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Real Business Cycle Realizations

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Abstract

Much recent business cycle research focuses on *moments* of macroeconomic aggregates. We construct examples of real business cycle *sample paths* for output, consumption, and employment for the U.S. economy. Annual sample paths are generated from an initial condition in 1925, measured technology and government spending shocks since then, and a standard, calibrated, one-sector model of the business cycle. Quarterly sample paths are generated similarly, from an initial condition in 1955. The law of motion for shocks is not parametrized and so decision-rules are estimated by GMM. We compare the paths with actual history graphically and by spectral methods.

Keywords: real business cycles, Solow residuals; US business cycle history

JEL Classification Code: E32, E27

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1. Introduction

One reason that policy evaluations using real business cycle models are interesting is that the models reproduce some key characteristics of historical cycles, such as the differential volatility of output and its components. Implications of a model have credence only if the model fits historical data in dimensions like this and if it improves upon competing or existing models. A traditional way to examine the predictions of a business-cycle model is to compare the time series sample paths it predicts for macroeconomic variables with the historical data. That is the subject of this paper.

Comparing predicted and actual values is standard in econometrics, but is uncommon in real business cycle analysis, as Hansen and Heckman (1996) have noted. In contrast, studies which compare moments of historical series with those from a business-cycle model are very numerous. Some comparisons of moment-like features have been made by Smith (1993) and Cogley and Nason (1995), who compared VAR estimates from simulated models with those from historical data, and by Simkins (1994) and King and Plosser (1994), who assessed whether Burns-Mitchell methods yield similar characterizations of cycles in historical and simulated data. A subsidiary area of work outlines various metrics for comparing these facts, and provides the resulting statistical evaluation. Examples of such studies include those by Gregory and Smith (1991), Christiano and Eichenbaum (1992), Watson (1993), Smith (1993), Canova (1994), Diebold, Ohanian, and Berkowitz (1995), and DeJong, Ingram, and Whiteman (1995).

Focusing on moments provides generality (for example if shock processes are similar across countries) but rules out some interesting questions. For example, does the business-cycle model describe recessions and booms equally well? Is it consistent with business cycle turning points? Simulated and historical series could have identical moments, VAR representations, and cycle durations and yet be uncorrelated with each other. Having a zero variance of the difference between two series is a much stricter criterion than having a zero difference of their variances. Making in-sample predictions seems particularly worthwhile because most real business cycle models calibrate stochastic processes for shocks (to technology or fiscal policy) using historical series. It is a natural step then to use the actual sequence of such measured shocks, rather than just their moments, and so produce a sequence of predicted values for output, capital, and so on.

Main precursors to this study are by Christiano (1988) and Plosser (1989). Christiano derived sample paths for output, consumption, investment, and hours predicted by a business cycle model in response to estimated Solow residuals. He also compared these to actual paths in quarterly U.S. data for 1953 to 1984. Plosser graphed predicted and actual series for output, consumption, investment, and real wages in annual U.S. data for 1955 to 1985, assuming that technology shocks followed a logarithmic random walk. In a similar vein, Hansen and Prescott (1993) studied the U.S. business cycle from 1984 to 1992 using a calibrated model. Altig and Carlstrom (1991) studied an overlapping-generations model with 55-period lives, subject to actual shocks to inflation and technology in the U.S. economy from 1955 to 1988. They plotted the sample paths of macroeconomic variables predicted by this model and compared them to actual values. In these studies the law of motion for shocks is parametrized; in contrast, we produce sample paths without modeling the process followed by shocks.

Section 2 outlines the solution method, which can be thought of as 'econometric guess-and-verify,' and the construction of sample paths. The paper's title contains a plural because we investigate some variants of a standard business-cycle model, although each model produces a single realization of a vector of variables. The solution method is semi-parametric: it requires parametrization of the production and utility functions, but it does not require a parametrization of the processes generating exogenous shocks. It is often the specification and quantification of the shock processes that generate the largest differences between models that are 'calibrated' and those that are 'estimated.' For example, Hansen (1997) shows that the predictions of the stochastic growth model are sensitive to the process followed by the technology shock even if it is required to be highly autocorrelated. The sample-path diagnostics that we examine in this paper, therefore, are robust to the researcher's choice of empirical methods.

Sections 3 and 4 construct some examples of sample paths for U.S. output and its components in annual data since 1925 and quarterly data since 1955 respectively. We compare these paths with actual paths graphically and using spectral methods. Section 5 offers some suggestions for further refinement and research, while section 6 lists preliminary conclusions.

2. The Method and Model Economy

To derive sample paths, we follow these steps:

(a) Measure technology shocks as Solow residuals, using a calibrated Cobb-Douglas production function and historical series for output, capital, and labor. We also measure shocks to government spending.

(b) Calibrate a theoretical economy, and write the implied Euler equations.

(b) Propose a functional form for approximate decision rules for investment, consumption, and hours. This requires us to assume that the shocks follow a Markov process of known order, but we do not need to calibrate the parameters of that process. We substitute the approximate decision rules in the Euler equations, and then estimate their coefficients by GMM.

(d) Generate paths using these rules and initial conditions.

To describe this method in detail requires some notation. Let Y_t be real output. Then y_t is detrended real output, and $\hat{y}_t$ is standardized y_t. The index t counts historical observations, while the index s counts predicted values from a business-cycle model. The aim is to compare paths such as $\{\hat{y}_t\}$ and $\{\hat{y}_s\}$.

2.1 Measuring Shocks

Most studies of real business cycles simulate a stochastic process for technology shocks in order to compute the moments predicted by a model. The stochastic process usually is calibrated as a first-order autoregression with mean and variance matching those in detrended, post-war, U.S. Solow residuals. In this tradition, we use detrended Solow residuals from a Cobb-Douglas production function (with capital exponent 0.33) but we use the realized sample path. Retaining the path retains more information which may be helpful in evaluating or modifying the model.

To measure shocks, we specify a technology:

$$Y_t = z_t K_t^{\alpha} (N_t \gamma_x^t)^{1-\alpha}, \tag{1}$$

with $\alpha = 0.33$. We detrend to find z_t, with a linear regression of Solow residuals on a time trend:

$$log(\frac{Y_t}{K_t^{\alpha} N_t^{1-\alpha}}) = (1-\alpha) log(\gamma_x) t + log(z_t). \tag{2}$$

This gives a sequence of shocks (which do not have mean zero) and also an estimate of γ_x.

Figure 1 shows the detrended technology shocks, in annual data since 1925. In addition, the cross-dashed line shows fiscal shocks, measured as log-linearly detrended government consumption spending. The solid line is detrended real GNP per capita. Data sources are given at the end of the paper. Figure 8 shows the same three series in quarterly data since 1955. Here there is much less variation, which will make identifying decision rules more challenging than in the long span of annual data.

In either set of data the technology shock series is less volatile than output. The business-cycle model explains the variation in capital and labor input, which account for the additional variation in output, as endogenous responses to the two shocks. Our aim is to find the predicted paths for those inputs and hence also a benchmark predicted path for output. The stationarity evident in Figures 1 and 8 allows us to use the solution method – DP by GMM – described below.

2.2 Model Economy

The framework for measurement is the basic neoclassical model introduced by Kydland and Prescott (1982), with a trend arising from labour-augmenting technical progress as examined by King, Plosser, and Rebelo (1988). The model also includes a demand-side shock, as introduced by Christiano and Eichenbaum (1992). We measure this shock with public expenditure, and so denote it g_s. There is one sector and no distortions. Technology is Cobb-Douglas, as in the shock measurements, and utility is logarithmic in consumption, additively separable between consumption and leisure, and additively separable over time.

The model economy consists of a large number of infinitely-lived households, which at each time s choose sequences of private consumption $\{C_s\}$ and a fraction of time spent working ('hours') $\{n_s\}$ to maximize:

$$E_s \sum_{i=0}^{\infty} \beta^i \left[log(C_{s+i} + \mu G_{s+i}) + \theta log(1 - n_{s+i})\right] \tag{3a}$$

where β is the discount factor and G_s is government spending. We refer to this as the 'divisible labor' case. We also consider the 'indivisible labor' case:

$$E_s \sum_{i=0}^{\infty} \beta^i \left[log(C_{s+i} + \mu G_{s+i}) + \theta(1 - n_{s+i})\right] \tag{3b}$$

which is a reduced-form preference ordering for an economy with indivisible labor, as described by Rogerson (1988) and Hansen (1985).

Each household has access to a production technology

$$Y_s = z_s K_s^{\alpha} (\gamma_x^s n_s)^{1-\alpha}, \tag{4}$$

where K_s is the capital stock, z_s is a stationary technology shock, γ_x^s is a trend in labor productivity, α is a positive fraction, and Y_s is output. As is well known, this combination of preferences and technology is consistent with balanced growth in which Y_s, C_s, G_s, and K_s grow at rate γ_x, while n_s does not grow. Output can be consumed or stored, with stored output adding to the capital stock next period. The capital stock depreciates at rate δ. Labor and the single good are traded on competitive markets. From market clearing, the aggregate constraint is

$$G_s + C_s + K_{s+1} = z_s K_s^{\alpha} (\gamma_x^s n_s)^{1-\alpha} + (1 - \delta) K_s. \tag{5}$$

Assume that the exogenous variables, the technology shock z_s and detrended public spending g_s, follow a stationary Markov process. Then, as is well known, the competitive equilibrium can be found as the solution to a dynamic programme in detrended variables in which expected utility is maximized subject to the sequence of accumulation constraints. The state vector is (z_s, g_s, k_s), where $k_s = K_s/\gamma_x^s$.

Equilibrium paths for detrended consumption, c_s, and for hours, n_s, satisfy the intertemporal Euler equation

$$\frac{1}{c_s + \mu g_s} = E_s \frac{\beta}{\gamma_x(c_{s+1} + \mu g_{s+1})}[1 + z_{s+1}\alpha k_{s+1}^{\alpha-1} n_{s+1}^{1-\alpha} - \delta] \tag{6}$$

and the intratemporal Euler equation, either

$$\frac{\theta}{1-n_s} = \frac{1}{c_s + \mu g_s}[z_s(1-\alpha)n_s^{-\alpha}k_s^{\alpha}] \tag{7a}$$

in the divisible-labor case, or

$$\theta = \frac{1}{c_s + \mu g_s}[z_s(1-\alpha)n_s^{-\alpha}k_s^{\alpha}] \tag{7b}$$

in the indivisible-labor case. The paths also satisfy the transformed constraint

$$\gamma_x k_{s+1} = (1-\delta)k_s + z_s k_s^{\alpha} n_s^{1-\alpha} - c_s - g_s \tag{8}$$

and a transversality condition.

While we have claimed that we avoid modeling the shock process, the discussion so far involves a blatant assumption that shocks are first-order Markov. As a check on this assumption, we also provide sample paths from models in which the state vector contains an additional lag in the technology shock, to allow for further dependence. We refer to the two cases as first-order and second-order models.

2.3 Calibration

To focus on the sample paths from a standard model, we adopt standard parameter values. We do not vary these parameters to improve the fit of the sample paths. Hansen and Heckman (1996) and Gregory and Smith (1993) discuss econometric alternatives to calibration. One of the aims of this study is to show that a standard tool for assessing models – in-sample prediction – can be useful even if no parameters are estimated.

The discount factor is $\beta = 0.95$ in annual data and 0.99 in quarterly data. The depreciation rate is $\delta = 0.10$ in annual data and 0.021 in quarterly data, while the Cobb-Douglas parameter is $\alpha = 0.33$. The sample gross growth rate of the Solow residual,

γ_x, is 1.0188 in annual data and 1.00264 in quarterly data. The weight on leisure in the utility function is $\theta = 3.0$, in the divisible-labor case, and $\theta = 3.9$ in the indivisible-labor case. These weights are those of Christiano and Eichenbaum (1992), rescaled to reflect our measurement of n as a fraction of time rather than as hours. In fact, in the indivisible-labor case the sample paths are invariant to θ, which simply affects the scale of the variables.

We consider four models: with divisible and indivisible labor, and with z_t following a first or second-order Markov process. To avoid a proliferation of graphs, we therefore examine only the case in which government spending does not directly influence utility, so that $\mu = 0$.

2.4 Solution

For simplicity, we focus on first-order models in describing the solution method. The solution can be written as a set of decision rules

$$\begin{aligned} c_s &= \Lambda(z_s, g_s, k_s) \\ n_s &= \Omega(z_s, g_s, k_s), \end{aligned} \tag{9}$$

which also imply a decision rule for investment. As these rules cannot be found analytically, we use an approximate solution method, using historical sequences of shocks $\{z_t\}$ and $\{g_t\}$, and an initial value for capital, k_0. We adopt decision rules which are log-linear in the state variables:

$$\begin{aligned} c_s &= exp[\lambda_0 + \lambda_z log(z_t) + \lambda_g log(g_t) + \lambda_k log(k_s)] \\ n_s &= exp[\omega_0 + \omega_z log(z_t) + \omega_g log(g_t) + \omega_k log(k_s)] \end{aligned} \tag{10}$$

where λ_0, λ_z, λ_g, λ_k, ω_0, ω_z, ω_g, and ω_k are as yet undetermined coefficients. This form has the advantage of ensuring positive values for c_s and n_s. The decision rules also are invariant to scale changes, in that λ_0 and ω_0 are chosen endogenously. Log-linear rules also are used in the widely applied method of King, Plosser, and Rebelo. More complicated functional forms could be estimated readily.

We next substitute these decision rules in the constraint, to get

$$\gamma_x k_{s+1} = (1-\delta)k_s + z_t k_s^{\alpha}[\Omega(z_t, g_t, k_s)]^{1-\alpha} - \Lambda(z_t, g_t, k_s) - g_t. \tag{11}$$

We then use the decision rules (10) and this expression for k_{s+1} to replace c_s, n_s, c_{s+1}, n_{s+1}, and k_{s+1} in the Euler equations. These substitutions give:

$$E_s \frac{\beta}{\gamma_x \Lambda(z_{t+1}, g_{t+1}, k_{s+1})} \cdot \Bigg[1 + z_{t+1}\alpha\Big(\big[(1-\delta)k_s + z_t k_s^\alpha [\Omega(z_t, g_t, k_s)]^{1-\alpha} - \Lambda(z_t, g_t, k_s) - g_t\big]\gamma_x^{-1}\Big)^{\alpha-1} \big[\Omega(z_{t+1}, g_{t+1}, k_{s+1})\big]^{1-\alpha} - \delta\Bigg] - \frac{1}{\Lambda(z_t, g_t, k_s)} = 0 \quad (12)$$

and either

$$\frac{\theta}{1 - \Omega(z_t, g_t, k_s)} - \frac{1}{\Lambda(z_t, g_t, k_s)}\big[z_t(1-\alpha)[\Omega(z_t, g_t, k_s)]^{-\alpha} k_s^\alpha\big] = 0 \quad (13a)$$

in the divisible-labor case, or

$$\theta - \frac{1}{\Lambda(z_t, g_t, k_s)}\big[z_t(1-\alpha)[\Omega(z_t, g_t, k_s)]^{-\alpha} k_s^\alpha\big] = 0, \quad (13b)$$

in the indivisible-labor case. Equations (12) and (13) depend on the measured shocks and their forecasts but not on expected future endogenous variables.

