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Land-lake Dynamics: Are there Welfare Gains from Targeted Policies in a Heterogeneous Landscape

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Abstract

Water quality within a watershed is strongly linked to human behavior. Nutrient runoff from agricultural lands affects water quality through eutrophication and the emergence of harmful algal blooms (NOAA), causing serious concerns for policy makers, scientists, and residents. Understanding dynamic feedbacks between economic decisions and water resources is essential for long-term policies that balance agricultural management and ecosystem services. We develop a spatial-dynamic model of agricultural decisions in a simulated watershed to examine the welfare implications of targeted policies in landscapes with different levels of spatial heterogeneity. At the steady state, we show that shadow prices of increased phosphorous loading are equal for all farms. We find the socially optimal steady state fertilizer input for a representative farm and derive the optimal tax to that would achieve the socially optimal outcome. Results show that the welfare gains from spatially targeted policies increase with higher levels of heterogeneity in both soil quality index and distance to the lake. When policy implementation costs are proportional to the heterogeneity in the landscape, we show that the welfare gains from targeted policies decrease beyond a certain threshold. This study provides policy insight to help design cost-effective long-term optimal agricultural policies in spatially heterogeneous landscapes and an essential first step towards a coupled model that explores interactions between economic activities and ecosystem services.

Key Words: dynamic optimal control; water quality; agricultural policy; heterogeneity.
JEL Codes: H23, Q51, Q52, Q53

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1. Introduction

Water quality and access to drinking water significantly impact human lives. Rivers, lakes, and reservoirs provid important ecosystem services, recreational amenities, tourism, and employment opportunities. However, non-point source pollution poses a growing concern in freshwater lakes in the U.S. According to Environmental Protection Agency (EPA)'s National Water Quality Assessment, agricultural nonpoint source (NPS) pollution is the leading source of water quality impacts on rivers and streams and the third largest source for freshwater lakes.

Agricultural runoff from heterogeneous landscapes affects water quality, leading to potential eutrophication and harmful algal blooms. Spatial heterogeneity plays an important role in the nutrient runoff process. With the help of improved technology in remote sensing, soil testing, and monitoring, we can gather more information on heterogeneous farmland based on their location, and view the nonpoint source pollution as point source pollution (Khanna et al., 1998; Goetz and Zilberman, 2000; Xabadia, Goetz, and Zilberman, 2008). However, the relationship between fertilizer input and the actual addition of phosphorus stock to receiving water body is not straightforward. Hydrologic models such as the Soil and Water Assessment Tool (SWAT) can help predict the relationship between agricultural decisions and total phosphorus outcome in the lake, but other required input information including land use patterns, elevation, temperature, and precipitation, may not be readily available. Generally, phosphorus is the limiting nutrient in freshwater aquatic systems (Correll, 1999; Michigan Department of Environmental Quality on phosphorus), and total phosphorus loading can be used to predict magnitude of HABs (Stumpf et al. 2012). Therefore, in this work, we focus on phosphorus as agricultural fertilizer in the model, and use phosphorus stock as the indicator for water quality damage in the lake. In this paper, we construct a simplified function to link the addition to phosphorus stock in the lake with soil quality and distance to the lake, which represents the absorption ability of the land and the absorption of nutrient along the transportation process.

Understanding dynamic feedbacks between economic decisions and water quality is essential for long-term policies that balance agricultural productivity and the land-lake dynamic system. Complex tradeoffs between agricultural decisions and water quality emerge because of the inter-temporal nature of phosphorus accumulation from heterogeneous farms that vary in location and soil quality. Spatial heterogeneity not only affects farmers' agricultural decisions (Antle et al. 2003; Antle and Stoorvogel 2006; Matthews et al. 2007), but also the nutrient runoff impact on receiving water (Gassman et al. 2007; Goetz and Zilberman 2000). The nutrient runoff from agricultural land accumulates in lakes and reservoirs, and has long-term and even irreversible impact, imposing huge costs on ecosystem services, water treatment, tourism, and more activities that depend on clean water resources (Carpenter, Ludwig, and Brock 1999; Maler et al. 2003). Optimal management of externalities from agricultural decisions therefore presents a spatial dynamic problem. Forward-looking economic models of agricultural decisions that incorporate costs of nutrient runoff and associated damages to soil and water quality have been used to determine optimal fertilizer input and best management practices (BMP) that maximize social welfare (Barrett, 1991, De Haen, 1982, Taylor et al., 1992, Matthews et al. 2007). Empirical analyses show that spatially differentiated land management policies—for example, fertilizer tax or subsidies for buffer strips — are more efficient than semi-uniform instruments

(Lankoski and Ollikainen, 2003), and spatial targeting of conservation tillage can jointly improve water quality and carbon retention (Yang, Wilson, and Voroney 2005). However, dynamic modeling of optimal management policies that account for spatial heterogeneity in land quality and the potential impact on NPS pollution remains limited.

Goetz and Zilberman (2000) set up a framework to determine the optimal management of production externalities using a two-stage modeling approach. They solve the spatial problem in the first stage and optimize over time in the second stage, and suggest zonal management if differentiated optimal policies are not available. In the application to water irrigation system problem in California, they further investigate welfare differences between non-differentiated policies and differentiated policies with heterogeneous economic agents, and find that welfare gains from spatially targeted policy depends on the initial pollution stock and land heterogeneity (Xabadia, Goetz, and Zilberman, 2008). However, these studies did not explicitly model distance to the receiving water body, implementation costs associated with landscape and policy heterogeneity, empirically calibrate the model with agricultural nutrient runoff problem, nor simultaneously solve for optimal decisions. Other studies that develop optimization models to investigate specific policies and best management practices (BMP), including precision fertilizer application, vegetative filter strip, and gypsum amendment, to reduce phosphorus at the source and from the stock, find thresholds for economically feasible amendment methods (Iho and Laukkanen 2009, 2012). However, these studies fail to capture the spatial features of landscape and soil.

Spatially targeted policies share similarities with precision agriculture: both implement field specific farming management practices based on observing, measuring and responding to inter and intra-field variability (McBratney et al., 2005). In the past three decades, precision agriculture has advanced tremendously as a means to increase efficiency by enabling farmers to optimize agricultural inputs, and improve water quality by reducing excess nutrient runoff (Dixon and McCann, 1997). However, the development and spread of precision agriculture have been slow due to socio-economic barriers for the new adopters (Robert, 2002), insufficient recognition of temporal variations, and lack of farm-level focus among other problems (McBratney et al., 2005). High costs of technology adoption and implementation can offset part, or even all, of the benefits gained from precision agriculture depending on the size of the farm and the extent of heterogeneity in soil quality. Understanding the costs and benefits associated with the inter- and intra- farm spatial heterogeneity is essential to inform agricultural decisions and policies that can effectively reduce nutrient runoff and control NPS pollution.

While residents near Lake Erie suffer most from the water quality deterioration impacts, the society as a whole also has to pay the price. To incentivize upstream individual profit-maximizing farmers take water quality damages into account when making the agricultural management decisions, policy makers need simple yet practical policy tools to design long-term management practices. To this goal, we develop this platform for policy makers, where the model derives policy schemes based on inputs of physical landscape features and economic parameters.

In this paper, we build on existing literature and develop a spatial-dynamic model of agricultural decisions in a simulated watershed to look for optimal management policies. A social planner chooses optimal fertilizer input levels across multiple farms or patches of land to maximize social welfare over an infinite time horizon, taking into account both agricultural profits and water quality damages. We show that, at the steady state, shadow costs of small changes in phosphorus loading are equalized across farms, which is consistent with economic theory of stock pollutants (Conrad and Olson 1992; Gotez and Zilberman 2000; Karp and Zhang 2012; Titenberg and Lewis 2016). Furthermore, at the optimal steady state, the marginal net benefit of agricultural production from applying one more unit of fertilizer is equal to the perpetuity value of damage from an additional unit of phosphorus stock in the lake adjusted by the individual farm's contribution share. In a simulated landscape, we find the socially optimal fertilizer input based on spatially heterogeneous features for a representative farm and derive the optimal tax scheme that would lead to the socially optimal outcome under a private decision-making model. Comparative analyses show that the social planner's decision coincides with private farmer's behavior when damages from water pollution are zero or when the private farmer care about water quality as much as the social planner, but in other cases, the socially optimal fertilizer amount is always less than the private farmer's optimal decision.

We simulate landscapes that are heterogeneous in soil characteristics and spatial features and find the welfare gains from spatially differentiated taxes on fertilizer input increase with increasing level of heterogeneity compared with uniform tax policy that assumes all indices are at the mean. In the case where policy implementation costs are proportional to the level of heterogeneity of policy, we show that there is an optimal level for implementing spatially targeted policy.

This work contributes to the literature in three ways. We develop a platform to look for long-term policy instruments to maximize social welfare and empirically parameterize the optimal control model to find welfare gains from spatially targeted policies. We contribute to the growing literature that captures the spatial and dynamic features of natural resource management models. Examining the optimal level of implementing spatially targeted policies based on level of heterogeneity in physical features provides policy insight that helps in designing long-term cost-effective agricultural policies in spatially heterogeneous landscapes. Our findings also provide insight into potential bargaining opportunities and payment schemes in the presence of strategic interactions between farmers within heterogeneous landscapes.

2. Spatial-Dynamic Model of Agricultural Decisions

We set up a social planner's problem to maximize the present value of social welfare, which incorporates agricultural profits, production costs, and the associated water quality damage costs. We develop a dynamic optimal control model to look for policy instruments to balance the tradeoffs between agricultural decisions and water quality and compare the welfare gains from spatially targeted policies versus uniform policy on a heterogeneous landscape.

