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# Academia or the Private Sector? Sorting of Agents into Institutions and an Outside Sector

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# Academia or the Private Sector? Sorting of Agents into Institutions and an Outside Sector\*

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## Abstract

This paper develops an equilibrium sorting model with utility maximizing agents (researchers) on one side of the market, and on the other side institutions (universities) and an outside sector. Researchers are assumed to care about peer effects, their relative status within universities, and salary compensation. They differ in their concern for salary compensation as well as in their ability. We derive the unique stable equilibrium allocation of researchers and investigate the effects on the academic sector of changes in the outside option as well as the interaction between the outside option and the researchers' concern for relative status. In any equilibrium, the right hand side of the ability distribution is allocated to the academic sector, while the left hand side of the ability distribution is allocated to the outside sector, with possible overlap between sectors and within the academic sector. The universities' qualities are determined endogenously, and we show that an increase in the value of the outside option decreases the difference in quality between the higher and lower ranked universities. Furthermore, differences in average salaries between the institutions arise endogenously.

Keywords: Sorting, Universities, Researchers, Academia, Ranking, Peer effects.

JEL classification: D02, C78, D83

## 1 Introduction

The academic sector has a substantial influence on welfare, both through educating the workforce and through the innovations and insights that are the products of research car-

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ried out at academic institutions. It is therefore imperative to understand the special issues that pertain to this sector. One factor that makes academic institutions different from a standard profit maximizing firm is that their most important input, i.e. academics, is far from being a standard homogeneous good that is passively supplied. Instead, academics are highly differentiated and optimizing agents, who actively choose where to work. Importantly in this respect, the academic sector is not an isolated island; rather it is part of the larger economy.

Since individuals with doctoral degrees are not only employed by the academic sector but also by the private sector, an important determinant of labor market outcomes for these individuals is the attractiveness of the outside sector. Thus, the private sector affects the equilibrium properties of the academic sector. Some who exit from academia do so voluntarily, while others would have preferred to work in the academic sector. Whether a particular individual is employed by the academic sector or has exited to the outside sector voluntarily or involuntarily typically depends on both the individual's ability to do research and on his trade-off between salary compensation and an academic career.

This paper takes these ideas and develops an equilibrium sorting model with utility maximizing agents on one side of the market, and institutions and an outside sector on the other side. Agents are assumed to care about peer effects, their relative status within institutions, and pecuniary compensation. They differ in their concern for pecuniary compensation and in their ability. Subsequently, we use the interpretation of the agents as researchers and the institutions as universities. The model however applies to any setting where peer effects and ranking concerns matter. Thus, what we refer to as institutions could also be employers like Google or NASA, where peer effects are presumably strongly prevalent as well. Likewise, what we denote as the private sector could include any employer for which peer effects and ranking are not of significant concern to the employees. Other alternative interpretations of the model could, for example, be one of the agents as musicians and the institutions as orchestras, or one of the institutions as country clubs and the agents as potential patrons.

The central question we seek to answer is how the presence of the outside sector and the value of the outside option affect the academic sector. We analyze the extent to which the outside sector constrains the institutions in their hiring and how changes in the outside option affect the qualities of universities and thus the quality difference between them. Put differently, we investigate how competition with the outside (private) sector affects competition within the academic sector. We also make predictions about the effect on the difference in mean salaries at the institutions and investigate the interaction between the outside option and the researchers' concern for relative status. The analysis enables comparative statics analysis between fields of research with different attractiveness of the outside option.

We solve for the allocation of researchers' ability across sectors and within the academic sector, as well as for how changes in the outside option and in the researchers' concerns for relative status affect this allocation and the universities' qualities. In both the academic

and the private sector, we allow for researchers' pecuniary compensation to be a function of ability. We can therefore analyze how both changes in the general level of outside sector salary and changes in the salary progression in the outside sector influence the academic sector.

In any equilibrium, the top of the ability distribution is allocated to the academic sector, while the bottom is allocated to the outside sector. In general, there can be overlap both between the institutions within the academic sector and between the academic and outside sectors.