Given an initial condition k_0 and parameters $(\gamma_x, \alpha, \delta, \beta, \theta)$, imagine guessing values of the decision-rule coefficients. Then one can recursively generate sequences for capital, consumption, and hours, given the shock sequences, by alternating between the decision rules and the accumulation constraint. We set $z_s = z_t$ and $g_s = g_t$, thus using the historical shock sequences. Next, instead of using an arbitrary guess for the decision-rule coefficients, we optimize over them by the criterion that the resulting sequence $\{k_s, c_s, n_s\}$ satisfies the Euler equations as closely as possible.

We estimate the coefficients of the decision rules using a generalized method of moments estimator. The left-hand side of the Euler equations (12) and (13) are replaced by their *ex post* counterparts which differ from their *ex ante* values of zero by an error. This is a pure forecast error which the model predicts has a mean of zero and, at date $s+1$, is uncorrelated with any variable known at date s. When we substitute the approximate decision rules from equations (10) for the actual but unknown decision rules, we add an approximation error to the standard forecast error. We assume that this approximation error also has zero mean and is also unforecastable.

Denote the two Euler equation errors ϵ_{1t+1} and ϵ_{2t}. Given this structure, we choose as instruments a constant, ϵ_{1t}, ϵ_{1t-1}, ϵ_{2t-1}, and ϵ_{2t-2}. Use of lagged Euler equation errors as instruments means that the moments are autocovariances of those errors. This selection reduces scaling problems because the instruments and errors both change scale when the estimated decision-rule coefficients do. These five instruments and two equations provide ten restrictions, because the product of each instrument and error is restricted to have mean zero. Consequently, the eight coefficients in the decision rules in equations (10) are over-identified. In the case with a lagged value of the technology shock included in the state vector, there are two additional parameters, so that the rules are exactly identified, given calibrated values of the parameters $(\gamma_x, \alpha, \delta, \beta, \theta)$. The weighting matrix is the identity matrix in all cases, which Monte Carlo evidence suggests is a prudent choice in small samples. We use a numerical simplex search algorithm (`fmins` in MATLAB). Asymptotic standard errors for the decision-rule coefficients are based on numerical derivatives, and are re heteroskedasticity-consistent. In several cases, the estimated variance matrix of the sample moments was not well-conditioned, so some standard errors are large.

This solution method – dynamic programming by GMM – is described by Letendre and Smith (1996), who document its application to the Merton-Samuelson savings and portfolio problem with risky income. It can be thought of as a semi-parametric version of the projection methods described by Judd (1992). Its appealing features include that it is easy to combine with a stationary shock path with unknown properties, it does not require discretization of the state space, it does not impose certainty equivalence, and it is a natural extension of the guess-and-verify method used in analytical solutions. The GMM metric also gives standard errors and allows a test based on overidentification, as well as stability tests.

2.5 Initial Condition

The annual historical data begin in 1925, and the quarterly data begin in 1955. There is no reason to suppose that k_{1925}, for example, was at its steady-state value. We detrend the capital stock series K_t and then measure how far k_{1925} was from zero in standard deviation units. We then set the initial condition for k_s so that $\hat{k}_s = \hat{k}_t$ for 1925; each series begins the same number of standard deviations from its mean. The values used are $k_{1925} = 2.47397$ in annual data and $k_{1955} = -1.91989$ in quarterly data.

With this initial condition, the realized shocks, and the estimated decision rules, predicted values for capital, consumption, labor input, and output are constructed. This method can be thought of as a dynamic simulation, for we use only the initial value of the actual capital stock. While this is a stringent way to assess the models, an alternative would be to plot the results of decision rules applied to the historical sequence of capital stocks.

2.6 Graphical and Spectral Methods

Business-cycle studies often provide information on how the model amplifies or propagates shocks. A good example is Table 11 of Cochrane's (1994) study of shocks. We do not present such statistics because the model and history are based on the same shocks, so we directly compare their paths for output, capital, employment, and consumption. To focus on the dynamics, each variable is standardized in the graphs of sample paths. The vertical axis, in standard deviation units, is the same for each variable and model.

The graphs which follow do not contain confidence bands around the predicted sample paths. There is a logical reason for this apparent omission. While we can sample from the asymptotic distribution of the estimated decision-rule parameters, we cannot sample from the distribution of z_t and g_t, for we do not specify the distribution of these shocks. As a result, we have only one realization of the shocks – the historical one – and so only one sample path for each model, state vector, and time period, which is conditional on the shock realization.

In addition to graphing predicted sample paths from the business-cycle models, we compare the predicted and historical paths by spectral methods. Spectral smoothing is done with the Hanning window, which has slightly fatter tails than the triangular window but a similar shape. The formula for the weights of an N-point Hanning window is $w_i = .5[1 - \cos((2\pi i)/(N+1))]$, $i = 1, 2, \ldots, N$. For example, a 3-point window has weights [.5 1 .5] and a 5-point window has weights [.25 .75 1 .75 .25]. We used a 10-point window for annual data and a 20-point window for quarterly data. The windows are constant across models and variables.

The horizontal scale of spectral plots is in fractions of π. Time-domain periods are therefore $2/x$, where x is the horizontal axis of the plot (the π's cancel from $(2\pi)/(x\pi)$). For example, in annual data, the point .5 on the horizontal axis corresponds to a cycle of 4 years. In quarterly data the same point corresponds to a cycle of 4 quarters.

We graph two properties of the cross spectrum of the predicted and actual series: the coherence and phase. The coherence is bounded between 0 and 1 and measures the degree to which the actual and predicted series share common fluctuations at each frequency. It also may be interpreted R^2-like, as the fraction of the variance of one series which can be linearly explained by the other, frequency by frequency. The phase is the fraction of a cycle by which one series leads the other at each frequency. If the predicted series leads the actual series the phase is positive, while in the opposite case it is negative. Thus a perfect fit between the two series would be reflected in a coherence of 1.00 and a phase of 0.00 at all frequencies.

3. Annual Realizations, 1925-1994

In this section we plot predicted and actual time series for U.S. output, capital, labor input, and consumption per capita in annual data for the period from 1925 to 1994. Measures of the aggregate capital stock as historically measured are not available for 1995, because the Bureau of Economic Analysis will replace the existing measure with a chain-weighted series during 1997. We consider four models, with divisible and indivisible labor and of first and second order in the technology shock.

Table 1 contains the estimated decision rule coefficients and their standard errors. For the most part, the estimates are consistent with theory. First, an increase in the capital stock leads to a rise in consumption and a fall in labor supply. Second, consumption and labor supply move in opposite directions in response to a technology shock. Third, a shock to government spending leads to lower consumption and higher labor supply in the divisible-labor model, while these effects are insignificant in the indivisible-labor model.

With $\mu = 0$, the fiscal shock is a pure resource drain. It is financed by lump-sum taxation which has a wealth effect on labour supply.

Table 2 shows the coefficients of variation of output, capital, labor, and consumption as predicted by the model and found in the data. The output coefficient of variation predicted by the models is from 1.52 to 4.51 times the coefficient of variation in the historical data. However, we do not argue that technology and fiscal shocks account for $152 - 451$ percent of output volatility, because: (i) these statistics have sampling variability and our solution has approximation error (see Eichenbaum, 1991); (ii) the statistics are sensitive to detrending (see Cochrane, 1994); (iii) we cannot say what percent of variation is caused by a certain shock unless we reproduce the variation (and, more strictly the sample path) of the historical data (see Aiyagari, 1994); and (iv) the Solow residual does not purely measure technology change. We discuss alternate ways of measuring shocks in section 5.

However, Table 2 can be used to compare relative volatilities in the models with those in the historical data. All four models reproduce the volatilities of employment and consumption relative to output relatively well. All four models underpredict the relative volatility of the capital stock.

Figures 2-4 summarize predictions of the model with divisible labor. Figure 2A shows actual output cycles (the solid line) and cycles predicted from the first-order and second-order models (the dashed lines), all standardized. The model incorrectly predicts a recession in the late 1920s. As a result, it produces a fairly great depression, though it cannot match the fall in output from 1929 to 1933 or the depth of the actual Depression. It matches the 1940-1945 boom quite closely, and in fact over predicts output then. For the postwar period, the model matches output cycles to some extent, but produces a path smoother than the actual series.

The discrepancies in the output paths can be accounted for by discrepancies in input paths. This also is a stricter test, because the model could reproduce the output path with offsetting errors in the two inputs. Figures 2B and 2C show that this is roughly what happens. The model underpredicts capital and labor inputs in the 1920s, and overpredicts them in the early 1940s. During the last 30 years the predicted labor input is less volatile than the actual series, and hence so is output. The only notable difference between the first and second-order models appears in Figure 2B, in the predictions for the capital stock.

Matching output dynamics is relatively easy, because both predicted and actual series are driven mainly by the sequence of Solow residuals. Matching the dynamics of aggregate consumption is more difficult. Figure 2D graphs the predicted and actual consumption of nondurables and services. The match again is poor in the 1920s. Combined with Figure 2C, this feature reflects the sample-path version of the syndrome described by Rotemberg and Woodford (1996). In response to a negative productivity shock (shown in Figure 1), output and hours move in opposite directions in the model but the same direction in the data. The model also erroneously predicts a large increase in private consumption during World War II, but has greater success after 1960.

Figure 3 plots coherences between predicted and actual series, while Figure 4 plots the phase of the predicted series relative to the actual. For output, the coherence in Figure 3A is high at low frequencies, but it is very low at frequency 0.65, corresponding to cycles of 3 years. The output phase in Figure 4A shows the two series in phase at low frequencies, the model leading the actual series at business cycle frequencies, and the actual series leading the model at high frequencies. The other panels in Figures 3 and 4 provide spectral evidence on capital, employment, and consumption. Like Figure 2, they show that the model's predictions are less accurate for inputs and consumption than for output.

It is well-known that this business-cycle model has relatively little internal propagation. For that reason, the dynamics of the indivisible labor model are quite similar to those of the divisible labor model. Figures 5-7 present predictions from the model with indivisible labor. Once again there is little role for z_{t-1} in the state vector, as the first and second-order models are very similar. Overall, the divisible labor model does slightly better at predicting annual capital and labor inputs, while the indivisible labor model does slightly better at predicting output and consumption.

Predicted and actual consumption have similar autocorrelation, correlation with output, and volatility relative to output (as Table 2 shows). Yet the D panels of figures 2-7 show some large discrepancies, notably during the 1940s where the model greatly overpredicts consumption. Our hope is that sample path discrepancies can be informative about modifications to the model, just as moment discrepancies have been. While we do not have a modification to propose which would improve the predictions, we emphasize the pitfalls of focusing on unconditional moment-matching as a diagnostic.

Studying the annual data suggests three facts to focus on. First, the model does relatively poorly for the 1920s. Second, the model cannot reproduce the procyclical behavior of both consumption and employment. Third, the model does relatively poorly at high frequencies. To provide further information on this third finding, we next document post-war quarterly realizations.

4. Quarterly Realizations, 1955-1992

Table 3 shows the coefficients of the quarterly decision rules, along with their standard errors. Table 4 gives coefficients of variation for output, capital, labor input, and consumption in the data and in the theory. Identifying the decision rules was more difficult than in the annual data, because the technology and fiscal shocks, shown in Figure 8, are highly correlated (a fact which might indicate demand shocks) and because there was no fiscal event as dramatic as World War II.

Table 3 generally is consistent with theory. An increase in the capital stock leads to more consumption and less labor supply. A fiscal shock lowers consumption and raises labor supply. In response to a technology shock, consumption and hours move in opposite directions in the divisible-labor model but in the same direction in the indivisible-labor model. However, several of these effects are estimated imprecisely. Moreover, table 4 shows that the model produces a volatility in the capital stock relative to that of output that is much greater than in the data. Also, consumption is too volatile relative to output, in comparison with the historical ratio.

We again study the divisible labor model first. Figures 9-11 present the results, some of which are similar to findings of Christiano (1988). In the plot for output in Figure 9A, the model captures much of the 1960s expansion, and fits business-cycle turning points in the early 1970s very well. In contrast, it does poorly at fitting oil-shock-era recessions, and dramatically underestimates the scale of the 1981–1982 and 1990–1991 recessions. The sample path for output is too smooth after 1975, relative to the historical path.

Figure 9B shows that the model has much more success in predicting the capital stock dynamics than it did in annual data. The second-order model in particular is quite accurate until 1984. Figure 9C shows that employment is not well predicted though. The model greatly overpredicts employment during the 1960s boom, and its employment series is too smooth since then. The chronic problem of matching the procyclicality of consumption and employment simultaneously is manifest again in Figure 9D, where consumption is under-predicted during the 1960s.

Figures 10 and 11 present the spectral evidence. They are difficult to summarize, except that again it is clear that matching inputs is more difficult than matching output. The predicted and actual output series have greater coherence than other variables, and there is little evidence of a phase shift in the output series.

Figures 12-14 give evidence on the indivisible-labor version of the model. As in the annual data, this version does better than the divisible-labor version in predicting output and consumption, but now it also does no worse at predicting capital and labor. In Figure 12A, the model is quite successful in matching the historical output dynamics. While the predicted path still is too smooth from 1980 to 1990, the model closely matches the historical data during the 1990-1991 recession. Hansen and Prescott (1993) also studied the ability of a model driven by technology shocks to reproduce that recession. Our results show that the basic real business cycle model can fit output in the early 1990s even with an initial condition thirty years earlier than the one used by Hansen and Prescott, though the fit is much poorer in the previous two recessions.

5. Extensions

In the examples so far we do not impose the common-trend (balanced growth) restrictions that the theory implies. It would be straightforward to jointly, log-linearly detrend the historical output, capital, consumption, and technology shock processes, requiring balanced growth, and to generate trending paths from the model. King, Plosser, Stock, and Watson (1991) have shown that the cycles from a common-trend model do not resemble those from the stochastic growth model. Gregory and Smith (1996) reach a complementary conclusion: if cycles are restricted to resemble those from the stochastic growth model then a common-trend restriction can be rejected. From these studies we know that imposing this trend would detract from the model's ability to fit historical cycles.

A second way to provide more information from these experiments would be to tabulate the moments of predicted and actual series, such as autocorrelations and correlation with output. We also could study additional variables such as investment or real wages. Our emphasis on realized sample paths and graphical methods is intended to complement this standard statistical evidence.

By producing in-sample predictions, our examples show that this stochastic, general equilibrium model can be studied much like econometric models. Given that analogy, it is natural to ask whether the shocks we use really should be viewed as exogenous variables. We have followed convention in calling z_t a technology shock, whereas it is well-known that it is neither a shock nor solely affected by technology. We refer to it as a shock even though it is not unpredictable, as pure changes in technology perhaps should be. Pioneers of productivity measurement knew well that it is tenuous to view Solow residuals as pure measures of technological progress, for example due to varying capital utilization or labor hoarding (see Jorgenson and Griliches (1967) and Griliches (1996)). Basu (1996) demonstrates that much postwar variation in U.S. Solow residuals is due to varying utilization. However, our method could readily be used with alternative production technologies and shock measurements.

The model also could be extended by the introduction of additional shocks. In this model there are other ways to back out combinations of the technology shock and fiscal shock, from the accumulation constraint or the consumption-leisure trade-off. Ingram, Kocherlakota, and Savin (1994) noted that these different measurements will be inconsistent and, equivalently, that there are singularities in the model. The presence of measurement error is sometimes argued to explain why there are no singularities in the data. But in fact there surely are multiple shocks, and so they should be measured and introduced also, to enhance the sample-path predictions.

