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Patent Length, Investment and Social Welfare

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Abstract

The intent of the patent system is to encourage innovation by granting the innovator exclusive rights to a discovery for a limited period of time: with monopoly power, the innovator can recover the costs of creating the innovation which otherwise might not have existed. And, over time, the resulting innovation makes everyone better off. This presumption of improved social welfare is considered here. The paper examines the impact of patents on welfare in an environment where there are large numbers of (small) innovators. With patents, because there is monopoly for a limited time the outcome is necessarily not socially optimal, although social welfare may be higher than in the no-patent state. Patent acquisition and ownership creates two opposing incentives at the same time: the incentive to acquire monopoly rights conferred by the patent spurs innovation, but subsequent ownership of those rights inhibits innovation (both own innovation and that of others). On balance, which effect will dominate? In the framework of this paper separate circumstances are identified under which patents are either beneficial or detrimental to innovation and welfare; and comparisons are drawn with the socially optimal level of investment in innovation.

1 Introduction.

Conventional wisdom holds that the granting of the right of exclusive use of a discovery for a limited period of time encourages innovation, and the value of this in the long run outweighs the inefficiencies associated with temporary monopoly power over the discovery. This reasoning provides a primary justification for the patent system and the protection of intellectual property. Yet, despite the fact that the patent system has been in operation for hundreds of years, this basic view is still the subject of ongoing debate.

This comparison of costs due to monopoly relative to the benefit of increased innovation abstracts from many other incentives arising from the rights granted by a patent and which may affect the efficacy of the patent system. While the problem of monopoly is well understood, there are many ways in which the assignment of monopoly rights through a patent generate unforeseen consequences. For example, overlap of patent rights to technologies combined in a product may create "holdup" problems where individual patents holders can make competing demands; or rent-seeking behavior may arise where a patent holder seeks to extract revenue from a competitor through rights claims and the threat of litigation; or undeserved patents may be issued, and so on. Such issues have attracted considerable attention recently, but will not be examined here.

The focus of this paper is on comparison of the benefit to increased incentive to innovate and acquire patent rights against the attendant monopoly costs in an "ideal world" where the system functions perfectly, identifies all true innovations, rejects all false claims and operates in a manner that completely eliminates any abuse, manipulation, or gaming of the system. Of course, in practice such issues of abuse and operational weakness limit the efficacy of the patent system, imposing additional operating burdens. So, as a benchmark, consideration of the value of the system in the theoretically ideal environment provides a best case scenario benchmark. In subsequent paragraphs some historical context is given followed by a brief outline of the paper and summary of the main results.

From a historical perspective, these considerations (of conflict between monopoly rights and the incentive to innovate) have been present since patents were first issued in the late Middle Ages. In England, Edward III granted a patent for Woollen weaving in 1331 to John Kempe of Flanders; Henry VI granted a patent for the manufacture of colored glass in 1449. At this time, one major purpose in issuing patents was to stimulate growth of new manufacturing;[1] but the potential for patents to encourage innovation was also understood. The Venetian Senate voted a patent law governing all classes of invention into existence in 1474, giving patent protection for 10 years, with free access to the government. According to the preamble to the law [5]: "We have among us men of great genius, apt to invent and discover ingenious devices... Now, if provisions were made for the works and devices discovered by such persons, so that others who may see them could not build them and take the inventor's honor [sic] away, more men would then apply

[1] Henry VI granted a stained glass making patent to a glass maker from Flanders, John of Utynam, with a view to developing glass making in England. In this case, the purpose of the patent was to encourage local development of a known procedure.

their genius, would discover, and would build devices of great utility to our commonwealth."

Thus, by the end of the Middle Ages, at least, the use of patents to encourage the creation of new inventions and discoveries and to promote general welfare was recognized. In the New World, patents were granted by colonial governments. Massachusetts granted a patent for a new way of making salt in 1641. That year the colonial legislature of Massachusetts enacted a law asserting that no monopolies (exclusive rights) would be granted [10], with the exception of new inventions "profitable to the country, and that for a short time." The first American patent was granted in 1790, and the first French patent in 1791. By the late 1800's, most European countries had patent laws in place. The importance attached to patents in the United States can be seen from the fact that the first ones issued were signed by George Washington, Thomas Jefferson and Edmund Randolph (the first Attorney General). Article 1, section 8 of the United States Constitution (ratified in 1788) gave Congress the power "To promote the progress of science and useful arts, by securing for limited times to authors and inventors the exclusive right to their respective writings and discoveries." After some chaos in developing procedures, the Patent Act of 1836 established the Patent Office as a separate bureau of the State Department, later (1926) to become a bureau of the Commerce Department. See [12] for a broad review of the history and patent literature to the present.

Still, concerns regarding the balance between granting monopoly power and protecting societal welfare were present, even from the earliest days in the United States. At the time of drafting the constitution, Jefferson expressed concern to Madison in a letter dated July 31, 1788 [14]:

> "...... The saying there shall be no monopolies lessens the incitements to ingenuity, which is spurred on by the hope of a monopoly for a limited time, as of 14 years; but the benefit even of limited monopolies is too doubtful to be opposed to that of their general suppression."

Madison replied:

> "With regard to monopolies they are justly classed among the greatest nuisances in government. But is it clear that as encouragements to literary works and ingenious discoveries, they are not too valuable to be wholly renounced? Would it not suffice to reserve in all cases a right to the public to abolish the privilege at a price to be specified in the grant of it? Is there not also infinitely less danger of this abuse in our governments than in most others? Monopolies are sacrifices of the many to the few. Where the power is in the few it is natural for them to sacrifice the many to their own partialities and corruptions. Where the power, as with us, is in the many not in the few, the danger can not be very great that the few will be thus favored. It is much more to be dreaded that the few will be unnecessarily sacrificed to the many."

Thus, careful reflection led to an equivocal view of patents involving an inseparable mixture of good and bad — with the belief that patents sped up the rate of innovation and the benefits from this outweighed the monopoly cost associated with the creation of temporary monopoly.

Putting aside questions of abuse, inefficient operation and so on, many consider the basic presumption of net benefit a largely unproven belief. In the area of medical research the assessment of the value of patent protection is mixed (see [13], [6], and the references cited therein.) Concerning software patents, Bessen and Hunt [2] conclude that in the software industry there appears to be little correlation between the rate at which firms invest in R&D and the rate of innovation. A recent and broad ranging critique of the patent system, and the notion of intellectual property more generally is given in [4]. In fact, from a variety of perspectives, one might argue that patent protection slows innovation. One such view sees competition as the key force behind innovation [11]: innovators gain market share, so the need to survive places continuous pressure on firms to innovate. In this Schumpeterian view, it is the absence of protection that drives firms to innovate.

The impact of patents may depend substantially on the field of application: Bessen and Hunt identify the chemical industry as one where patents may be important in the decision to conduct R&D. Thus the balance of costs and benefits is still the subject of intense debate; and the intent of this paper is to address this matter, comparing the benefit from spurred invention against the temporary monopoly costs incurred.

The impact of patents in advanced industries (such as software and semiconductors) has been studied by Bessen and Maskin [3] who consider environments where innovation is sequential and complementary — successive innovation builds on what has gone before in a sequential way and innovation is complementary in the sense that the probability of success in discovery is improved when more firms pursue research. Different innovators follow different routes of research. In this setting, in certain circumstances, patents can actually inhibit innovation by limiting imitation that spurs the development of further innovation. They focus on the case where there are a small number of competing innovators. In contrast, here the environment is one with a large population.

While the use of (voluntary) licensing potentially permits broad application of new discovery immediately, this depends on the feasibility of such agreements. One may even appeal to the Coase theorem to assert that where gains are possible, licensing agreements will be reached. However, in practice, with a multiplicity of "players", unclear ownership rights or unclear breadth of patent; with the potential use of hold-up tactics to extract rents, the complexity, time and expense of legal resolution, and so forth, the expectation of a Coase-style resolution may be wishful thinking. The study of such issues will not be taken up here. This paper considers the question of whether the welfare cost from temporary monopoly is greater or less than the welfare gains from increased innovation spurred by the monopoly rights granted by patents. And, in examining this tradeoff, the research here is exclusively concerned with the impact of patents in an ideal world where the complications just mentioned are absent.

An outline of the paper is as follows. Section 2 develops the model. In the framework developed here a firm may have monopoly power in the sense that it has ownership over some component of the total technology set, and can limit use of this particular innovation by others. A patent gives the holder the right to exclusive use of the owned component. Firms have two choice variables, the current decision and an investment decision to improve its future technology. The current decision variable is subsumed in a profit function, only the firm's investment decision is considered. Section 3 considers the consequences of changing patent length and highlights the impact of access to the innovation of others on the marginal product of investment and the manner in which such knowledge correlates with a firms' own technology in terms of payoff impact. Depending on these features, patents may either reduce or raise social welfare (although they never lead to the socially optimal level of innovation). The welfare implications of the patent system in this environment are considered in detail in section 4. When the primary users of innovations entering the public domain after patent expiry are firms that are relatively weak technologically, then lengthening the patent period forces weaker firms to invest and create their own technology. This in turn puts pressure on better firms to stay ahead with increased investment and the overall effect is to raise social welfare. When these effects are not present, the case for patents is weaker or non-existent. Section 5.1 briefly discusses a variety of issues not considered in the paper, section 6 concludes.

2 The Model.

The environment is characterized by a collection of many competing firms. Firms are differentiated by their own technology and profitability depends on this and aggregate variables. We assume that firms protect own technology improvements through patenting, so that only the technology of a firm that is beyond the patent life is available for use by competitors. Only legitimate innovations are patented, and all such innovations are patented. Patent holders do not license innovations but exploit them as monopolists until patent expiry. These assumptions abstract from many important issues surrounding the issuance and acquisition of patents, some of which are discussed further in section 5.1. In this framework, the merits of patenting stand or fall on whether or not monopoly for a limited period of time has the overall effect of encouraging innovation, relative to the no-patent environment. Of course, because the market does not internalize fully the benefits of innovation there is scope for welfare improvement through incentives to encourage innovation.

2.1 The Model: Main Features

The technology of each firm evolves over time. At any point in time, a firm has its own technology, has access to technologies that are in the public domain, and makes investment decisions that affect its future technology. Aggregating individual behavior gives the aggregate distribution over technology and investment, and determines the evolution of the aggregate distribution of technologies over time. These details are described next.

2.1.1 Technology.

A firm is characterized by its technology $\alpha \in A$, where A is the set of all possible technologies. To allow for the possibility that one firm may be better than another in some respects, and worse in others, technology is multidimensional. Reflecting this, and to permit comparison of different firms technologies, take A to be an ordered space, with order $\succeq$ (see the appendix for discussion). This formulation permits a large set of technologies and, in particular, allows different firms to have different strengths and weaknesses.