Consider a watershed (Figure 1), which is an area where all rivers draining ultimately to particular water body, with heterogeneous landscape and one lake. Within the watershed, all nutrient runoff that is not absorbed by the soil will end up in this same receiving lake. For simplicity, we consider each unit of farmland to be one acre in size, with spatial features including soil and land surface characteristics index s ($0<s<1$) and location index d (km) ($0 < d < \bar{d}$) (See Table 1). s represents the soil fertility and absorption ability related physical features, where bigger s represents higher productivity and better absorption ability, thus more yield and less nutrient runoff. In Figure 1, different shades of color represent different values of soil indices. s is generated by combining information on soil test, physical features including slope and soil type, and soil erosion index. To simplify notation, we will name s the "soil index" from now on and it is normalized between 0 and 1. Location index d denotes the distance from farmland to the receiving lake. We denote the maximum distance from farm to the lake in this watershed is $\bar{d}$, that is, any farm that is located farther away would be outside of this watershed. The distance negatively affects the amount of nutrient added to the lake phosphorus stock because more nutrient is absorbed along the transportation process in longer distance. Heterogeneity is determined by the variance in the distribution of these two indices, reflecting the differences in spatial characteristics.

In a simulated agricultural landscape, we assume for simplicity that in the watershed a single crop is produced – corn, which is a major crop and the largest contributor to nutrient runoff in the Midwest. The amount of fertilizer input in the agricultural production for farm i, denoted as u_i (>0), is the control variable. We assume the unit cost of fertilizer is c_u. The state variable that represents the state of this land-lake dynamic system is the phosphorus stock x (x>0) in the lake. Every time period, the stock increases with the phosphorus runoff added to the lake and decreases at rate γ that represents all natural decrease processes including decay, decomposition, sedimentation, and carried out by flow. To simplify notation, we call it "natural decay rate" in this paper.

The social planner makes socially optimal management decisions in the watershed with n heterogeneous farms to maximize the present value net benefits from agricultural production while accounting for the water quality damage associated with nutrient runoff. The agricultural profits occur at the farm level but the water quality damage occur at the aggregate level at the receiving lake. The value function $V(x)$ for the social planner's problem is defined as:

$$V(x) = max_{u_i(t)} \int_0^{+\infty} e^{-\delta t} \left\{ \sum_{i=1}^{n} \{[pQ(u_i(t), s_i) - c_u u_i(t)]\} - D\big(x(t)\big) \right\} dt \qquad [1]$$

subject to

$$\dot{x}(t) = \sum_{i=1}^{n} R(s_i, d_i) u_i(t) - \gamma x(t) \qquad [2]$$

where *p* is the exogenous market price for the agricultural output (corn). $u_i(t)$ denotes the quantity of fertilizer input applied on each farm at time period t, the soil index and location index are represented by s_i and d_i, *i=1,2 ... n*. $D\big(x(t)\big)$ denotes the damage function associated with phosphorus stock *x*, which is determined at the lake level. To ensure an interior solution, we assume the damage function to be convex in phosphorus stock $x(t)$. $R(s_i, d_i)$ represents a simplified nutrient runoff function, which reflects the relationship between fertilizer input at the farm level, farm-specific soil and location characteristics, and the addition to phosphorus stock $x(t)$. The phosphorous runoff decreases with high soil index s_i (which could reflect a higher absorptive capacity, for example) and distance to the lake d_i. The agricultural yield function is quadratic in fertilizer input, and increase with better soil quality.

Production, damage, and runoff functions, *Q*(*,*), *D*(*), and *R*(*,*) have the following features:

$$\frac{\partial Q}{\partial u_i} > 0, \qquad \frac{\partial^2 Q}{\partial {u_i}^2} < 0 \qquad [3]$$

$$\frac{dD}{dx} > 0, \qquad \frac{d^2 D}{dx^2} > 0 \qquad [4]$$

$$\frac{dR}{ds_i} < 0, \qquad \frac{dR}{dd_i} < 0 \qquad [5]$$

To be more specific, we assume the production function, damage function, and runoff functions take the following forms and are parameterized by my empirical estimates and existing studies:

$$Q(u_i, s_i) = \rho s_i(-a{u_i}^2 + bu_i - c) \quad [6]$$

$$D(x) = mx^2 \quad [7]$$

$$R(s_i, d_i) = \theta(1 - s_i)\left(1 - \frac{d_i}{\bar{d}}\right) \quad [8]$$

To solve the optimal control model, the corresponding Current Value Hamiltonian (CVH) function is given by[3]:

$$H(u_i, x, \lambda) = \sum_{i=1}^{n}\{[pQ(u_i, s_i) - c_u u_i]\} - D(x) + \lambda[\sum_{i=1}^{n} R(s_i, d_i)u_i - \gamma x] \quad [9]$$

The costate variable $\lambda(t)$ is interpreted as the shadow cost of water quality damage from an additional unit of phosphorus stock in the receiving lake. The solution of this problem has to satisfy the corresponding first order conditions:

$$\frac{\partial H}{\partial u_i}(u_i, x, \lambda) = p\frac{\partial Q}{\partial u_i} - c_u + R(s_i, d_i)\lambda = 0 \quad [10]$$

$$\dot{\lambda} - \delta\lambda = -\frac{\partial H}{\partial x} = \frac{\partial D}{\partial x} + \gamma\lambda \quad [11]$$

and the transition function:

$$\dot{x} = \sum_{i=1}^{n} R(s_i, d_i)u_i - \gamma x \quad [12]$$

where Eq.[11] can be simplified as:

[3] To simplify the notation, the arguments *t* are suppressed.

$$\dot{\lambda} = \frac{\partial D}{\partial x} + (\gamma + \delta)\lambda \tag{13}$$

We solve for the optimal steady state, where all the decision variables and state variable are stabilized:

$$\dot{u}_i = 0, \quad \dot{x} = 0 \tag{14}$$

The shadow price will also remain constant at steady state:

$$\dot{\lambda} = 0 \tag{15}$$

Combining Eq. [13] and Eq. [15] we have

$$\lambda = \frac{-\frac{\partial D}{\partial x}}{(\gamma + \delta)} \tag{16}$$

Optimality condition Eq. [16] implies that the shadow price of an additional unit to the phosphorus stock equals to the discount and decay rate adjusted marginal damage of phosphorus stock in the lake. It can also be interpreted as the perpetuity value of damage from an additional unit of phosphorus.

From first order condition Eq. [10] we find the shadow value for each farm is equalized:

$$\lambda = -\frac{\left(p\frac{\partial Q}{\partial u_i} - c_u\right)}{R(s_i, d_i)} \tag{17}$$

Combing Eq. [16] and Eq. [17] we find

$$\lambda = -\frac{\left(p\frac{\partial Q}{\partial u_i} - c_u\right)}{R(s_i, d_i)} = \frac{-\frac{\partial D}{\partial x}}{(\gamma + \delta)} \tag{18}$$

If we arrange the terms of Eq. [18], we have:

$$p\frac{\partial Q}{\partial u_i} - c_u = \frac{\frac{\partial D}{\partial x}}{(\gamma + \delta)} R(s_i, d_i) \tag{19}$$

On the left hand side of the equation: $p\frac{\partial Q}{\partial u_i} - c_u$ is the marginal agricultural net benefit from applying one more unit of fertilizer. On the right hand side, $\frac{\frac{\partial D}{\partial x}}{(\gamma+\delta)}$ is the marginal damage of additional phosphorus in the stock adjusted by discounted rate and natural decay rate, and $R(s_i, d_i)$ is the contribution share of each farm to the phosphorus stock in the lake. Eq. [19] shows the fundamental tradeoff between agricultural productivity and water quality. It shows that, at the steady state, for each farm, the forgone agricultural net benefit is equal to the perpetuity damage of additional phosphorus in the lake adjusted by the farm's contribution to the phosphorus stock. This value is equalized for all farms, where each farm adjusts its fertilizer input to equate its marginal net benefit to the marginal adjusted damage, thus ensuring the socially optimal outcome.

3. Model Calibration

We parameterize the model with corn production data in Ohio using USDA National Agricultural Statistics Service (NASS) quickstats corn yield data from 1990 to 2005, and the corresponding fertilizer usage from EPA agricultural fertilizer report. The yield function is fitted using ordinary least squares (OLS) regression model and the fertilizer price is obtained from Agricultural Prices, USDA NASS. Runoff coefficient θ, represents the share of fertilizer applied on the farmland that will eventually end up in the lake, is estimated based on mass calculation, the annual total amount of fertilizer applied in Maumee River watershed divided by the total phosphorus addition to Lake Erie in a year. The damage coefficient is based on estimates from existing studies, but these estimates can vary significantly across different lakes and reservoirs (Bingham et al. 2015). Natural decay rates also vary across different geographical locations and climates (Maler et al. 2003; Reitzel et al. 2007). In the following analysis, we assume a moderate decay rate of 0.3. The discount rate is assumed to be 0.1. We normalized the soil index to [0, 1]. The maximum distance from watershed boundary to the lake is denoted as $\bar{d}$ based on existing literature. The parameters are shown in Table 1. More robustness analysis of the parameters is in the steady state analysis section.

4. Numerical Analysis

4.1 Representative Farm Problem Steady States Analysis

We solve the model, first focusing on a representative farm to determine the optimal outcome of the dynamic system when it reaches the steady state, where constant policies induce constant choices and the state remains constant level as in Eq. [14]. We plug the functions Eq. [6], Eq. [7], and Eq. [8] into the Hamiltonian function Eq. [9] to solve for the analytical solutions of the optimal combination of fertilizer input amount and phosphorus stock in the lake. We derive how fertilizer inputs change over time based on the shadow price from Eq. [10] and Eq. [13]:

$$\dot{u}_i = (\gamma+\delta)u_i + \frac{(\gamma+\delta)}{2p\rho a s_i}(c_u - p\rho b s_i) + \frac{\theta m(1-s_i)(1-\frac{d_i}{\bar{d}})}{p\rho a s_i}x \qquad [20]$$

Transition function Eq. [12] shows how the stock changes over time in response to fertilizer inputs at the farm level. Combining the two equations, we solve for the optimal steady state fertilizer amount and phosphorus stock level, denoted as u_i^* and x^* respectively, for *i*=1, 2,…*n*. We first solve the problem with a representative farm with soil index (s_i) and location index (d_i). Analytical solution for the representative farm determines the optimal fertilizer input (u_i^*) and phosphorus stock level (x^*):

$$x^* = \frac{\theta(1-s_i)\left(1-\frac{d_i}{\bar{d}}\right)(\delta+\gamma)(p\rho b s_i - c_u)}{2\theta^2(1-s_i)^2\left(1-\frac{d_i}{\bar{d}}\right)^2 m + 2(\delta+\gamma)\gamma p\rho a s_i} \qquad [21]$$

$$u_i^* = \frac{(\gamma+\delta)\gamma(p\rho b s_i - c_u)}{2\theta^2(1-s_i)^2\left(1-\frac{d_i}{\bar{d}}\right)^2 m + 2(\delta+\gamma)\gamma p\rho a s_i} \qquad [22]$$

The first and second order derivatives of the optimal fertilizer amount on soil index and location index shows how the optimal choice should change with farm physical features:

$$\frac{\partial u_i^*}{\partial s_i} > 0, and \; \frac{\partial^2 u_i^*}{\partial {s_i}^2} < 0 \qquad [23]$$

$$\frac{\partial u_i^*}{\partial d_i} > 0, and \; \frac{\partial^2 u_i^*}{\partial {d_i}^2} < 0 \qquad [24]$$

The optimal quantity of fertilizer input increases with better soil quality and with distance from the receiving lake but at a decreasing rate. In Figure 2, we show the 3-D surface of optimal fertilizer input for combinations of s_i and d_i. We find that the impact of soil index on optimal fertilizer input decisions is larger than the impact of distance from the lake. Farms with low soil quality, located close to the lake should apply the lowest amount of fertilizer, and farms with high soil quality and located far away from the lake could apply more fertilizer. The complete table of optimal fertilizer amount for every combination of soil index and location index is in Appendix A.

