**Modeling Quality Differentiation: Does the Choice of Model Matter?**

**Nathalie Lavoie, University of Massachusetts Amherst, lavoie@resecon.umass.edu**

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# Modeling Quality Differentiation: Does the Choice of Model Matter?

*Nathalie Lavoie*
Department of Resource Economics, University of Massachusetts Amherst

## Introduction and Motivation

With a trend towards a healthier lifestyle, food quality, nutrition, and safety are increasingly important to consumers today. At the same time, obesity and diabetes rates continue to rise and there is significant interest in developing and implementing public policies to address these problems. Food manufacturers and retailers respond to these trends and policies by differentiating products and providing more information with labels and claims.

To analyze these issues, industrial organization research in food markets has studied food quality with models of vertical differentiation (Saitone and Sexton, 2010). The model by Mussa and Rosen (MR) (1978) has been privileged, while the model attributed to Gabszewicz and Thisse (GT) (1979) and Shaked and Sutton (1982) has been largely ignored, with few exceptions (e.g., Roe and Sheldon, 2007; Xia and Sexton, 2010). When authors have acknowledged the existence of an alternative model to MR, they argued that results would be similar. Motta (1993) states that the model attributed to MR "is the analog of the models where consumers differ by their incomes rather than by their tastes (Gabszewicz and Thisse (1979, 1980), Shaked and Sutton (1982, 1983, 1984), Bonanno (1986) and Ireland (1987)." This research challenges this view.

## Objectives

- To provide a comparison of the two models under various assumptions.
- To inform practitioners about the different predictions that can be generated depending on the model choice and assumptions.
- To encourage practitioners to ensure that the model choice does not drive the results.

## Model Set-Up

### *Supply*

- Follows Motta (1993)
- Two firms – one producing the low-quality product and the other, the high-quality product.
- Two-stage game:
  - 1st stage: firms choose quality
  - 2nd stage: firms choose price (Bertrand competition) or quantity (Cournot competition)
- Cost of production assumed to be zero.
- The results of the MR and GT models are compared at the second stage because in the short-run, quality is often considered exogenous.
- Table 1 shows the scenarios considered.

### *Demand*

Consumers buy only one unit of the product. Table 2 summarizes the demand set-up for both models.

**Table 1. Scenarios examined**

| Fixed Cost of Quality Improvement $(c_i = 0.5k_i^2, i = H, L)$ | Variable Cost of Quality Improvement $(c_i = 0.5k_i^2, i = H, L)$ |
|---|---|
| o Bertrand duopoly | o Bertrand duopoly |
| ▪ Uncovered market | ▪ Uncovered market |
| ▪ Covered market | ▪ Covered market |
| o Cournot duopoly | o Cournot duopoly |
| ▪ Uncovered market | ▪ Uncovered market |

**Table 2. Two models of Vertical differentiation**

| | Mussa and Rosen (MR) | Gabszewicz and Thisse (GT) |
|---|---|---|
| **Indirect Utility** | $V = k\theta - p$ | $V = k(y - p)$ |
| **Consumers' distribution** | $\theta \in U[a, b]$ with unit density | $y \in U[a, b]$ with unit density |
| **Indirect utilities with two qualities:** | $\begin{cases} \theta k_H - p_H & \text{if buy good } H \\ \theta k_L - p_L & \text{if buy good } L \\ 0 & \text{if buy nothing} \end{cases}$ | $\begin{cases} k_H(y - p_H) & \text{if buy good } H \\ k_L(y - p_L) & \text{if buy good } L \\ 0 & \text{if buy nothing} \end{cases}$ |
| **Indifferent consumers** | $\theta_{HL} = \frac{p_H - p_L}{k_H - k_L}$ <br> $\theta_{L0} = \frac{p_L}{k_L}$ | $y_{HL} = \frac{k_H p_H - k_L p_L}{k_H - k_L}$ <br> $y_{L0} = p_L$ |
| **Demand equations ($q_i$)** | $q_H = b - \frac{p_H - p_L}{k_H - k_L}$ <br> $\begin{cases} q_L = \frac{p_H - p_L}{k_H - k_L} - \frac{p_L}{k_L} & \text{if } \theta_{L0} > a \\ q_L = \frac{p_H - p_L}{k_H - k_L} - a & \text{if } \theta_{L0} \le a \end{cases}$ | $q_H = b - \frac{k_H p_H - k_L p_L}{k_H - k_L}$ <br> $\begin{cases} q_L = k_H\left(\frac{p_H - p_L}{k_H - k_L}\right) & \text{if } p_L > a \\ q_L = \frac{k_H p_H - k_L p_L}{k_H - k_L} - a & \text{if } p_L \le a \end{cases}$ |

Note: $k$=quality of product, $p$=price of product, $\theta$=willingness to pay for quality, $y$=level of income, $H$=high quality, $L$=low quality