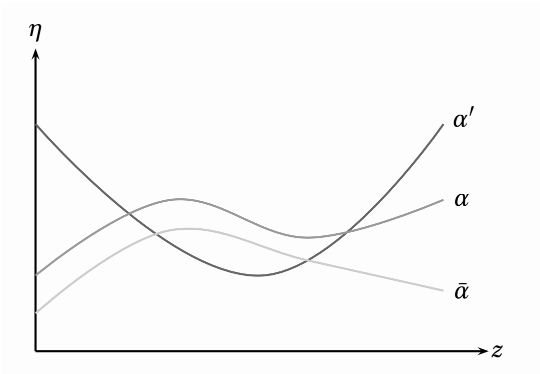

Figure 1: Technologies

Example 1 Figure 1 illustrates the case where a firm's technology is characterized by a function α, attaching a quality η to each feature z. Each feature is indexed as a real number in some range, $z \in [0, \bar{z}]$, and quality, η, a real non-negative number. Comparing technologies (or firms) α and α', technology α' is superior on low and high indexed features while technology α is superior on mid-ranged features. Two technologies are comparable if $\alpha \succeq \alpha'$ or if $\alpha' \succeq \alpha$. If $\alpha \succeq \alpha'$ then α is unequivocally a better firm than α' in every way; but in general technologies may not be comparable (neither $\alpha \succeq \alpha'$ or $\alpha' \succeq \alpha$). In the figure, if we order the functions according the rule, $\alpha' \succeq \alpha$ if and only if $\alpha'(z) \geq \alpha(z)$ for all z, then in this example, neither $\alpha \succeq \alpha'$ or $\alpha' \succeq \alpha$, whereas $\alpha \succeq \bar{\alpha}$. $\diamondsuit$

The distribution of technologies in the market is denoted μ, or μ_t to denote the distribution of technologies at time t: a probability measure on A.[2] There are a continuum of firms. This implies that each firm is negligible and eliminates strategic considerations from the model.[3,4]

[2] Assume that the measure has no atoms: each firm has probability or measure 0, so that no firm has strategic market power.
[3] See section 5.1 for a brief discussion of some of these issues.
[4] Furthermore, since the model is concerned with the dynamics of investment over time, this assumption makes state variables independent of individual behavior and simplifies the computations.

If α and α' are in the support of μ_t, they represent two technologies in operation at time t. Let

$$\boldsymbol{\mu}_t = \{\mu_\tau\}_{\tau=t}^{-\infty} = (\ldots, \mu_{t-1}, \mu_t) = (\ldots, \mu_{t-\ell-1}\,; \mu_{t-\ell}, \mu_{t-\ell+1} \ldots \mu_t)$$

denote the sequence of past technology distributions over time. With patent length ℓ (the length for which a patent is granted), technologies $\{\mu_{t-\ell-j}\}_{j\geq 0}$ are available for public use at time t: any technology in the support of $\mu_{t-\ell-j}$ may be used by a firm, and apart from technology α, only technologies in the support of distribution $\mu_{t-\ell}$ or older may be used by firm α. Finally, in considering market efficiency, it is necessary to compare the quality of technology distributions — throughout the paper technology distributions are compared in terms of first order stochastic dominance.[5,6]

2.1.2 The Firm

Firms are defined in terms of their profit functions. The profit of a firm at time t depends on its technology α, current competition represented by the distribution of prevailing technologies, μ_t, and technologies in the public domain, $\mu_{t-\ell}$, $\pi(\mu_t, \mu_{t-\ell}, \alpha)$. This may be written alternatively as $\pi(\boldsymbol{\mu}_t, \alpha, \ell)$ to highlight the length of technology protection ℓ.[7,8] Finally, over time firms invest from profit to improve future technology: the level of investment i costs $r(i)$, where r is assumed to satisfy $r' \geq 0$ and $r'' \geq 0$. With investment level i, the firm's current net revenue is $\pi(\mu_t, \mu_{t-\ell}, \alpha) - r(i)$.

Example 2 Let a market have demand given at time t by $P_d(Q, \mu_t)$ and suppose that $\mu'_t \succcurlyeq \mu_t$ implies $P_d(Q, \mu'_t) \geq P_d(Q, \mu_t)$, so that better technology raises demand. Suppose also that firm α has cost given by $\frac{1}{2}c(\alpha, \mu_{t-\ell})q^2$. Assume that for any $(\mu_{t-\ell}, \alpha)$, $c(\alpha, \mu_{t-\ell}) \geq c(\alpha', \mu_{t-\ell})$ if $\alpha' \succeq \alpha$ and $c(\alpha, \mu'_{t-\ell}) \leq c(\alpha, \mu_{t-\ell})$ if $\mu'_{t-\ell} \succcurlyeq \mu_{t-\ell}$. Therefore, firm α's profit is given by $\max_q P_d(Q, \mu_t)q - \frac{1}{2}c(\alpha, \mu_{t-\ell})q^2$. The solution to this has $P_d(Q, \mu_t) - c(\alpha, \mu_{t-\ell})q = 0$, determining q: $q(\alpha) = \frac{P_d(Q,\mu_t)}{c(\alpha,\mu_{t-\ell})}$, and aggregate is $Q = \int q(\alpha)\mu_t(d\alpha) = P\int \frac{1}{c(\alpha,\mu_{t-\ell})}\mu_t(d\alpha)$, giving inverse supply curve $P_s(Q, \mu_t, \mu_{t-\ell}) = Q\frac{1}{\int \frac{1}{c(\alpha,\mu_{t-\ell})}\mu_t(d\alpha)}$. To simplify, let $\varphi(\alpha)$ be a quality-efficiency index of technology α. Suppose that cost is given by $c(\alpha, \mu_{t-\ell}) = [\varphi(\alpha)g(\mu_{t-\ell})]^{-1}$ where φ and g are increasing in α and $\mu_{t-\ell}$ respectively. Finally, set $f(\mu) = \int \varphi(\alpha)d\mu$ with demand specified as $P_d(Q, \mu_t) = f(\mu)Q^{-\beta}$, $\beta > 0$, so the elasticity of demand is $\frac{1}{\beta}$. Thus,

$$\begin{aligned}
\int q(\alpha)\mu_t(d\alpha) &= f(\mu_t)Q^{-\beta}\int \frac{1}{c(\alpha, \mu_{t-\ell})}\mu_t(d\alpha) = f(\mu_t)Q^{-\beta}g(\mu_{t-\ell})\int \varphi(\alpha)d\mu_t(\alpha) \\
Q &= f(\mu_t)^2 Q^{-\beta} g(\mu_{t-\ell})
\end{aligned}$$

[5] Given two measures $\mu, \nu \in \mathscr{P}(\Lambda)$, μ first order dominates ν, written $\mu \succcurlyeq \nu$ if and only if for all measurable increasing functions $g : \Lambda \to \Re$, $\int g d\mu \geq \int g d\nu$.

[6] Note that "$\succeq$" is an ordering on technologies, Λ, whereas "$\succcurlyeq$" is an ordering on distributions over technologies $\mathscr{P}(\Lambda)$. See the appendix for further discussion.

[7] If the patent length, ℓ, lies between two periods, τ and $\tau - 1$, interpolate between the two periods: for example, let $(\ell - [\tau - 1])\mu_\tau + (\tau - \ell)\mu_{\tau-1}$ be the most recent publicly available technology, so that the patent length can be treated as a non-negative real number.

[8] In fact we may let π depend on the entire sequence $\mu_t, \mu_{t-1}, \ldots$ so that in principle recent technical developments affect current profit.

Thus, $Q^{1+\beta} = f(\mu_t)^2 g(\mu_{t-\ell})$, $Q = f(\mu_t)^{\frac{2}{1+\beta}} g(\mu_{t-\ell})^{\frac{1}{1+\beta}}$ so that the equilibrium price is:

$$\begin{aligned} P^* &= f(\mu_t)[f(\mu_t)]^{\frac{-2\beta}{1+\beta}} g(\mu_{t-\ell})^{\frac{-\beta}{1+\beta}} \\ &= [f(\mu_t)]^{\frac{1-\beta}{1+\beta}} g(\mu_{t-\ell})^{\frac{-\beta}{1+\beta}} \end{aligned}$$

Profit of α is

$$\begin{aligned} \pi(\mu_t, \mu_{t-\ell}, \alpha) &= P^* q(\alpha) - \frac{1}{2} c(\alpha, \mu_{t-\ell}) q(\alpha)^2 = [P^* - \frac{1}{2} c(\alpha, \mu_{t-\ell}) q(\alpha)] q(\alpha) \\ &= [P^* - \frac{1}{2} c(\alpha, \mu_{t-\ell}) \frac{P^*}{c(\alpha, \mu_{t-\ell})}] q(\alpha) \\ &= \frac{1}{2} P^* \frac{P^*}{c(\alpha, \mu_{t-\ell})} \\ &= \frac{1}{2} (P^*)^2 \varphi(\alpha) g(\mu_{t-\ell}) \\ &= \frac{1}{2} [f(\mu_t)]^{\frac{2(1-\beta)}{1+\beta}} g(\mu_{t-\ell})^{\frac{-2\beta}{1+\beta}} \varphi(\alpha) g(\mu_{t-\ell}) \\ &= \frac{1}{2} [f(\mu_t)]^{\frac{2(1-\beta)}{1+\beta}} g(\mu_{t-\ell})^{\frac{1-\beta}{1+\beta}} \varphi(\alpha) \\ \pi(\mu_t, \mu_{t-\ell}, \alpha) &= \frac{1}{2} [f(\mu_t)^2 g(\mu_{t-\ell})]^{\frac{(1-\beta)}{1+\beta}} \varphi(\alpha) \end{aligned}$$

In this context, improvements in technology have three effects: demand rises, cost decreases as all firms avail of technology improvements; and each firm becomes more efficient as advances in technology raise the firms ability to make technology improvements. The first effect unambiguously benefits all firms, the second effect increases both a firm's ability to compete and the competitiveness of its competitors. If the overall net effect is to raise a firm's profit, say that profit improves with aggregate technology. ◊

Definition 1 *Profit increases with technology improvement if* $\pi(\boldsymbol{\mu}_t, \alpha, \ell)$ *is increasing in* $\boldsymbol{\mu}_t$.

The profit function in example 2 satisfies this condition provided $\beta < 1$. Later, it is assumed that profit does increase with technological improvement. This assumption is not innocuous since, as mentioned, there are conflicting effects: while technological improvements may raise demand and lower the cost structure of a firm — thus tending to raise profit, they also strengthen competition between firms which works in the opposite direction.

2.1.3 The Evolution of Technology

A firm may invest each period in research and technological improvement. Technological improvement is represented by a transition kernel, $P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell)$, where i is investment. This formulation covers many possibilities: for example, if only technology available at the patent date is relevant, then the transition kernel may be written as $P(d\tilde{\alpha} \mid \mu_{t-\ell}, \alpha, i)$ — a firm with technology α may use investment and the knowledge of technologies in the support of $\mu_{t-\ell}$ to develop its technology next period. The formulation allows for the possibility that a firm may require investment to achieve the standards represented by $\mu_{t-\ell}$ — it may not be possible for a firm to effortlessly implement the best technologies in the support of $\mu_{t-\ell}$. At the other extreme, it may be possible for the firm to costlessly implement any technology in the support of $\mu_{t-\ell}$. A firm owns it technology, no licensing of technology occurs. Obviously, licensing introduces new possibilities for a firm to improve its technology and could potentially mitigate some of the monopoly creating aspects of a patent. However, the intent here is to study the merits of patenting on the primary comparison of an enhanced rate of innovation against the creation of temporary monopoly power.

