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# The CCAPM Meets Euro-Interest Rate Persistence, 1960-2000

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# The CCAPM Meets Euro-Interest Rate Persistence, 1960-2000

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## Abstract

Euro-interest rates are well-known to be persistent, as are their differentials across countries for a given maturity. The international CCAPM implies that the rates are persistent because forecasts of national consumption growth or inflation are persistent too. We examine this prediction for a panel of countries. The standard CCAPM with power utility is augmented to allow for external habit, government consumption, and adaptive learning. In all cases, we find little evidence that the persistence in Euro-rates is consistent with the CCAPM.

*Keywords:* Euro-interest rates, CCAPM

*JEL classification:* F30, G12

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## 1. Introduction

With the removal of capital controls in many countries and the increasing deregulation of domestic interest rates, differentials between onshore and offshore interest rates have declined over time. This trend adds to the need to understand Euro market differentials, because Euro interest rates increasingly represent firms' financing costs.

A number of studies – surveyed by Marston (1995, chapter 6) – have examined Euro-rates by testing whether real interest rate parity holds. They find that it does not, and that international real interest rate differentials may be quite persistent. But as Marston's (1995) comprehensive study makes clear, there have been few attempts to model these departures from real interest rate parity. In contrast, numerous studies test whether a consumption-based asset-pricing kernel can explain departures from uncovered interest parity (the foreign exchange risk premium). There also have been important investigations of international bond and equity returns using the latent variable model of Hansen and Hodrick (1983) and Gibbons and Ferson (1985), based on constant, relative, consumption betas. However, those studies typically reject the restrictions implied by the model, and they do not tie returns directly to aggregate consumption data.

This paper investigates a natural explanation for the persistence in cross-country, real interest differentials: differences in national consumption growth rates. We study Euro-rates in levels rather than differentials, because the restrictions on levels are stronger: an asset-pricing model could reproduce differentials but not levels, but the reverse is not true. And we study nominal interest rates, because they are directly observable. We model the combinations of expected inflation and consumption growth whose time-series properties match those of interest rates, according to various versions of the CCAPM. We study all countries for which Euro-rates and measures of consumption excluding durables are available (a total of eight) so as to assemble as large a panel as possible. The data begin as early as 1960 and end in 2000.

Generalized method of moments tests of the CCAPM reveal some evidence in favor of the theory in that some sub-panels yield positive point estimates of the intertemporal elasticity of substitution. However, these estimates usually are insignificant and are not stable across countries or instrument sets. Modifications to the CCAPM to include ex-

ternal habit or learning do not redeem these shortcomings. There remains a great deal of persistence in Euro-rates (and hence their differentials) that cannot be matched with forecasts of either consumption growth or inflation.

Stulz (1981, 1987) theorized that differences in consumption sets across countries might account for interest rate differentials. Surprisingly, we have not encountered any empirical study which has used consumption growth as a benchmark to evaluate the persistence in panels of international interest rates. This strikes us as an important exercise because cross-country studies of a single type of asset – Euro-deposits – may be as informative as cross-asset studies within a single country. An advantage of pooling international data is that consumption growth in some countries (*e.g.* the U.S.) displays very little persistence. In that case, Euler equations cannot readily be used to identify the degree of intertemporal substitution. Since consumption growth in several countries is predictable to a greater extent than in the U.S., the use of data from multiple countries facilitates identification.

The rest of the paper is organized as follows. Section 2 briefly describes the CCAPM and notation. Section 3 estimates and tests the power-utility version of the CCAPM by the generalized method of moments (GMM). Sections 4, 5, 6, and 7 look at extensions to the power-utility CCAPM to allow for external habit ('catching up with the Joneses'), a conditional factor representation, public consumption (measured by government spending), and adaptive learning rather than rational expectations. Section 8 uses the fact that interest rates are known at the start of each quarter to provide a graphical summary, while section 9 contains conclusions.

## 2. Pricing Model and Notation

Consider an international economy with  $I$  countries, indexed by  $i$ . In country  $i$ , a representative household seeks to maximize:

$$U_i = \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_{it}, \nu_{it}, g_{it}) \right], \quad (1)$$

subject to a budget constraint. Here  $u$  is period utility and  $\beta$  is a discount factor. Utility depends on private consumption,  $c_{it}$ , and possibly on an external benchmark level of

consumption  $\nu_{it}$  or public consumption  $g_{it}$ . Denote the marginal utility of consumption by  $u_{c_i}$ , and the intertemporal marginal rate of substitution from  $t$  to  $t+1$ ,  $u_{c_{it+1}}/u_{c_{it}}$ , by  $m_{it+1}$ . Let  $\pi_{it+1}$  be the gross inflation rate from period  $t$  to  $t+1$ , so that the nominal intertemporal marginal rate of substitution is  $M_{it+1} \equiv m_{it+1}/\pi_{it+1}$ .