Our key contribution lies in our comparative statics results. For low values of the outside option, the academic sector is unaffected by the outside sector and exit from academia happens involuntarily. In contrast, the academic sector is affected by the outside option and by changes in its value when its value is sufficiently high. In particular, the universities are unable to satisfy their demands for researchers and all exit from academia is voluntary. The universities' qualities are determined endogenously, and as long as both universities are active, then an increase in the value of the outside option decreases the difference in quality between the higher and lower ranked universities. This decrease in the quality difference happens regardless of whether the value of the outside option increases due to a higher general level of salaries or due to stronger salary progression. These results suggest that we should observe smaller differences in average quality between departments in fields with high outside options, such as accounting, than in fields with low outside options, such as English. Economics would fall somewhere in between these. The results also suggest that exit from academia in fields with high outside options (accounting) is voluntary, while a greater fraction of researchers in low outside option fields (English) are allocated involuntarily to the outside sector. We furthermore establish that differences in average salaries between the institutions arise endogenously, such that the average salary difference between departments should be higher in fields with low outside options than in fields with high outside options.<sup>1</sup>

There is an important interaction between the researchers' concern for ranking and the outside option. The more they care about ranking, the higher is the value of the outside option for which the outside sector starts to constrain the academic sector and the lower is the value of the outside option for which we start to get overlap between the universities. The concern for ranking is also a measure of how researchers weigh rank relative to peer effects in their payoffs. Therefore, it also affects the extent of overlap within the academic sector for a given value of the outside option. A higher concern for relative ranking is associated with a larger overlap of types between the institutions. When the concern for ranking is low enough, the peer effects at the better university dominate holding a higher ranking at the less good university for all researchers, and there is no overlap between the institutions.

The present paper builds on Damiano, Li, and Suen (2010), who assume that researchers

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<sup>1</sup>These comparisons between fields assumes that the concern for ranking is equivalent across fields.

care about both peer effects and their relative status within institutions. We assume that a researcher’s utility is a weighted sum of relative status, peer effect, and salary. Researchers may care about peer effect for a number of reasons. For example, higher ability colleagues improve the intellectual environment and graduate programs, attract better visiting speakers, and potentially provide the opportunity to produce better joint work. The weight researchers assign to status relative to peer effect need not be interpreted purely as stemming from the researchers’ personality; it can also be interpreted as incorporating other factors, such as technological innovation. For example, if the fact that the internet facilitates joint work with researchers at other institutions has reduced the importance of peer effects, this change is equivalent to an increase in the weight on status relative to peer effect.

For young researchers, one reason to care about ranking within institutions is that ranking is a proxy for the likelihood of getting tenure. For senior researchers, someone who is ranked higher within his institution is more likely to hold a Chair with research funds and teaching reductions than someone who ranks lower. Thus, relative ranking matters for senior researchers as well.<sup>2</sup>

We make the assumption that academic salary is non-decreasing in ability and that for each ability level the salary is the same across the academic sector. Given this assumption, salary differences between academic institutions are due to differences in their demographics, that is, differences in the ability of those who work there. That academic salary is non-decreasing in ability (within a given field) is a reasonable assumption since higher ability researchers tend to receive more job offers, which typically push up salaries. In reality, a researcher’s relative rank within his institution may also be relevant for his salary compensation. However, making salary a function of ability directly has the theoretical appeal that the effects of ranking and salary can be separated from each other. Thus, in the present paper, researchers’ ranking concerns are separate from their salary concerns, which results in endogenous salary differences between the institutions. The predictions of our model in this respect are consistent with observed differences in average salary between research universities and teaching colleges, c.f. Cawley (2009, section 7).

We abstain from explicitly modeling the universities’ optimization problems. Instead, each university is a passive agent who simply posts a demand, i.e. a quantity, or measure, of researchers it would like to hire. This is taken as exogenous. However, the equilibrium concept we employ implicitly assumes that the universities hire the best possible researchers.

The sorting equilibrium concept for the allocation of researchers used in this paper is a modified version of the one applied in Damiano et al. (2010). We extend their equilibrium concept to account for the outside option and for the fact that universities may not operate at capacity because they may be constrained by the outside sector. Damiano et al. do not

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<sup>2</sup>In a broader interpretation of the model, ranking can be interpreted as a proxy for job security in general, or, with the country club interpretation, higher ranked patrons may enjoy special benefits if the country club recognizes the positive externality they generate.

allow for an outside option, and thus they cannot address the questions that lie at the heart of the present paper. This distinction will be discussed further after the presentation of our model and results.