Example 3 One simple way to model the transition kernel is to have it be a weighted average of two distributions. Let F and G be two distributions on Λ with $F \succcurlyeq G$, and let $\rho(\boldsymbol{\mu}, \alpha, i, \ell) \in [0,1]$. Define the transition kernel:

$$P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell) = \rho(\boldsymbol{\mu}, \alpha, i, \ell) F(d\tilde{\alpha}) + [1 - \rho(\boldsymbol{\mu}, \alpha, i, \ell)] G(d\tilde{\alpha}) \tag{1}$$

◊

Throughout the discussion some basic assumptions on technology are maintained. In particular, having a better technology or investing more raises the probability of drawing a better technology, while having less access to technology (through longer patent life) lowers the probability of drawing a good technology. Formally, $P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell)$ is weakly increasing in α, and i; and weakly decreasing in ℓ (in terms of first order stochastic dominance).[9] This latter assumption captures the impact of patent length on the firm's ability to innovate. Furthermore, $P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell)$ is increasing in $\boldsymbol{\mu}_t$ — in the sense that if $\boldsymbol{\mu}'_t$ dominates $\boldsymbol{\mu}_t$ coordinate-wise, written $\boldsymbol{\mu}'_t \succcurlyeq \boldsymbol{\mu}_t$, then other things equal, a better distribution is drawn conditional on $\boldsymbol{\mu}'_t$ than $\boldsymbol{\mu}_t$. Better current technology and better technology in the public domain improves the firms success in innovation. Finally, a firm's technology cannot dis-improve over time: if drawing α' is possible for α, then $\alpha' \succeq \alpha$.[10,11]

The investment strategies of firms in conjunction with the transition kernel, P, move the state of the system forward over time. Firms investment strategies are represented by a joint distribution, τ, on $(i, \alpha) \in I \times \mathscr{A}$, written $\tau \in \mathscr{M}(I \times \mathscr{A})$. Conditioning on α, $\tau(di \mid \alpha)$, gives the distribution over investment of firm α. Given the extant distribution over technologies is μ, for consistency, if $\tau \in \mathscr{M}(I \times \mathscr{A})$ the marginal distribution of τ on $\mathscr{A}$ should coincide with μ: $\text{marg}_{\mathscr{A}} \tau = \mu$. Let $\mathscr{C}(\mu) = \{\tau \mid \text{marg}_{\mathscr{A}} \tau = \mu\}$, the set of

[9]For example, $\alpha' \succeq \alpha$ implies first order stochastic dominance: $P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha', i, \ell) \succcurlyeq P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell)$.

[10]If $\alpha' \in \operatorname{supp} P(\cdot \mid \boldsymbol{\mu}, \alpha, i, \ell)$, then $\alpha' \succeq \alpha$, where given a measure v on Λ, $\operatorname{supp} v$ is the support of v.

[11]If $P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell)$ has support $\{\alpha\}$ when $i = 0$, then the firm cannot improve without investment. Next period, without investment, the firm can avail of technologies in $\mu_{t-\ell+1}$.

distributions on $I \times \mathscr{A}$ with marginal μ on $\mathscr{A}$. The distribution of technologies evolves as:

$$\mu_{t+1}(\cdot) = \int_{i_t,\alpha_t} P(\cdot \mid \boldsymbol{\mu}_t, \alpha_t, i_t, \ell)\tau_t(di_t \mid \alpha_t)\mu_t(d\alpha_t) = \int_{i_t,\alpha_t} P(\cdot \mid \boldsymbol{\mu}_t, \alpha_t, i_t, \ell)\tau_t(di_t \times d\alpha_t). \quad (2)$$

So, given the current distribution on technologies, μ_t, if α_t invests according to the strategy $\tau_t(\cdot \mid \alpha_t)$, then next period the aggregate distribution on technologies is given by μ_{t+1}. If we fix a sequence of strategies $\tau^t = \{\tau_{t+j}(\cdot \mid \alpha_{t+j})\}_{j\geq 0}$, given μ_t, we may determine the sequence of distributions:

$$\mu_{t+1}(\cdot) = \psi_1(\cdot \mid \mu_t, \tau^t, \ell) \quad = \quad \psi(\cdot \mid \boldsymbol{\mu}_t, \tau_t, \ell) \quad (3)$$

$$\mu_{t+2}(\cdot) = \psi_2(\cdot \mid \mu_t, \tau^t, \ell) \quad = \quad \psi(\cdot \mid (\boldsymbol{\mu}_t, \psi_1(\cdot \mid \mu_t, \tau^t, \ell)), \tau_{t+1}, \ell) \quad (4)$$

and so on. (See the appendix for some additional discussion.)[12]

2.2 Investment Decisions.

Over time, firms make period by period decisions on production, and in addition, make investment decisions to develop future technology. The current production decision arises in the period by period market equilibrium and determines current profit, π. The investment decision generates current cost but improves the competitive position of the firm in subsequent periods. For some of the discussion, it is useful to write the present value at time t of the payoff flow to a firm, α, optimizing in each period from this point on — given the distribution up to the present, $\boldsymbol{\mu}_t$, and a sequence of aggregate distributions $\boldsymbol{\tau}^t = \{\tau_s\}_{s=t}^{\infty}$ as $v(\boldsymbol{\mu}_t, \boldsymbol{\tau}^t, \alpha, \ell)$. The individual optimization problem may expressed in a Bellman equation as:

$$v(\boldsymbol{\mu}_t, \boldsymbol{\tau}^t, \alpha, \ell) = \max_i \{\pi(\boldsymbol{\mu}_t, \alpha, \ell) - r(i) + \delta \int v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \tilde{\alpha}, \ell) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell)\} \quad (5)$$

where $\boldsymbol{\mu}_{t+1} = (\boldsymbol{\mu}_t, \mu_{t+1})$ with μ_{t+1} given by equation (3). The function v is increasing in α: a firm with higher α can imitate the investment strategy of one with lower α but enjoy lower cost and stochastically better technology draws. Note that the positive externality from investment does not appear in the individual firm's investment decision, equilibrium is inefficient. These observations are clarified in the following calculations.

Equilibrium. Assuming an interior solution, the first order condition at the solution is:

$$-r'(i) + \delta \lim_{i' \to i} [\frac{1}{i' - i}] \int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \tilde{\alpha}, \ell)[P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i', \ell) - P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell)] = 0$$

[12]In this environment, improvement in technology overall results from the flow of individual discoveries — with each individual discovery insignificant relative to the overall volume of discovery. One possible extension of this model is to allow for "paradigm shift" discoveries which revolutionize an industry — in the way, for example, that the *Mosaic* web browser revolutionized internet use. The formulation used here can accommodate such an extension provided that big breakthroughs are unanticipated. In such a formulation, there is positive probability of a breakthrough discovery in any period (some firm will have a major discovery or development), but no single firm can guarantee that it will have such a discovery with positive probability. In this case, revolutionary innovations are unanticipated and hence don't directly affect the investment incentives of firms.

The first order condition for i is:[13]

$$-r'(i)+\delta\int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1},\boldsymbol{\tau}^{t+1},\tilde{\alpha},\ell)\Delta_i P(\tilde{\alpha}\mid\boldsymbol{\mu}_t,\alpha,i,\ell)=0. \tag{6}$$

The second order condition for an optimum is then:[14]

$$-r''(i)+\delta\int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1},\boldsymbol{\tau}^{t+1},\tilde{\alpha},\ell)\Delta_{ii} P(d\tilde{\alpha}\mid\boldsymbol{\mu}_t,\alpha,i,\ell)<0. \tag{7}$$

Considering equation (6), since v is increasing in $\tilde{\alpha}$ and P in α (in first order stochastic dominance terms), the optimal value of i increases in α. A sequence of strategies $\bar{\tau}^t=\{\bar{\tau}_{t+j}(\cdot\mid\alpha_{t+j})\}_{j\geq 0}$ is an equilibrium if for any t, for each $j\geq 0$, $\bar{\tau}_{t+j}\otimes\mu_{t+j}$ has support $\{(i_t(\alpha),\alpha)\mid\alpha\in A\}$, where $i_t(\alpha)$ solves (6) at $\bar{\tau}^t$. Establishing the existence of equilibrium is straightforward using arguments from [1] or [8].

Efficiency. In any equilibrium, the level of investment is inefficient because there are positive externalities from investment. Consider the aggregate expected payoff:

$$\begin{aligned}\int v(\boldsymbol{\mu}_t,\boldsymbol{\tau}^t,\alpha,\ell)\tau_t(di\mid d\alpha)\mu_t(d\alpha)=\int \pi(\boldsymbol{\mu}_t,\alpha,\ell)\mu_t(d\alpha)-\int r(i)\tau_t(di\mid d\alpha)\mu_t(d\alpha)\\ +\delta\int v(\boldsymbol{\mu}_t,\psi_1,\boldsymbol{\tau}^{t+1},\tilde{\alpha},\ell)P(d\tilde{\alpha}\mid\boldsymbol{\mu}_t,\alpha,i,\ell)\tau_t(di\mid d\alpha)\mu_t(d\alpha)\}.\end{aligned}$$

Perturbing $\tau_t(di\mid d\alpha)$ in the direction $\tilde{\tau}_t$ to $\tau_t(di\mid d\alpha)+\epsilon\Delta\tau_t(di\mid d\alpha)$, $\Delta\tau_t(di\mid d\alpha)=\tilde{\tau}_t-\tau_t$, and considering the variation in the right side and dividing by ϵ gives:

$$\begin{aligned}&\Big\{-\int r(i)+\delta\int v(\boldsymbol{\mu}_t,\psi_1,\boldsymbol{\tau}^{t+1},\tilde{\alpha},\ell)P(d\tilde{\alpha}\mid\boldsymbol{\mu}_t,\alpha,i,\ell)\Big\}\Delta\tau_t(di\mid d\alpha)\mu_t(d\alpha)\\ &\quad+\int\frac{1}{\epsilon}\big[v(\boldsymbol{\mu}_t,\tilde{\psi}_1,\boldsymbol{\tau}^{t+1},\tilde{\alpha},\ell)-v(\boldsymbol{\mu}_t,\psi_1,\boldsymbol{\tau}^{t+1},\tilde{\alpha},\ell)\big]P(d\tilde{\alpha}\mid\boldsymbol{\mu}_t,\alpha,i,\ell)\tau_t(di\mid d\alpha)\mu_t(d\alpha)\end{aligned}$$

where $\tilde{\psi}_1$ is the $t+1$ period distribution given the strategy $\tau_t(di\mid d\alpha)+\epsilon\Delta\tau_t(di\mid d\alpha)$ at time t. At the market equilibrium, the first term is approximately 0, but the second term is non-zero and captures the (positive) externalities from improving the aggregate distribution.