4.2 Private Farmers' Problem and Optimal Fertilizer Tax

Although private farmers are profit maximizers, they may also care about the water quality, especially those who are more environmental conscious (Kalcic 2013). We assume there is a coefficient for water quality consciousness (ζ_i) associated with individual's perception of water quality damage for private farmers ($0 \leq \zeta_i \leq 1$), where higher value means the private farmer cares more about water quality. The private farmer's profit maximization problem is then defined as:

$$max_{u_i(t)} \int_0^{+\infty} e^{-\delta t}\{pQ(u_i(t), s_i) - c_u u_i(t) - \zeta_i D(x(t))\}dt \qquad [25]$$

subject to

$$\dot{x}(t) = R(s_i, d_i)u_i(t) - \gamma x(t) \qquad [26]$$

where the private famer chooses the fertilizer amount (u_i) to maximize his/her net benefit, while partially taking into account his/her nutrient runoff impact on the receiving water body. The solution of this optimization problem is denoted as $\bar{u}_i$:

$$\bar{u}_i = \frac{(\gamma+\delta)\gamma(p\rho b s_i - c_u)}{2\zeta_i m\theta^2(1-s_i)^2\left(1-\frac{d_i}{\bar{d}}\right)^2 + 2(\delta+\gamma)\gamma p\rho a s_i} \qquad [27]$$

This water quality conciseness coefficient is likely to vary among individual farmers (Burnett et al. 2015) but we assume it to be the same ($\zeta_i = 0.1$) for simplicity for now and will relax this assumption later. When private farmers' optimal decisions deviate from the social planner's optimal outcome, policy instruments, such as fertilizer tax, are necessary to incentivize farmers to reduce fertilizer input. A fertilizer tax, or fertilizer usage fee, places an additional cost to the price of fertilizer thus decreases farmer's fertilizer usage. The goal of a fully differentiated tax (τ_i) is to correct the private farmers' decisions to achieve the socially optimal outcome, which requires making the socially optimal fertilizer usage amount (u_i^*) the solution to the following private farmer profit maximization problem:

$$max_{u_i(t)} \int_0^{+\infty} e^{-\delta t}\{pQ(u_i(t), s_i) - (c_u + \tau_i)u_i(t) - \zeta_i D(x(t))\}dt \qquad [28]$$

subject to

$$\dot{x}(t) = R(s_i, d_i)u_i(t) - \gamma x(t) \qquad [29]$$

We solve for the optimal tax for every combination of soil index (s_i) and location index (d_i) and show the optimal tax as percentage increase in unit cost of fertilizer in Figure 3:

$$\tau_i = p\rho b s_i - c_u - \frac{2\theta^2 \zeta_i (1 - s_i)^2 \left(1 - \frac{d_i}{\bar{d}}\right)^2 m + 2(\delta + \gamma)\gamma p \rho a s_i u_i^*}{(\gamma + \delta)\gamma} \qquad [30]$$

The optimal tax scheme (Figure 3), shown as percentage increase in fertilizer usage fee, results in highest tax on farms with poor soil quality and are located close to the lake. The tax decreases as the soil quality increases, or as the distance increases. We find the optimal tax varies from 0 to 35%, which is in line with other tax policy suggestions (Sohngen et al. 2015). The complete table of optimal tax for every combination of soil index and location index is shown in Appendix B.

We also solve for optimal transition paths and find that whether with high or low initial values, the dynamic system reaches steady state in about 10 time periods (years).

4.3 Comparative Static Analysis of Fertilizer Application Rates to Changes in Parameters

We conduct comparative static analyses to examine optimal fertilizer usage in response to changes in one parameter and how it may affect the private farmer and social planner's decisions differently. We develop a baseline scenario where the soil index and distance to the lake are set at the mean: s_i=0.5, d_i=2.5 km, and other parameters are set as in Table 1.

1) Optimal Response to Changes in Agricultural Output (corn) Price (*p*).

Analytically taking the first and second order derivatives of output price p from Eq. [22], we find:

$$\frac{\partial u_i^*}{\partial p} > 0, and\ \frac{\partial^2 u_i^*}{\partial p^2} < 0 \qquad [31]$$

Steady state optimal fertilizer amount for both private farmer and social planner increases with output price . Higher output price increases agricultural profits, making agricultural production more favorable in the tradeoff between agricultural decisions and water quality. However, this marginal impact diminishes as output price increases. At any given output price, the private farmer always applies more fertilizer than socially optimal to maximize his/her profits.

2) Optimal Response to Changes in Water Damage Coefficient (m)

Similarly, we derive the first and second order derivatives of the water quality coefficient from Eq. [22]:

$$\frac{\partial u_i^*}{\partial m} > 0, and\ \frac{\partial^2 u_i^*}{\partial m^2} < 0 \qquad [32]$$

When the water damage coefficient is set at zero, which means there is no cost associated with water quality deterioration, the private farmer and the social planner will make the same decision on fertilizer usage. In this case, the social welfare coincides with the sum of private agricultural profits. However, as the water damage coefficient increases, there is higher water quality damage cost associated with phosphorus stock in the lake, the socially optimal and private farmer level of fertilizer amount decrease (Figure 4). As long as the coefficient is bigger than zero, the private farmer always applies more fertilizer than socially optimal.

3) Optimal Response to Changes in Natural Decay Rate (γ)

For natural decay rate, the first and second order derivatives are:

$$\frac{\partial u_i^*}{\partial \gamma} > 0, and \ \frac{\partial^2 u_i^*}{\partial \gamma^2} < 0 \qquad [33]$$

Natural decay rate represents the self-restoration ability of the lake. With higher natural decay rate, the phosphorus stock from last time period can be decomposed or carried out of the phosphorus cycle in the lake, which allows farmers in this watershed to apply more fertilizer (Figure 5). As the natural decay rate increases, the social planner's decisions will converge to the private farmer's decision. In contrast, if natural decay rate is close to zero, which means the phosphorus in the lake does not recover by itself and has accumulating impact in the lake, the socially optimal and environmentally conscious private farmer's choice of fertilizer usage will go to zero.

4) Optimal Response to Changes in Water Quality Consciousness Coefficient (ζ_i)

The first and second order derivatives of the water quality consciousness coefficient are:

$$\frac{\partial u_i^*}{\partial \zeta_i} < 0, and \ \frac{\partial^2 u_i^*}{\partial {\zeta_i}^2} > 0 \qquad [34]$$

Water quality consciousness coefficient represents how much a private farmer cares about water quality in the lake and higher coefficient means higher value associated with water quality. In the extreme cases: $\zeta_i = 0$ is for private farmers maximizing profits without taking into account their impact on water quality, which makes the private farmer's decision a static profit-maximizing problem that does not change over time. As the coefficient increases, the private farmer's decision will converge to the social planner's problem because the private farmer internalizes the social cost of water quality damage. When $\zeta_i = 1$, the private farmer fully internalizes the water quality deterioration cost, and the private farmer's decision will be the same as social planner's.

4.4 Welfare Analysis of a Spatially Targeted Tax in a Heterogeneous Landscape

Finally we expand the analysis to a watershed with heterogeneous landscape. Eq. [12] establishes that the aggregate impact on phosphorus stock comes from every individual farm, implying the interactions among heterogeneous farms. In this case, it is very hard to display every possible combination of watersheds when each farm's optimal fertilizer decision depends on other farms. We define the uniform policy here as the uniform tax designed assuming every farm's soil quality index is equal to the mean of all farms. In this example to illustrate the welfare gains from spatially targeted policies versus uniform policy, we simulate a watershed with 10 farms that vary in soil quality, where each farm is 200 acres large, which is the average farm size in Ohio. We assume all farms are located 2.5 km from the lake, and we simulate different levels of heterogeneity in soil quality by changing the variance of the soil index distribution while holding the mean at 0.5. We compare a fully differentiated tax policy with a uniform fertilizer tax and results show that compared with the uniform tax, the welfare gains from spatially targeted policy increases with increasing level of heterogeneity (Figure 6).

4.5 Optimal Level of Policy Heterogeneity

We see the welfare gains from spatially targeted policies, however, in real world situations, spatially targeted policies are expensive. The implementation costs are higher with higher level of heterogeneity in spatial features because of the additional costs of soil testing, implementation, and enforcement, which turns policy makers to second best zonal policies (Xabadia, Goetz, and Zilberman 2008). Therefore, we examine if there is an optimal level for implementing spatially targeted policy.