In the present paper, a researcher derives non-pecuniary utility from being matched to a university, a feature that can also be found in Besley and Ghatak (2005), who consider incentives for motivated agents. Lach and Schankerman (2009) provide empirical evidence that scientists have both pecuniary and intrinsic research motives, while the importance of peer effects is, for example, demonstrated empirically in Sacerdote (2001) and Sarpca (2010) who consider peer effects between students. Hansmann (1986) provides a theoretical model where an individual's utility from membership of an organization depends on the average type among the members and the membership fee charged, while Postlewaite (1998) provides an argument for including relative status in reduced form utility functions. Some of these ideas can also be found in Hartwick and Kanemoto (1984).

Chade, Lewis, and Smith (2013) present a model with two colleges and students who differ in ability. Chade et al. solve the students' problem of finding the optimal set of colleges to apply to given the admission chances at the colleges and the application costs. In contrast to our model, in their model, an agent's payoff from attending a specific institution is exogenously given.

Epple and Romano (1998) present a model of school choice where utility is a function of the student's ability, mean ability of the student body in the school attended, and numeraire consumption. In contrast, in our model, the utility function depends on the agent's own ability through the agent's ranking and is therefore affected by the actions of other agents through the entire distribution of agents within his institution. In Epple and Romano the only interaction between own ability and other agents' actions is through the mean ability. Therefore, the models yield substantially different results. In their model of college choice, Epple, Romano, and Sieg (2006) also allow, among other things, own ability, mean ability of the student body, and numeraire consumption to affect the agent's utility.

Damiano, Li, and Suen (2012) present a model in which two institutions of fixed and equal size choose salary schedules that map researchers' relative rank into salary. In their model, academics derive utility from peer-effects and pecuniary compensation, and there is no outside sector. Note that in the present paper, salaries are functions of ability. It is, of course debatable whether salary should be a function of ranking or ability, and there undoubtedly exists empirical or at least anecdotal evidence in favor of either option. The theoretical appeal of making salary a function of ability directly is discussed above.

Prüfer and Walz (2011) compare different potential governance structures for institutions when researchers care about peer-effects and pecuniary compensation. Various aspects of the problems pertaining to universities and researchers have been addressed by other authors, including incentives for researchers (Lazear, 1997), dynamic incentives (Carayol, 2008), the tenure system (Carmichael, 1988), stratification into research vs. teaching institutions (Del Rey, 2001), optimal class size (Lazear, 2001), and the game between administration and faculty (Ortmann and Squire, 2000). These issues are thus

not the focus of the present paper. Instead we focus on the outside option and on how it affects researchers' behavior and the quality differences between academic institutions.

The paper is organized as follows. In section 2, we describe the sorting model. In section 3, we characterize the allocation of researchers between the two universities and the outside sector, and present the comparative statics results. Section 4 concludes. Technical details and proofs can be found in the appendices.

## 2 Model

Consider a sorting model with a continuum of researchers on one side and two universities and an outside sector on the other side. Each researcher seeks to maximize his utility by choosing the job among his offers which is the best match. Given that outside options vary greatly between fields of education, the model presented below is best thought of as describing a single field, for example economics. The comparative statics results can be used to compare different fields.

There is a continuum of researchers of mass 1, each characterized by a two-dimensional type  $(\beta, \theta)$ . The variable  $\theta$  measures the researcher's ability while the variable  $\beta$  measures the weight he assigns pecuniary compensation relative to other components of his utility, to be made precise shortly. The two dimensions of researchers' types are distributed independently of each other. The variable  $\theta$  is distributed on the interval  $[\underline{\theta}, \bar{\theta}]$  with  $F(\theta)$  denoting the cumulative distribution function. We assume that the density  $f(\theta)$  exists and is positive for all  $\theta$  in  $[\underline{\theta}, \bar{\theta}]$ . The variable  $\beta$  is distributed on the interval  $[\underline{\beta}, \bar{\beta}]$  with  $G(\beta)$  denoting the cumulative distribution function. If all researchers assign the same weight to pecuniary compensation, i.e. are homogenous with respect to  $\beta$ , then  $\bar{\beta} = \underline{\beta}$ , or  $\bar{\beta} > \underline{\beta}$  with  $G(\beta)$  degenerate, that is,  $G(\beta) \in \{0, 1\}$  for all  $\beta \in [\underline{\beta}, \bar{\beta}]$ . If instead they are heterogenous with respect to  $\beta$ , then  $\bar{\beta} > \underline{\beta}$  and  $G(\beta)$  is non-degenerate, that is, there exists  $\hat{\beta} \in [\underline{\beta}, \bar{\beta})$  such that  $G(\hat{\beta}) \in (0, 1)$ .