In example 2, these externalities arise from two considerations. Improving the aggregate distribution improves the knowledge base that individual firms can use, and therefore raises their investment success. Second, improvement in technology improves demand, so that, in that environment the sum of producer and consumer surplus increases in each period. This implies,

Theorem 1 *In equilibrium, investment is below the efficient level, since the positive externalities of investment through improved technology distributions are not internalized.*

[13] Assume that $\frac{1}{i'-i}[P(d\tilde{\alpha}\mid\boldsymbol{\mu}_t,\alpha,i',\ell)-P(d\tilde{\alpha}\mid\boldsymbol{\mu}_t,\alpha,i,\ell)]$ converges weakly to a signed measure $\Delta_i P(d\tilde{\alpha}\mid\boldsymbol{\mu}_t,\alpha,i,\ell)$ as $i'\to i$.

[14] For the second order condition, assume that $\left[\frac{1}{i'-i}\right][\Delta_i P(B\mid\boldsymbol{\mu}_t,\alpha,i',\ell)-\Delta_i P(B\mid\boldsymbol{\mu}_t,\alpha,i,\ell)]$ converges weakly to a signed measure, $\Delta_{ii}P(\cdot\mid\boldsymbol{\mu}_t,\alpha,i,\ell)$, as $i'\to i$.

3 The Impact of Patent Length on Investment.

The effect of lengthening patent life is to reduce the publicly available technology. What is the impact of such a change on welfare? Because the socially optimal level of investment is higher than that arising in competitive equilibrium, whether lengthening patent length is beneficial or not depends on the impact such changes have on investment. The results to follow identify two cases.

When low technology firms are more dependent than high technology firms on the use of technology protected by patent, then, subject to conditions, the impact of lengthening patent life is to force those firms to greater research effort (by depriving them of access to previously unrestricted technology.) And this has a knock-on effect of increasing the competitive pressure on good firms, forcing them to also raise investment. As a result, overall investment in R&D increases and this raises social welfare. Put differently, technically weak firms are making greater use of publicly available technology: old technology is a substitute for investment. Reducing access to older technology forces those firms to invest. In the following discussion, this is described as the "substitutes" case.

In the second case, low technology firms make relatively less use of technology beyond the patent period: they are less dependent on public technology. Then, lengthening the patent period has relatively greater impact on better technology firms. To the extent that patented technology is used, lengthening patent length reduces the benefits from and incentives to being "good" (relatively more than in the substitutes case). Call this case the complements case, since good technology is complemented by knowledge outside the patent period than is poor technology. In this case the overall impact is to lower social welfare.

These results suggest that patents are beneficial when, *as a result of the need to compete*, they spur R&D and hence innovation. To the extent that disallowing a firm to use the discovery of others ultimately forces that firm to greater investment in R&D the effect of patents is beneficial.

3.1 Investment and Patented Knowledge as Substitutes.

When the (negative) impact of lengthening patent life is greatest on low technology firms, good technology may be considered a substitute for the patented technology, and since investment improves technology, investment becomes a substitute for patented technology. Furthermore, if increasing patent length raises the marginal product of investment and the marginal value of being a better firm, then investment also serves as a substitute for patented information. These assumptions are formalized in the following conditions:

(I-i) Increasing patent life, ℓ, or worsening technology, $\boldsymbol{\mu}$:

(a) impacts lower technology firms profits more, lowering current profits of better firms less than weaker firms (in terms of α):[15]

$$\pi(\boldsymbol{\mu}', \alpha, \ell') - \pi(\boldsymbol{\mu}, \alpha, \ell), \text{ is increasing in } \alpha, \text{ for } \ell' \geq \ell,\ \boldsymbol{\mu}' \preccurlyeq \boldsymbol{\mu}$$

(b) and worsens the technology draw of weaker firms more. For g increasing:

$$\int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}', \alpha, i, \ell') - \int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}, \alpha, i, \ell), \text{ is increasing in } \alpha, \text{ for } \ell' \geq \ell,\ \boldsymbol{\mu}' \preccurlyeq \boldsymbol{\mu}$$

(I-ii) Increasing patent life, ℓ, or improving technology, $\boldsymbol{\mu}$, raises the marginal productivity of investment. For g increasing:

$$\int_{\tilde{\alpha}} g(\tilde{\alpha}) \Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}', \alpha, i, \ell') \geq \int_{\tilde{\alpha}} g(\tilde{\alpha}) \Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}, \alpha, i, \ell), \quad \text{for } \ell' \geq \ell,\ \boldsymbol{\mu}' \succcurlyeq \boldsymbol{\mu}$$

Thus, increased ambient technology or less access to protected technology raises the marginal productivity of investment.

Figure 2 illustrates these assumptions (Since α is not a real number, "α" denotes an axis of ordered α's).

For a firm, the impact of increasing ℓ is to reduce access to patented technology, so the firm will be worse off. Assumption (I–$i(a)$) requires that the impact is less for firms with better technologies. Similarly, if the ambient aggregate distributions are worse, the effect is greater on weaker firms. Informally, limited access to technology or poorer aggregate technology impacts weaker firms more negatively. Assumption ($I-i(b)$) likewise assumes that better firms are more advantaged in these terms in the technology draw. So, for example, increasing patent length has a more detrimental effect on lower quality firm's in terms of success in drawing a new technology. Assumption ($I-ii$) describes how the marginal productivity of investment for all firms is impacted by changes in patent life or prevailing technology. If the ambient technology improves (comparing $\boldsymbol{\mu}'$ and $\boldsymbol{\mu}$), this has synergies with a firm's investment and raises the marginal productivity of investment. Similarly, the marginal productivity of investment is raised when access to existing technologies is withdrawn by a increasing of patent length.

To repeat, (I-$i(a)$) implies that the loss of technologies excluded by patents has a greater negative impact on weak or low technology firms. While increasing patent life worsens lowers every firm's productivity, according to I-$i(b)$ the impact is less for better technology firms. Together, these conditions imply that there is greater pressure on weak firms to improve in terms of profitability; and there is greater payback to investment after improvement. Condition (I-ii) implies that reducing the access of any firm, α, to patented technology, raises the marginal productivity of investment by α.

When increases in patent life affects good firms less than bad firms and when increased length of patent protection raises the marginal value of investment then increased patent protection raises welfare as a

[15] Note that $\pi(\boldsymbol{\mu}', \alpha, \ell') - \pi(\boldsymbol{\mu}, \alpha, \ell)$ is negative, so that larger values correspond to smaller profit reduction.

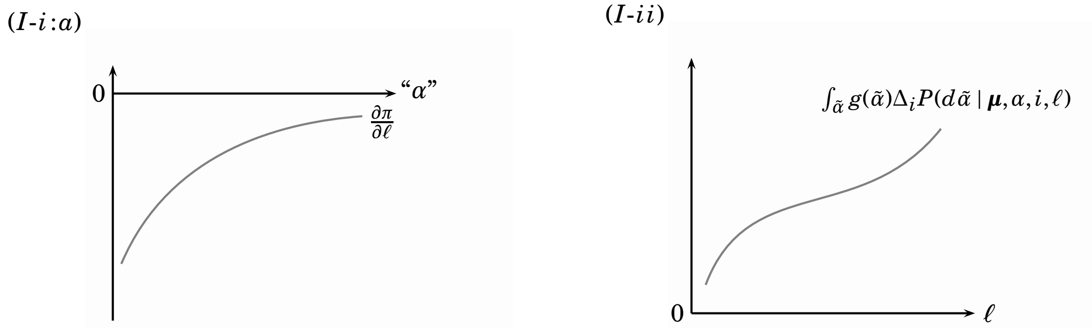

Figure 2: PATENTED KNOWLEDGE SUBSTITUTABLE FOR TECHNOLOGY AND INVESTMENT

result of raising investment. These observations are summarized in the next theorem.[16]

Theorem 2 *Suppose that profit increases with technology improvement and assumptions* $(I\text{-}i)-(I\text{-}ii)$ *are satisfied. Then lengthening patent life improves the aggregate distribution of technologies in successive periods.*

Let V_S or $V_S(\boldsymbol{\mu},\ell)$ be the welfare surplus in competitive equilibrium (in the substitutes case.) As ℓ varies, so does the value of surplus, and, when the substitutes condition is satisfied, an increase in ℓ leads to an increase in investment and hence raises welfare, *since, from the social welfare perspective*, investment is too low in equilibrium. The curve V_S in figure 5 depicts this case. The next section considers the opposite case — where knowledge is complementary to innovation and cost reduction.

3.2 Investment and Patented Knowledge as Complements.

The effect of lengthening patent life is to reduce the publicly available technology, and when the impact of this is greatest on high technology firms, good technology is complemented by the patented information. Furthermore, if increasing patent length reduces the marginal product of investment, then that information is also a complement to investment.

So, in contrast to the previous assumptions $(I\text{-}i)-(I\text{-}ii)$, suppose instead that better firms are more dependent on patented information to generate current profit and support innovation, so that such information is a complement to the quality of a firms' technology. Suppose also that increasing patent length removes from use information which raises the marginal product of investment (such information is complementary to investment). These conditions are formalized next.

(II-i) Increasing patent life, ℓ, or worsening technology, $\boldsymbol{\mu}_t$:

(a) impacts better technology firms profits more, lowering current profits of weaker firms less than

[16]Proofs are in the appendix.

stronger firms (in terms of α):

$$\pi(\boldsymbol{\mu}', \alpha, \ell') - \pi(\boldsymbol{\mu}, \alpha, \ell), \text{ is decreasing in } \alpha, \text{ for } \ell' \geq \ell,\ \boldsymbol{\mu}' \preccurlyeq \boldsymbol{\mu}$$

(b) and worsens the technology draw of better firms more. For g increasing:

$$\int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t', \alpha, i, \ell') - \int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell), \text{ is decreasing in } \alpha, \text{ for } \ell' \geq \ell,\ \boldsymbol{\mu}' \preccurlyeq \boldsymbol{\mu}$$

(II-ii) Increasing patent life or worsening the aggregate distribution, $\boldsymbol{\mu}$, lowers the marginal productivity of investment. For g increasing:

$$\int_{\tilde{\alpha}} g(\tilde{\alpha}) \Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}', \alpha, i, \ell') \leq \int_{\tilde{\alpha}} g(\tilde{\alpha}) \Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}, \alpha, i, \ell), \quad \text{for } \ell' \geq \ell,\ \boldsymbol{\mu}' \preccurlyeq \boldsymbol{\mu}$$

These assumptions are depicted in figure 3. Assumption $(II-i(a))$ asserts that better technology firms are more negatively impacted by an extension in patent length, and would be more negatively impacted by a worsening of the aggregate technology. Assumption $(II-i(b))$ expresses the same features, but in terms of the impact on innovation of variations in patent length or aggregate distribution quality. Assumption $(II-ii)$ says that increasing the patent length or lowering the quality of the aggregate distribution raises the marginal productivity of investment.

