M)] - (1-\phi)[AR_H - D(1+\tilde{r}_{M,g}) + L_{CB}(\tilde{r}_{M,g} - r_{CB})] > 0 \quad (73)$$

It is obvious to see that an L-type bank's deviation payoff is lower than an H-type bank. As a result, an H-type bank has a stronger incentive to deviate than an L-type bank.

Second, consider the case of $\tilde{\lambda} < \tilde{\lambda}^{CBS}_{freeze}$. In this case, banks face a market freeze when deviating. An H-type bank's deviation payoff is the same as in the basic model (equation (59)), which we replicate here:

$$\tilde{e}_{H,freeze} = AR_H - \frac{D}{\gamma_H}R_H + L_{CB}(\frac{R_H}{\gamma_H} - 1 - r_{CB})$$

where $l_H = \frac{D-L_{CB}}{\gamma_H}$. As a result, given that there is a market freeze, H-type banks' no-deviation condition is the same as in the basic model (condition (14)), which we replicate here:

$$e_{H,NCB} - \tilde{e}_{H,freeze} = D\left[\frac{R_H}{\gamma_H} - 1 - r_M\right] - L_{CB}(\frac{R_H}{\gamma_H} - 1 - r_{CB}) > 0$$

which is condition (22).

If an L-type bank deviates, its expected equity value is given by:

$$\begin{aligned} \tilde{e}_{L,devi} &= 0 \times (1-p) + p(1-\phi)\tilde{e}^u_{L,freeze} + p\phi \times 0 \\ &= p(1-\phi)\tilde{e}^u_{L,freeze} \end{aligned} \quad (74)$$

where $\tilde{e}^u_{L,freeze}$ is given by equation (61). As a result, its no-deviation condition changes into $p\tilde{e}^u_{L,Acc} > \tilde{e}_{L,devi}$, or

$$\tilde{e}^u_{L,Acc} - (1-\phi)\tilde{e}^u_{L,freeze} = e_{H,NCB} - (1-\phi)\tilde{e}^u_{L,freeze} > 0 \quad (75)$$

Recall that $\tilde{e}^u_{L,freeze} \leq \tilde{e}_{H,freeze}$ because H-type banks have a higher liquidation rate ($\gamma_H \geq \gamma_L$). Meanwhile, $0 < \phi < 1$. As a result, H-type banks have a stricter no-deviation condition. That is, H-type banks are more likely to deviate.

Next we will argue that in both cases with and without market freeze, the essential no-deviation condition is the one for H-type banks. Again, we will also always use the intuitive criterion to argue that once H-type banks deviate, L-type banks will always deviate to mimic H-type ones. Otherwise, they will be identified as L-type and suffer the highest loss of zero equity.

Note that once condition (21) is satisfied, condition (22) will always be satisfied, because $\frac{R_H}{\gamma_H} - 1 > \tilde{r}_{M,g}$ and $D > L_{CB}$. Thus, similar to the basic model, a necessary condition for this equilibrium to exist is that when $\tilde{\lambda} < \tilde{\lambda}^{CBS}_{freeze}$, condition (22) is satisfied. If condition (22) is not satisfied, such an equilibrium can never exist. ■

H Proof of proposition 7

First, we find the threshold level of h_i, h_1. A bank with a private benefit of $PB(h_1)$ must be indifferent between investing in safe and risky assets. Its expected date 2 equity value from investing in the safe asset is given by:

$$Ee^s = \pi e^s_{H,noshock} + (1-\pi)e^s_{H,shock} \tag{76}$$

Here $e^s_{H,noshock} = AR_H - D$ denotes the bank's equity value at date 2 when no crisis hits the economy. In this case, all the assets will mature with a return of R_H, and the banks will roll over their debts at the riskless rate of zero. $e^s_{H,shock}$ denotes the bank's equity value at date 2 when a crisis hits the economy. In this case, a proportion $\lambda = h_1$ of banks will be H-type, and a proportion $1-\lambda = 1-h_1$ of banks will be L-type. The equity value at date 2 for the bank to choose the safe asset is given by equation (64), which we replicate here:

$$\begin{aligned} e^s_{H,shock} &= AR_H - L_{CB}(1+r_{CB}) - (D - L_{CB})(1+r_{M,g}) \\ &= AR_H - D(1+r_{M,g}) + L_{CB}(r_{M,g} - r_{CB}) \end{aligned}$$

where $r_{M,g}$ is given by equation (16), which we replicate here:

$$r_{M,g} = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{g+(1-g)p} - 1\right)$$