One of the available assets is a nominally riskless, one-period deposit with gross, nominal return  $R_{it}$  from period  $t$  to  $t+1$ . The notation reflects the fact that this return is known at the start of the period. This asset pays one unit of the domestic currency in all states at period  $t+1$ . The gross, real return on this asset is  $R_{it}/\pi_{it+1}$ . Absent liquidity constraints, a first-order condition for utility maximization then is:

$$\begin{aligned} 1 &= E_t [M_{it+1} R_{it}] \\ &= E_t \left[ \frac{m_{it+1} R_{it}}{\pi_{it+1}} \right] \\ &= E_t \left[ \beta \frac{u_c(c_{it+1}, \nu_{it+1}, g_{it+1})}{u_c(c_{it}, \nu_{it}, g_{it})} \frac{R_{it}}{\pi_{it+1}} \right]. \end{aligned} \tag{2}$$

A widely-used example makes utility depend only on current, private consumption, with power functional form:

$$u(c_t) = \begin{cases} c_t^{1-\gamma}/(1-\gamma) & \gamma > 0, \quad \gamma \neq 1; \\ \ln c_t & \gamma = 1. \end{cases} \tag{3}$$

Let  $x_t = c_t/c_{t-1}$  be the gross, growth rate of consumption. In this case, then

$$\frac{1}{R_{it}} = E_t \left[ \beta \frac{x_{it+1}^{-\gamma}}{\pi_{it+1}} \right]. \tag{4}$$

As the notation suggests, we focus on cases in which the preference parameters, such as  $\beta$  and  $\gamma$ , are common across countries, although we also test this restriction in section 3.

According to this CCAPM, the difference between the nominal interest rate and the forecast of the nominal, intertemporal marginal rate of substitution should be unpredictable. Thus it is not enough to find an  $M$  whose forecasts have autocorrelation properties similar to those of persistent returns. The two series also must match observation by observation. We study several models of  $M$  to see if this criterion can be satisfied, while theoretical restrictions on  $M$ , such as concavity of utility, also hold.

### 3. GMM Estimates of the Benchmark CCAPM

This section begins our tests of pricing models for nominal Euro-deposit rates. The focus on these rates, rather than international differentials, should lead to demanding tests, for unexplained persistence in rates may cancel in differentials. Studying nominal interest rate also avoids the difficulties associated with using *ex post* real rates as proxies for *ex ante* rates, which include modeling the conditional covariance of consumption growth and inflation.

Data are for three-month interest rates from the Bank for International Settlements and on corresponding quantity measures from the *OECD Quarterly National Accounts*. The panel includes the U.S., U.K, Japan, Canada, Italy, France, Sweden, and Denmark. These eight countries were selected based on the availability of data on consumption of nondurables and services. The data are described in the appendix. Evidence of nonstationarity in inflation and *ex post* real interest rates for a similar panel of countries is reviewed by Gregory and Watt (1995). They conclude that it is reasonable to treat these series as stationary, and so our statistical inferences are based on that assumption. To allow formal tests, we estimate the Euler equation linking nominal interest rates and consumption growth by the generalized method of moments, and test the over-identifying restrictions.

The Euler equation that we consider is that of the power-utility CCAPM (4). The moment conditions used for estimation and testing are:

$$E \left[ z_{it} \cdot \left( \beta x_{it+1}^{-\gamma} \pi_{it+1}^{-1} R_{it} - 1 \right) \right] = 0, \quad (5)$$

where  $z_{it}$  is a vector of instruments for the equation corresponding to country  $i$ . Estimation is by iterated GMM, which Hansen, Heaton, and Yaron (1996) found to have the best finite-sample properties among alternative GMM estimators. The covariance matrix of the moment conditions is estimated using the quadratic-spectral kernel introduced by Andrews (1991). Over-identification stems from our imposition of a common  $\beta$  and  $\gamma$  across countries, and from the use of lagged variables as instruments. Stock and Wright (2000) have shown that a lack of persistence in consumption growth and returns may hinder the search for relevant instruments for the CCAPM. But precisely the feature we are trying to explain – the persistence in Euro-rates – makes lagged rates useful instruments here.

Table 1 contains the estimation results. The top two rows contain results from a balanced panel of 285 observations for three countries, the U.S., the U.K., and Japan, for the period 1975:1 to 1999:1. This sample was selected for its long time span. We consider two different sets of instruments. The bottom two rows come from a wide panel, based on 504 observations for 7 countries beginning in 1980. Econometric software for unbalanced panels with nonlinearity in the parameters is not well developed, so table 1 studies only balanced panels.

Depending on the set of countries and time period used,  $\hat{\beta}$  ranges from 0.987 to 0.996 and  $\hat{\gamma}$  ranges from  $-0.156$  to  $0.455$ . A positive, significant point estimate for  $\hat{\gamma}$  is found only for the three-country panel involving the U.S., the U.K., and Japan and with a large instrument set. The  $J$ -test readily rejects the model with the instrument sets shown in Table 1, though. Results from other instrument sets (not shown) also yielded rejections of the over-identifying restrictions across countries. The general finding is that we cannot identify a  $\hat{\gamma}$  which is stable across countries, stable across instruments, and positive. Formal tests for parameter stability developed by Sowell (1996) reject the hypothesis that the preference parameters are constant across countries.

To provide more information, Table 2 contains results of country-by-country GMM estimation, without the cross-country restrictions on the parameters but with the same instruments. All estimates of  $\beta$  are significant and of the magnitude one would predict from the theory. However, in no country do we find a significant, positive value for  $\hat{\gamma}$  at the same time that the  $J$ -test does not reject the over-identifying restrictions. For Canada and Italy  $\hat{\gamma}$  is negative and significant at the five percent level, while in the other six countries it is insignificantly different from zero. Given the disparate findings in Table 2, it is not surprising that the evidence from the pooled estimation in Table 1 also was mixed.