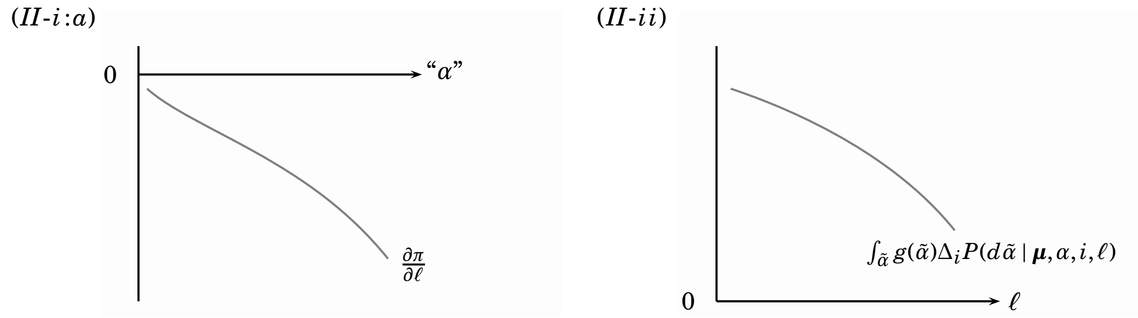

Figure 3: PATENTED KNOWLEDGE COMPLEMENTARY TO TECHNOLOGY AND INVESTMENT

Under these circumstances, lengthening patent life reduces investment.

Theorem 3 *Suppose that profit increases with technology improvement and assumptions* $(II\text{-}i)-(II\text{-}ii)$ *are satisfied. Then lengthening patent life worsens the aggregate distributions in successive periods.*

As in the earlier discussion write V_C or $V_C(\boldsymbol{\mu}, \ell)$ to denote the surplus in competitive equilibrium (in the complements case.) In contrast to the substitutes case, here an increase in ℓ leads to a decrease in investment and hence reduces welfare. The curve V_C in figure 5 depicts this case.

4 Welfare and Efficiency.

The previous discussion focused on investment incentives. Here, that discussion is placed in the context of overall welfare. Assume an environment where in each period there is a measure of consumer and producer welfare (surplus) which depends on the distribution of characteristics and the patent policy (length). Let total welfare generated at time t be denoted $TS(\boldsymbol{\mu}_t, \ell)$ and assume that:

$$TS(\boldsymbol{\mu}'_t, \ell') - TS(\boldsymbol{\mu}_t, \ell) \geq 0, \quad \boldsymbol{\mu}'_t \succcurlyeq \boldsymbol{\mu}_t,\ \ell' \leq \ell$$

so that improving technologies or reducing patent protection on existing technologies raises current welfare. With this the social welfare maximizing problem may be defined:

$$V(\boldsymbol{\mu}, \ell) \quad = \quad \max_{\tau \in C(\mu)} \{TS(\boldsymbol{\mu}, \ell) - \int r(i) d\tau + \delta V(\boldsymbol{\mu}', \ell)\} \tag{8}$$

where $\boldsymbol{\mu}' = (\boldsymbol{\mu}, \mu')$, with μ' determined from $\mu'(\cdot) = \int P(\cdot \mid \alpha, \boldsymbol{\mu}_t, i, \ell)\tau_t(di \times d\alpha)$ The effect of improving the aggregate distributions is to raise demand and lower supply (marginal cost).

Example 4 Consider a demand and supply model represented by $P_d(Q, \boldsymbol{\mu})$ and $P_s(Q, \boldsymbol{\mu}, \ell)$ respectively. Surplus may be written:

$$T(\boldsymbol{\mu}, \ell) = \max_Q \int_0^Q [P_d(\tilde{Q}, \boldsymbol{\mu}) - P_s(\tilde{Q}, \boldsymbol{\mu}, \ell)] d\tilde{Q}\ ,$$

Write $CS(\boldsymbol{\mu}_t, \ell)$ for consumer welfare at time t and $PS(\boldsymbol{\mu}_t, \ell)$ for producer welfare.

$$TS(\boldsymbol{\mu}_t, \ell) = PS(\boldsymbol{\mu}_t, \ell) + CS(\boldsymbol{\mu}_t, \ell)$$

Total surplus may be decomposed: for the firm, profit is $\pi(\boldsymbol{\mu}, \alpha, \ell)$ so total firm surplus is $PS(\boldsymbol{\mu}_t, \ell) = \int \pi(\boldsymbol{\mu}, \alpha, \ell)\mu(d\alpha)$ and consumer surplus is then $T(\boldsymbol{\mu}, \ell) - PS(\boldsymbol{\mu}, \ell)$. This case is illustrated in figure 4, along with the impact of reducing ℓ to ℓ' or improving $\boldsymbol{\mu}$ to $\boldsymbol{\mu}'$. Improving the distribution increases both supply and demand, reducing ℓ increases technological availability for firms and raises supply. In either case the overall effect is positive (as depicted by the crosshatched lines in figure 4.) ◊

The social welfare optimization problem must respect the same intellectual property rights as in the individual firm problem. However, this optimization problem does address the externality issues — with access to technology restricted according to the patent length, ℓ. (For this reason the function is depicts as declining in ℓ in figure 5).

Theorem 4 *Social welfare, $V(\boldsymbol{\mu}, \ell)$, is decreasing in ℓ.*

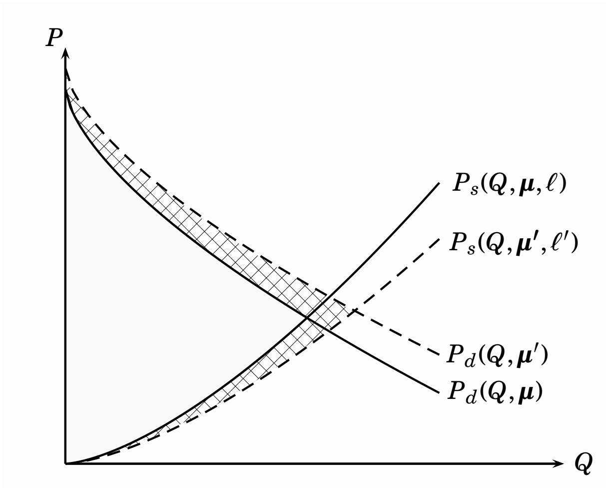

Figure 4: Surplus Variation

4.1 Welfare comparisons

Summarizing the previous discussion, when patented information is a substitute for both technology and (the need for) investment then increasing patent life reduces access to that information and forces firms to greater investment, raising social welfare. In the complementary case, the opposite is true: these contrasting cases are shown in figure 5.[17]

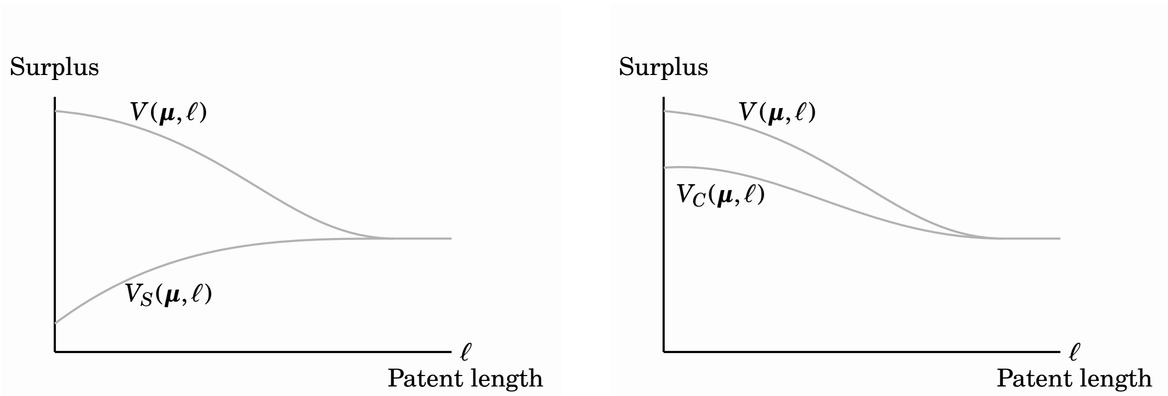

Figure 5: WELFARE AS PATENT LENGTH VARIES

In the substitutes case, increasing patent life raises the value of having good technology (hence encouraging investment indirectly), and raises the direct value of investment in improving one's own technology. Hence, increasing patent length raises welfare. In contrast, with complements, it is advantageous to have good technology at the firm level to benefit from synergies with the available public technology: capital-

[17]When ℓ is very large, patent protection extends beyond obsolescence and changing ℓ has no impact on societal welfare.

izing on this synergy encourages investment to improve one's own technology.[18] There, the less available is public technology, the less benefit from private investment. In addition, in the complements case, the direct value of investment is lower since the improvement in own technology is lower when publicly available technology is older. Thus, the benefit of investment is reduced and these effects together imply that lengthening patent life reduces welfare.

When these effects conflict the consequence of increasing patent life is ambiguous. This occurs, for example, when increasing patent life has greater (negative) impact on better technology firms but at the same time increases the marginal productivity of investment (in generating innovation). In this case, there is less incentive to improve, but it is easier to do so.

Remark 1 One special case of interest is that where the impact of changing patent length on the marginal productivity is small or 0, so that varying patent length has (essentially) no impact in the marginal productivity of investment:

$$\frac{1}{\ell' - \ell}\Big[\int_{\tilde{\alpha}} g(\tilde{\alpha})\Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}, \alpha, i, \ell') - \int_{\tilde{\alpha}} g(\tilde{\alpha})\Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}, \alpha, i, \ell)\Big] \approx 0.$$

In this case, if increasing patent length has a more adverse effect on weaker (low technology) firms, then it is welfare improving; and if the effect is greater on stronger firms, then it is welfare reducing. In particular, patent policy plays a beneficial role when it *forces* less technologically advanced firms to invest more (by depriving them of the use of others ideas). So, patents are beneficial not because they *encourage* reward seeking behavior, but because to survive a firm is compelled to invest and innovate: competitive pressure rather than the prize of monopoly spurs research. ◊

5 Remarks on the Model.

The model developed in this paper is an idealized model with the entire focus on incentive aspects of access to technology among competitive firms. This abstracts from a host of issues, currently the subject of intense discussion. Some of these are discussed next, in section (5.1).

The assumptions appearing in the "substitutes" and "complements" cases identify two sets of circumstances under which the impact of patent length variation on investment can be determined. There are many other possibilities, when one or more of the conditions fail in either case. In such circumstances, the impact of varying patent length on investment is ambiguous since it creates competing influences on a firm's decision to invest.

[18] In the figures, when ℓ becomes large, the patent is protecting old and obsolete technology so there is no welfare impact.

5.1 Other Issues

The framework here assumes a specific market structure (large numbers of agents) and competitive firm behavior. However, in reality there are only a finite number of firms, and in practice a small number may exercise substantial power.[19] This concentration of "patent power" in the hands of small numbers of firms alters dramatically the impact of patents on welfare. With small numbers of firms dominant in each field, often with a large fringe of small or marginal firms, the environment is more typical of competition between small numbers of firms providing differentiated products and having overlapping patent rights, so that competition is both at the level of the produce and at the level of technology holdings in terms of patents.