## Selective Results

### *Second Stage (Exogenous quality)*

- Similar results are obtain under Bertrand and Cournot competition:

$$\begin{array}{ll} if\ k_H > 1.25 & CS^{MR}, PS^{MR} > CS^{GT}, PS^{GT} \\ if\ k_H \le 1 & CS^{MR}, PS^{MR} < CS^{GT}, PS^{GT} \end{array}$$

$$if\ k_H \in\ ]1; 1.25] \begin{cases} sign\ depends\ on\ k_H \gtrless \frac{5}{4+\alpha}\ under\ Bertrand \\ sign\ depends\ on\ k_H \gtrless \frac{(2-\alpha)^2+1}{(2-\alpha)^2+\alpha}\ under\ Cournot \end{cases}$$

- Figures 1 and 3 show that when $k_H$=1, the consumer and producer surplus (CS, PS) obtained using the MR model are smaller than under the GT model. The discrepancy decreases with less differentiation (i.e., as $\alpha$ increases). For example, at $\alpha$=0.5, CS and PS are understated by 11% (Bertrand) and 18% (Cournot) relative to the GT model.
- Figure 2 and 4 show that the size of the discrepancy (in %) increases as $\left|k_H - \frac{5}{4+\alpha}\right|$ (Bertrand) or $\left|k_H - \frac{(2-\alpha)^2+1}{(2-\alpha)^2+\alpha}\right|$ increases. Example, at $\alpha$=0.5, the further away $k_H$ is from 1.11 (Bertrand, 1.18 for Cournot) the greater will be the difference between welfare under the MR and GT model. The MR model generates greater values of welfare than the GT model with larger values of $k_H$ and necessarily for $k_H$>1.25 (when $b$=1).

**Figure 1. Total consumer and producer surplus (Bertrand competition, FC, $k_H$=1, b=1)**

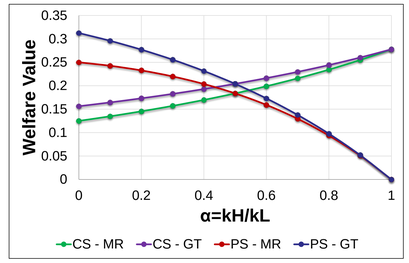



**Figure 2. Percentage difference in consumer and producer surplus between the GT and MR models (Bertrand competition, FC, b=1)**

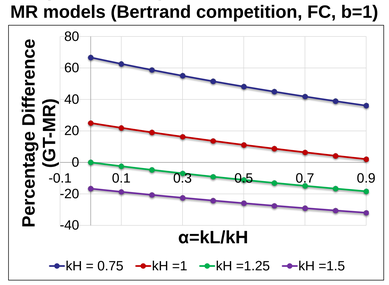



**Figure 3. Total consumer and producer surplus (Cournot competition, FC, $k_H$=1, b=1)**

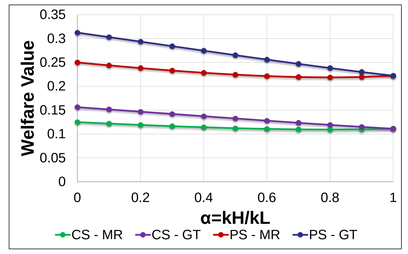
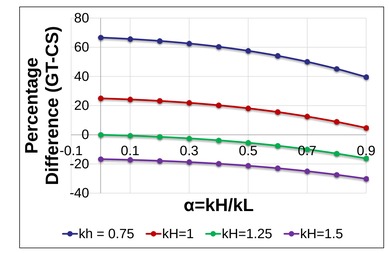



**Figure 4. Percentage difference in consumer and producer surplus between the GT and MR models (Cournot competition, FC, b=1)**




### *First Stage (Endogenous quality)*

- Under Bertrand competition (fixed cost and uncovered market), the MR model provides the same solution is as in Motta (1993) where firms choose to differentiate their product in equilibrium. However, the GT model leads to the principle of minimum differentiation.
- Under Cournot competition, total consumer surplus (profits) is larger under MR than GT if $b$>2.1 ($b$>3). The size of the discrepancy decreases with $b$. At $b$=1, consumer surplus (profit) is 342% (794%) larger under the GT than MR model. At $b$=3, consumer surplus (profit) is 51% (1%) smaller under the GT than MR model.

## Conclusions and Undergoing Work

- Choice of model affects the size of welfare measures.
- Under exogenous quality, assuming $k_H$=1, $b$=1 (i.e., $\theta \in U[0,1]$), using the Mussa and Rosen (MR) model leads to more conservative welfare measures than the Gabszewicz and Thisse (GT) model.
- However, the impact of model choice when quality is endogenous is important. Under Bertrand competition, the GT model leads to the principle of minimum differentiation, a strikingly different result than Motta (1993) who uses the MR specification.
- Under Cournot competition, the upper bound on consumer distribution has a large impact on welfare measures -- smaller values of $b$ lead to vast understatements of welfare under MR.
- It is likely that the ranking of different policy scenarios is affected under different model (MR vs GT) choice.
- Undergoing work replicates previous results to analyze this possibility.