We investigated the robustness of the findings in Table 2 in three different ways. First, we studied whether the findings depend on capital controls. The pricing model holds that the Euro-rate in a given currency is determined by inflation and the real IMRS in that country. While these offshore rates are free of tax and liquidity effects that appear in some onshore rates, one might wonder whether controls break their links with onshore fundamentals. We followed Marston's (1995, chapter 3) history of international capital controls,

and studied time periods when no controls were in place. We re-estimated equation (5) for the U.K. after 1979, the U.S. after 1974, Japan after 1981, and France after 1987. Only for Japan were the results more favourable to the CCAPM. For Japan,  $\hat{\gamma} = 2.388(0.830)$  and the  $p$ -value for the  $J$ -test was 0.53. For no other country did we find a positive  $\hat{\gamma}$  and large  $p$ -value.

Second, we recalculated Table 2 using point-in-time interest rates, rather than time-averaged ones. The rejections by the  $J$ -test in Table 2 show that the Euler equation residuals are persistent and predictable, unlike forecast errors. Time aggregation is a classic statistical explanation of persistence. However, estimates with end-of-quarter Euro rates were similar to those in Table 2.

Third, the CCAPM sometimes is defended with the argument that even if the conditional mean of consumption growth or inflation is not persistent, their conditional volatilities or conditional covariance are persistent. But the nonlinearity in the power-utility asset-pricing kernel already allows for this effect of these variances on the level of the interest rate. By using GMM and a panel of countries to add to the number of observations, we have avoided parametric modeling of the conditional variances. Such parametric modeling could add to the efficiency of estimates, but it is unlikely to alter our verdict on the CCAPM.

As a check on this approach, we modeled the conditional mean of each consumption growth series  $\{x_{it}\}$  with a constant and one lag, then looked at the autocorrelation in the squared residuals. The autocorrelation functions died away quite quickly. The highest first-order autocorrelation coefficients were found for Japan, Canada, and Denmark, at about 0.25, while the value for the U.S. was 0.048 and other countries' values also were near zero. How much persistence we find in the second moment of course depends on our modeling of the first moment, but some experimentation with the conditional mean had little effect on these findings. Our findings were similarly negative when we studied volatility dynamics in inflation rates. Meanwhile, all these countries have persistent interest rates. We conclude that modeling the volatility dynamics of consumption or inflation at quarterly frequency also is unlikely to lead to a match with the properties of Euro-rates.

Our conclusion is that it is worth studying other pricing kernels, whose forecasts may

match interest rates, be consistent with theoretical restrictions on preferences, and be stable across countries. The next three sections look at three alternative kernels.

#### 4. Catching up with the Joneses

We next examine the predictions of the CCAPM under a utility function featuring ‘catching up with the Joneses’ (sometimes called external habit), introduced by Abel (1990). This utility function implies that current consumption growth – as well as expected future consumption growth – enters the Euler equation. Thus, it has the potential to rationalize the previous section’s finding that  $x_{it}$ , an instrument in Tables 1 and 2, often helps predict the difference between  $R_{it}$  and  $M_{it+1}$  in the power-utility CCAPM.

This characterization of utility has been fruitfully applied by Campbell and Cochrane (1999) and Abel (1999) to a variety of features of asset prices, including the equity premium, the cyclical behavior of stock returns and stock market volatility, long horizon predictability of stock returns, and the term structure of interest rates, but not to the persistence of interest rates. We provide new information on this asset-pricing model by estimating preference parameters in our international panel. Unlike these authors, we do not assume that consumption growth is serially uncorrelated.

We use the formulation of Abel (1999), in which the period utility function is:

$$u(c_t, \nu_t) = \frac{1}{1-\gamma} \left( \frac{c_t}{\nu_t} \right)^{1-\gamma} \quad (6)$$

where  $\nu_t$  is a benchmark level of consumption exogenous to the individual consumer. The benchmark level follows:

$$\nu_t = c_t^{\delta_0} c_{t-1}^{\delta_1} \omega^{t^{\delta_2}}, \quad (7)$$

where  $\omega \geq 1$  and  $0 \leq \delta_j \leq 1$  for  $j = 0, 1, 2$ . The presence of  $\omega^t$  allows the benchmark level of consumption to grow over time. The presence of  $c_t$  and  $c_{t-1}$  allows for catching up with the Joneses, in that the benchmark is an increasing, homogeneous function of current and recent levels of aggregate consumption.