Apart from the incentive to invest, the existence of patent rights creates a host of other strategic incentives, unintended or unanticipated in the abstract conception of the role of a patent. Along with legitimate incentives to innovate, incentives exist for individuals or companies to abuse the system in a variety of ways: for example, by seeking patents for ideas already know and for trivial blocking inventions which may then be used to obtain unearned rents from other firms. Historically, the nature of patentable discovery was narrowly circumscribed, limiting scope for abuse, but in recent years the range of patentable innovation has been dramatically expanded, covering business methods, communication protocols, medical procedures and so on. As a result, the rate at which patents are granted has increased dramatically, generating a complex web of intellectual property ownership. This in turn has intensified a variety of issues in the use of patents as strategic tools in competition. Examples are plentiful. When an application makes use of a variety of patents (such as in video compression or communications protocols), individual patent holders can exert holdup-power. Or, when a patent is complex and it's contribution difficult to determine, an undeserving patentee may use the court system to extract rents via the threat of lengthy litigation. A patent can be used to impede competition. A patent may be obtained in anticipation of inventions by others from which the patentee can then extract unearned rent; and so on. Or a patent may be obtained for defensive purposes — to assert ownership of ideas in case of a legal challenge, or to have tradeable patents usable a bargaining chips. All these concerns are exacerbated by the fact that a dramatic increase in the volume of applications makes it more difficult to assess the merit of each patent application raising the variability in the quality of the screening process and the true merit of patents granted. The study of such issues is complex, each particular issue requiring individual analysis. See [7], for example, for a discussion of a range of current problems facing the patent system. Finally, in many environments it is difficult to unravel competing ownership rights and so the legal rights of the patentee may be clouded, or there may be multiple entities with rights to various components of a program or implementation of an idea. In such circumstances, the detailed operation of the patent office and the court

[19] Apple, IBM, Microsoft and a handful of firms are dominant patent holders in the field of computer technology; in cellular phone service a small number of firms are dominant; in video technology the MPEG LA consortium consists of the major patent holders of video compression technology, and so on.

system is central to any attempt to disentangle rights and claims.

While such problems may hinder the functioning of the patent system or reduce the benefit provided by the system, the basic presumption remains that the patent system, “warts and all”, promotes innovation and increases social welfare in the long run. From this perspective, the basic principle is sound, and therefore examination of the performance of the system should focus on improvement and “fine-tuning”. In practice this has proved difficult. In 2002, the European Commission issued a directive (“Directive on the Patentability of Computer Implemented Inventions”, with the intent of providing a standard for software patents across Europe. However, the proposals were ultimately rejected by the European Parliament after a protracted struggle over the content of the proposal. With a view to re-evaluation of the USPTO patent policy, the FTC prepared a detailed report on patent policy in 2003[20]. Concern over patent granting in the United States led Congress to introduce a bill[21] in July 2005 with the intent of improving patent quality and restricting “legal gamesmanship”; the Senate introduced a bill in 2006 also with the aim of reforming the patent system. Eventually a patent act[22] was passed by both House and Senate in 2011. Among the main new features are the switch from a first-to-invent to first-to-file system, a post patent-grant review system and a study (through the General Accounting Office) of patent litigation by non-practicing entities and the impact of such entities on the overall functioning of the system. The basic premise is that with greater care in the Patent Office and a more finely tuned judiciary handling patents, standards will be corrected and enforced.

6 Conclusion

The traditional argument for patents is that they encourage innovation by giving the innovator monopoly power for a period of time: monopoly rights create the incentive to invest so that innovation in the aggregate is greater that it would be in the absence of patents. In the environment here, individual innovations are important to the firm but alone not significant in the overall pool of discovery from period to period. The synergy from the pooling of innovation is what creates the externality value in discovery. But the patent blocks the innovator from benefiting from that pool of discovery. While the patent system cannot achieve an efficient outcome for the market — because some of the positive externalities cannot be internalized, this blocking effect can have either a positive or a negative impact on overall innovation and welfare. Whether patents improve or worsen overall welfare depends on the exact way in which innovation interacts with investment and firm quality (as discussed in sections (3.1) and (3.2)).

This discussion focuses entirely on the incentive effects that arise from obtaining monopoly rights on innovation when there are large numbers of innovators. In particular, the prospect of licensing is not

[20]To Promote Innovation: The Proper Balance of Competition and Patent Law and Policy, Report by the FTC, October 2003.
[21]Patent Reform Act of 2005, HR 2795.
[22]The Leahy-Smith America Invents Act.

considered. One might argue that somehow the positive externalities might be internalized by the firms in the market. However, as mentioned earlier, there are good reasons to expect that this might not happen. Large numbers of innovators make reaching consensus on sharing of innovation difficult, but beyond this, the overlap in discovery in any period and the dependent evolution of discovery over time make it difficult if not impossible to price individual innovations and hence allocate value to ideas. So, while the possibility exists that a market might develop to price each innovator's discoveries, it is questionable as to whether or not this would occur.

Appendix: Orderings and Technology Evolution

Ordering on Technology

$\mathscr{A}$ is an ordered topological space where the relation $\succeq$ is reflexive ($\alpha \succeq \alpha$), transitive ($\alpha \succeq \alpha'$ and $\alpha' \succeq \alpha''$ imply $\alpha \succeq \alpha''$), and antisymmetric ($\alpha \succeq \alpha'$ and $\alpha' \succeq \alpha$ imply $\alpha = \alpha'$). (For example: $\mathscr{A} = \{\alpha \mid \alpha : [a,b] \to \Re,\ \alpha \text{ measurable}\}$ where $\alpha' \succeq \alpha$ if $\alpha'(x) \geq \alpha(x), x \in [a,b]$.) If technology were characterized by a real number, the firm with the largest α would be the best firm, unequivocally, eliminating the possibility for different firms to have area specific strengths.

Ordering on Distributions over Technology

For the following review, take as given: (a1) Λ, a completely regular topological space (for example, Λ a metric space), (a2) $\mathscr{B}_\Lambda$ the Borel field on Λ, (b) $\succeq$, an order on Λ (reflexive, transitive and antisymmetric), (c) $C_b(\Lambda)$, the set of continuous bounded real-valued functions on Λ, (d) $\mathscr{M}_+(\Lambda)$, the set of non-negative measures on Λ, and (e) $\mathscr{P}(\Lambda)$ the set of probability measures on Λ. (A topological space Λ is completely regular if and only if when A is closed in Λ and $\alpha \notin A$, there is a continuous function, f, $f : \Lambda \to [0,1]$ such that $f(\alpha) = 0$ and $f(A) = 1$.)

Definition 2 *A real valued function $f : \Lambda \to \Re$ is called increasing if $\alpha' \succeq \alpha$ implies that $f(\alpha') \geq f(\alpha)$ (and decreasing if $\alpha' \succeq \alpha$ implies that $f(\alpha') \leq f(\alpha)$). Write $\mathscr{I}_m(\Lambda)$ for the set of increasing measurable functions on Λ.*

A set $B \subseteq \Lambda$ is called increasing if $x, y \in \Lambda$, $x \in B$ and $y \succeq x$ imply that $y \in B$.

Definition 3 *Given $\mu, \nu \in \mathscr{P}(\Lambda)$, define a pre-ordering (reflexive and transitive relation) on $\mathscr{P}(\Lambda)$:*

$$\mu \succeq \nu \quad \textit{if and only if} \quad \int f(\alpha)\mu(d\alpha) \geq \int f(\alpha)\nu(d\alpha),\ \forall f \in \mathscr{I}_m(\Lambda)$$

The natural generalization of a result on dominance in $\Re$ is (see Torres [15]):

Theorem 5 *$\mu \succeq \nu$ if and only if $\mu(A) \geq \nu(A)$ for every increasing measurable set A.*

The Evolution of Technology

The distribution $\mu_{t+1}(\cdot)$ depends on $\boldsymbol{\mu}_t$, ℓ and τ_t. This may be made explicit by writing:

$$\mu_{t+1}(\cdot) = \psi(\cdot \mid \boldsymbol{\mu}_t, \tau_t, \ell) \stackrel{\text{def}}{=} \int_{i_t, \alpha_t} P(\cdot \mid \boldsymbol{\mu}_t, \alpha_t, i_t, \ell)\tau_t(di_t \times d\alpha_t), \quad \tau_t \in \mathscr{M}(I \times \mathscr{A}), \tag{9}$$

where $\tau_t \in \mathscr{C}(\mu_t)$ and ψ is the function determining the one-period ahead distribution over technologies. With $\boldsymbol{\mu}_{t+1} = (\boldsymbol{\mu}_t, \mu_{t+1})$, and $\tau_{t+1} \in \mathscr{C}(\mu_{t+1})$, $\mu_{t+2}(\cdot) = \psi(\cdot \mid \boldsymbol{\mu}_{t+1}, \tau_{t+1}, \ell)$, and so on.

If we fix a sequence of strategies $\tau^t = \{\tau_{t+j}\}_{j\geq 0}$, given μ_t, we may determine the sequence of distributions:

$$
\begin{aligned}
\psi_1(\cdot \mid \mu_t, \tau^t, \ell) &= \psi(\cdot \mid \boldsymbol{\mu}_t, \tau_t, \ell) \\
\psi_2(\cdot \mid \mu_t, \tau^t, \ell) &= \psi(\cdot \mid (\boldsymbol{\mu}_t, \psi_1(\cdot \mid \mu_t, \tau^t, \ell)), \tau_{t+1}, \ell) \\
\psi_3(\cdot \mid \mu_t, \tau^t, \ell) &= \psi(\cdot \mid (\boldsymbol{\mu}_t, \psi_1(\cdot \mid \mu_t, \tau^t, \ell), \psi_2(\cdot \mid \mu_t, \tau^t, \ell)), \tau_{t+2}, \ell) \\
\vdots \quad &= \quad \vdots \\
\psi_j(\cdot \mid \mu_t, \tau^t, \ell) &= \psi(\cdot \mid (\boldsymbol{\mu}_t, \psi_1, \psi_2, \ldots, \psi_j), \tau_{t+j}, \ell)
\end{aligned}
$$

Given $\boldsymbol{\mu}_t$ and τ_t, the history is determined in $t+j$ as $(\boldsymbol{\mu}_t, \psi_1, \ldots, \psi_j)$. Then profit to α in period $t+j$ is $\pi((\boldsymbol{\mu}_t, \psi_1, \ldots, \psi_j), \alpha, \ell)$ and the conditional distribution $P(\cdot \mid (\boldsymbol{\mu}_t, \psi_1, \ldots, \psi_j), \alpha, i, \ell)$. When $j > \ell$, variations in ℓ affect both the innovations in the public domain (from periods prior to $t+j-\ell$ and the current distribution $t+j$.) Since $(\psi_1, \ldots, \psi_j)$ is determined by $\boldsymbol{\mu}_t$ and τ^t, let $\pi_j(\boldsymbol{\mu}_t, \tau^t, \alpha, \ell) \stackrel{\text{def}}{=} \pi((\boldsymbol{\mu}_t, \psi_1, \ldots, \psi_j), \alpha, \ell)$ and $P_j(\cdot \mid \boldsymbol{\mu}_t, \tau^t, \alpha, i, \ell) \stackrel{\text{def}}{=} P(\cdot \mid (\boldsymbol{\mu}_t, \psi_1, \ldots, \psi_j), \alpha, i, \ell)$.

In this formulation, a variation in ℓ at time t impacts both the length of time for which patented discovery stays out of the public domain, but also impacts the aggregate distributions from period t onward, through the updating rule ψ_j, with the aggregate distributions, $\{\tau_{t+j}\}$ fixed.

Appendix: Proofs

The first lemma, lemma 1, confirms that the monotonicity of π in $\boldsymbol{\mu}$ carries over to the value function.