Readers who prefer to estimate parameters, rather than calibrate them, also can make straightforward extensions to the exercises in this paper. The GMM estimation we use to solve for decision rules also can estimate parameters of the economic model, subject to identification. Alternately, those parameters can be estimated using first and second moments of the data, as was done by Christiano and Eichenbaum (1992).

Our solution method – DP by GMM – naturally involves approximation error. In some applications it may be interesting to explore more flexible functional forms for decision rules. GMM tests based on over-identification or stability may be used to assess the accuracy of the approximation.

It is well-known that it is difficult to estimate the zero-frequency spectral density of GMM errors with a small sample of persistent data and hence hard to construct accurate Wald tests (see the symposium in the *Journal of Business and Economic Statistics* (1996)). In our examples, there was little persistence in GMM residuals, and so this difficulty did not arise. In other applications, one might decide instead to parametrize the shock processes and then use a standard solution method. Once again, sample paths could be found.

Finally, we hope to investigate these realizations by comparing them with those from other models. Leading candidates include the models of Taylor (1993, chapter 2) and Leeper and Sims (1994).

6. Conclusion

The findings in this paper may be of interest whether one chooses to calibrate or estimate model parameters. While we have been deliberately conservative in calibrating the model, the realizations would be very similar if we estimated parameters using macroeconomic information. Values for the parameters of utility and production functions usually are not controversial, and our method is agnostic about the law of motion of the shocks because we use actual, historical shocks. Thus our solution method and sample paths are consistent with either calibration or estimation.

Our main aim has been to show that in-sample prediction, a standard statistical tool, is straightforward and informative in dynamic general equilibrium models. Like discrepancies in moments, the sample-path discrepancies here are designed to suggest modifications to the theory. Real business cycle realizations for the U.S. show that the model fits output relatively well, with the exception of the 1920s and some postwar recessions. The sample paths for labor input are quite different from the historical series though, and so the labor-market component of the model may be a worthwhile focus for reformulation.

Data Sources

Annual Data:

population: population aged 16 and over, millions: 1925-1946, *Historical Statistics of the United States*, series A39; 1946-1994 Bureau of the Census web site.

capital: constant cost net stock of fixed private non-residential capital, billions of 1987 dollars: 1925-1988, Musgrave (1992); 1989-1993, *Survey of Current Business*, August 1994; 1994 value provided by the Bureau of Economic Analysis.

output: real gross national product, billions of 1987 dollars: 1925-1929, Balke and Gordon (1989); 1930-1958, National Income and Product Accounts; 1959-1994, Department of Commerce, Bureau of Economic Analysis, stat-usa.

consumption: real consumption of nondurables and services, billions of 1987 dollars: 1925-1928, Backus and Kehoe (1992); 1929-1958, National Income and Product Accounts; 1959-1994, Department of Commerce, Bureau of Economic Analysis, stat-usa.

hours: total employment aged 16 and over, thousands, multiplied by average weekly hours in manufacturing, and multiplied by 50: 1925-1947, Historical Statistics of the United States (1976), series D5 and D803; 1947-1994, *Statistical Abstract of the United States.*

government consumption: total government expenditure: 1925-1929, Kendrick; 1929-1958, National Income and Product Accounts; 1959-1994, Department of Commerce, Bureau of Economic Analysis, stat-usa. minus government gross non-military investment: 1925-1979; Bureau of Economic Analysis (1982), *Fixed reproducible tangible wealth in the United States, 1925-1979*; 1979-1994, stat-usa.

Quarterly Data:

population: civilian, noninstitutional population aged 16 and over from Citibase (Mnemonic P16).

capital: the sum of the net constant dollar stocks of consumer durables, producer structures and equipment, and government and private residential capital plus government nonresidential capital. These are from Bureau of Economic Analysis: *Fixed Reproducible Tangible Wealth in the United States in the United States, 1925-89* Washington D.C., 1993 and from "Fixed Reproducible Tangible Wealth in the United States: Revised Estimates for 1991-93 and Summary Estimates for 1925-93," *Survey of Current Business*, August 1994, pp 54-62. These data are annual and so were rendered quarterly using a smoother provided with RATS which is the optimal filter if the true data are a random walk with drift. An alternative procedure would be to estimate the quarterly capital stock using observations on investment, as Levy and Chen (1994) do.

output: real GDP from NIPA.

consumption: the sum of real private expenditures on nondurable goods plus services plus the imputed service flow from the real stock of consumer durable goods. The first two measures are directly from the NIPA. The third is from the Board of Governors and is derived from their quarterly model of the US economy.

hours: the seasonally adjusted household hours series obtained from CITIBASE (mnemonic LHOURS). This series has marked remaining seasonality and so was passed through the Census X11 program by Estima (makers of RATS).

government consumption: real government (federal, state and local) purchases of goods and services (from NIPA) minus government investment, measured as the change in the gross stock of government capital (same sources as above).

The quarterly data were used by Burnside and Eichenbaum (1996), and provided by those authors.

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Table 1: Annual Decision-rule Coefficients

Consumption

	Divis-1	Divis-2	Indivis-1	Indivis-2
constant	-3.3668	-7.1635	0.1683	0.2022
	(0.2624)	(0.5088)	(0.7169)	(0.1331)
z_t	2.1148	1.8636	-1.1414	-1.1891
	(0.3413)	(1.3777)	(0.0684)	(3.3695)
z_{t-1}	—	0.9626	—	0.0941
		(1.8869)		(2.0383)
g_t	-0.5912	-0.7181	0.0837	0.0940
	(0.1632)	(0.3150)	(0.1821)	(12.5051)
k_s	0.5345	1.9370	0.8947	0.8470
	(0.3678)	(2.3470)	(0.2616)	(3.0194)

Labor Supply

	Divis-1	Divis-2	Indivis-1	Indivis-2
constant	0.5024	2.9457	-5.8476	-5.9504
	(0.3546)	(1.2635)	(2.1724)	(12.5652)
z_t	-0.8878	-1.1879	6.4889	6.6336
	(0.3173)	(0.9019)	(0.2073)	(0.0279)
z_{t-1}	—	-0.3547	—	-0.2852
		(0.5127)		(86.9649)
g_t	0.4921	0.6442	-0.2535	-0.2847
	(0.4449)	(0.3822)	(0.5518)	(4.1573)
k_s	-0.2234	-1.0119	-1.7113	-1.5666
	(0.3761)	(1.0334)	(0.7926)	(86.9649)

Notes: Log-linear decision rules are estimated in U.S. annual data from 1925 to 1994. Divis and Indivis refer to models with divisible and indivisible labor respectively, while 1 and 2 refer to first and second-order Markov processes in the technology shock.

Table 2: Annual Coefficients of Variation

variable	history	Divis-1	Divis-2	Indivis-1	Indivis-2
y_t	0.1222	0.1859	0.2008	0.5508	0.5428
k_t	0.1758	0.0944	0.0456	0.3577	0.3556
n_t	0.1017	0.1255	0.1327	0.4632	0.4499
c_t	0.0721	0.0871	0.0955	0.3335	0.2511

Notes: Divis and Indivis refer to models with divisible and indivisible labor respectively, while 1 and 2 refer to first and second-order Markov processes in the technology shock.

Table 3: Quarterly Decision-rule Coefficients

Consumption

	Divis-1	Divis-2	Indivis-1	Indivis-2
constant	-2.3897	-2.2819	-1.7752	-1.7617
	(0.3057)	(0.5382)	(0.0114)	(0.0129)
z_t	1.1839	1.3155	0.8422	0.8390
	(0.3157)	(0.4392)	(0.0317)	(32.9054)
z_{t-1}	—	-0.1232	—	-0.0119
		(1.3971)		(4.7909)
g_t	-0.4373	-0.5172	-0.1581	-0.1689
	(1.6402)	(2.3489)	(0.3260)	(8.3310)
k_s	0.5157	0.6034	0.4766	0.4771
	(0.0741)	(0.1607)	(0.0038)	(9.5703)

Labor Supply

	Divis-1	Divis-2	Indivis-1	Indivis-2
constant	-0.0478	-0.2735	0.0191	-0.0012
	(7.3376)	(4.9287)	(0.1209)	(10.1575)
z_t	-0.3098	-0.2996	0.4937	0.5433
	(9.7190)	(2.8188)	(0.0657)	(0.0001)
z_{t-1}	—	0.0511	—	-0.0165
		(0.6658)		(10.5009)
g_t	0.5880	0.5551	0.4627	0.5114
	(14.6377)	(7.7494)	(0.9489)	(1.1731)
k_s	-0.2060	-0.2805	-0.4432	-0.4460
	(0.4906)	(0.3060)	(0.0085)	(10.4492)

Notes: Log-linear decision rules are estimated in U.S. quarterly data from 1955 to 1992. Divis and Indivis refer to models with divisible and indivisible labor respectively, while 1 and 2 refer to first and second-order Markov processes in the technology shock.

Table 4: Quarterly Coefficients of Variation

variable	history	Divis-1	Divis-2	Indivis-1	Indivis-2
y_t	0.0401	0.0540	0.0555	0.0621	0.0646
k_t	0.0276	0.0590	0.0712	0.0736	0.0778
n_t	0.0344	0.0307	0.0256	0.0361	0.0391
c_t	0.0316	0.0562	0.0638	0.0561	0.0574

Notes: Divis and Indivis refer to models with divisible and indivisible labor respectively, while 1 and 2 refer to first and second-order Markov processes in the technology shock.

Figure 1A: Annual Output, Solow Residual, Fiscal Shock

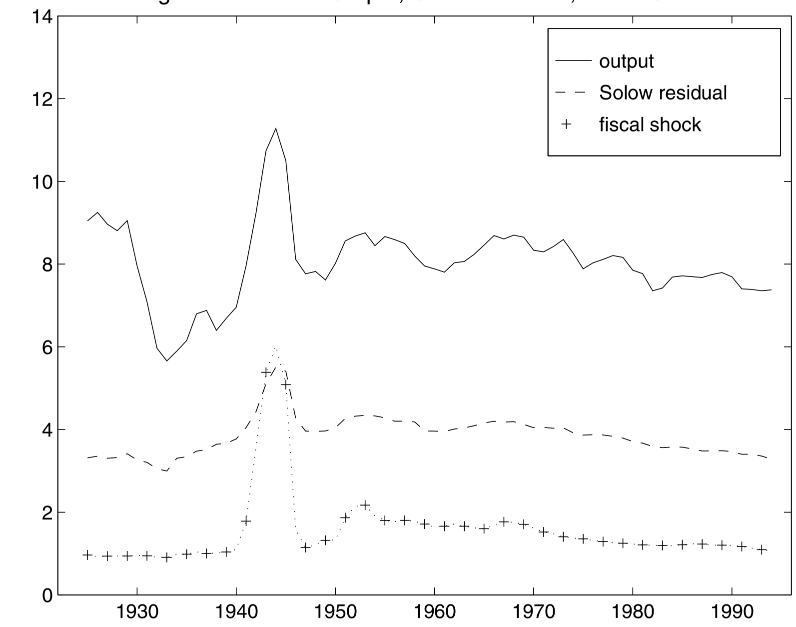

Figure 1B: Annual Output, Solow Residual, Fiscal Shock (standardized)

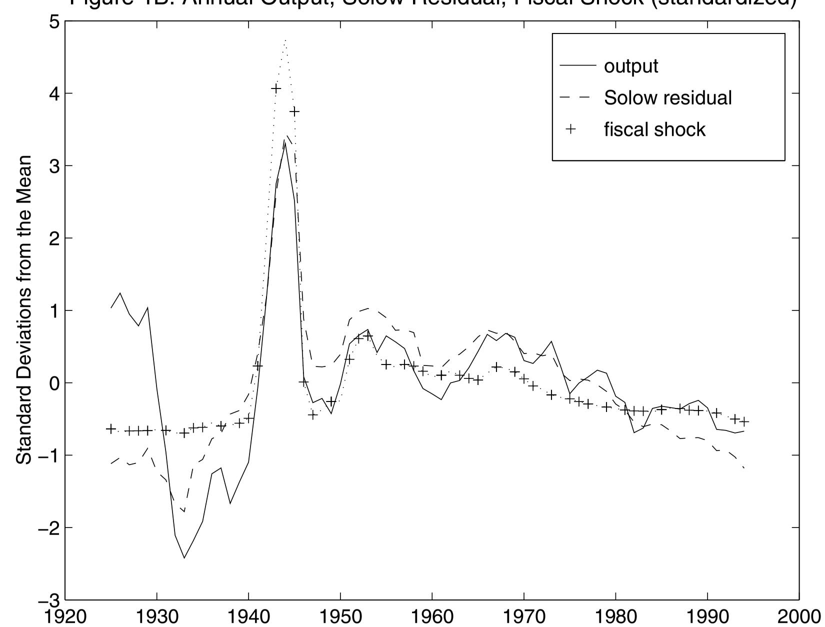

Figure 2A: Annual Output and Predicted (divisible labor)

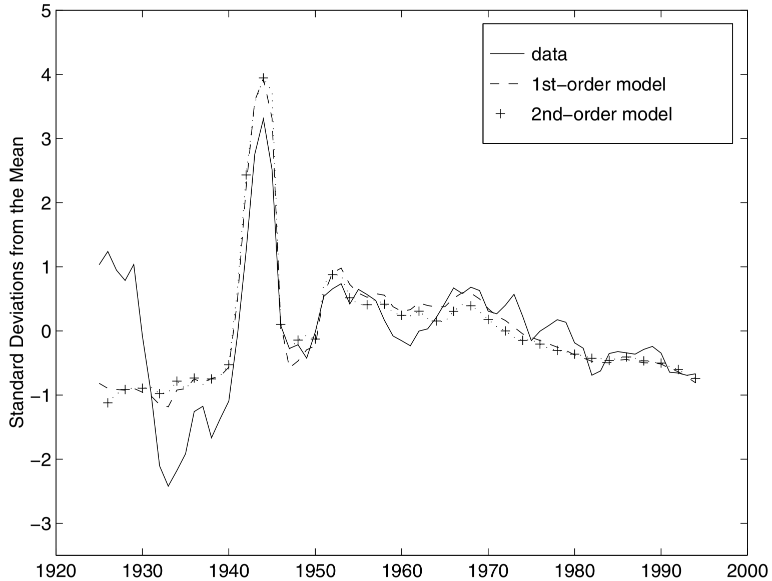

Figure 2B: Annual Capital Stock and Predicted (divisible labor)