In this simulation, we assume the watershed is 2000 acres large with varying soil quality indices drawn from a uniform distribution with mean 0.5 and variance 0.3. We test dividing the watershed into 1 to 10 zones and assume the policy implementation cost is proportional to the number of zones we divide into. Here we represent level of policy heterogeneity by the number of zones. We compare the social welfare of differentiated management policies that manage based on zonal specific soil quality with optimal uniform policy that is based on mean soil quality of the watershed. Figure 7 shows that there is an optimal level of dividing the zones, or policy heterogeneity, for spatially targeted policy. In this specific example, implementing targeted policy by dividing the watershed into 3 zones generates the highest social welfare. We test for different watershed sizes and different implementation costs and find similar peak in welfare gains with different levels of policy heterogeneity.

5. Discussion and Conclusion

Agriculture and water quality are two important components of the social welfare. This paper develops a dynamic optimal control model that balances the tradeoffs between agricultural productivity and the associated nutrient runoff impact on the water quality based on heterogeneous spatial characteristics. We develop a platform to solve the social planner's problem of maximizing social welfare on heterogeneous landscapes. We show that the shadow price is equalized for all farms and is equal to the marginal damage of additional unit of phosphorus in the stock adjusted by the discount rate and natural decay rate. The marginal net benefit of each farm is equal to the perpetuity damage of additional unit of phosphorus stock in the lake adjusted by the farm's contribution to the phosphorus stock.

We derive the optimal fertilizer choice for every combination of soil index and location index, and the corresponding tax scheme to achieve the social optimal outcome for a representative farm. Further on, we compare the welfare gains from spatially targeted policies versus the optimal uniform fertilizer tax on a watershed with 10 heterogeneous farms, and show that the welfare gains increase with increasing level of heterogeneity. We also show that there is an optimal level of policy heterogeneity in implementing spatially targeted policies when the policy implementation cost increases with number of zones.

We also analyze the interactions between farms when their fertilizer usages have aggregate impact on the lake. In the case of two farms where we can find analytical solution, we show the tradeoff between their fertilizer input and economic profits, and the spillover effect of improving soil quality in Appendix C. We will examine these interactions to look for strategic behavior and optimal payment schemes among farms to maximize private farmer's profits in future work.

This work contributes to environmental and resource economics literature in three major ways. 1) We are pushing forward the frontier of empirically grounded modeling of optimal resource management by linking valuation with dynamic optimal control models (Iho and Laukkanen 2009, 2012; Xabadia, Goetz, and Zilberman, 2008). 2) We contribute to the growing literature that captures the spatial and dynamic features of natural resource management models (Smith, Sanchirico, Wilen 2009). 3) We show the role of heterogeneity in optimal policy design, which provides insight into optimal zonal policies. The platform we build in this paper can be extended in many ways to better simulate the real world situations of agricultural production: 1) add BMP as a method to improve soil quality at farm level, and or to reduce phosphorus stock in the lake; 2) add an exogenous variable of precipitation with stochastic processes that represent climate change that affects the probability of having an algal bloom; 3) empirically test the model with farmer behavior of fertilizer management practice adoption data in Western Lake Erie water basin. This framework can also be applied to other resource management problems such as forest management, invasive species control, and test for different climate change scenarios.

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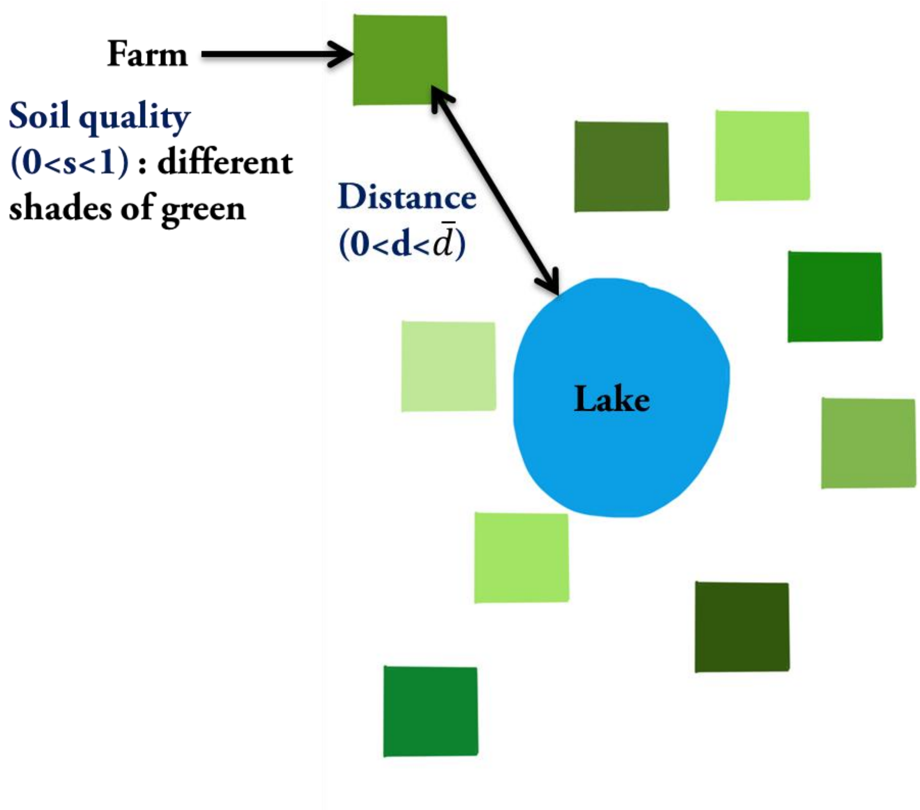