Lemma 1 *$v(\boldsymbol{\mu}_t, \tau^t, \alpha, \ell)$ is increasing in $\boldsymbol{\mu}_t$: if $\bar{\boldsymbol{\mu}}_t = (\ldots, \bar{\mu}_{t-1}, \bar{\mu}_t)$ dominates $\hat{\boldsymbol{\mu}}_t = (\ldots, \hat{\mu}_{t-1}, \hat{\mu}_t)$ componentwise, then $v(\bar{\boldsymbol{\mu}}_t, \tau^t, \alpha, \ell) \geq v(\hat{\boldsymbol{\mu}}_t, \tau^t, \alpha, \ell)$ for all α.*

Proof: Since $\pi(\bar{\boldsymbol{\mu}}_t, \alpha, \ell) \geq \pi(\hat{\boldsymbol{\mu}}_t, \alpha, \ell)$ from $(I-i(\alpha))$ and $P(d\tilde{\alpha} \mid \bar{\boldsymbol{\mu}}_t, \alpha, i, \ell) \succcurlyeq P(d\tilde{\alpha} \mid \hat{\boldsymbol{\mu}}_t, \alpha, i, \ell)$ for each t, the result follows directly. ■

When patent length, ℓ, varies then apart from the direct effect on the payoff function and the transition kernel, there is the indirect effect of varying future distributions which impact the profit in those subsequent periods. Considering profit k periods on after the increasing of patent length in period t, the following calculations show that profit increases with ℓ, given any fixed sequence of strategies $\boldsymbol{\tau}^t = \{\tau_{t+j}\}_{j\geq 0}$.

Lemma 2 *For all k,*

$$\frac{\partial \pi((\mu_t, \psi_1, \ldots, \psi_k), \alpha, \ell)}{\partial \ell}$$

is increasing in α.

Proof: Let $\psi_1, \ldots, \psi_k$ be the distribution sequence determined by ℓ and $\psi'_1, \ldots, \psi'_k$ the sequence determined by ℓ'. Consider

$$\begin{aligned}
\pi((\mu_t, \psi'_1, \ldots, \psi'_k), \alpha, \ell') - \pi((\mu_t, \psi_1, \ldots, \psi_k), \alpha, \ell) \quad = \\
\pi((\mu_t, \psi'_1, \ldots, \psi'_k), \alpha, \ell') - \pi((\mu_t, \psi'_1, \ldots, \psi'_k), \alpha, \ell) \\
+\pi((\mu_t, \psi'_1, \ldots, \psi'_k), \alpha, \ell) - \pi((\mu_t, \psi_1, \ldots, \psi_k), \alpha, \ell)
\end{aligned}$$

Considering terms, since $(\mu_t, \psi'_1, \ldots, \psi'_k) \preccurlyeq (\mu_t, \psi_1, \ldots, \psi_k)$,

$$\pi((\mu_t, \psi'_1, \ldots, \psi'_k), \alpha, \ell') - \pi((\mu_t, \psi'_1, \ldots, \psi'_k), \alpha, \ell)$$

is increasing in α, using $I-i(\alpha)$. And, since $\ell' \geq \ell$,

$$\pi((\mu_t, \psi'_1, \ldots, \psi'_k), \alpha, \ell) - \pi((\mu_t, \psi_1, \ldots, \psi_k), \alpha, \ell)$$

is increasing in α. Therefore, with $\alpha^* \geq \alpha$

$$\begin{aligned}
\frac{1}{\ell' - \ell}[\pi((\mu_t, \psi'_1, \ldots, \psi'_k), \alpha^*, \ell') - \pi((\mu_t, \psi_1, \ldots, \psi_k), \alpha^*, \ell)] \\
\geq \frac{1}{\ell' - \ell}[\pi((\mu_t, \psi'_1, \ldots, \psi'_k), \alpha, \ell') - \pi((\mu_t, \psi_1, \ldots, \psi_k), \alpha, \ell)]
\end{aligned}$$

and

$$\frac{1}{\ell' - \ell}[\pi((\mu_t, \psi'_1, \ldots, \psi'_k), \alpha^*, \ell) - \pi((\mu_t, \psi_1, \ldots, \psi_k), \alpha, \ell)]$$
$$\geq \quad \frac{1}{\ell' - \ell}[\pi((\mu_t, \psi'_1, \ldots, \psi'_k), \alpha^*, \ell) - \pi((\mu_t, \psi_1, \ldots, \psi_k), \alpha, \ell)]$$

and in the limit, for $\alpha^* \geq \alpha$:

$$\frac{\partial \pi((\mu_t, \psi_1, \ldots, \psi_k), \alpha^*, \ell)}{\partial \ell} \geq \frac{\partial \pi((\mu_t, \psi_1, \ldots, \psi_k), \alpha, \ell)}{\partial \ell}$$

■

Lemma 3 *For all k,*

$$\frac{\partial}{\partial \ell} \int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi^k, \alpha, i, \ell)$$

is increasing in α.

Proof: With $\ell' \geq \ell$,

$$\int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi'^k, \alpha, i, \ell') - \int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi^k, \alpha, i, \ell) =$$
$$[\int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi'^k, \alpha, i, \ell') - \int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi'^k, \alpha, i, \ell)]$$
$$+[\int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi'^k, \alpha, i, \ell) - \int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi^k, \alpha, i, \ell)]$$

where $\psi^k = (\psi_1, \psi_2, \ldots, \psi_k)$. So, as before, using $I - i(b)$, and with $\alpha^* \geq \alpha$,

$$\int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi'^k, \alpha^*, i, \ell') - \int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi^k, \alpha^*, i, \ell)$$
$$\geq \quad \int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi'^k, \alpha, i, \ell') - \int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi^k, \alpha, i, \ell)$$

Passing to the limit,

$$\frac{\partial}{\partial \ell} \int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi^k, \alpha^*, i, \ell) \geq \frac{\partial}{\partial \ell} \int g(\tilde{\alpha}) P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \psi^k, \alpha, i, \ell)$$

■

Lemma 4 *For each t:*

$$v_\ell(\boldsymbol{\mu}_t, \boldsymbol{\tau}^t, \alpha', \ell) \equiv \frac{\partial v(\boldsymbol{\mu}_t, \boldsymbol{\tau}^t, \alpha', \ell)}{\partial \ell} \geq \frac{\partial v(\boldsymbol{\mu}_t, \boldsymbol{\tau}^t, \alpha, \ell)}{\partial \ell}, \ \alpha' \geq \alpha.$$

Proof: Consider a two period problem where:

$$v^2(\boldsymbol{\mu}_t,\boldsymbol{\tau}^t,\alpha,\ell) \stackrel{\text{def}}{=} \max_{i_t}\{\pi(\boldsymbol{\mu}_t,\alpha,\ell)-r(i_t)+\delta\int \pi(\boldsymbol{\mu}_t,\psi_1,\tilde{\alpha},\ell)P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\alpha,i_t,\ell)\}$$

Differentiating with respect to ℓ (and using the optimality condition for i_t):

$$\begin{aligned}\frac{\partial v^2(\boldsymbol{\mu}_t,\boldsymbol{\tau}^t,\alpha,\ell)}{\partial \ell} &= \frac{\partial \pi(\boldsymbol{\mu}_t,\alpha,\ell)}{\partial \ell}+\delta\int \frac{\partial \pi(\boldsymbol{\mu}_t,\psi_1,\tilde{\alpha},\ell)}{\partial \ell}P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\alpha,i_t,\ell)+ \\ &\qquad \delta\int \pi(\boldsymbol{\mu}_t,\psi_1,\tilde{\alpha},\ell)\Delta_\ell P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\alpha,i_t,\ell)\end{aligned}$$

Therefore,

$$\begin{aligned}&\frac{\partial v^2(\boldsymbol{\mu}_t,\boldsymbol{\tau}^t,\alpha',\ell)}{\partial \ell}-\frac{\partial v^2(\boldsymbol{\mu}_t,\boldsymbol{\tau}^t,\alpha,\ell)}{\partial \ell} \\ &\quad = [\frac{\partial \pi(\boldsymbol{\mu}_t,\alpha',\ell)}{\partial \ell}-\frac{\partial \pi(\boldsymbol{\mu}_t,\alpha,\ell)}{\partial \ell}]+ \\ &\qquad \delta\int \frac{\partial \pi(\boldsymbol{\mu}_t,\psi_1,\tilde{\alpha},\ell)}{\partial \ell}[P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\alpha',i_t,\ell)-P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\alpha,i_t,\ell)]+ \\ &\qquad \delta\int \pi(\boldsymbol{\mu}_t,\psi_1,\tilde{\alpha},\ell)[\Delta_\ell P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\alpha',i_t,\ell)-\Delta_\ell P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\alpha,i_t,\ell)]\end{aligned}$$

Considering the three terms on the right, the first term is positive since since π_ℓ is increasing in α, using $(I-i(a))$; the second term is positive since $P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\alpha',i_t,\ell)\succcurlyeq P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\alpha,i_t,\ell)$. Finally, from $(I-i(b))$ the third term is positive — since $\pi(\boldsymbol{\mu}_{t+1},\alpha,\ell)$ is increasing in α and $\Delta_\ell P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\alpha',i_t,\ell)$ first order stochastically dominates $\Delta_\ell P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\alpha,i_t,\ell)$]. Hence, $\frac{\partial v^2(\boldsymbol{\mu}_t,\boldsymbol{\tau}^t,\alpha,\ell)}{\partial \ell}$ is increasing in α.

For notational convenience, observe that the k period valuation function, $v^k(\boldsymbol{\mu}_t,\tau^t,\alpha,\ell)$ may be expressed in terms of $\psi_1,\psi_2,\ldots,\psi_k$, the aggregate distribution in each of the k periods from t, determined by τ^t: $v^k(\boldsymbol{\mu}_t,\psi_1,\ldots,\psi_k,\alpha,\ell)$. Write $v^k_\ell(\boldsymbol{\mu}_t,\psi_1,\ldots,\psi_k,\alpha,\ell)$ for the partial derivative with respect to ℓ. Also, for convenience, let $(\psi_1,\ldots,\psi_k)=\psi^k$.

Suppose that $v^k_\ell(\boldsymbol{\mu}_t,\psi^k,\alpha,\ell)$ is increasing in α, then so is $v^{k+1}_\ell(\boldsymbol{\mu}_t,\psi^{k+1},\alpha,\ell)$.

$$\begin{aligned}\frac{\partial v^{k+1}(\boldsymbol{\mu}_t,\psi^{k+1},\alpha,\ell)}{\partial \ell} &= \frac{\partial \pi(\boldsymbol{\mu}_t,\alpha,\ell)}{\partial \ell}+\delta\int \frac{\partial v^k(\boldsymbol{\mu}_t,\psi^{k+1},\tilde{\alpha},\ell)}{\partial \ell}P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\psi^{k+1},\alpha,i_t,\ell)+ \\ &\qquad \delta\int v^k(\boldsymbol{\mu}_t,\psi^k,\tilde{\alpha},\ell)\Delta_\ell P(d\tilde{\alpha}\mid \boldsymbol{\mu}_t,\psi^k,\alpha,i_t,\ell)\end{aligned}$$

Thus, for any k, and $(\boldsymbol{\mu}_t,\tau^t,\ell)$, $v^k_\ell(\boldsymbol{\mu}_t,\tau^t,\alpha,\ell)$ is increasing in α.