In equilibrium individual consumption is proportional to aggregate consumption, so our notation does not distinguish between the two. But  $\nu_t$  is held constant in calculating

marginal utility, since it represents external habit. The real, intertemporal marginal rate of substitution then can be written as:

$$m_{t+1} = \kappa x_{t+1}^{-\phi} x_t^\theta, \quad (8)$$

where

$$\begin{aligned} \kappa &\equiv \beta \omega^{\delta_2(\alpha-1)} > 0 \\ \phi &\equiv \gamma - \delta_0(\gamma - 1) > 0 \\ \theta &\equiv \delta_1(\gamma - 1). \end{aligned} \quad (9)$$

This includes the standard specification as a special case when  $\delta_j = 0$ . As Abel (1999) noted, only the three composite parameters  $\kappa$ ,  $\phi$ , and  $\theta$  are identifiable, so that relative to the standard iso-elastic specification only one parameter has been added. He suggested backing out preference parameters using this identification scheme:  $\delta_0 = 0$ ;  $\delta_0 + \delta_1 + \delta_2 = 1$ ; and  $\omega$  equal to the sample mean of  $x$ .

Given this pricing kernel, the nominal return on a one-period riskless Euro-deposit is given by:

$$\frac{1}{R_{it}} = E_t \left[ \kappa x_{it+1}^{-\phi} x_t^\theta \pi_{it+1}^{-1} \right] \quad (10)$$

for country  $i$ . Notice that  $x_{it}$  is in the information set and in the marginal rate of substitution. Thus the model with external habit can potentially rationalize the role for current consumption growth – along with the interest rate – in predicting inflation and consumption growth. That role was detected for some countries in the  $J$ -test of section 3. The catch is that the coefficient on consumption growth will be required to be the same across countries.

Table 3 contains the results of estimating equation (10) country by country using Euro-rates alone. The panels and instruments are the same as in Table 1. In the long panel of the US, UK, and Japan, estimation with a large set of instruments yields a coefficient on future consumption growth,  $\hat{\phi}$ , which is negative and insignificant. Although the coefficient on current consumption growth,  $\hat{\theta}$ , is positive and significant, the  $J$ -test still strongly rejects the restrictions on the panel. A shorter set of instruments, in the second row of Table 3, does not yield a rejection of the restrictions, but now both consumption

terms are insignificant. The findings are the same for the broad panel of seven countries in the third and fourth rows.

Thus, adding first-order external habit does not rescue the CCAPM in Euro-rate dynamics. Higher-order lags in the external habit stock,  $\nu_t$ , might improve the results for the CCAPM, but we would like to use the same modification of the CCAPM that has been used in explaining equity premia, for example. The finding in this section is not that no model of habit can explain Euro-rate persistence, but rather that the standard model of external habit, used in other applications, cannot resolve this puzzle.

## 5. Conditional Factor Models

Cochrane (1996) and Lettau and Ludvigson (2001) present evidence that conditional factor models can do well in explaining the cross section of U.S. asset returns. A conditional, linear, factor version of the CCAPM begins with:

$$m_{t+1} = a_t + b_t \ln x_{t+1}. \quad (11)$$

Next, Lettau and Ludvigson model the time variation in the parameters by interacting consumption growth with an estimate of the lagged, log consumption-wealth ratio, which they denote  $\widehat{cay}_t$ . The model then is:

$$m_{t+1} = \gamma_0 + \gamma_1 \widehat{cay}_t + \gamma_2 \ln x_{t+1} + \gamma_3 \widehat{cay}_t \ln x_{t+1}. \quad (12)$$

They argue that this may approximate models of habit persistence with unobservable habit stocks, as well as other versions of the CCAPM with time variation in risk premia. Since the parametric model of habit we studied directly in section 4 did not improve on the power-utility CCAPM, we next explore this alternative model of the pricing kernel.

Instead of using the parameters estimated by Lettau and Ludvigson from a cross-section of average returns in the U.S., we estimated them to maximize the fit with nominal U.S. Euro-deposit rates. The estimating equations are

$$E \left[ z_{it} \cdot \left( m_{it+1} \pi_{it+1}^{-1} R_{it} - 1 \right) \right] = 0, \quad (13)$$

with  $m_{it+1}$  in (12). We used the series  $\widehat{cay}_t$  provided by Lettau and Ludvigson for the U.S., and examined this kernel with U.S. dollar Euro-rates from 1960:3 to 1999:2. Five instruments – a constant,  $R_{it}$ ,  $R_{it-1}$ ,  $x_{it}$ , and  $\pi_{it}$  – were used, so that the pricing kernel's four parameters were over-identified. Only  $\hat{\gamma}_0$  was significant at conventional levels. The  $J$ -test readily rejected the over-identifying restriction. We conclude that this pricing kernel does not fit the dynamics of the U.S. interest rate series, and so we did not build it for other countries.

## 6. Public Consumption

The CCAPM also may be modified so that the marginal utility of private consumption depends on the scale of public consumption,  $g$ . To investigate this possibility empirically, we modified the the period utility function as:

$$u(c_t, g_t) = \frac{1}{1-\gamma} (c_t^\eta + \mu g_t^\eta)^{\frac{1-\gamma}{\eta}}, \quad (14)$$

so that utility is a power of a CES aggregator of private and public consumption spending. The nominal Euro-deposit rate in country  $i$  is then:

$$\frac{1}{R_{it}} = \beta E_t \left[ \left( \frac{c_{it+1}^\eta + \mu g_{it+1}^\eta}{c_{it}^\eta + \mu g_{it}^\eta} \right)^{\frac{1-\gamma-\eta}{\eta}} \left( \frac{c_{it+1}}{c_{it}} \right)^{\eta-1} \pi_{it+1}^{-1} \right]. \quad (15)$$

Identification of the four preference parameters was difficult, and there was no evidence that this modification improved the model. Again parameters were unstable across countries and instrument sets, and often violated theoretical restrictions.