Figure 1: Illustration of Farms and the Receiving Lake

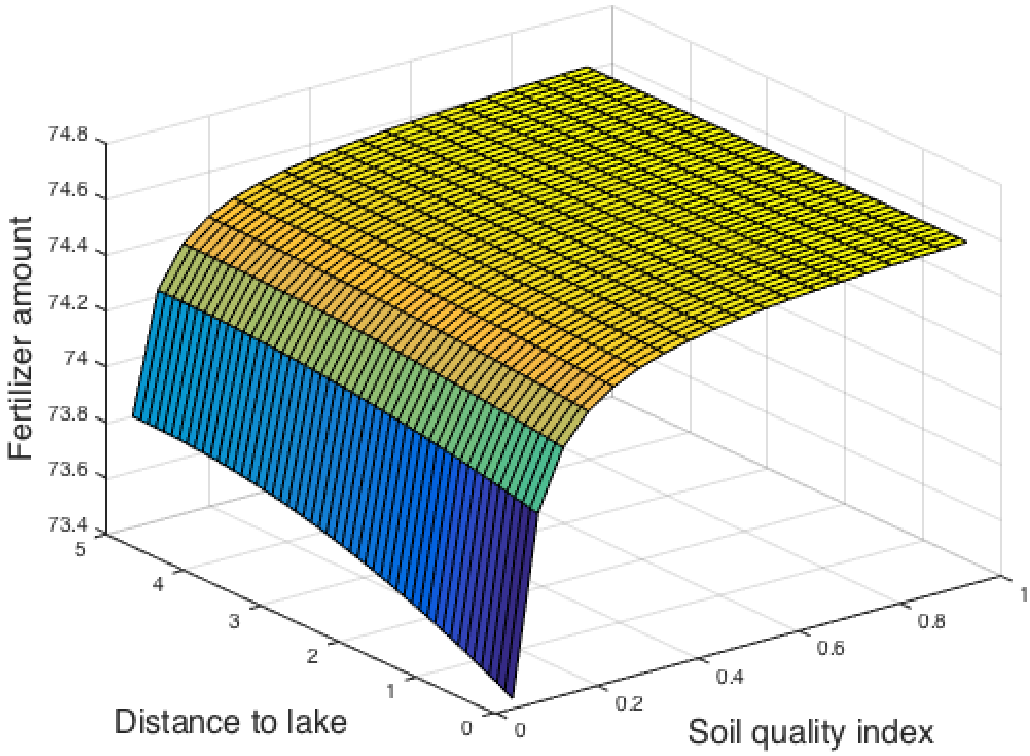

Figure 2 Steady State Optimal Fertilizer Amounts

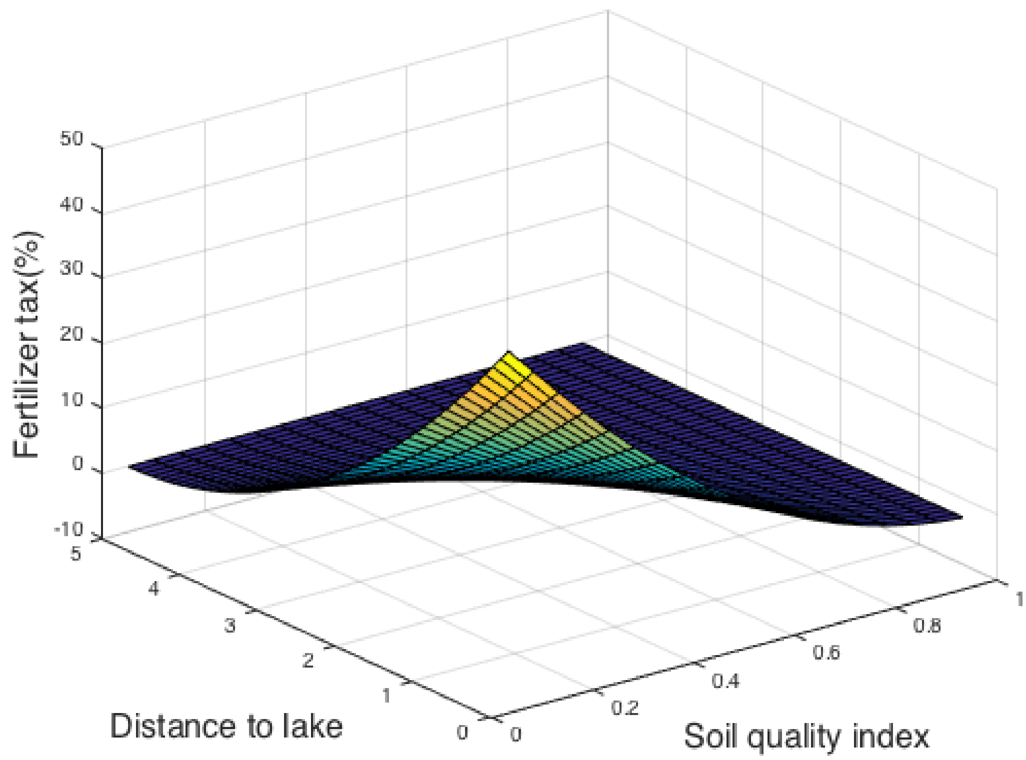

Figure 3 Optimal Differentiated Tax for Private Farmers

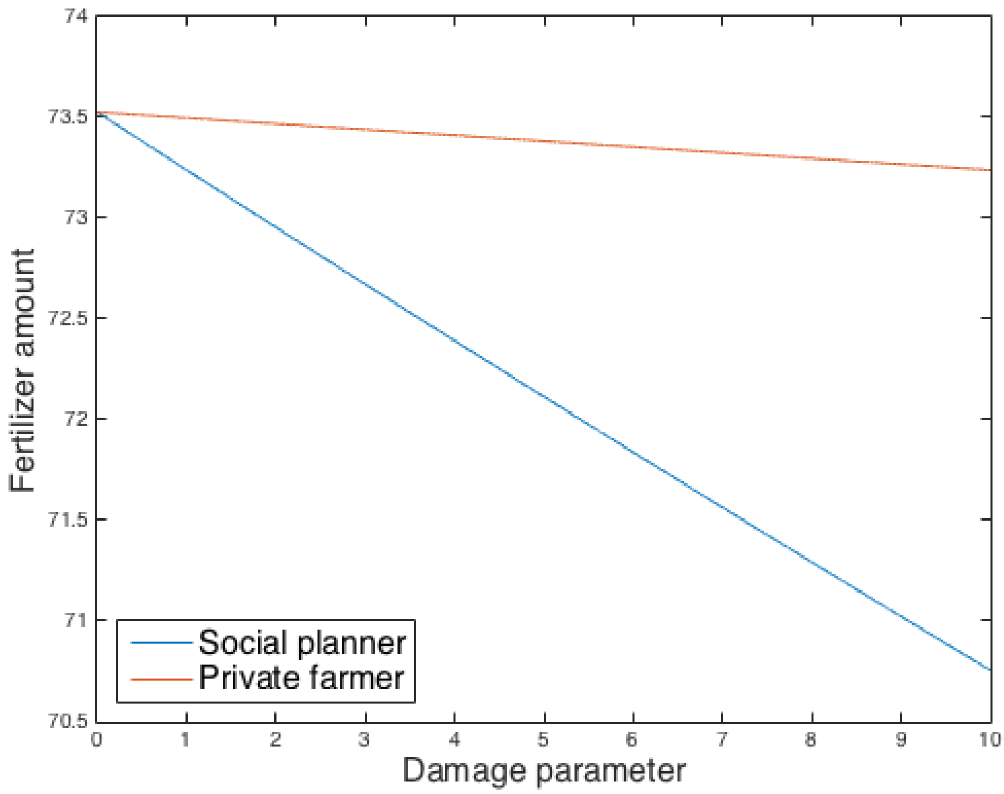

Figure 4 Comparative Analysis of Water Quality Damage Parameter on Fertilizer Amount

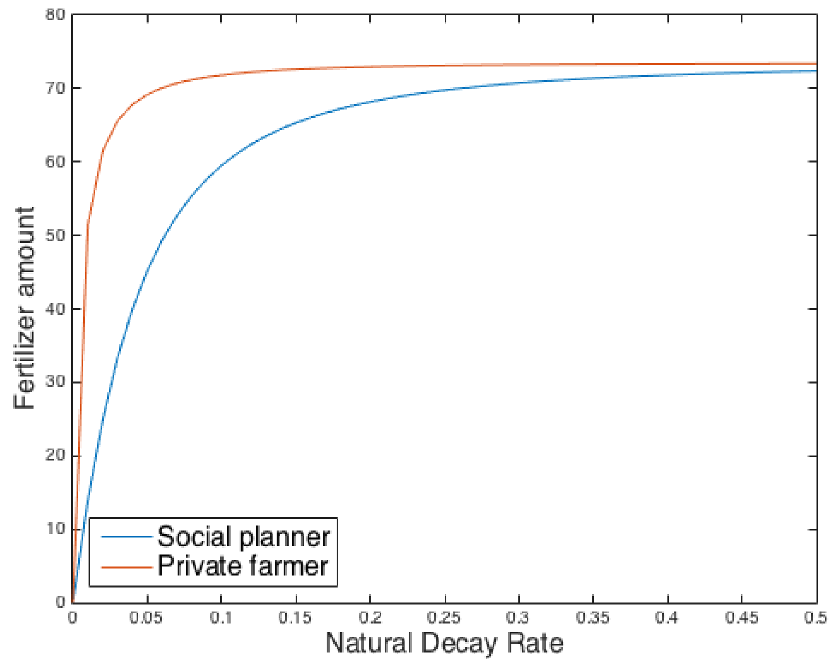

Figure 5 Comparative Analysis of Natural Decay Rate on Fertilizer Amount

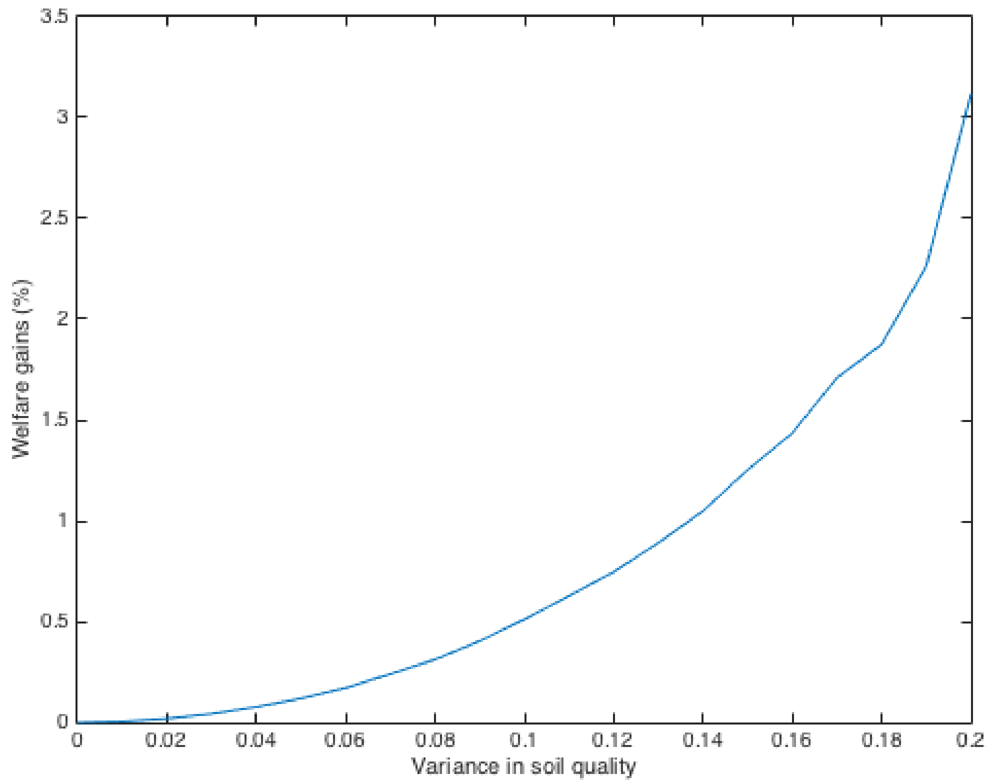

Figure 6 Welfare Gains from Spatially Targeted Policies with Variance in Soil Quality

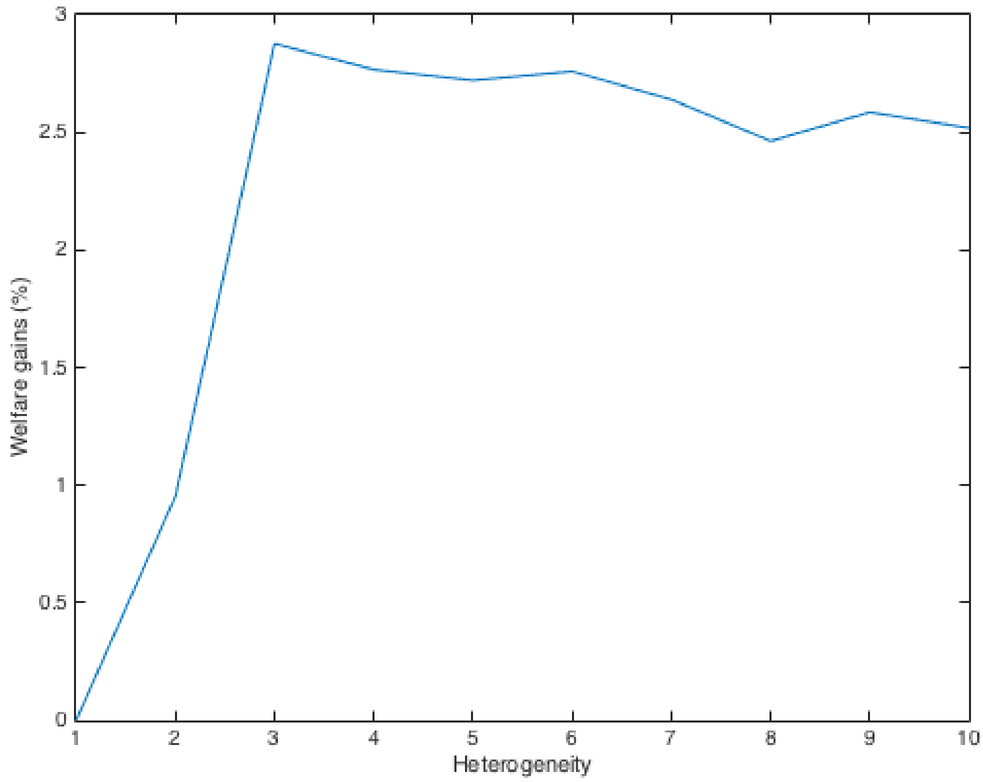

Figure 7 Welfare Gains from Spatially Targeted Policies with Heterogeneity Level in Policies