Salary in the academic sector is a function of ability and is given by  $W(\theta)$ , with  $W(\theta) \geq 0$  and first-order derivative  $W'(\theta) \geq 0$  for all  $\theta$ . A researcher's academic salary is therefore non-decreasing in his ability.<sup>3</sup> The salary function is assumed to give the market wage in academia (within a given field) for each particular level of ability and is therefore the same for the two universities.

The salary in the non-academic sector is given by  $C(\theta)$ , with  $C(\underline{\theta}) \geq W(\underline{\theta})$  and first-order derivative  $C'(\theta) \in [0, W'(\theta)]$  for all  $\theta$ . Outside sector salary is therefore also non-decreasing in ability, but there is a (weakly) smaller salary progression with respect to ability in the outside sector than in the academic sector.<sup>4</sup> Note that outside sector salary is assumed to be at least as high as academic sector salary for the lowest level of ability, and may be higher for any level of ability.

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<sup>3</sup>Note that constant salary is a special case.

<sup>4</sup>Constant salary is again a special case.



The assumptions about the salary functions are well justified if we think of the set of researchers as a particular cohort. The assumption that the base salary is higher in the outside sector than in the academic sector appears well justified considering the difference between the lowest salaries offered by private sector firms and the salaries offered by lower ranked teaching colleges. Higher ability researchers get more job offers and can use these to negotiate a higher salary, so wage progression certainly exists in academia. If one thinks of the salary functions as giving the salaries offered in initial positions, the assumption that progression is larger in academia than in the private sector also appears reasonable, since progression in the private sector is to a large part due to bonuses and promotions or salary increases during the career. More generally, since researchers in this model do not differ in dimensions such as seniority, one can think of the salary functions as those remaining after controlling for such other dimensions as seniority and year-to-year fluctuations (or drift) in starting salaries.

For ease of notation, let  $c_0 \equiv C(\underline{\theta})$  and  $w_0 \equiv W(\underline{\theta})$ . We will refer to these as the base salaries in the two sectors. We can then write the two salary functions as  $C(\theta) \equiv c_0 + c(\theta)$  and  $W(\theta) \equiv w_0 + w(\theta)$ , respectively, where  $c(\theta) \equiv C(\theta) - c_0$  and  $w(\theta) \equiv W(\theta) - w_0$ . Hence,  $c(0) = w(0) = 0$  and  $0 \leq c'(\theta) \leq w'(\theta)$  for all  $\theta \in [\underline{\theta}, \bar{\theta}]$ . Finally, define  $\Delta(\theta) \equiv c(\theta) - w(\theta)$  and note that  $\Delta'(\theta) = c'(\theta) - w'(\theta) \leq 0$ .

It is assumed that there are two universities,  $A$  and  $B$ . We follow Damiano et al. (2010, 2012) and assume that the peer effect in university  $i$ , denoted  $m_i$ , is given by the average ability of those employed by university  $i$ . The rank within university  $i$  of a researcher of type  $(\beta, \theta)$  is given by  $r_i(\theta)$ .

Researchers derive utility from their relative ranking within their institution, the peer effect in their institution, and their pecuniary compensation. A type- $(\beta, \theta)$  researcher's utility from working in academia at university  $i \in \{A, B\}$  is given by

$$V_i(\beta, \theta) = \alpha r_i(\theta) + m_i + \beta W(\theta).$$

The parameter  $\alpha > 0$  measures the weight researchers assign to status relative to peer effect, while  $\beta$  captures the weight researchers assign to pecuniary compensation relative to peer effect.