## 7. Adaptive Learning

So far we have studied several parametric models of the nominal IMRS, to see whether forecasts of these models fit with the time paths of nominal interest rates. Our success has been very limited. Finally, then, instead of reformulating the object being forecast, we consider changing the forecasting method attributed to market participants. Earlier we used the restrictions of rational expectations to identify and estimate parameters. We now suppose that agents do not know the law of motion for the IMRS but instead learn

adaptively. Our choice is based on Evans and Honkapohja's (2001, chapter 14) assessment of the constant-gain model of learning. Those authors argue that persistent dynamics – which appear to be missing from the models of the IMRS so far – can be accounted for by constant-gain learning but not by learning that converges to rational expectations.

Recall that the asset-pricing relationship first tested was:

$$\frac{1}{R_{it}} = E_t [M_{it+1}] = E_t \left[ \beta \frac{x_{it+1}^{-\gamma}}{\pi_{it+1}} \right]. \quad (16)$$

We now model the expectation using adaptive learning with a constant gain  $\lambda$ :

$$\begin{aligned} E_t[M_{it+1}] &= E_{t-1}[M_{it}] + \lambda_i (M_{it} - E_{t-1}[M_{it}]) \\ &= \frac{\lambda_i M_{it}}{1 - (1 - \lambda_i)L} \end{aligned} \quad (17)$$

where  $L$  is the lag operator. With a constant gain,  $0 < \lambda_i \leq 1$ , past observations are exponentially weighted. This is simply 1950's-style adaptive expectations. This learning scheme is not fully rational but it may be rationalized if the parameters of the law of motion change over time. Examples might include a changed behavior of inflation in the U.S. between 1979 and 1982 or a shift in the mean of consumption growth in Japan during the 1990s. In these circumstances, a constant-gain rule may track changes relatively well, at the expense of higher variance when the law of motion is stable. A second interpretation of this scheme is that agents have a misspecified model of the time series properties of  $M$ . Specifically, if they believe the time series follows an IMA(1,1) process then the evolution of expectations (17) is rational given this belief. Evans and Honkapohja (2001, section 14.1) describe this possible observational equivalence between ‘misspecified learning’ and ‘persistent learning dynamics.’ Constant-gain learning is studied in detailed applications by Cho and Sargent (1999) and Sargent (1993, 1999).

An alternative model of adaptive learning has a declining gain, such as  $\lambda_i = t^{-1}$ , which is equivalent to recursive least squares estimation of the law of motion of  $M$ . This scheme converges to rational expectations if the form of the law of motion is not misspecified. Timmermann (1993, 1996) shows that learning with a declining gain series can explain several anomalies in asset-pricing theory. However, this E-stable model of learning introduces few additional dynamics for large time series.

Combining the learning scheme (17) with the asset-pricing model and parametric model of  $M$  (16) gives:

$$\frac{1}{R_{it}} = \lambda_i \left[ \beta \frac{x_{it}^{-\gamma}}{\pi_{it}} \right] + \frac{(1 - \lambda_i)}{R_{it-1}}. \quad (18)$$

We estimate parameters  $\{\beta, \gamma, \lambda_i\}$  by GMM, using instruments  $z_{it}$  as before. The panels allow us to test whether  $\beta$  and  $\gamma$  are positive and whether their values are stable across countries. We do not test for stability of  $\lambda_i$  across countries, for the constant gain may vary depending on the time series properties of national consumption growth and inflation.

The top part of Table 4 contains estimation results for the same panels studied previously. The estimates for  $\beta$  are near one and significant, while the estimates of  $\gamma$  are positive and insignificant. Most values for the learning parameter  $\hat{\lambda}_i$  were near zero and insignificant. Positive, significant values were found only for Japan (0.363 in the panel of three countries) and for Canada, Italy, and France (0.149, 0.418, and 0.304 in the panel of seven countries).

The  $J$ -test results show that the cross-country restrictions now cannot be rejected at conventional significant levels. However, the estimates of  $\gamma$  show that we still have not identified a significant role for consumption growth. The often insignificant estimates of  $\lambda_i$  also show that there is often little connection between interest rate dynamics and the model of the IMRS.

The bottom part of Table 4 combines adaptive learning with the model of external habit from section 4, which uses a pricing kernel (8) that nests the standard case. There is a slight improvement in fit relative to the power-utility CAPM at the top of the table. However, again we do not find a positive, significant  $\hat{\phi}$  that satisfies the restrictions (9) of theory. Estimates  $\hat{\lambda}_i$  were almost all near zero. Thus the addition of external habit leads to only a marginal statistical improvement. We also experimented with the pricing model (15) involving public consumption, using both the CES and Cobb-Douglas aggregators, but found no role for government spending in the learning model.

We attributed this learning scheme to market participants because it maximizes the added persistence due to learning. Yet Table 4 shows that the CCAPM with this modification still does not match the persistence in Euro-rates. Other learning models of course

could be applied to model  $E_t M_{it+1}$ , including ones in which agents learn the parameters of a law of motion of known form (as studied by Lewis, 1989, part III) or learn which of two regimes is in place (as in Lewis, 1989 part II). Evans and Honkapohja (2001, chapter 15) discuss other examples.