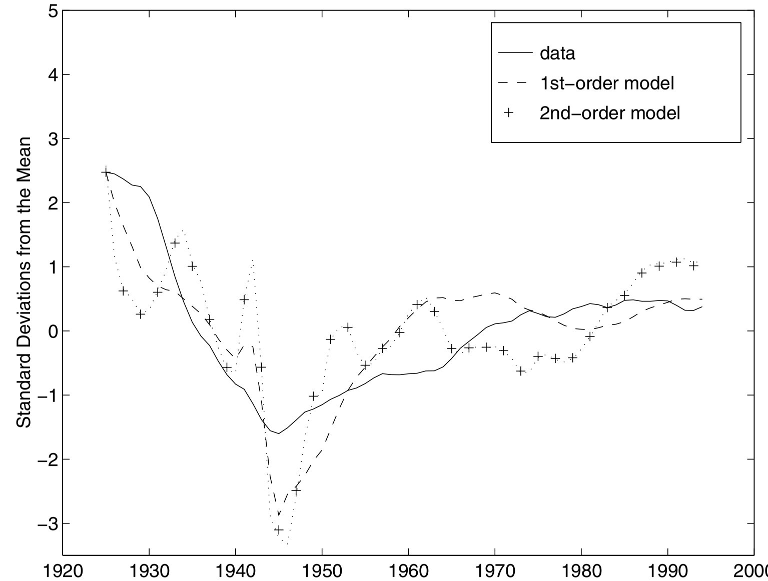

Figure 2C: Annual Employment and Predicted (divisible labor)

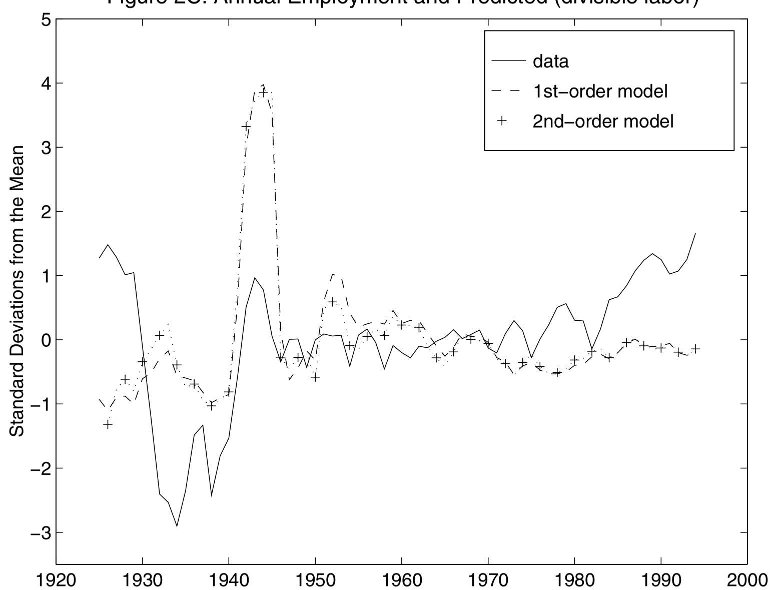

Figure 2D: Annual Consumption and Predicted (divisible labor)

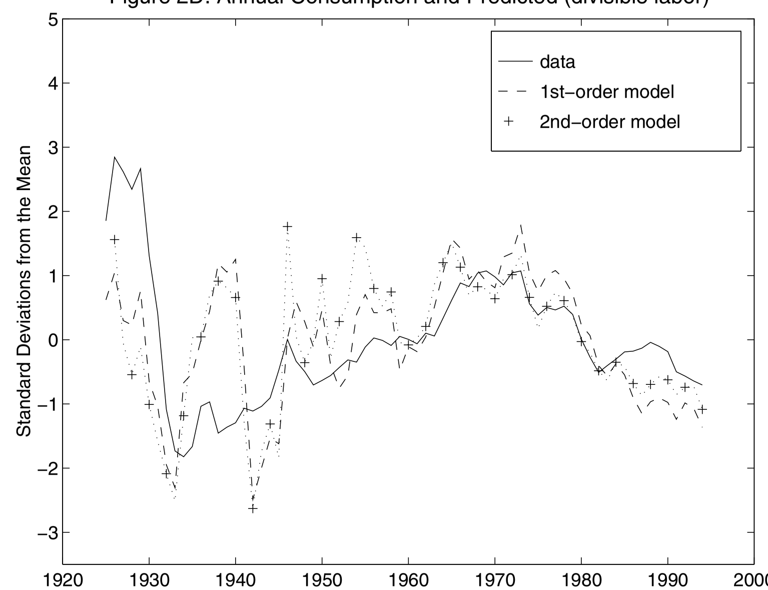

Figure 3A: Annual Output Coherence with Predicted (divisible labor)

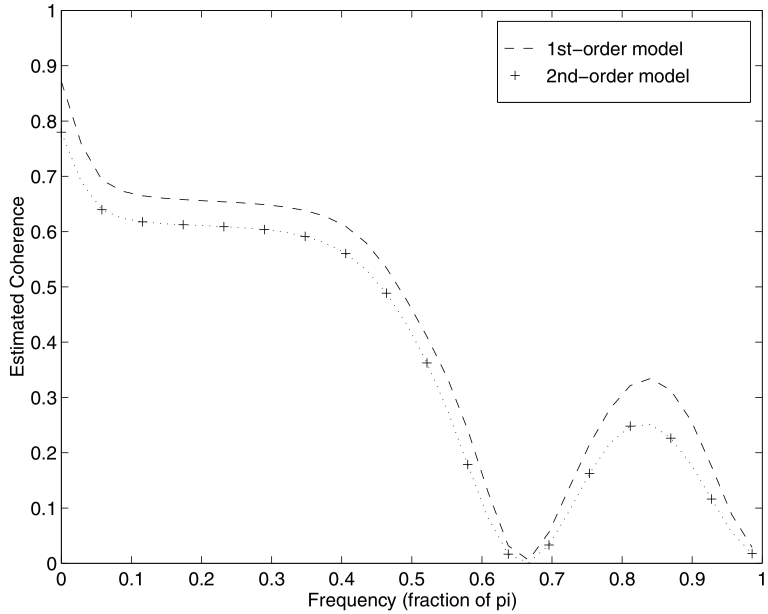

Figure 3B: Annual Capital Stock Coherence with Predicted (divisible labor)

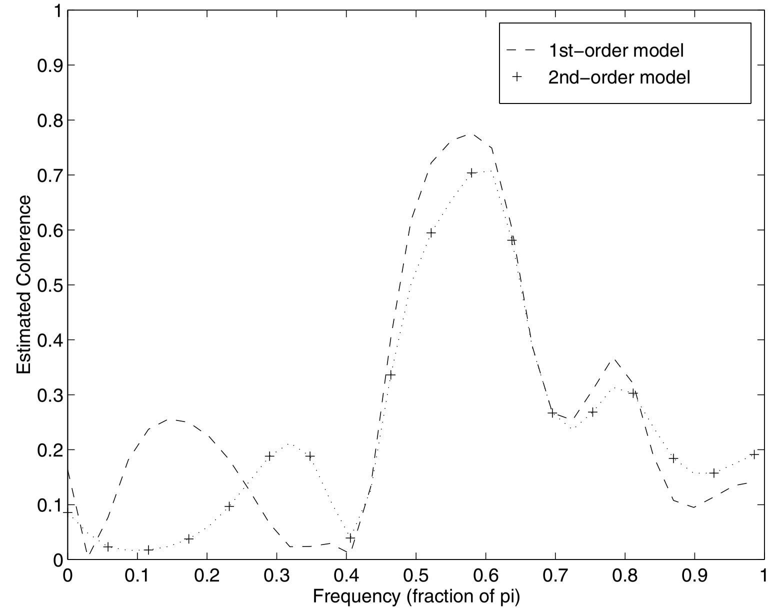

Figure 3C: Annual Employment Coherence with Predicted (divisible labor)

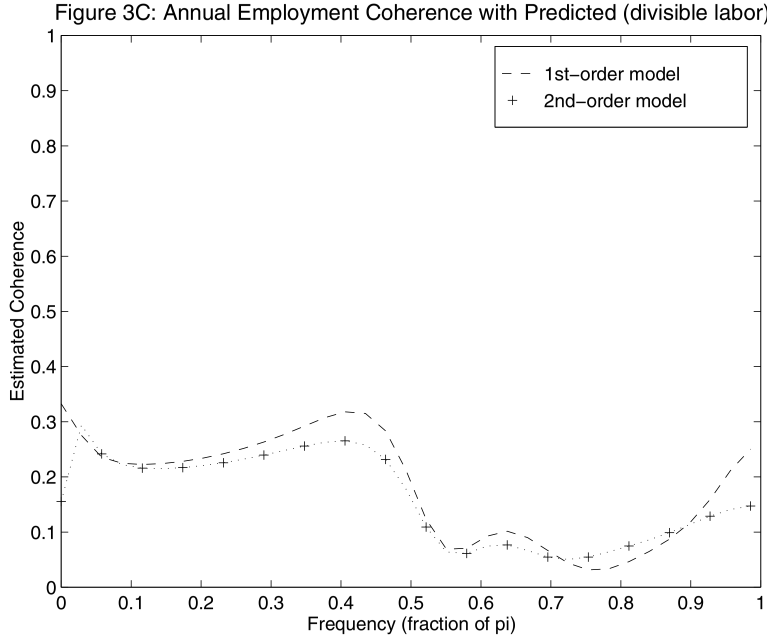

Figure 3D: Annual Consumption Coherence with Predicted (divisible labor)

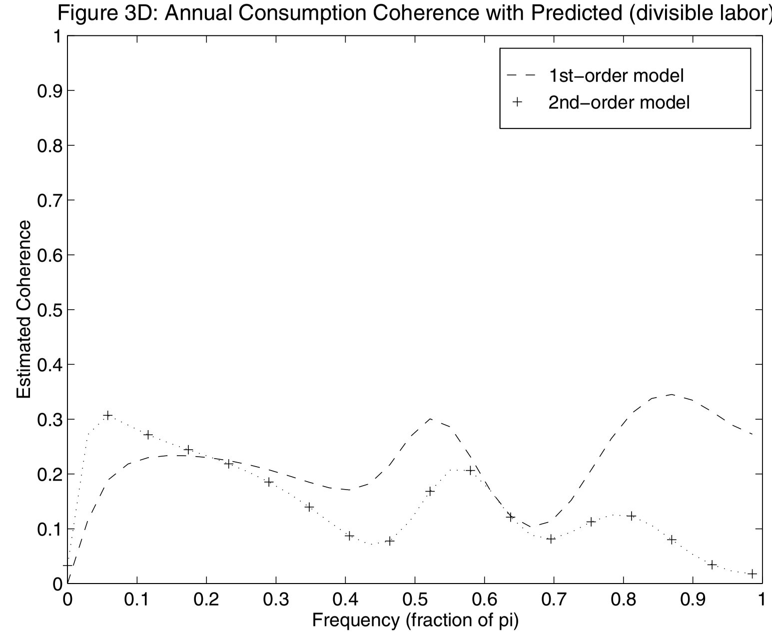

Figure 4A: Annual Output Phase with Predicted (divisible labor)

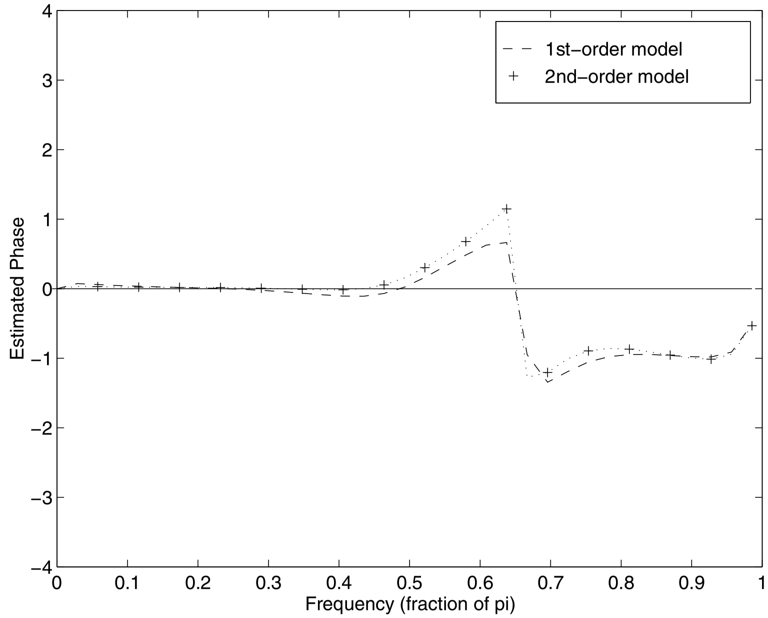

Figure 4B: Annual Capital Stock Phase with Predicted (divisible labor)

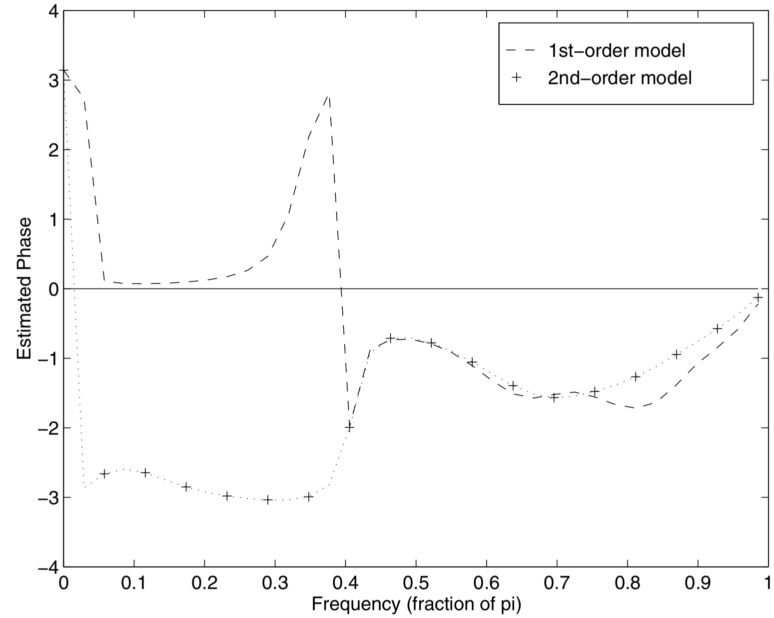

Figure 4C: Annual Employment Phase with Predicted (divisible labor)

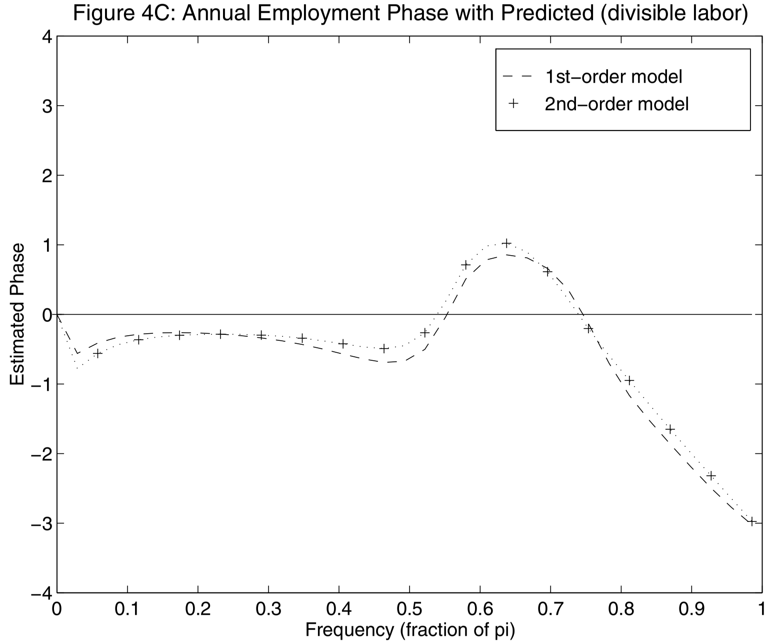

Figure 4D: Annual Consumption Phase with Predicted (divisible labor)