Table 1 Model Parameter Values

Parameter	Symbol	Value
	a	0.133
Yield function	b	19.858
	c	611.06
	ρ	5
Corn price	p	6.15 ($/bu)
Fertilizer price	c_u	0.351 ($/lb)
Runoff coefficient	θ	0.1
Damage coefficient	m	0.0148 ($/kg)
Natural decay rate	γ	0.3
Discount rate	δ	0.1
Soil quality index	s	$0<s<1$
Maximum distance to the lake	$\bar{d}$	5 (km)
Distance to the lake	d	$0<d<\bar{d}$(km)

Appendix A: Optimal Fertilizer Amount Table

s \ d	0.05	0.1	0.15	0.2	0.25	0.3	0.35
0.1	73.4133	74.0519	74.2649	74.3710	74.4342	74.4761	74.5057
0.2	73.4288	74.0589	74.2690	74.3737	74.4362	74.4775	74.5067
0.3	73.4439	74.0658	74.2731	74.3765	74.4381	74.4789	74.5077
0.4	73.4587	74.0725	74.2771	74.3791	74.4400	74.4802	74.5087
0.5	73.4732	74.0791	74.2811	74.3817	74.4418	74.4816	74.5097
0.6	73.4873	74.0855	74.2849	74.3843	74.4436	74.4829	74.5107
0.7	73.5012	74.0918	74.2887	74.3868	74.4454	74.4842	74.5116
0.8	73.5148	74.0979	74.2923	74.3892	74.4471	74.4854	74.5126
0.9	73.5280	74.1039	74.2959	74.3916	74.4488	74.4866	74.5135
1	73.5409	74.1098	74.2994	74.3939	74.4504	74.4878	74.5143
1.1	73.5535	74.1155	74.3028	74.3962	74.4520	74.4890	74.5152
1.2	73.5658	74.1210	74.3061	74.3984	74.4536	74.4901	74.5160
1.3	73.5778	74.1265	74.3094	74.4006	74.4551	74.4912	74.5168
1.4	73.5895	74.1318	74.3125	74.4027	74.4565	74.4923	74.5176
1.5	73.6008	74.1369	74.3156	74.4047	74.4580	74.4933	74.5184
1.6	73.6118	74.1419	74.3185	74.4067	74.4594	74.4943	74.5192
1.7	73.6225	74.1467	74.3214	74.4086	74.4607	74.4953	74.5199
1.8	73.6329	74.1514	74.3242	74.4105	74.4620	74.4963	74.5206
1.9	73.6430	74.1560	74.3270	74.4123	74.4633	74.4972	74.5213
2	73.6528	74.1604	74.3296	74.4140	74.4645	74.4981	74.5219
2.1	73.6622	74.1647	74.3321	74.4157	74.4657	74.4990	74.5226
2.2	73.6714	74.1688	74.3346	74.4174	74.4669	74.4998	74.5232
2.3	73.6802	74.1728	74.3370	74.4189	74.4680	74.5006	74.5238
2.4	73.6887	74.1766	74.3393	74.4205	74.4691	74.5014	74.5244
2.5	73.6968	74.1803	74.3415	74.4219	74.4701	74.5021	74.5249
2.6	73.7047	74.1839	74.3436	74.4233	74.4711	74.5028	74.5255
2.7	73.7122	74.1873	74.3456	74.4247	74.4720	74.5035	74.5260
2.8	73.7194	74.1905	74.3476	74.4260	74.4729	74.5042	74.5265
2.9	73.7263	74.1937	74.3494	74.4272	74.4738	74.5048	74.5269
3	73.7329	74.1966	74.3512	74.4284	74.4746	74.5054	74.5274
3.1	73.7392	74.1995	74.3529	74.4295	74.4754	74.5060	74.5278
3.2	73.7451	74.2021	74.3545	74.4306	74.4762	74.5065	74.5282
3.3	73.7507	74.2047	74.3560	74.4316	74.4769	74.5071	74.5286

3.4	73.7560	74.2071	74.3574	74.4325	74.4776	74.5075	74.5289
3.5	73.7610	74.2093	74.3587	74.4334	74.4782	74.5080	74.5293
3.6	73.7656	74.2114	74.3600	74.4342	74.4788	74.5084	74.5296
3.7	73.7700	74.2134	74.3612	74.4350	74.4793	74.5088	74.5299
3.8	73.7740	74.2152	74.3622	74.4357	74.4798	74.5092	74.5301
3.9	73.7777	74.2169	74.3632	74.4364	74.4803	74.5095	74.5304
4	73.7811	74.2184	74.3641	74.4370	74.4807	74.5098	74.5306
4.1	73.7841	74.2198	74.3650	74.4376	74.4811	74.5101	74.5308
4.2	73.7868	74.2210	74.3657	74.4380	74.4814	74.5104	74.5310
4.3	73.7892	74.2221	74.3663	74.4385	74.4817	74.5106	74.5312
4.4	73.7913	74.2230	74.3669	74.4388	74.4820	74.5108	74.5313
4.5	73.7931	74.2238	74.3674	74.4392	74.4822	74.5109	74.5314
4.6	73.7945	74.2245	74.3678	74.4394	74.4824	74.5111	74.5315
4.7	73.7957	74.2250	74.3681	74.4396	74.4825	74.5112	74.5316
4.8	73.7965	74.2253	74.3683	74.4398	74.4827	74.5112	74.5317
4.9	73.7970	74.2256	74.3684	74.4399	74.4827	74.5113	74.5317
5	73.7971	74.2256	74.3685	74.4399	74.4827	74.5113	74.5317
d / s	0.4	0.45	0.5	0.55	0.6	0.65	0.7
0.1	74.5276	74.5444	74.5576	74.5683	74.5770	74.5841	74.5901
0.2	74.5284	74.5450	74.5581	74.5686	74.5772	74.5843	74.5903
0.3	74.5291	74.5456	74.5585	74.5689	74.5774	74.5845	74.5904
0.4	74.5299	74.5461	74.5589	74.5692	74.5776	74.5846	74.5905
0.5	74.5306	74.5467	74.5593	74.5695	74.5779	74.5848	74.5906
0.6	74.5313	74.5472	74.5597	74.5698	74.5781	74.5849	74.5907
0.7	74.5320	74.5477	74.5601	74.5701	74.5783	74.5851	74.5908
0.8	74.5327	74.5482	74.5605	74.5704	74.5785	74.5852	74.5909
0.9	74.5334	74.5487	74.5609	74.5707	74.5787	74.5854	74.5910
1	74.5341	74.5492	74.5612	74.5709	74.5789	74.5855	74.5911
1.1	74.5347	74.5497	74.5616	74.5712	74.5791	74.5856	74.5912
1.2	74.5353	74.5502	74.5619	74.5714	74.5793	74.5858	74.5912
1.3	74.5359	74.5506	74.5623	74.5717	74.5794	74.5859	74.5913
1.4	74.5365	74.5511	74.5626	74.5719	74.5796	74.5860	74.5914
1.5	74.5371	74.5515	74.5629	74.5722	74.5798	74.5861	74.5915
1.6	74.5377	74.5519	74.5632	74.5724	74.5799	74.5863	74.5916
1.7	74.5382	74.5523	74.5635	74.5726	74.5801	74.5864	74.5917
1.8	74.5387	74.5527	74.5638	74.5728	74.5803	74.5865	74.5917

1.9	74.5392	74.5531	74.5641	74.5730	74.5804	74.5866	74.5918
2	74.5397	74.5535	74.5644	74.5732	74.5806	74.5867	74.5919
2.1	74.5402	74.5538	74.5647	74.5734	74.5807	74.5868	74.5919
2.2	74.5407	74.5542	74.5649	74.5736	74.5808	74.5869	74.5920
2.3	74.5411	74.5545	74.5652	74.5738	74.5810	74.5870	74.5921
2.4	74.5415	74.5548	74.5654	74.5740	74.5811	74.5871	74.5921
2.5	74.5419	74.5551	74.5656	74.5742	74.5812	74.5872	74.5922
2.6	74.5423	74.5554	74.5658	74.5743	74.5813	74.5872	74.5923
2.7	74.5427	74.5557	74.5661	74.5745	74.5814	74.5873	74.5923
2.8	74.5431	74.5560	74.5663	74.5746	74.5816	74.5874	74.5924
2.9	74.5434	74.5562	74.5664	74.5748	74.5817	74.5875	74.5924
3	74.5438	74.5565	74.5666	74.5749	74.5818	74.5875	74.5925
3.1	74.5441	74.5567	74.5668	74.5750	74.5819	74.5876	74.5925
3.2	74.5444	74.5570	74.5670	74.5752	74.5819	74.5877	74.5925
3.3	74.5447	74.5572	74.5671	74.5753	74.5820	74.5877	74.5926
3.4	74.5449	74.5574	74.5673	74.5754	74.5821	74.5878	74.5926
3.5	74.5452	74.5576	74.5674	74.5755	74.5822	74.5878	74.5927
3.6	74.5454	74.5577	74.5676	74.5756	74.5822	74.5879	74.5927
3.7	74.5456	74.5579	74.5677	74.5757	74.5823	74.5879	74.5927
3.8	74.5458	74.5580	74.5678	74.5757	74.5824	74.5880	74.5928
3.9	74.5460	74.5582	74.5679	74.5758	74.5824	74.5880	74.5928
4	74.5462	74.5583	74.5680	74.5759	74.5825	74.5880	74.5928
4.1	74.5464	74.5584	74.5681	74.5760	74.5825	74.5881	74.5928
4.2	74.5465	74.5585	74.5681	74.5760	74.5826	74.5881	74.5928
4.3	74.5466	74.5586	74.5682	74.5761	74.5826	74.5881	74.5929
4.4	74.5467	74.5587	74.5683	74.5761	74.5826	74.5881	74.5929
4.5	74.5468	74.5588	74.5683	74.5761	74.5827	74.5882	74.5929
4.6	74.5469	74.5588	74.5684	74.5762	74.5827	74.5882	74.5929
4.7	74.5469	74.5589	74.5684	74.5762	74.5827	74.5882	74.5929
4.8	74.5470	74.5589	74.5684	74.5762	74.5827	74.5882	74.5929
4.9	74.5470	74.5589	74.5684	74.5762	74.5827	74.5882	74.5929
5	74.5470	74.5589	74.5684	74.5762	74.5827	74.5882	74.5929
d s	0.75	0.8	0.85	0.9	0.95		
0.1	74.5952	74.5995	74.6032	74.6063	74.6090		
0.2	74.5953	74.5995	74.6032	74.6063	74.6090		
0.3	74.5953	74.5996	74.6032	74.6063	74.6090		