## 8. Graphical Summary

We next exploit the fact that nominal interest rates are known at the start of each quarter in order to provide a graphical summary of some of the results. According to the theory, the inverse of the gross, nominal interest rate is the best predictor of the nominal IMRS. Examples of the nominal IMRS include the power utility (4) and external habit (10) versions.

We graph the nominal Euro-deposit rate  $R_{it}$ , which is expressed in percent per year so that the scale is familiar. Beside it, we graph the inverse of the next period's fitted IMRS for the power utility and external habit models:  $1/\hat{M}_{it+1}$ . The parameters of the IMRS are estimated by country-specific GMM, rather than a panel, so as to give the theory an opportunity to fit the individual return series. The interest rate and fitted, inverse IMRS should not coincide in the diagrams, even if the theory could not be rejected statistically, for three reasons. First, there is sampling variability in the parameters. Second, Jensen's inequality introduces an error when we compare  $R_{it}$  and  $1/M_{it+1}$  rather than their inverses. Third,  $1/R_{it}$  of course is supposed to be a forecast of  $M_{it+1}$  so the two may differ by a forecast error.

However, the  $J$ -statistics have already provided formal tests of the model. The aim of the figures is to show the scale of the differences between returns and the inverse nominal IMRS and the fact that they are predictable. That fact was shown in the tables; rejections with the  $J$ -test show that instruments  $z_{it}$  can predict the Euler equation errors. For example, if the persistence in  $R_{it}$  were much different from that of the IMRS then additional variables could be found which would significantly improve the forecast.

Figure 1 graphs the results for the U.S.. The figure shows  $R_{it}$  (the heavy line) and the next quarter's inverse, nominal IMRS for the cases with power utility and external habit (the light lines). Figure 1 shows that the Euler equation errors are as large as 5 percentage

points in the early 1980s and that they are quite persistent, even though the IMRS is parametrized to maximize the predictive ability of the nominal interest rate. Generally, there are long swings in the interest rate which do not appear in the IMRS, so that the former cannot be thought of as a good predictor of the latter.

Figure 2 shows the results for Japan. These are quite similar, in that again the interest rate is a smooth series, while the fitted, inverse IMRS has much more variation at high frequencies. As Table 2 showed, the inverse nominal interest rate generally can be improved upon as a forecast of the nominal IMRS. For some countries – such as France and Italy – the inverse interest rate is a better predictor of the IMRS than it is for Japan or the U.S., but non-concave utility is required to achieve a non-rejection using the  $J$ -test.

For ease of reading, Figures 1 and 2 do not include sample paths from the cases with adaptive learning (16). Those cases lead to a closer fit with the interest-rate time series. But recall that most estimates of  $\lambda_i$  are insignificantly different from zero. As a result, most of the work in the fitting of the return series is done by lagged returns rather than the pricing kernel.

## 9. Conclusion

This goal of this paper was to see whether standard asset-pricing models could link macroeconomic variables with the properties of quarterly interest rates, in an international panel. The cross-country approach provides over-identifying information to aid estimation and testing. Earlier research – surveyed by Marston (1995, chapter 6) – found that real Euro-rates, and their differentials, are quite persistent. We have studied a variety of models of the real interest rate, without much success in fitting this persistence. By focusing on nominal rates, we also have allowed for a persistent, inflation risk premium, the conditional covariance between  $m_{t+1}$  and  $\pi_{t+1}$ .

The CCAPM based on power utility did not yield parameter estimates which were stable across countries and instrument sets and consistent with theory. We considered two main extensions to the model which allow for additional dynamics. First, allowing for external habit admits lagged consumption growth into the pricing model. Second, modeling the forecast of the intertemporal marginal rate of substitution using adaptive learning

admits a lagged interest rate into the pricing relationship. Neither of these extensions provided stable, significant estimates of the parameters of the CCAPM.

## Data Appendix

### 1. Interest rates

Rates are on three-month Euro-deposits. Rates for the U.S., U.K., France, and Japan prior to 1977 were generously provided by Richard Marston. The original sources are Morgan Guaranty Trust, *World Financial Markets* and *OECD Financial Statistics* and are described in detail by Marston (1995). Data are averages of monthly rates. All other rates were provided by the Bank for International Settlements, and are averages of daily bid rates.

### 2. Consumption, prices, government spending

Consumption is quarterly, real consumption spending on nondurables and services, from the *OECD Quarterly National Accounts*. The price level is the corresponding implicit deflator. Government spending is from the same source and in some cases includes purchases of durables.

Data use the fixed-weight standard of the 1993 SNA, with base years varying by country. The exception is the US where the data are chain-weighted. For the US, consumption is measured by summing nominal expenditures on nondurables and services then dividing by the deflator for total consumption, because the chain-weighted real components are not additive.

Data are seasonally adjusted for all countries except Japan and Sweden. Data for these countries were adjusted with the `esmooth` function in RATS<sup>TM</sup> which chooses between additive and multiplicative models of seasonality based on goodness of fit.