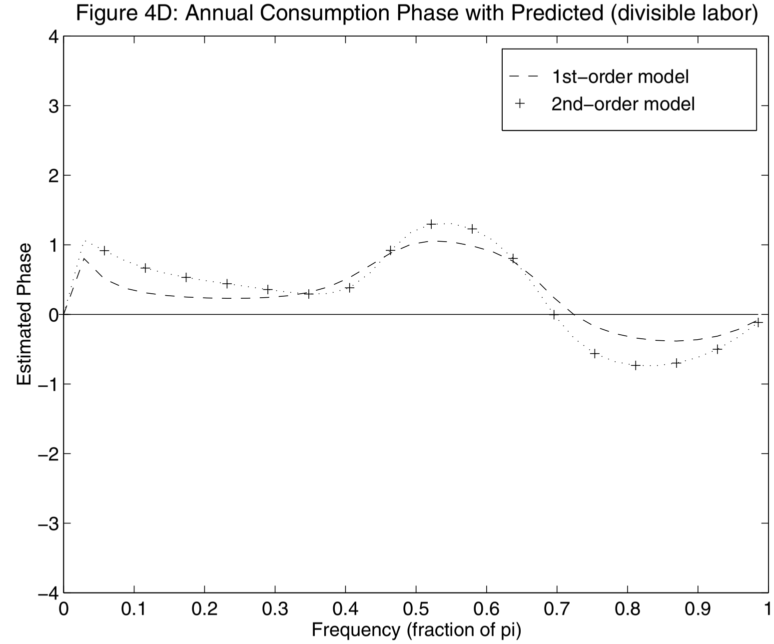

Figure 5A: Annual Output and Predicted (indivisible labor)

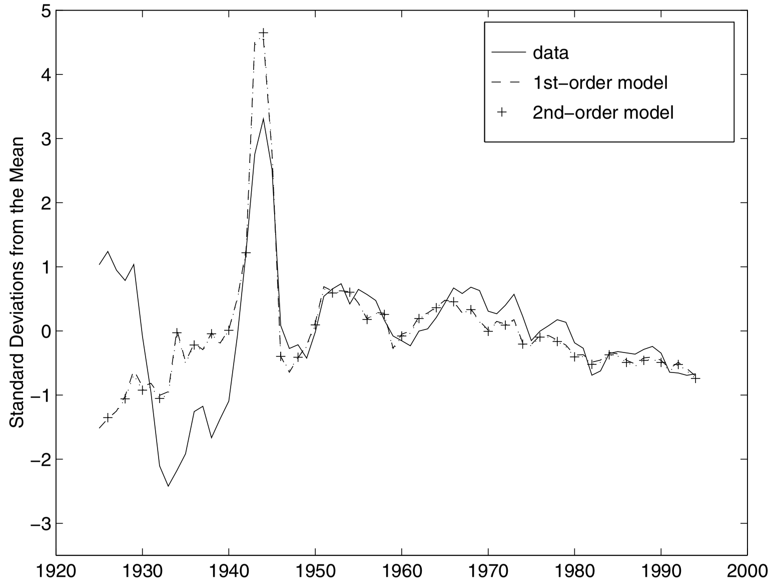

Figure 5B: Annual Capital Stock and Predicted (indivisible labor)

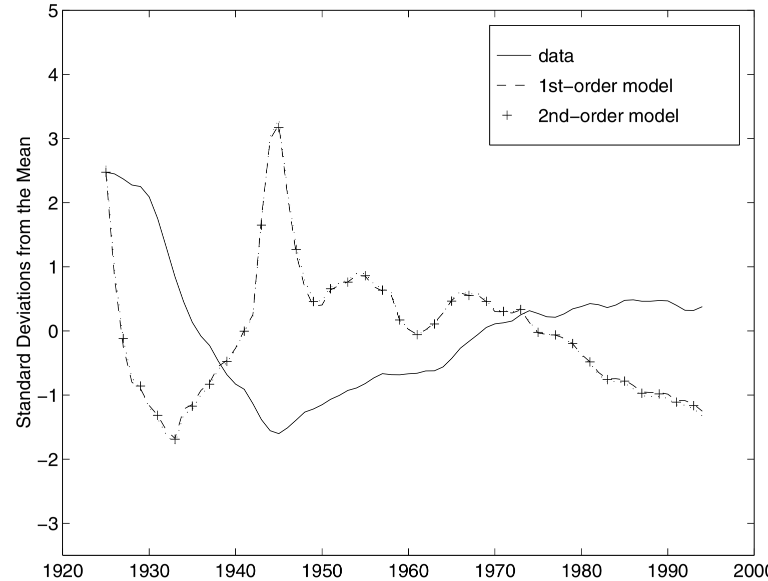

Figure 5C: Annual Employment and Predicted (indivisible labor)

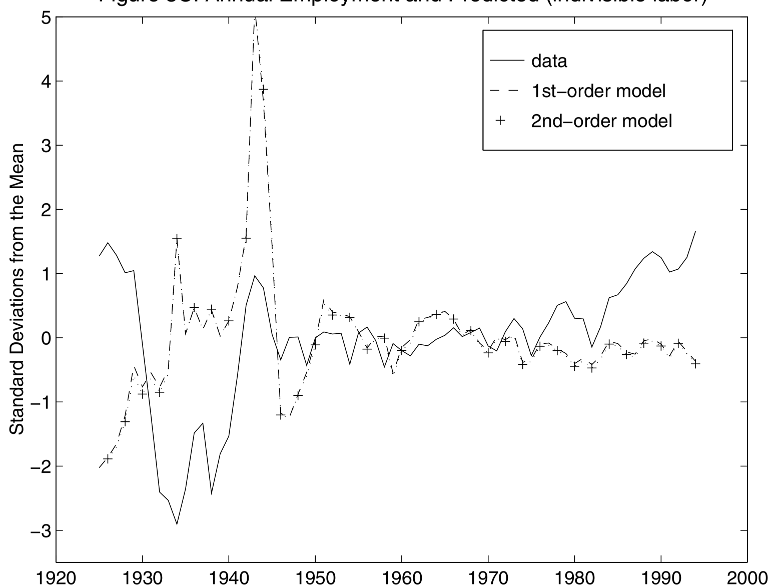

Figure 5D: Annual Consumption and Predicted (indivisible labor)

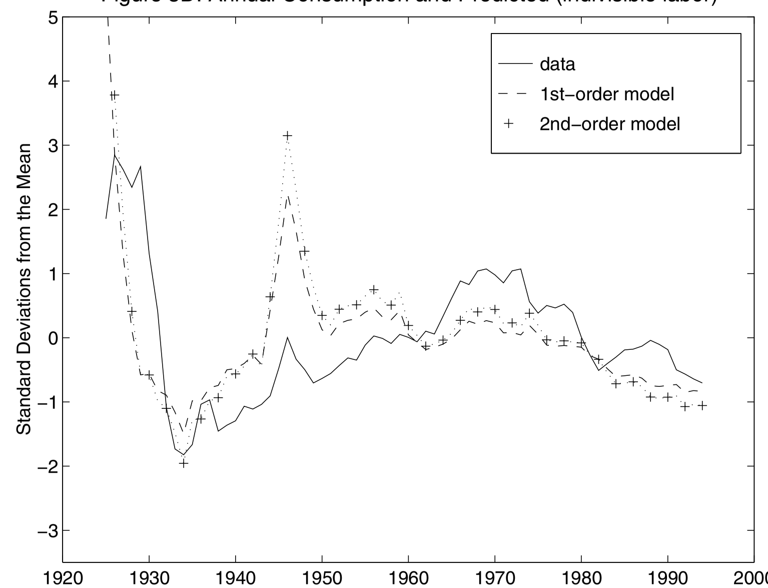

Figure 6A: Annual Output Coherence with Predicted (indivisible labor)

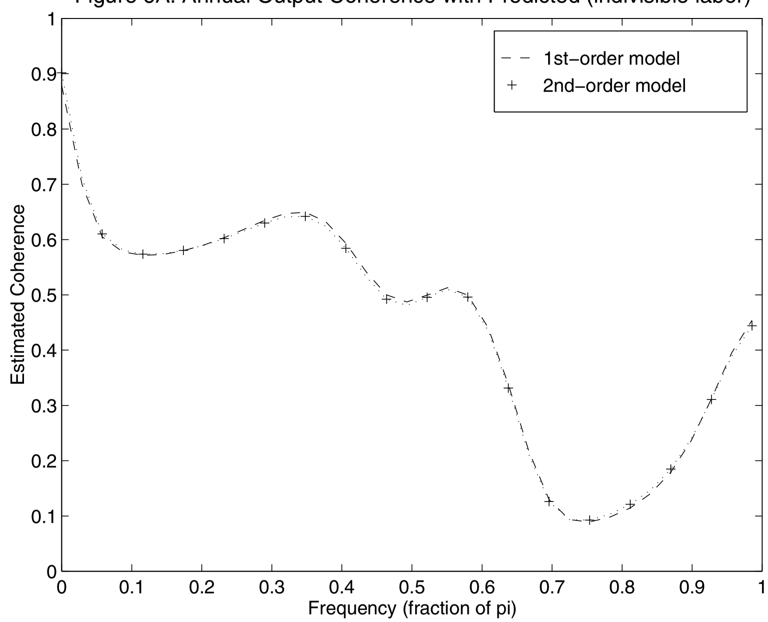

Figure 6B: Annual Capital Stock Coherence with Predicted (indivisible labor)

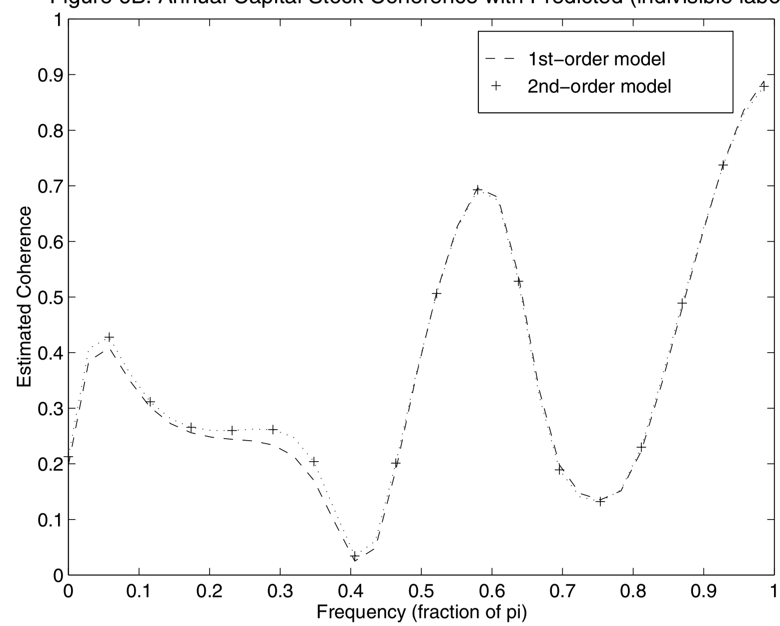

Figure 6C: Annual Employment Coherence with Predicted (indivisible labor)

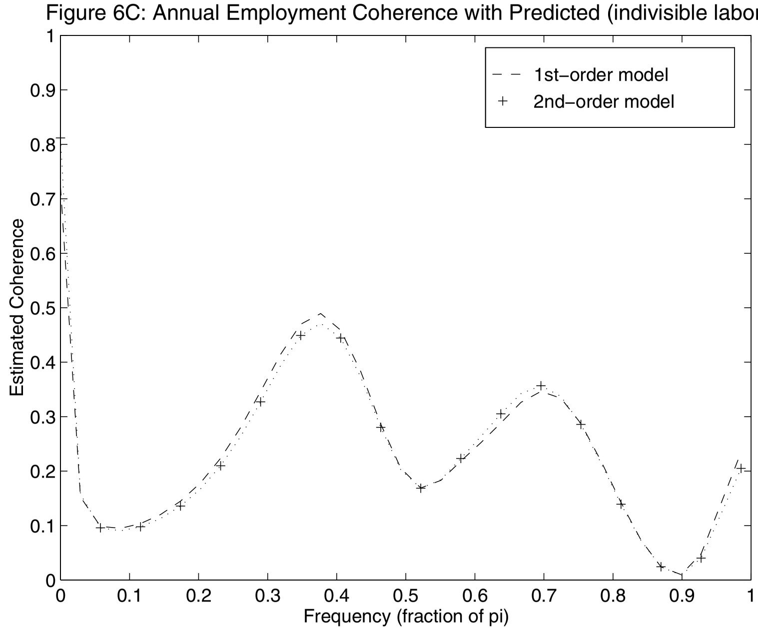

Figure 6D: Annual Consumption Coherence with Predicted (indivisible labor)

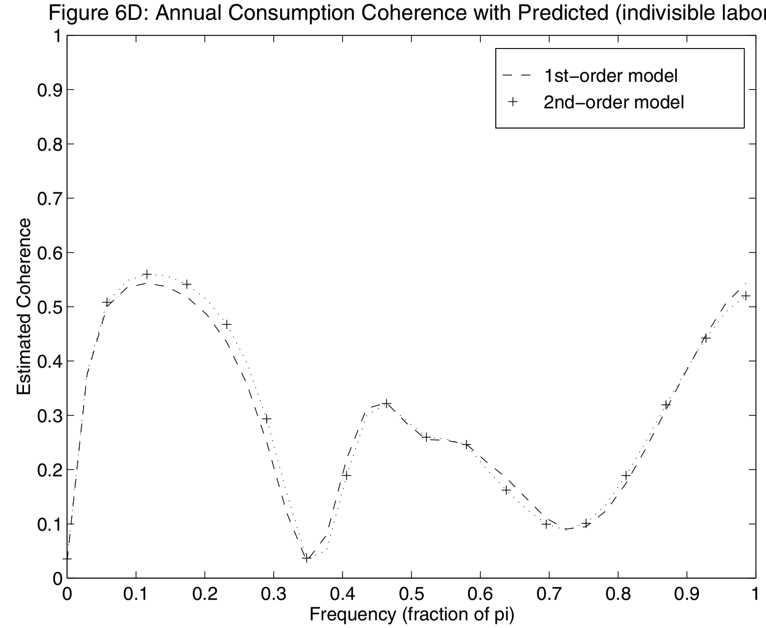

Figure 7A: Annual Output Phase with Predicted (indivisible labor)