The utility from taking a job outside of academia equals the salary in the non-academic sector weighted by  $\beta$ . A researcher of type  $(\beta, \theta)$  enjoys utility

$$V_C(\beta, \theta) = \beta C(\theta)$$

if he is employed in the outside sector. Researchers always have the possibility of choosing the outside option and they prefer this to unemployment.

A researcher's objective is to maximize his utility by choosing among available jobs in academia and the outside option. Thus, if a researcher of type  $(\beta, \theta)$  has an offer from both  $A$  and  $B$ , he will choose  $A$  if  $V_A(\beta, \theta) \geq V_B(\beta, \theta)$  and  $V_A(\beta, \theta) \geq V_C(\beta, \theta)$ , he will choose  $B$  if  $V_B(\beta, \theta) \geq V_A(\beta, \theta)$  and  $V_B(\beta, \theta) \geq V_C(\beta, \theta)$ , and otherwise leave academia to work in

the outside sector. If the researcher has an offer from only university  $i$ , he will accept the offer if  $V_i(\beta, \theta) \geq V_C(\beta, \theta)$ , and otherwise work in the outside sector. If he has no academic offers, the researcher will work in the outside sector.

The universities are modeled as passive agents that simply post demands for researchers. A university's demand is a quantity, i.e. a measure of researchers it would like to hire. This is taken as exogenous. However, the equilibrium concept we employ implicitly assumes that universities hire the best possible researchers. Let  $\tilde{H}_A$  and  $\tilde{H}_B$  be the demands posted by universities  $A$  and  $B$  respectively and assume that  $\tilde{H}_A + \tilde{H}_B \leq 1$ . Note that either school can post the larger demand, or they may post equal demands. The outside sector is able to employ the full continuum of researchers, that is  $\tilde{H}_C = 1$ . Let  $H_A(\bar{\theta})$  and  $H_B(\bar{\theta})$  denote the actual sizes of the two universities, i.e. the measure of researchers they actually succeed in hiring, and let  $H_C(\bar{\theta})$  denote the measure of researchers who leave academia to work in the outside sector. Without loss of generality, let  $A$  denote the best university, i.e. the university with the highest average ability.

### 3 Allocation of researchers

We now solve for the allocation of researchers between the universities and the outside sector given the demands of the universities and the wage functions for the academic and outside sectors. A researcher's rank within university  $i$ ,  $r_i(\theta)$ , is given by the cumulative distribution function of types of researchers within university  $i$ . Since the size of university  $i$  is  $H_i(\bar{\theta})$ , we can define the function

$$H_i(\theta) = r_i(\theta)H_i(\bar{\theta}), \quad (1)$$

and abusing vocabulary we call this function,  $H_i(\theta)$ , the non-normalized type distribution for university  $i$ .

**Definition 1** (Feasible allocation). *A feasible allocation is a triple  $(H_A, H_B, H_C)$  of non-normalized type distributions such that  $H_A(\theta) + H_B(\theta) + H_C(\theta) = F(\theta)$  for all  $\theta \in [\underline{\theta}, \bar{\theta}]$ .*

Note that  $H_A(\bar{\theta}) + H_B(\bar{\theta}) + H_C(\bar{\theta}) = 1$ . If both  $H_i$  and  $H_j$ ,  $i \neq j$ , are strictly increasing in a neighborhood of  $\theta$ , then types with ability around  $\theta$  are split between workplaces  $i$  and  $j$ . Let  $T_i$  be the support set of workplace  $i$ , defined as the closure of the set of types at which  $H_i$  is strictly increasing.

We apply a modified version of the sorting equilibrium concept in Damiano et al. (2010):

**Definition 2** (Sorting equilibrium). *A sorting equilibrium is a feasible allocation  $(H_A, H_B, H_C)$  such that for each  $i, j = A, B, C$  and  $i \neq j$ , the following two conditions are satisfied:*

- (i) if  $(\beta, \theta) \in T_i$  and  $\theta > \inf T_j$ , then  $V_i(\beta, \theta) \geq V_j(\beta, \theta)$
- (ii) if  $(\beta, \theta) \in T_i$  and  $H_j < \tilde{H}_j$ , then  $V_i(\beta, \theta) \geq V_j(\beta, \theta)$ .