### 3. Panel

The panel includes all countries for which both Euro-rates and consumption excluding durables were available. Data apply to the following countries and time periods, listed in decreasing order of sample size:

Country	Time Period	<i>T</i>
U.S.	1960:2 – 2000:4	163
U.K.	1962:2 – 2000:4	155
Japan	1975:1 – 1999:1	97
Canada	1977:3 – 2000:4	94
Italy	1977:3 – 1998:3	85
France	1978:2 – 1998:4	83
Sweden	1980:2 – 1998:4	75
Denmark	1988:2 – 2000:4	51

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**Table 1 Panel GMM Estimation: Power Utility**

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$E[z_{it} \cdot (\beta x_{it+1}^{-\gamma} \pi_{it+1}^{-1} R_{it} - 1)] = 0$				
Panel	$z_{it}$	$\hat{\beta}$ (se)	$\hat{\gamma}$ (se)	$J$ (df) (p)
US, UK, Japan 1975:1–1999:1 285 observations	$\iota, x_{it}, \pi_{it}, R_{it-1}$	0.996 (0.001)	0.455 (0.166)	53.56 (10) (0.00)
	$\iota, R_{it-1}$	0.993 (0.002)	-0.108 (0.248)	26.34 (4) (0.00)
US, UK, Japan, Canada, Italy, France, Sweden 1980:2–1998:3 504 observations	$\iota, x_{it}, \pi_{it}, R_{it-1}$	0.987 (0.001)	-0.123 (0.064)	53.45 (26) (0.01)
	$\iota, R_{it-1}$	0.988 (0.001)	-0.156 (0.125)	41.52 (12) (0.00)

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*Notes:*  $x_{it+1}$  is the gross, real consumption growth rate,  $\pi_{it+1}$  the gross inflation rate,  $R_{it}$  the gross nominal interest rate from  $t$  to  $t+1$  in country  $i$ ;  $\iota$  is a vector of ones. Estimation is by iterated GMM using the Hansen-Heaton-Ogaki Gauss code.

**Table 2 Country-by-Country GMM Estimation: Power Utility**

$$E[z_{it} \cdot (\beta_i x_{it+1}^{-\gamma_i} \pi_{it+1}^{-1} R_{it} - 1)] = 0$$

$$z_{it} = \{\iota \ x_{it} \ \pi_{it} \ R_{it-1}\}$$

Country	$\hat{\beta}_i$ (se)	$\hat{\gamma}_i$ (se)	$J$ (2) (p)
U.S.A. 1960:2–2000:4	0.997 (0.002)	0.436 (0.251)	27.02 (0.00)
U.K. 1962:2–2000:4	1.005 (0.015)	2.020 (2.486)	7.15 (0.03)
Japan 1975:1–1999:1	0.996 (0.002)	0.317 (0.384)	18.01 (0.00)
Canada 1977:3–2000:4	0.982 (0.003)	-1.150 (0.497)	6.31 (0.04)
Italy 1977:3–1998:3	0.980 (0.003)	-2.081 (0.700)	8.00 (0.02)
France 1978:2–1998:4	0.974 (0.007)	-2.243 (1.159)	6.28 (0.04)
Sweden 1980:2–1998:4	0.991 (0.004)	0.425 (1.272)	2.84 (0.24)
Denmark 1988:2–2000:4	0.959 (0.048)	-4.211 (6.938)	0.43 (0.81)

Notes:  $x_{it+1}$  is the gross, real consumption growth rate,  $\pi_{it+1}$  the gross inflation rate,  $R_{it}$  the gross nominal interest rate from  $t$  to  $t+1$  in country  $i$ ;  $\iota$  is a vector of ones. Estimation is by iterated GMM using the Hansen-Heaton-Ogaki Gauss code. Countries are listed in decreasing order of sample size.

**Table 3** Panel GMM Estimation: External Habit

$E[z_{it} \cdot (\kappa x_{it+1}^{-\phi} x_{it}^\theta \pi_{it+1}^{-1} R_{it} - 1)] = 0$					
Panel	$z_{it}$	$\hat{\kappa}$ (se)	$\hat{\phi}$ (se)	$\hat{\theta}$ (p)	$J$ (df)
US, UK, Japan 1975:1–1999:1 285 observations	$\iota, x_{it}, \pi_{it}, R_{it-1}$	0.991 (0.002)	-0.436 (0.364)	0.355 (0.160)	45.60 (9) (0.00)
	$\iota, R_{it-1}$	0.986 (0.023)	10.88 (12.30)	-11.51 (12.86)	0.06 (4) (0.99)
US, UK, Japan, Canada, Italy, France, Sweden 1980:2–1998:3 504 observations	$\iota, x_{it}, \pi_{it}, R_{it-1}$	0.990 (0.001)	-0.079 (0.084)	0.192 (0.052)	53.46 (25) (0.00)
	$\iota, R_{it-1}$	0.987 (0.001)	1.564 (1.300)	-1.741 (1.398)	22.68 (11) (0.02)

Notes:  $x_{it+1}$  is the gross, real consumption growth rate,  $\pi_{it+1}$  the gross inflation rate,  $R_{it}$  the gross nominal interest rate from  $t$  to  $t+1$  in country  $i$ ;  $\iota$  is a vector of ones. Estimation is by iterated GMM using the Hansen-Heaton-Ogaki Gauss code.