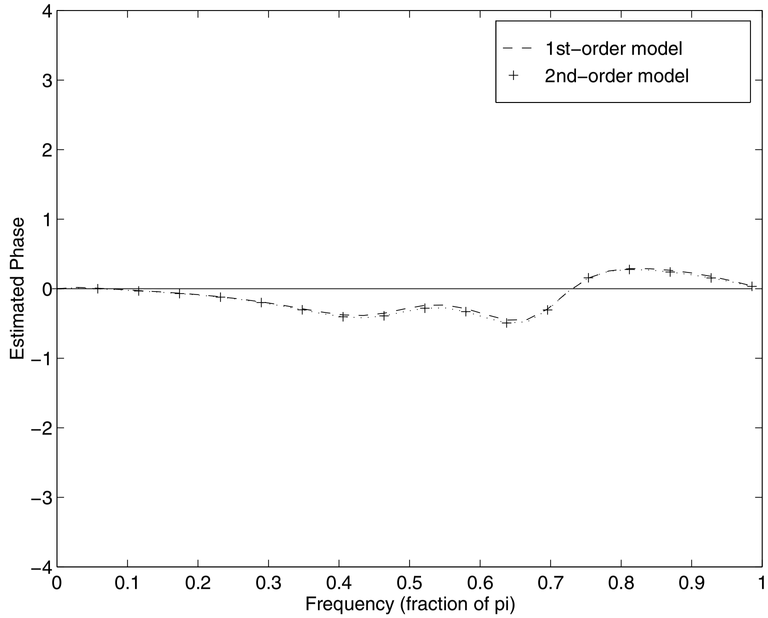

Figure 7B: Annual Capital Stock Phase with Predicted (indivisible labor)

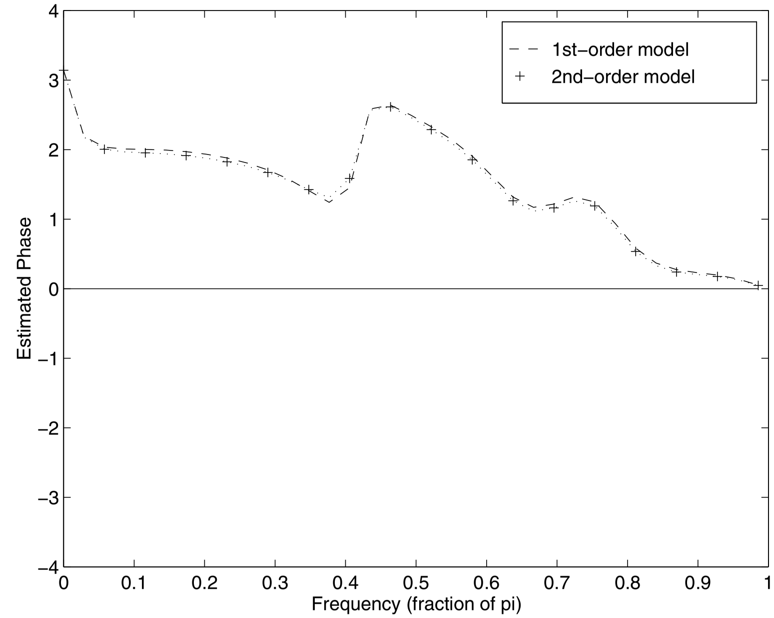

Figure 7C: Annual Employment Phase with Predicted (indivisible labor)

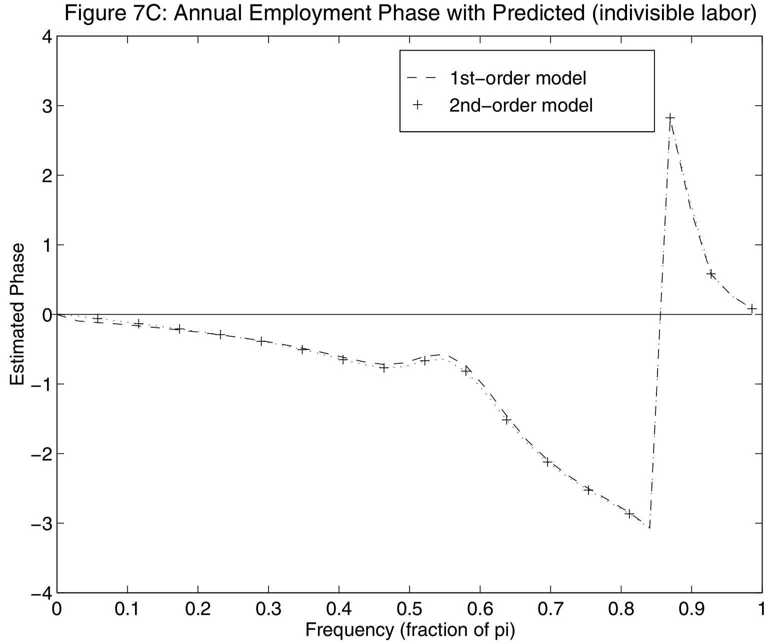

Figure 7D: Annual Consumption Phase with Predicted (indivisible labor)

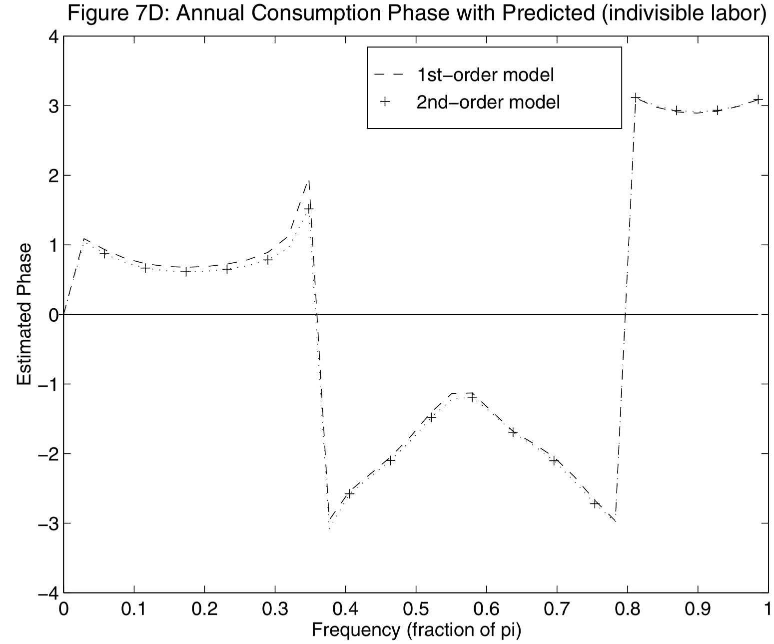

Figure 8A: Quarterly Output, Solow Residual, Fiscal Shock

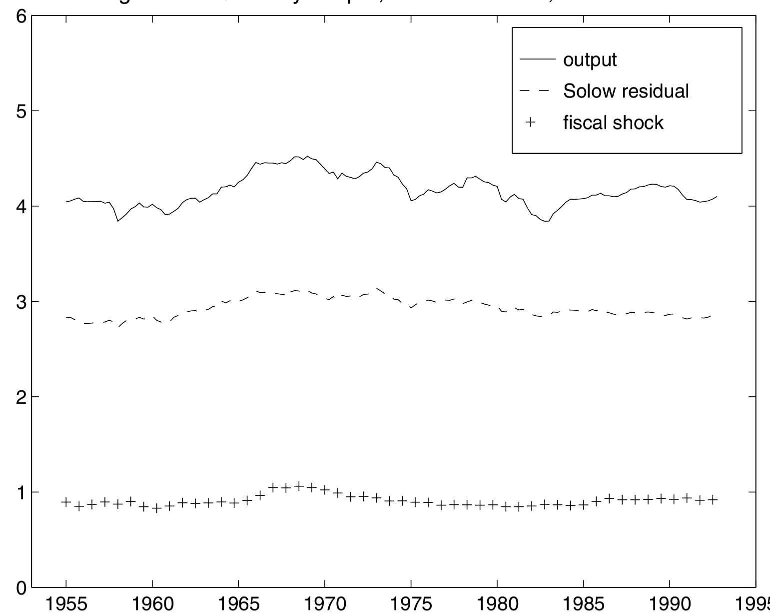

Figure 8B: Quarterly Output, Solow Residual, Fiscal Shock (standardized)

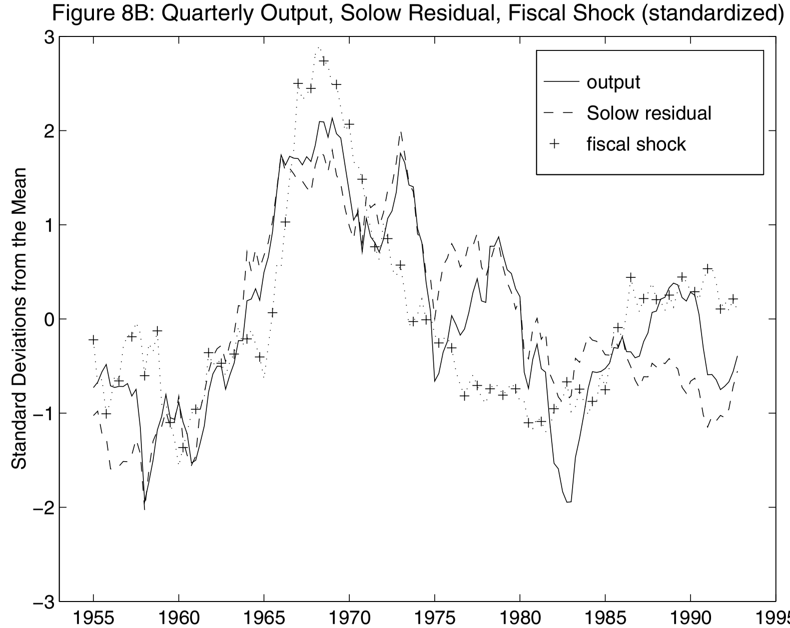

Figure 9A: Quarterly Output and Predicted (divisible labor)

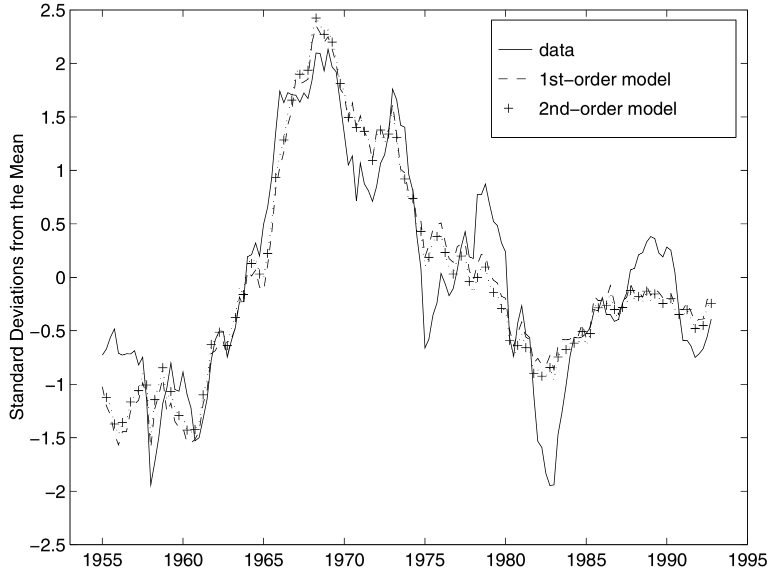

Figure 9B: Quarterly Capital Stock and Predicted (divisible labor)

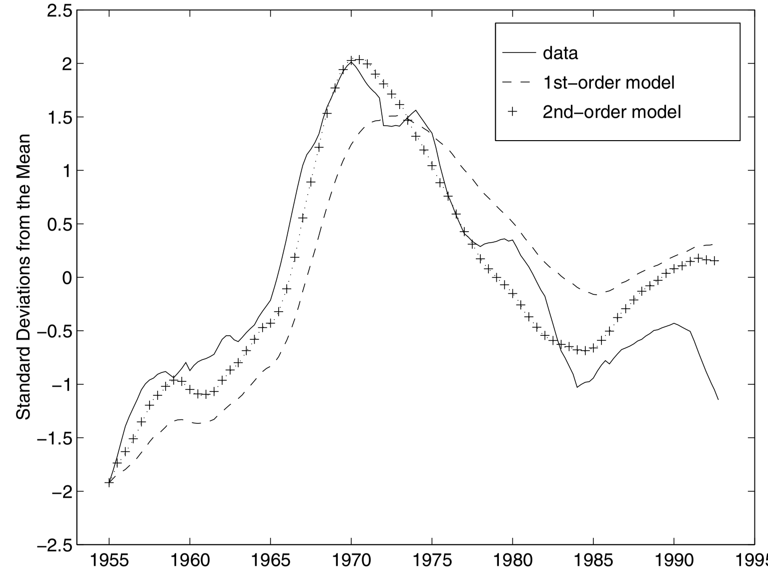

Figure 9C: Quarterly Employment and Predicted (divisible labor)

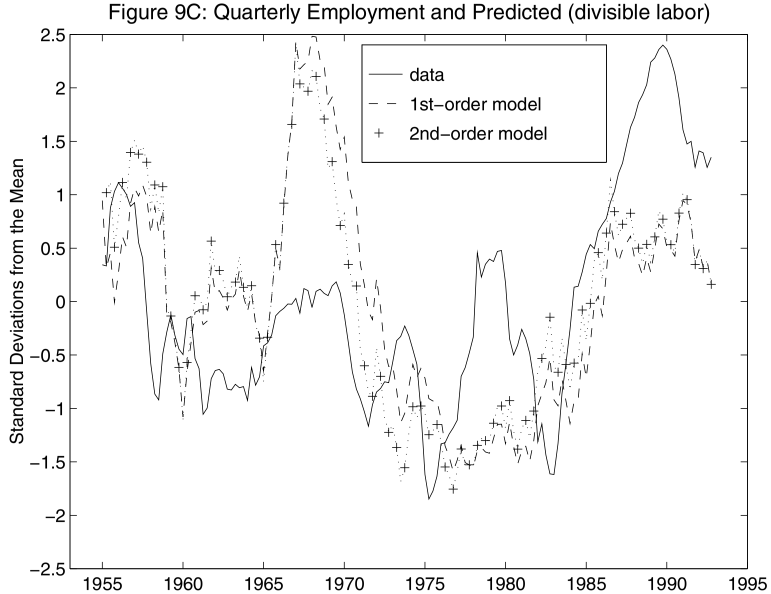

Figure 9D: Quarterly Consumption and Predicted (divisible labor)

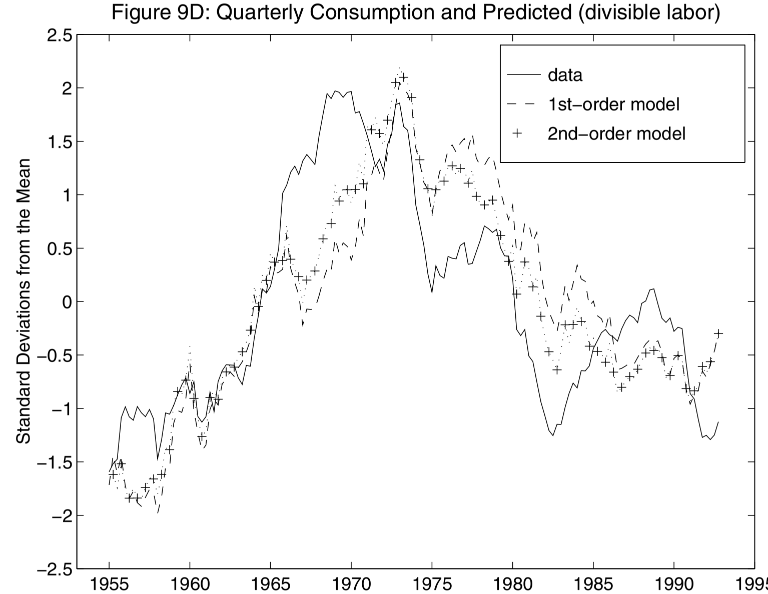

Figure 10A: Quarterly Output Coherence with Predicted (divisible labor)