Consider, for $\alpha'\geq\alpha$, $\ell'\geq\ell$

$$v^k(\boldsymbol{\mu}_t,\tau^t,\alpha',\ell')-v^k(\boldsymbol{\mu}_t,\tau^t,\alpha',\ell)\geq v^k(\boldsymbol{\mu}_t,\tau^t,\alpha,\ell')-v^k(\boldsymbol{\mu}_t,\tau^t,\alpha,\ell)$$

Passing to the limit,

$$v(\boldsymbol{\mu}_t,\tau^t,\alpha',\ell')-v(\boldsymbol{\mu}_t,\tau^t,\alpha',\ell)\geq v(\boldsymbol{\mu}_t,\tau^t,\alpha,\ell')-v(\boldsymbol{\mu}_t,\tau^t,\alpha,\ell)$$

Dividing by $\ell' - \ell$ and taking limits ($\ell' \to \ell$) gives:

$$\frac{\partial v(\boldsymbol{\mu}_t, \boldsymbol{\tau}^t, \alpha, \ell)}{\partial \ell} \quad \text{is increasing in} \quad \alpha. \qquad \blacksquare$$

Proof of theorem 2: Considering equation (6):

$$-r'(i) + \delta \int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \alpha, \ell) \Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell) \equiv 0$$

the marginal impact of a variation in ℓ on firm α's investment level, ignoring the impact on the equilibrium distribution $\boldsymbol{\tau}^t$, is obtained by differentiating 6 with respect to ℓ.

$$\begin{aligned} -r''(i)\frac{di_t}{d\ell} + \delta \int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \alpha, \ell) \Delta_{ii} P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell) \frac{di_t}{d\ell} \\ +\delta \int_{\tilde{\alpha}} v_\ell(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \alpha, \ell) \Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell) \\ +\delta \int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \alpha, \ell) \Delta_{li} P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell) \equiv 0 \end{aligned} \tag{10}$$

Therefore:

$$\frac{di_t}{d\ell} = \frac{\delta \int_{\tilde{\alpha}} v_\ell(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \alpha, \ell) \Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell) + \delta \int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \alpha, \ell) \Delta_{li} P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell)}{r''(i_t) - \delta \int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \alpha, \ell) \Delta_{ii} P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell)} \tag{11}$$

From the second order condition, the denominator is positive, so the sign of $\frac{di_t}{d\ell}$ is the same as that of the numerator. Since v is increasing in α and $\Delta_i P(\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell') \succcurlyeq \Delta_i P(\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell)$ for $\ell' \geq \ell$, from assumption $(I-iii)$, $\int_{\tilde{\alpha}} v(\boldsymbol{\mu}_t, \boldsymbol{\tau}^t, \alpha, \ell) \Delta_{li} P(\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell) > 0$. From lemma 4, $v_\ell(\boldsymbol{\mu}_t, \boldsymbol{\tau}^t, \alpha, \ell)$ is increasing in α so that $\int_{\tilde{\alpha}} v_\ell(\boldsymbol{\mu}(t), \tilde{\alpha}, \ell, t) \Delta_i P(\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell) > 0$, for each t. Consequently, $\frac{di_t}{d\ell} > 0$.

This expression gives the variation in firm α's investment level that would result if ℓ were increased, and the actions of the population, $(\tau_t, \boldsymbol{\tau}^{t+1})$, held constant. However, increased investment resulting from an increase in ℓ occurs for all agents, alters future aggregate distributions, and alters optimal decisions in future periods.

These calculations ignore the impact of changes in investment behavior on the aggregate distribution. Recall the first order condition for i at technology α:

$$r'(i) = \delta \int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \alpha, \ell) \Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell).$$

With the increase in ℓ, from equation (11), the variation in i is upward. So, for any α, with $\ell' > \ell$, the expression:

$$r'(i') = \delta \int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \tilde{\alpha}, \ell') \Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i', \ell').$$

has solution $i' > i$. However, τ^{t+1} cannot now be an equilibrium strategy as higher investment by each firm will impact μ_{t+j}, $j \geq 1$, raising the quality of the aggregate distribution in the next and subsequent periods. From lemma (1) and $(I-ii)$, $\int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \tilde{\alpha}, \ell')\Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i', \ell')$ increases so that the best response from α is to raise i further with consequent (further) impact on the aggregate distribution. Assuming $r'(x)$ is sufficiently large for large values of x, the iterative process will eventually converge to equilibrium. Consequently the impact of increasing ℓ is to raise the aggregate distribution quality in subsequent periods and hence the present value of surplus (welfare).

■

Proof of theorem 3: Assumption $(II-ii)$ implies that $\int_{\tilde{\alpha}} v(\boldsymbol{\mu}(t), \tilde{\alpha}, \ell)\Delta_{li} P(\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell) < 0$ and $(II-i(a))$ implies that $\int_{\tilde{\alpha}} v_\ell(\boldsymbol{\mu}(t), \tilde{\alpha}, \ell)\Delta_i P(\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell) < 0$. Consequently, $\frac{di_t}{d\ell} < 0$.

As before, these calculations ignore the impact of changes in investment behavior on the aggregate distribution. So, reconsider the first order condition:

$$r'(i) = \delta \int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \alpha, \ell)\Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i, \ell).$$

After raising ℓ, the variation in i_t, holding τ^{t+1} fixed, is downward. With $\ell' > \ell$, $i' < i$ satisfies:

$$r'(i') = \delta \int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \tilde{\alpha}, \ell')\Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i', \ell').$$

Again, this expression ignores the fact that lower investment will impact the future distributions: τ^{t+1} cannot now be an equilibrium strategy as lower investment by each firm will impact μ_{t+j}, $j \geq 1$, reducing the quality of the aggregate distribution in the next and subsequent periods.

From lemma (1) and $(II-ii)$, $\int_{\tilde{\alpha}} v(\boldsymbol{\mu}_{t+1}, \boldsymbol{\tau}^{t+1}, \tilde{\alpha}, \ell')\Delta_i P(d\tilde{\alpha} \mid \boldsymbol{\mu}_t, \alpha, i', \ell')$ decreases so that the best response from α is to reduce i further with consequent (further) impact on the aggregate distribution. Assuming $r'(x) \to 0$ as $x \to 0$, the iterative process will eventually converge to equilibrium. Consequently the impact of increasing ℓ is to worsen the aggregate distribution quality in subsequent periods and hence the present value of surplus (welfare).

□

Proof of theorem 4: Recall social welfare is measured as:

$$V(\boldsymbol{\mu}, \ell) \quad = \quad \max_{\tau \in C(\mu)} \{TS(\boldsymbol{\mu}, \ell) - \int r(i) d\tau + \delta V(\boldsymbol{\mu}', \ell)\} \tag{12}$$

where $\boldsymbol{\mu}' = (\boldsymbol{\mu}, \psi_1)$ and where ψ_1 depends on τ (where necessary, this may be made explicit by writing $\psi_1(\tau)$). The following discussion shows that

$$V(\boldsymbol{\mu}', \ell') - V(\boldsymbol{\mu}, \ell) \geq 0, \quad \boldsymbol{\mu}' \succcurlyeq \boldsymbol{\mu},\ \ell' \leq \ell \tag{13}$$

so that, in particular, V is decreasing in ℓ.

Define $V_1(\boldsymbol{\mu},\ell) = TS(\boldsymbol{\mu},\ell)$ and inductively,

$$V_n(\boldsymbol{\mu},\ell) \quad = \quad \max_{\tau\in C(\mu)} \{TS(\boldsymbol{\mu},\ell) - \int r(i)d\tau + \delta V_{n-1}((\boldsymbol{\mu},\psi_1),\ell)\} \tag{14}$$

Suppose that for some $n>1$, V_{n-1} satisfies (14), then V_n satisfies (13). $V_n(\boldsymbol{\mu},\ell) = V_{n-1}(\boldsymbol{\mu},\ell)$. To see this, let τ^n solve (14), and consider a variation in ℓ to $\ell' < \ell$. With τ^n fixed, apart from direct impact, ψ_1 shifts to $\psi_1' \succsim \psi_1$, so that $V_{n-1}((\boldsymbol{\mu},\psi_1'),\ell') \geq V_{n-1}((\boldsymbol{\mu},\psi_1),\ell)$ and similarly, $TS(\boldsymbol{\mu},\ell') \geq TS(\boldsymbol{\mu},\ell)$, so that

$$\begin{aligned} V_n(\boldsymbol{\mu},\ell') = \max_{\tau\in C(\mu)} \{TS(\boldsymbol{\mu},\ell) - \int r(i)d\tau + \delta V_{n-1}(\boldsymbol{\mu},\psi_1(\tau),\ell)\} \\ \geq \quad TS(\boldsymbol{\mu},\ell') - \int r(i)d\tau + \delta V_{n-1}(\boldsymbol{\mu},\psi_1(\tau^n),\ell')\} \\ \geq \; TS(\boldsymbol{\mu},\ell) - \int r(i)d\tau + \delta V_{n-1}(\boldsymbol{\mu},\psi_1(\tau^n),\ell)\} = V_n(\boldsymbol{\mu},\ell) \end{aligned}$$

Similar computations give a comparable result for $\boldsymbol{\mu}$ variation: $V_n(\boldsymbol{\mu}',\ell) \geq V_n(\boldsymbol{\mu},\ell)$, $\boldsymbol{\mu}' \succsim \boldsymbol{\mu}$. Thus, if V_{n-1} satisfies (13), so does V_n. Since V_1 satisfies (13), V_n also does for each n, and taking the limit as $n\to\infty$, gives the property for V. In particular, this implies that V is decreasing in ℓ.

An alternative proof can be give as follows. The optimization problem may be expressed in slightly different terms. Let:

$$\mathscr{C}^*(\mu) = \{\boldsymbol{\tau} = (\tau_1,\tau_2,\ldots) \mid \text{marg}_\Lambda \tau_t = \mu_t,\ \mu_1 = \mu,\ \mu_{t+1} = \int P(\cdot \mid \boldsymbol{\mu}_t,\alpha_t,i_t,\ell)\tau_t(di_t\times d\alpha_t),\ t\geq 1\}$$

giving those sequences $(\tau_1,\tau_2,\ldots)$ of feasible strategies — consistent with the initial aggregate distribution. With this notation, the value function may be written:

$$V(\boldsymbol{\mu},\ell) \quad = \quad \max_{\boldsymbol{\tau}\in C^*(\mu)} \sum_{t=1}^{\infty} \delta^{t-1}[T(\boldsymbol{\mu}_t,\ell) - \int r(i)d\tau_t]$$

where μ_t,μ_{t+1}, and so on are determined according to equation (3).

Given any $\boldsymbol{\tau}^t$, let $\{\boldsymbol{\mu}_{t+j}(\boldsymbol{\tau}^t,\ell)\}_{j=0}^{\infty}$ be the resulting sequence of technology distributions. Increasing ℓ lowers the sequence pointwise (in stochastic dominance terms), for all t, so that $T(\boldsymbol{\mu}_t,\ell)$ falls for each t. Hence the maximum over $\boldsymbol{\tau}^t$ is lower as ℓ increases.

□

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