0.4	74.5954	74.5996	74.6032	74.6063	74.6090
0.5	74.5955	74.5997	74.6032	74.6063	74.6090
0.6	74.5955	74.5997	74.6033	74.6063	74.6090
0.7	74.5956	74.5997	74.6033	74.6063	74.6090
0.8	74.5957	74.5998	74.6033	74.6063	74.6090
0.9	74.5957	74.5998	74.6033	74.6064	74.6090
1	74.5958	74.5999	74.6033	74.6064	74.6090
1.1	74.5959	74.5999	74.6034	74.6064	74.6090
1.2	74.5959	74.5999	74.6034	74.6064	74.6090
1.3	74.5960	74.6000	74.6034	74.6064	74.6090
1.4	74.5960	74.6000	74.6034	74.6064	74.6090
1.5	74.5961	74.6000	74.6034	74.6064	74.6090
1.6	74.5961	74.6001	74.6034	74.6064	74.6090
1.7	74.5962	74.6001	74.6035	74.6064	74.6090
1.8	74.5962	74.6001	74.6035	74.6064	74.6090
1.9	74.5963	74.6001	74.6035	74.6064	74.6090
2	74.5963	74.6002	74.6035	74.6064	74.6090
2.1	74.5964	74.6002	74.6035	74.6064	74.6090
2.2	74.5964	74.6002	74.6035	74.6064	74.6090
2.3	74.5965	74.6002	74.6035	74.6065	74.6090
2.4	74.5965	74.6003	74.6036	74.6065	74.6090
2.5	74.5965	74.6003	74.6036	74.6065	74.6090
2.6	74.5966	74.6003	74.6036	74.6065	74.6090
2.7	74.5966	74.6003	74.6036	74.6065	74.6090
2.8	74.5966	74.6004	74.6036	74.6065	74.6090
2.9	74.5967	74.6004	74.6036	74.6065	74.6090
3	74.5967	74.6004	74.6036	74.6065	74.6090
3.1	74.5967	74.6004	74.6036	74.6065	74.6090
3.2	74.5968	74.6004	74.6036	74.6065	74.6090
3.3	74.5968	74.6004	74.6037	74.6065	74.6090
3.4	74.5968	74.6005	74.6037	74.6065	74.6090
3.5	74.5968	74.6005	74.6037	74.6065	74.6090
3.6	74.5969	74.6005	74.6037	74.6065	74.6090
3.7	74.5969	74.6005	74.6037	74.6065	74.6090
3.8	74.5969	74.6005	74.6037	74.6065	74.6090
3.9	74.5969	74.6005	74.6037	74.6065	74.6090
4	74.5969	74.6005	74.6037	74.6065	74.6090

4.1	74.5969	74.6005	74.6037	74.6065	74.6090
4.2	74.5970	74.6005	74.6037	74.6065	74.6090
4.3	74.5970	74.6005	74.6037	74.6065	74.6090
4.4	74.5970	74.6006	74.6037	74.6065	74.6090
4.5	74.5970	74.6006	74.6037	74.6065	74.6090
4.6	74.5970	74.6006	74.6037	74.6065	74.6090
4.7	74.5970	74.6006	74.6037	74.6065	74.6090
4.8	74.5970	74.6006	74.6037	74.6065	74.6090
4.9	74.5970	74.6006	74.6037	74.6065	74.6090
5	74.5970	74.6006	74.6037	74.6065	74.6090

Appendix B: Optimal Tax Table

s \ d	0.05	0.1	0.15	0.2	0.25	0.3	0.35
0.1	0.1567	0.1419	0.1269	0.1126	0.0991	0.0863	0.0745
0.2	0.1504	0.1362	0.1218	0.1081	0.0951	0.0828	0.0715
0.3	0.1443	0.1306	0.1168	0.1036	0.0911	0.0794	0.0685
0.4	0.1382	0.1251	0.1119	0.0993	0.0873	0.0761	0.0656
0.5	0.1323	0.1197	0.1071	0.0950	0.0836	0.0728	0.0628
0.6	0.1265	0.1145	0.1024	0.0908	0.0799	0.0696	0.0601
0.7	0.1209	0.1093	0.0978	0.0867	0.0763	0.0665	0.0574
0.8	0.1153	0.1043	0.0933	0.0828	0.0728	0.0634	0.0547
0.9	0.1099	0.0994	0.0889	0.0789	0.0694	0.0605	0.0521
1	0.1046	0.0946	0.0846	0.0751	0.0660	0.0575	0.0496
1.1	0.0995	0.0900	0.0805	0.0714	0.0628	0.0547	0.0472
1.2	0.0945	0.0854	0.0764	0.0677	0.0596	0.0519	0.0448
1.3	0.0896	0.0810	0.0724	0.0642	0.0565	0.0492	0.0425
1.4	0.0848	0.0767	0.0686	0.0608	0.0535	0.0466	0.0402
1.5	0.0802	0.0725	0.0648	0.0575	0.0506	0.0441	0.0380
1.6	0.0757	0.0684	0.0612	0.0542	0.0477	0.0416	0.0359
1.7	0.0713	0.0644	0.0576	0.0511	0.0449	0.0392	0.0338
1.8	0.0671	0.0606	0.0542	0.0481	0.0423	0.0368	0.0318
1.9	0.0629	0.0569	0.0509	0.0451	0.0397	0.0346	0.0298
2	0.0589	0.0533	0.0476	0.0422	0.0371	0.0324	0.0279
2.1	0.0551	0.0498	0.0445	0.0395	0.0347	0.0303	0.0261
2.2	0.0514	0.0464	0.0415	0.0368	0.0324	0.0282	0.0243
2.3	0.0478	0.0432	0.0386	0.0342	0.0301	0.0262	0.0226
2.4	0.0443	0.0400	0.0358	0.0317	0.0279	0.0243	0.0210
2.5	0.0410	0.0370	0.0331	0.0293	0.0258	0.0225	0.0194
2.6	0.0378	0.0341	0.0305	0.0270	0.0238	0.0207	0.0179
2.7	0.0347	0.0313	0.0280	0.0248	0.0218	0.0190	0.0164
2.8	0.0317	0.0287	0.0256	0.0227	0.0200	0.0174	0.0150
2.9	0.0289	0.0261	0.0233	0.0207	0.0182	0.0159	0.0137
3	0.0262	0.0237	0.0212	0.0188	0.0165	0.0144	0.0124
3.1	0.0237	0.0214	0.0191	0.0169	0.0149	0.0130	0.0112
3.2	0.0212	0.0192	0.0172	0.0152	0.0134	0.0117	0.0101
3.3	0.0190	0.0171	0.0153	0.0136	0.0119	0.0104	0.0090

3.4	0.0168	0.0152	0.0136	0.0120	0.0106	0.0092	0.0079
3.5	0.0148	0.0133	0.0119	0.0106	0.0093	0.0081	0.0070
3.6	0.0129	0.0116	0.0104	0.0092	0.0081	0.0071	0.0061
3.7	0.0111	0.0100	0.0089	0.0079	0.0070	0.0061	0.0052
3.8	0.0094	0.0085	0.0076	0.0068	0.0059	0.0052	0.0045
3.9	0.0079	0.0072	0.0064	0.0057	0.0050	0.0044	0.0038
4	0.0066	0.0059	0.0053	0.0047	0.0041	0.0036	0.0031
4.1	0.0053	0.0048	0.0043	0.0038	0.0033	0.0029	0.0025
4.2	0.0042	0.0038	0.0034	0.0030	0.0026	0.0023	0.0020
4.3	0.0032	0.0029	0.0026	0.0023	0.0020	0.0018	0.0015
4.4	0.0024	0.0021	0.0019	0.0017	0.0015	0.0013	0.0011
4.5	0.0016	0.0015	0.0013	0.0012	0.0010	0.0009	0.0008
4.6	0.0010	0.0009	0.0008	0.0008	0.0007	0.0006	0.0005
4.7	0.0006	0.0005	0.0005	0.0004	0.0004	0.0003	0.0003
4.8	0.0003	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001
4.9	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
d s	0.4	0.45	0.5	0.55	0.6	0.65	0.7
0.1	0.4	0.45	0.5	0.55	0.6	0.65	0.7
0.2	0.0635	0.0533	0.0441	0.0357	0.0282	0.0216	0.0159
0.3	0.0609	0.0512	0.0423	0.0343	0.0271	0.0207	0.0152
0.4	0.0584	0.0491	0.0406	0.0329	0.0260	0.0199	0.0146
0.5	0.0559	0.0470	0.0389	0.0315	0.0249	0.0190	0.0140
0.6	0.0535	0.0450	0.0372	0.0301	0.0238	0.0182	0.0134
0.7	0.0512	0.0430	0.0356	0.0288	0.0228	0.0174	0.0128
0.8	0.0489	0.0411	0.0340	0.0275	0.0217	0.0166	0.0122
0.9	0.0466	0.0392	0.0324	0.0262	0.0207	0.0159	0.0117
1	0.0444	0.0374	0.0309	0.0250	0.0198	0.0151	0.0111
1.1	0.0423	0.0356	0.0294	0.0238	0.0188	0.0144	0.0106
1.2	0.0402	0.0338	0.0279	0.0226	0.0179	0.0137	0.0101
1.3	0.0382	0.0321	0.0265	0.0215	0.0170	0.0130	0.0095
1.4	0.0362	0.0304	0.0251	0.0204	0.0161	0.0123	0.0091
1.5	0.0343	0.0288	0.0238	0.0193	0.0152	0.0117	0.0086
1.6	0.0324	0.0272	0.0225	0.0182	0.0144	0.0110	0.0081
1.7	0.0306	0.0257	0.0212	0.0172	0.0136	0.0104	0.0076
1.8	0.0288	0.0242	0.0200	0.0162	0.0128	0.0098	0.0072