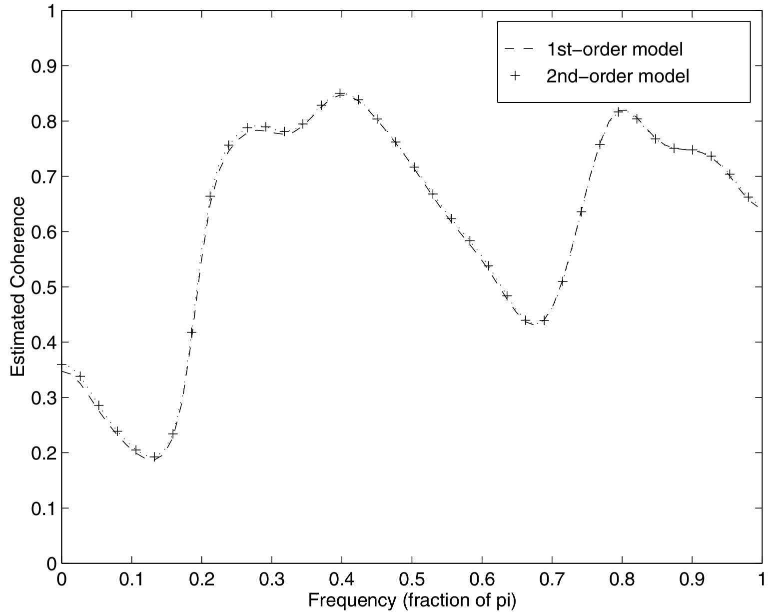

Figure 10B: Quarterly Capital Stock Coherence with Predicted (divisible labor)

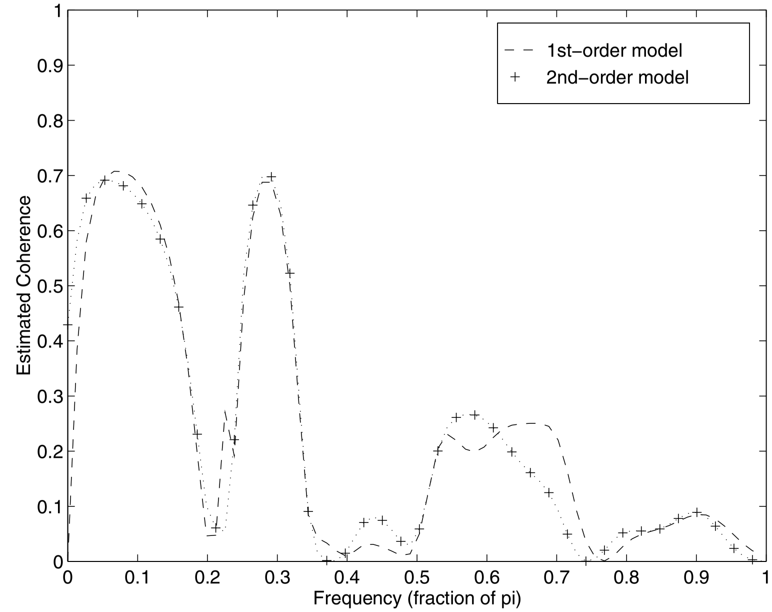

Figure 10C: Quarterly Employment Coherence with Predicted (divisible labor)

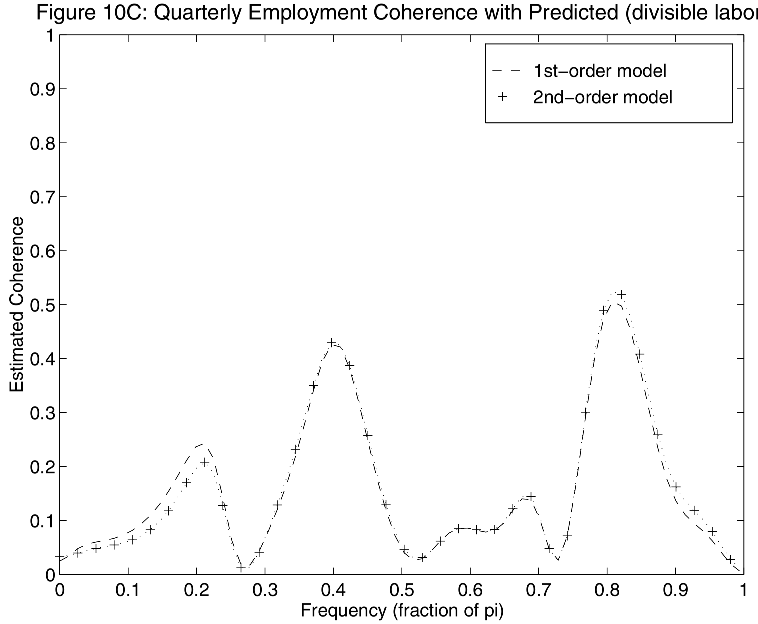

Figure 10D: Quarterly Consumption Coherence with Predicted (divisible labor)

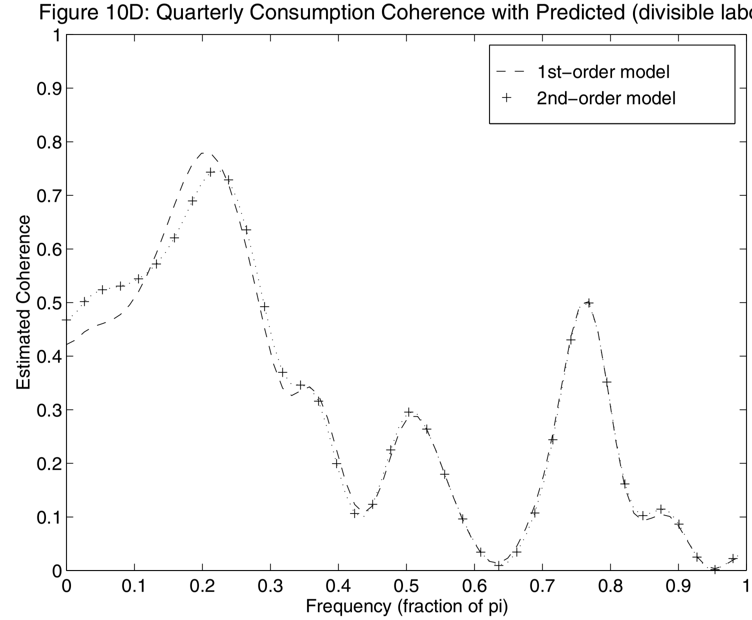

Figure 11A: Quarterly Output Phase with Predicted (divisible labor)

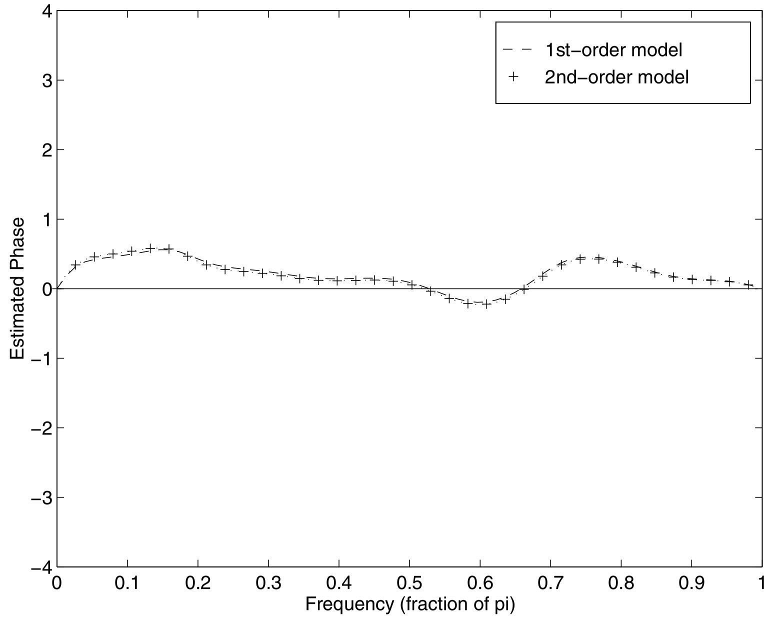

Figure 11B: Quarterly Capital Stock Phase with Predicted (divisible labor)

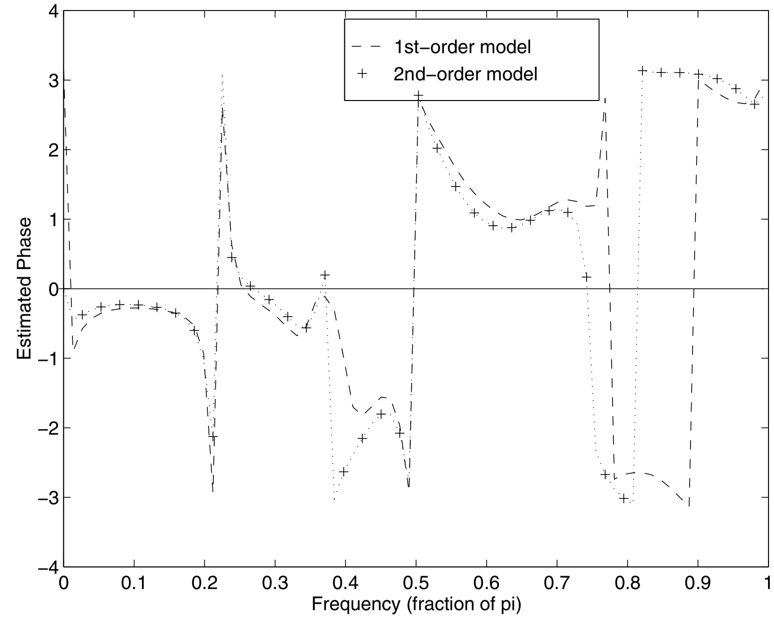

Figure 11C: Quarterly Employment Phase with Predicted (divisible labor)

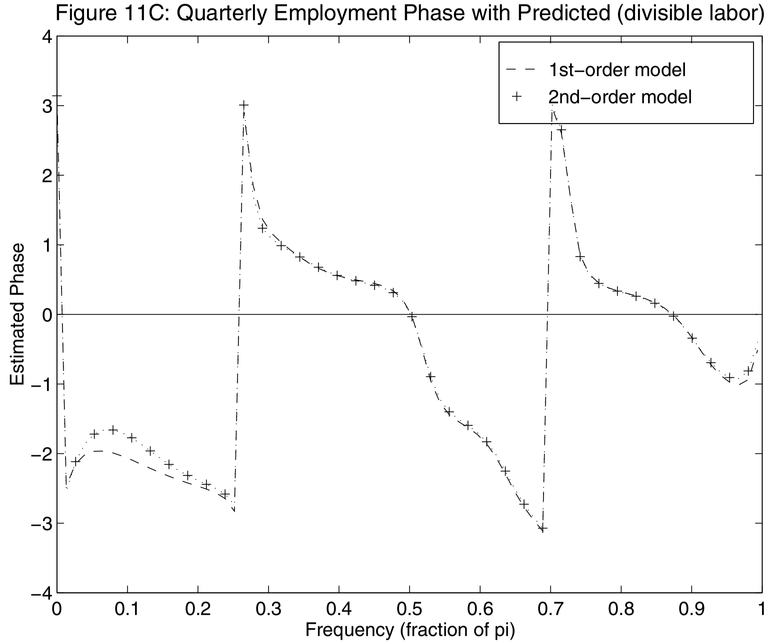

Figure 11D: Quarterly Consumption Phase with Predicted (divisible labor)

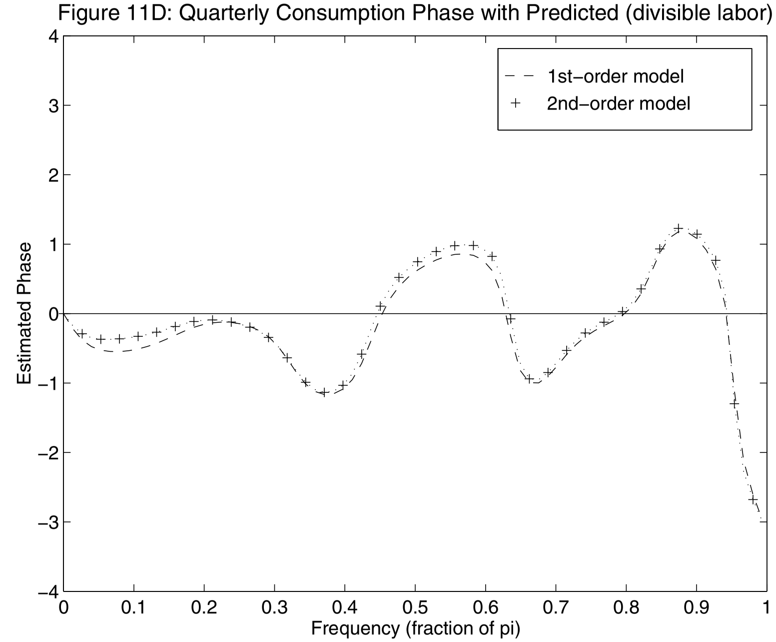

Figure 12A: Quarterly Output and Predicted (indivisible labor)

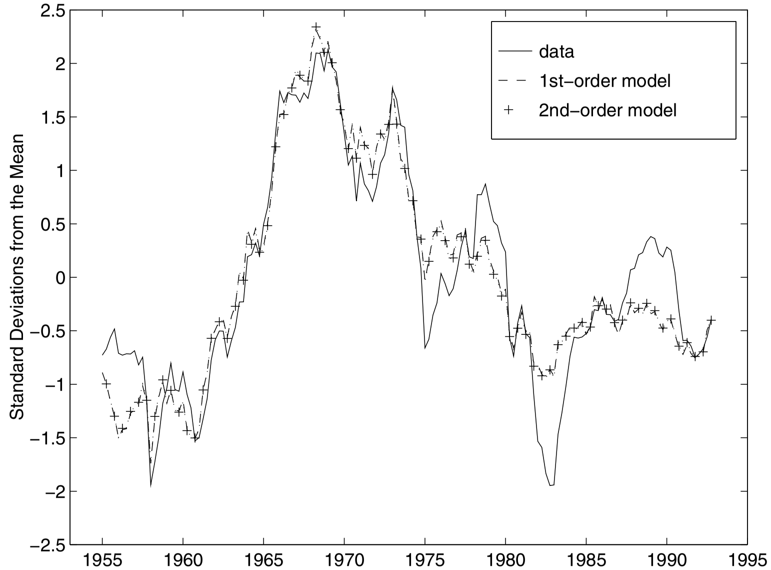

Figure 12B: Quarterly Capital Stock and Predicted (indivisible labor)