Here $g = \frac{\lambda}{\lambda+(1-\lambda)(1-\phi)}$ and $\lambda = h_1$. Note that here we focus on the case with no market freeze.

As a result, the bank's expected equity from choosing the safe asset is

$$Ee^s = \pi(AR_H - D) + (1-\pi)(AR_H - D(1+r_{M,g}) + L_{CB}(r_{M,g} - r_{CB})) \tag{77}$$

If the bank chooses the risky asset, its expected equity value at date 2 is given by:

$$Ee^r = \pi e^r_{H,noshock} + (1-\pi)e^r_{H,shock} \tag{78}$$

Here $e^r_{H,noshock} = e^s_{H,noshock}$ denotes the bank's equity value at date 2 when no crisis hits the economy. $e^r_{H,shock}$ denotes the bank's equity value at date 2 when a crisis hits the economy. We have

$$e^r_{H,shock} = (1-p) \times 0 + p\phi \times 0 + p(1-\phi)e^u_{L,Acc} \tag{79}$$

where $e^u_{L,Acc} = e^s_{H,shock}$ is given by equation (64). We have the above equation because when a crisis occurs, with a probability of $1-p$, the down state is realized and the bank's equity is zero. With a probability of $p\phi$, the up state is realized and the bank's application is rejected by the central bank. In this case, the bank's equity is zero too. With a probability of $p(1-\phi)$, the up state is realized and the bank's application is accepted by the central bank. In this case, the bank's equity is given by $e^u_{L,Acc}$.

The bank with $h_i = h_1$ must be indifferent between the two choices. Thus we have

$$Ee^s = Ee^r + (a_0 + a_1 h_1) \tag{80}$$

or

$$\begin{aligned} & \pi e^s_{H,noshock} + (1-\pi)e^s_{H,shock} = \pi e^r_{H,noshock} + (1-\pi)p(1-\phi)e^u_{L,Acc} + (a_0 + a_1 h_1) \\ \Rightarrow \quad & (1-\pi)[1-(1-\phi)p]e^s_{H,shock} = a_0 + a_1 h_1 \end{aligned}$$

with $r_{M,g}$ given at $\lambda = h_1$, which is equation (26).

Next we prove that this trigger strategy h_1 is optimal for each bank. It is straightforward to see that given that each bank follows this trigger strategy, any bank with $h_i > h_1$ will have the RHS of the above equation unchanged, and the LHS higher. Thus it is indeed optimal for it to choose the risky asset. On the other hand, any bank with $h_i < h_1$ will have the RHS unchanged and the LHS lower. Thus it is indeed optimal for it to choose the safe asset. Note that $e^s_{H,shock}$ is strictly increasing in h_1. We assume that the values of a_0 and a_1 are so that a unique solution between 0 and 1 to equation (26) is guaranteed when there is no market freeze. ■

I Proof of the expected payoff differences with and without the LOLR policy

We first examine the case without central bank loans. In this case, at date 1 the equilibrium is essentially the one where all the banks borrow from the market. We focus on the case where the market rate exits and there is no market freeze. The interest rate is r_M given by equation (7) as we found before, which we replicate here.

$$r_M = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{\lambda + (1-\lambda)p} - 1\right)$$