1.9	0.0271	0.0228	0.0188	0.0152	0.0120	0.0092	0.0068
2	0.0254	0.0214	0.0177	0.0143	0.0113	0.0087	0.0064
2.1	0.0238	0.0200	0.0165	0.0134	0.0106	0.0081	0.0060
2.2	0.0222	0.0187	0.0154	0.0125	0.0099	0.0076	0.0056
2.3	0.0207	0.0174	0.0144	0.0117	0.0092	0.0071	0.0052
2.4	0.0193	0.0162	0.0134	0.0108	0.0086	0.0066	0.0048
2.5	0.0179	0.0150	0.0124	0.0101	0.0079	0.0061	0.0045
2.6	0.0165	0.0139	0.0115	0.0093	0.0073	0.0056	0.0041
2.7	0.0152	0.0128	0.0106	0.0086	0.0068	0.0052	0.0038
2.8	0.0140	0.0118	0.0097	0.0079	0.0062	0.0048	0.0035
2.9	0.0128	0.0108	0.0089	0.0072	0.0057	0.0044	0.0032
3	0.0117	0.0098	0.0081	0.0066	0.0052	0.0040	0.0029
3.1	0.0106	0.0089	0.0073	0.0060	0.0047	0.0036	0.0026
3.2	0.0095	0.0080	0.0066	0.0054	0.0042	0.0032	0.0024
3.3	0.0086	0.0072	0.0060	0.0048	0.0038	0.0029	0.0021
3.4	0.0076	0.0064	0.0053	0.0043	0.0034	0.0026	0.0019
3.5	0.0068	0.0057	0.0047	0.0038	0.0030	0.0023	0.0017
3.6	0.0059	0.0050	0.0041	0.0033	0.0026	0.0020	0.0015
3.7	0.0052	0.0044	0.0036	0.0029	0.0023	0.0018	0.0013
3.8	0.0045	0.0038	0.0031	0.0025	0.0020	0.0015	0.0011
3.9	0.0038	0.0032	0.0026	0.0021	0.0017	0.0013	0.0010
4	0.0032	0.0027	0.0022	0.0018	0.0014	0.0011	0.0008
4.1	0.0026	0.0022	0.0018	0.0015	0.0012	0.0009	0.0007
4.2	0.0021	0.0018	0.0015	0.0012	0.0010	0.0007	0.0005
4.3	0.0017	0.0014	0.0012	0.0010	0.0008	0.0006	0.0004
4.4	0.0013	0.0011	0.0009	0.0007	0.0006	0.0004	0.0003
4.5	0.0010	0.0008	0.0007	0.0005	0.0004	0.0003	0.0002
4.6	0.0007	0.0006	0.0005	0.0004	0.0003	0.0002	0.0002
4.7	0.0004	0.0004	0.0003	0.0002	0.0002	0.0001	0.0001
4.8	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001
4.9	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
s \ d	0.75	0.8	0.85	0.9	0.95		
0.1	0.0110	0.0071	0.0040	0.0018	0.0004		
0.2	0.0106	0.0068	0.0038	0.0017	0.0004		
0.3	0.0102	0.0065	0.0037	0.0016	0.0004		

0.4	0.0097	0.0062	0.0035	0.0016	0.0004
0.5	0.0093	0.0060	0.0033	0.0015	0.0004
0.6	0.0089	0.0057	0.0032	0.0014	0.0004
0.7	0.0085	0.0054	0.0031	0.0014	0.0003
0.8	0.0081	0.0052	0.0029	0.0013	0.0003
0.9	0.0077	0.0049	0.0028	0.0012	0.0003
1	0.0073	0.0047	0.0026	0.0012	0.0003
1.1	0.0070	0.0045	0.0025	0.0011	0.0003
1.2	0.0066	0.0042	0.0024	0.0011	0.0003
1.3	0.0063	0.0040	0.0023	0.0010	0.0003
1.4	0.0060	0.0038	0.0021	0.0010	0.0002
1.5	0.0056	0.0036	0.0020	0.0009	0.0002
1.6	0.0053	0.0034	0.0019	0.0008	0.0002
1.7	0.0050	0.0032	0.0018	0.0008	0.0002
1.8	0.0047	0.0030	0.0017	0.0008	0.0002
1.9	0.0044	0.0028	0.0016	0.0007	0.0002
2	0.0041	0.0026	0.0015	0.0007	0.0002
2.1	0.0039	0.0025	0.0014	0.0006	0.0002
2.2	0.0036	0.0023	0.0013	0.0006	0.0001
2.3	0.0033	0.0021	0.0012	0.0005	0.0001
2.4	0.0031	0.0020	0.0011	0.0005	0.0001
2.5	0.0029	0.0018	0.0010	0.0005	0.0001
2.6	0.0026	0.0017	0.0010	0.0004	0.0001
2.7	0.0024	0.0016	0.0009	0.0004	0.0001
2.8	0.0022	0.0014	0.0008	0.0004	0.0001
2.9	0.0020	0.0013	0.0007	0.0003	0.0001
3	0.0018	0.0012	0.0007	0.0003	0.0001
3.1	0.0017	0.0011	0.0006	0.0003	0.0001
3.2	0.0015	0.0010	0.0005	0.0002	0.0001
3.3	0.0013	0.0008	0.0005	0.0002	0.0001
3.4	0.0012	0.0008	0.0004	0.0002	0.0000
3.5	0.0010	0.0007	0.0004	0.0002	0.0000
3.6	0.0009	0.0006	0.0003	0.0001	0.0000
3.7	0.0008	0.0005	0.0003	0.0001	0.0000
3.8	0.0007	0.0004	0.0002	0.0001	0.0000
3.9	0.0006	0.0004	0.0002	0.0001	0.0000
4	0.0005	0.0003	0.0002	0.0001	0.0000

4.1	0.0004	0.0002	0.0001	0.0001	0.0000
4.2	0.0003	0.0002	0.0001	0.0000	0.0000
4.3	0.0002	0.0001	0.0001	0.0000	0.0000
4.4	0.0002	0.0001	0.0001	0.0000	0.0000
4.5	0.0001	0.0001	0.0000	0.0000	0.0000
4.6	0.0001	0.0000	0.0000	0.0000	0.0000
4.7	0.0000	0.0000	0.0000	0.0000	0.0000
4.8	0.0000	0.0000	0.0000	0.0000	0.0000
4.9	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000

Appendix C: Interactions between Two Farms

We then examine potential strategic interactions between farm decisions with heterogeneous farm-specific characteristics. Consider a watershed with 2 different farms, each has its soil index (s_i) and location index (d_i), *i* = 1, 2. Decisions made by the two individual farms have aggregate impact on the same receiving lake, thus linking the farms through their shared social water quality damage costs. Combining the transition function Eq. [12], Eq. [20] and the steady state condition Eq. [14], we find the analytical solution of optimal steady state fertilizer amounts (u_1^*, u_2^*), and lake phosphorus stock (x^*):

$$u_1^* = -\frac{-B2 * C1 * D2 + B1 * C2 * D2 + A * B1 * E}{A(C1 * D1 + C2 * D2 + A * E)} \quad [35]$$

$$u_2^* = -\frac{B2 * C1 * D1 - B1 * C2 * D1 + A * B2 * E}{A(C1 * D1 + C2 * D2 + A * E)} \quad [36]$$

$$x^* = -\frac{B1 * D1 + B2 * D2}{C1 * D1 + C2 * D2 + A * E} \quad [37]$$

where

$$A = \delta + \gamma \quad [38]$$

$$Bi = \frac{(\delta + \gamma)}{2p\rho a s_i}(c_u - p\rho b s_i) \quad [39]$$

$$Ci = \frac{\theta(1 - s_i)\left(1 - \frac{d_i}{\bar{d}}\right) m}{p\rho a s_i} \quad [40]$$

$$Di = \theta(1 - s_i)\left(1 - \frac{d_i}{\bar{d}}\right) \quad [41]$$

$$E = \gamma \qquad [42]$$

$$for\ i = 1, 2$$

We also derive the optimal decision of farmer 1 and farmer 2 in response to the other farmer's behavior:

$$u_1^* = -\frac{C_1 D_2}{AE + C_1 D_1} u_2 + \frac{B_1 E}{AE + C_1 D_1} \qquad [43]$$

$$u_2^* = -\frac{C_2 D_1}{AE + C_2 D_2} u_1 + \frac{B_2 E}{AE + C_2 D_2} \qquad [44]$$

where u_i^* denotes the optimal fertilizer choice for farm *i* the fertilizer amount when other farm's fertilizer choice is u_{-i}, *i* =1,2. Because both $\frac{C_1 D_2}{AE+C_1 D_1}$ and $\frac{C_1 D_2}{AE+C_1 D_1}$ are positive, we find the inverse relationship between fertilizer input decisions in the two farms, reflecting the tradeoff between two contributors to the same stock pollutant. If one farm applies more fertilizer than socially optimal, the other farm has to reduce fertilizer input and therefore reduce its profit to maintain social welfare level. The optimal outcome is for the farm with better soil quality and located farther away from the lake to apply more fertilizer, while the other farm apply less.

We further explore the effect of one farm's physical characteristics on optimal fertilizer decisions in the second farm by deriving the first order and second order derivatives of the other farm's soil index (s_{-i}) and location index (d_{-i}):

$$\frac{\partial u_i^*}{\partial s_{-i}} > 0, and \ \frac{\partial^2 u_i^*}{\partial {s_{-i}}^2} > 0 \quad [45]$$

$$\frac{\partial u_i^*}{\partial d_{-i}} > 0, and \ \frac{\partial^2 u_i^*}{\partial {d_{-i}}^2} > 0 \quad [46]$$

Results show that higher soil quality in one farm can increase the optimal fertilizer application rate in the other farm, and thus increase its agricultural profits. Similarly, larger distance of the farm to the lake can also increase the other farm's optimal fertilizer. This result implies spillover effects from improving farmland soil quality in the long term. If farms adopt Best Management Practices (BMP) and other soil management practices to improve soil quality, they not only benefit their own agricultural production and increase their profits, but also improve other farm's agricultural profits by increasing their socially optimal fertilizer input level. These results justify government spending on policy incentives for farmers to adopt BMP to improve farmland soil quality, and show potential welfare gains from payment transfer between farms.