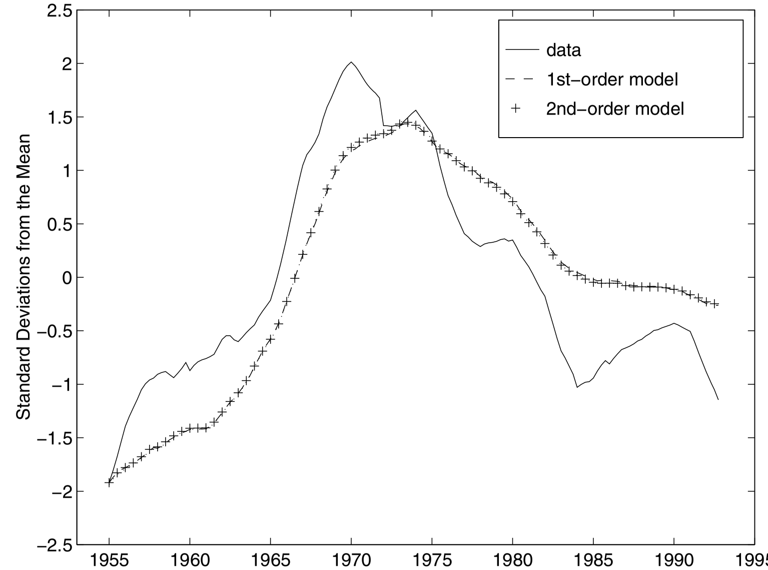

Figure 12C: Quarterly Employment and Predicted (indivisible labor)

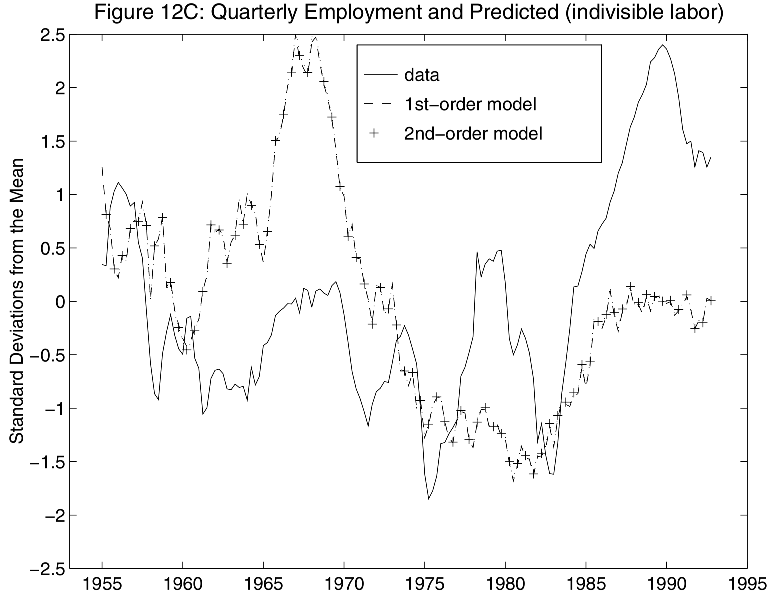

Figure 12D: Quarterly Consumption and Predicted (indivisible labor)

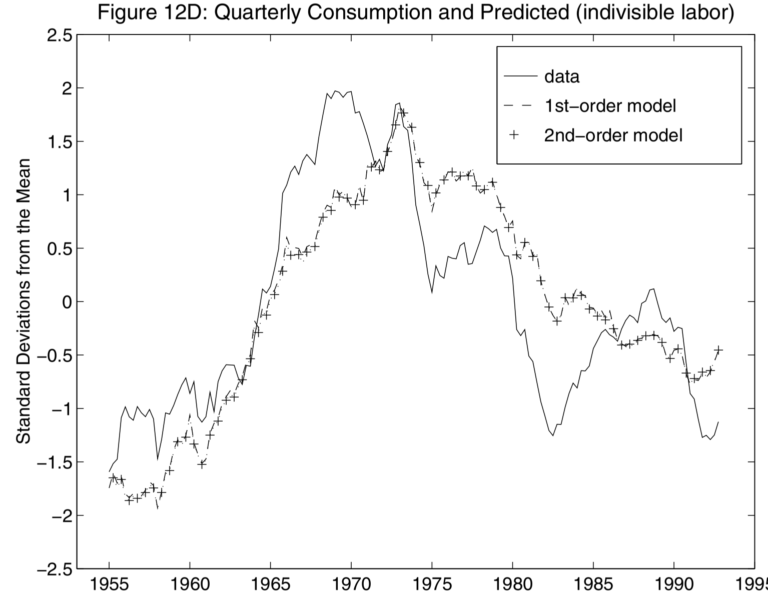

Figure 13A: Quarterly Output Coherence with Predicted (indivisible labor)

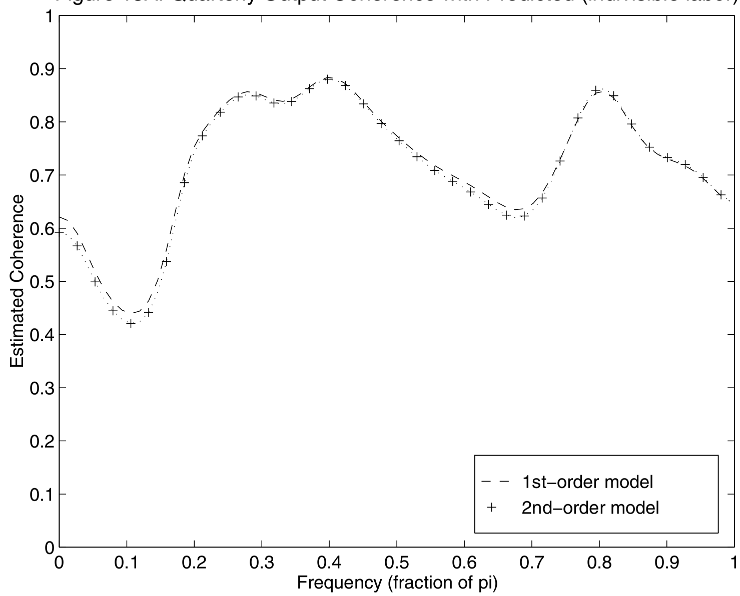

Figure 13B: Quarterly Capital Stock Coherence with Predicted (indivisible labor)

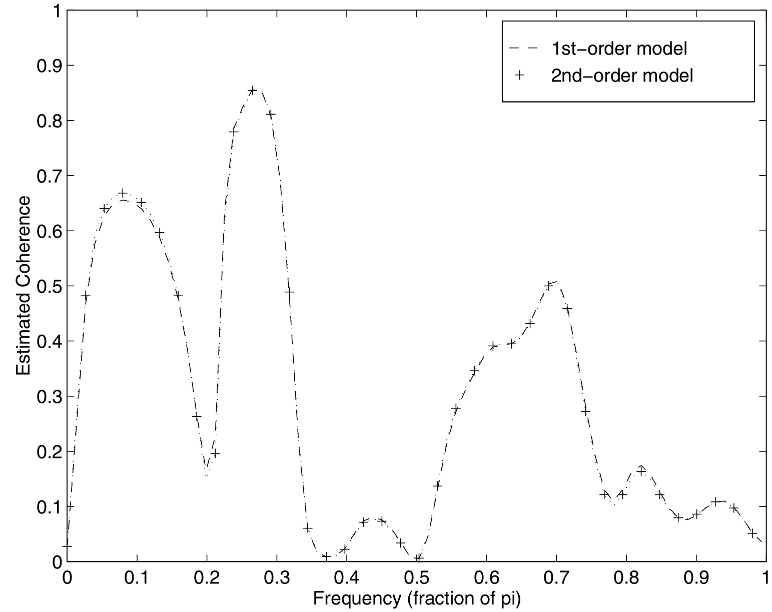

Figure 13C: Quarterly Employment Coherence with Predicted (indivisible labor)

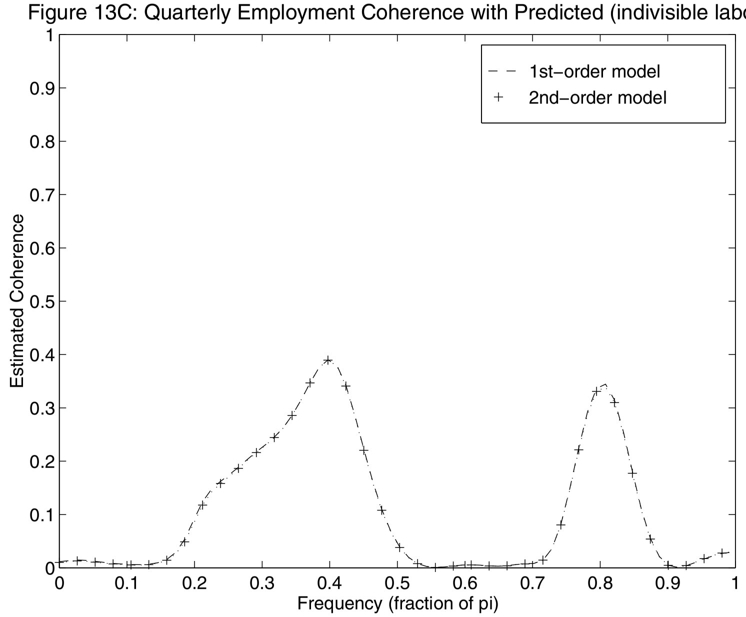

Figure 13D: Quarterly Consumption Coherence with Predicted (indivisible labor)

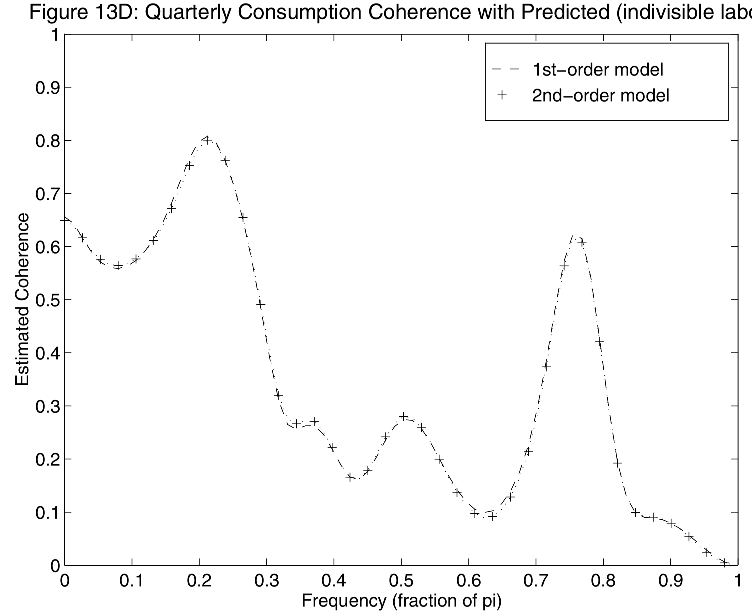

Figure 14A: Quarterly Output Phase with Predicted (indivisible labor)

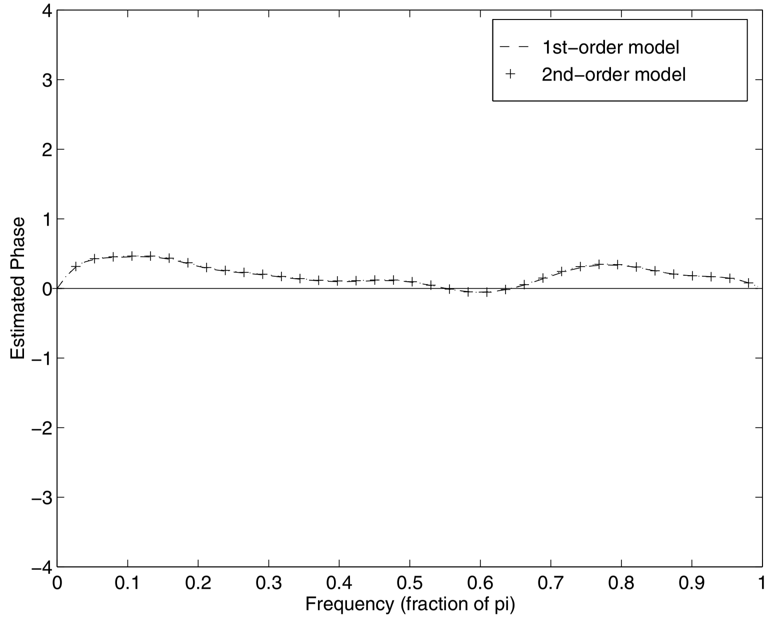

Figure 14B: Quarterly Capital Stock Phase with Predicted (indivisible labor)

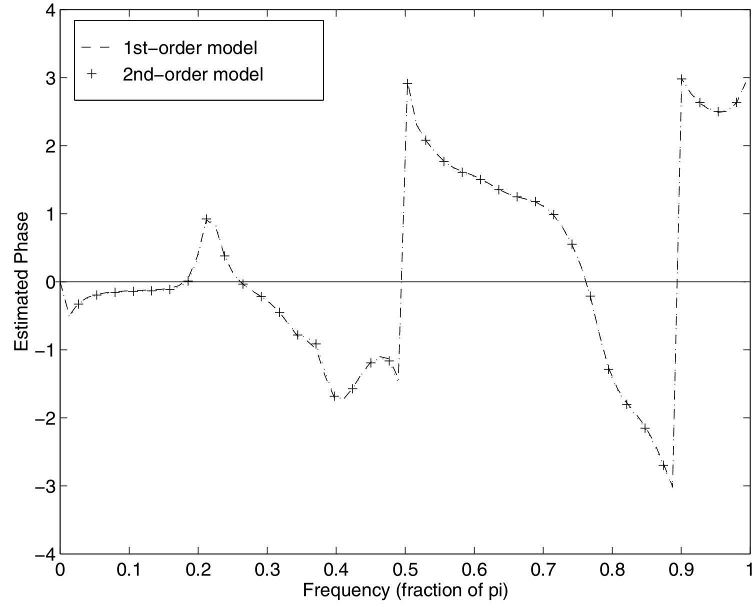

Figure 14C: Quarterly Employment Phase with Predicted (indivisible labor)

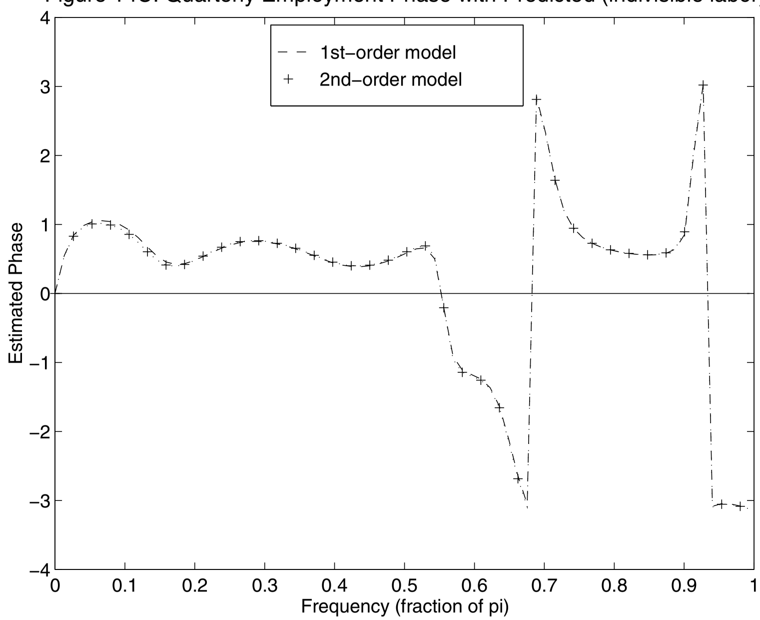

Figure 14D: Quarterly Consumption Phase with Predicted (indivisible labor)