**Table 4 Panel GMM Estimation: Adaptive Learning**


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$$E[z_{it} \cdot (\lambda_i \beta \frac{x_{it}^{-\gamma}}{\pi_{it}} R_{it} + (1 - \lambda_i) \frac{R_{it}}{R_{it-1}} - 1)] = 0$$


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Panel	$z_{it}$	$\hat{\beta}$ (se)	$\hat{\gamma}$ (se)	$J$ (df) (p)
US, UK, Japan 1975:1–1999:1 285 observations	$\iota, x_{it-1}, \pi_{it-1}, R_{it-1}$	0.977 (0.006)	0.902 (0.713)	13.83(7) (0.05)
US, UK, Japan, Canada, Italy, France, Sweden 1980:2–1998:3 504 observations	$\iota, x_{it-1}, \pi_{it-1}, R_{it-1}$	0.990 (0.001)	0.240 (0.234)	17.09 (19) (0.58)

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$$E[z_{it} \cdot (\lambda_i \kappa \frac{x_{it}^{-\phi} x_{it-1}^{\theta}}{\pi_{it}} R_{it} + (1 - \lambda_i) \frac{R_{it}}{R_{it-1}} - 1)] = 0$$

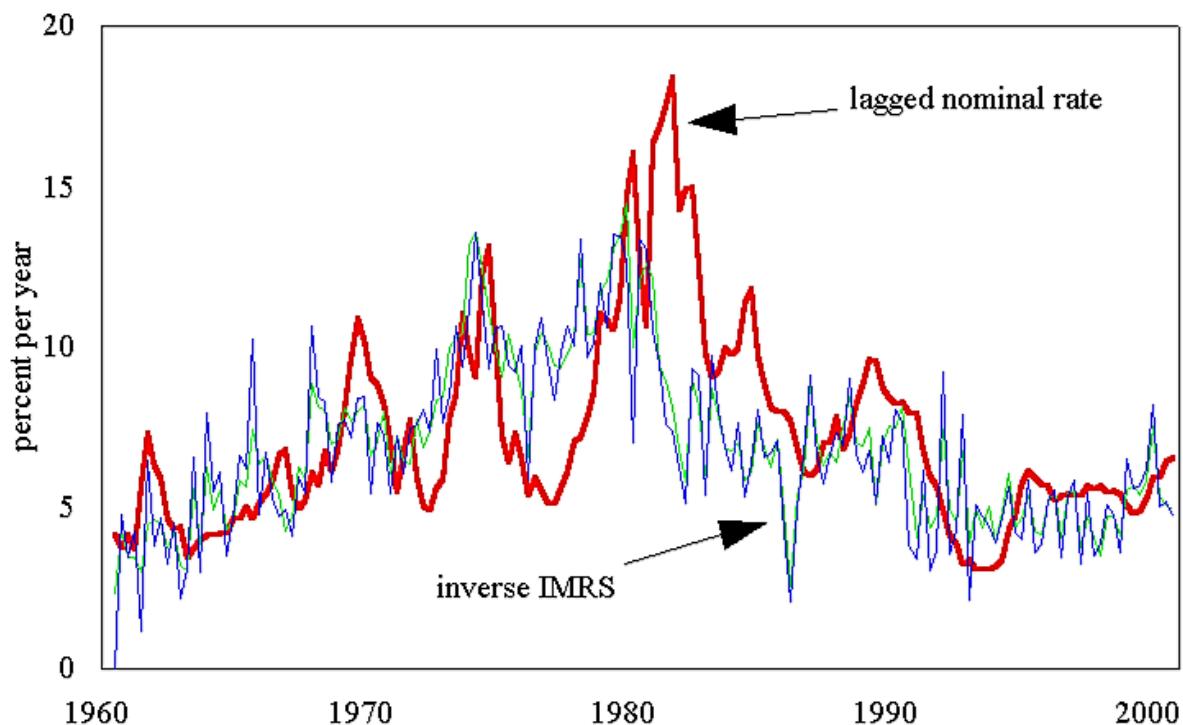

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Panel	$z_{it}$	$\hat{\kappa}$ (se)	$\hat{\phi}$ (se)	$\hat{\theta}$ (p)	$J$ (df)
US, UK, Japan 1975:1–1999:1 285 observations	$\iota, x_{it-1}, \pi_{it-1}, R_{it-1}$	0.973 (0.010)	0.499 (1.513)	-0.145 (0.424)	13.51(6) (0.04)
US, UK, Japan, Canada, Italy, France, Sweden 1980:2–1998:3 504 observations	$\iota, x_{it-1}, \pi_{it-1}, R_{it-1}$	0.993 (0.006)	-2.314 (1.114)	-2.253 (0.966)	14.02 (18) (0.73)

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Notes:  $x_{it+1}$  is the gross, real consumption growth rate,  $\pi_{it+1}$  the gross inflation rate,  $R_{it}$  the gross nominal interest rate from  $t$  to  $t+1$  in country  $i$ ;  $\iota$  is a vector of ones. Estimation is by iterated GMM using the Hansen-Heaton-Ogaki Gauss code.

**Figure 1: Euro-rates and Inverse IMRS**  
United States



**Figure 2: Euro-rates and Inverse IMRS**  
Japan