After the shock, an H-type bank's payoff from following the equilibriums strategy is

$$e_{H,NCB} = AR_H - D(1 + r_M) \tag{81}$$

An L-type bank's payoffs in the up and down states from following the equilibriums strategy are the following:

$$\begin{aligned} e^u_{L,NCB} &= AR_H - D(1 + r_M) = e_H \\ e^d_{L,NCB} &= 0 \end{aligned}$$

Thus, a bank's expected payoff from choosing the safe asset is

$$\begin{aligned} Ee^s &= \pi(AR_H - D) + (1-\pi)e_{H,NCB} && (82) \\ &= \pi(AR_H - D) + (1-\pi)(AR_H - D(1 + r_M)) && (83) \end{aligned}$$

A bank's expected payoff from choosing the risky asset is

$$\begin{aligned} Ee^r &= \pi(AR_H - D) + (1-\pi)[pe^u_L + (1-p)0] && (84) \\ &= \pi(AR_H - D) + (1-\pi)p(AR_H - D(1 + r_M)) && (85) \end{aligned}$$

So we get

$$Ee^s - Ee^r = (1-\pi)(1-p)e_{H,NCB} = (1-\pi)(1-p)(AR_H - D(1 + r_M)) \tag{86}$$

In equilibrium, bank h_1 is indifferent between these two assets. Thus we have

$$Ee^s = Ee^r + (a_0 + a_1 h_1) \tag{87}$$

which can be simplified to

$$(1-\pi)(1-p)(AR_H - D(1+r_M)) = a_0 + a_1 h_1 \tag{88}$$

Using equation (7) for r_M and the equilibrium condition $\lambda = h_1$, we have

$$(1-\pi)(1-p)\left[AR_H - D\left[1+\left(1-\frac{AR_L}{D}\right)\left(\frac{1}{\lambda+(1-\lambda)p}-1\right)\right]\right] = a_0 + a_1\lambda \tag{89}$$

This equation determines the equilibrium λ.

Now consider the case with central bank loans but no screening ($\phi = 0$). Again, we assume that the equilibrium where all the banks borrow from the central bank exists and the central bank always coordinates the banks towards this equilibrium.

After the shock, an H-type bank's payoff from following the equilibriums strategy is

$$\begin{aligned} e_H &= AR_H - L_{CB}(1+r_{CB}) - (D - L_{CB})(1+r_M) && (90)\\ &= AR_H - D(1+r_M) + L_{CB}(r_M - r_{CB}) && (91) \end{aligned}$$

An L-type bank's payoffs in the up and down states from following the equilibriums strategy are

$$\begin{aligned} e^u_{L,NCB} &= e_H \\ e^d_{L,NCB} &= 0 \end{aligned}$$

Thus, a bank's date 0 expected payoff from choosing the safe asset is

$$Ee^s = \pi(AR_H - D) + (1-\pi)e_H \tag{92}$$

A bank's date 0 expected payoff from choosing the risky asset is

$$Ee^r = \pi(AR_H - D) + (1-\pi)[pe^u_L + (1-p)0] \tag{93}$$

Thus we have

$$\begin{aligned} Ee^s - Ee^r &= (1-\pi)(1-p)e_H && (94)\\ &= (1-\pi)(1-p)[AR_H - D(1+r_M) + L_{CB}(r_M - r_{CB})] && (95) \end{aligned}$$

In equilibrium, the equilibrium λ is determined as follows:

$$(1-\pi)(1-p)[AR_H - D(1+r_M) + L_{CB}(r_M - r_{CB})] = a_0 + a_1 h_1 \tag{96}$$

Note that the RHS of equations (88) and (96) are the same. Since $\lambda = h_1$ in equilibrium, the LHS of both equations increases in λ (because r_M is decreasing in λ). Note that give the same λ and, consequently, the same r_M, the LHS of equation (96) is higher than that of equation (88) by $L_{CB}(r_M - r_{CB})$ (which is positive because we assume that $r_{CB} < r_M$). This implies that the expected payoff difference curve with central bank loans is always above the one without central bank loans. As a result, the equilibrium λ will be higher with central bank loans. ■

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