Figure 1: Overlapping intervals of researchers



An implicit assumption behind this equilibrium concept is that a researcher of type  $(\beta, \theta)$  gets an offer from university  $i$  if either  $i$  has not yet reached its desired size, or  $\theta$  is higher than the lowest ability at  $i$ . Given this assumption, in equilibrium no researcher with the option of changing workplace should strictly prefer to do so. Condition (ii) of the definition states that if a university has not reached its capacity then no researcher would be strictly better off by moving to that university.<sup>5</sup> One can think of the equilibrium as a result of an assignment procedure such as the one presented in Gale and Shapley (1962).

We will show that the model has a unique (up to the relative size of the universities if they are of equal quality) stable sorting equilibrium. A sorting equilibrium is defined in Definition 2, and to determine whether an equilibrium is stable, we apply Theorem 6.5 in Stokey and Lucas (1989).

The researchers face a tradeoff between rank and peer effect, the latter measured by the average type  $m_i$  in the researcher's institution. If a researcher works for the better university  $A$  the peer effect  $m_A$  will be higher than if he works for university  $B$ , but the researcher's rank  $r_A(\theta)$  will be lower than if he works for university  $B$ . Thus, there can be a range of researchers who are indifferent between working for the two universities. Also, since researchers differ in their weight  $\beta$  on pecuniary compensation, there can be a range of researchers, as measured by  $\theta$ , who are indifferent between working in the academic and the outside sector. This illustrates the intuition behind the following definition of an overlapping interval allocation:

**Definition 3** (Overlapping interval allocation). *An allocation of researchers is an overlapping interval allocation if there exist types  $s, z, v, y, x$ , and  $q$  with  $s \leq v \leq z, y \leq x \leq q$ , and  $v \leq x$ , such that the support sets for  $A, B$ , and  $C$  are  $[y, q], [v, x]$ , and  $[s, z]$ , respectively.*

Figure 1 illustrates an overlapping interval allocation with  $s = \underline{\theta}$  and  $q = \bar{\theta}$ . Here, researchers with  $\theta \in [\underline{\theta}, v)$  work exclusively in the non-academic sector, researchers with  $\theta \in [v, z]$  are indifferent between the non-academic sector and university  $B$ , researchers with  $\theta \in (z, y)$  work exclusively for  $B$ , researchers with  $\theta \in [y, x]$  are indifferent between  $A$  and  $B$ , and researchers with  $\theta \in (x, \bar{\theta}]$  work exclusively for  $A$ .

<sup>5</sup>We thank a referee for the suggestion to include condition (ii) in the sorting equilibrium definition.

Theorem 1 below shows that the allocation of researchers between universities  $A$  and  $B$  and the non-academic sector is of an overlapping interval form.

**Theorem 1.** *The equilibrium allocation of researchers is of an overlapping interval form. Furthermore,*

- (i) *If researchers are homogenous with respect to the weight they assign to pecuniary compensation relative to peer effect, then there exists  $v \in [\underline{\theta}, \bar{\theta}]$  such that the interval  $[\underline{\theta}, v]$  is allocated exclusively to the outside sector and the interval  $[v, \bar{\theta}]$  is allocated exclusively to the academic sector.*
- (ii) *If researchers are heterogenous with respect to the weight they assign to pecuniary compensation relative to peer effect, then it is possible to have overlap between the outside and academic sector for a positive measure of ability types, i.e. to have  $v < z$ .*

**Proof:** Please see Appendix A.

According to Theorem 1, the left end of the ability distribution always works in the outside sector, while the right end works in academia. If all researchers assign the same weight to pecuniary compensation relative to peer effect, then they sort themselves in such a way that there is no overlap between the two sectors. If, instead, researchers differ in the weights they assign to pecuniary compensation relative to peer effect, then it is possible to have overlap between the outside and academic sectors for a strictly positive measure of ability types. The intuition is that those who weight compensation more go to the non-academic sector if outside jobs pay more, while those who weight compensation less choose academic jobs. As ability rises, a declining fraction of types go to non-academic jobs, since higher ability types will have higher ranking within academia, resulting in the non-pecuniary components of their utility starting to dominate.<sup>6</sup>

For the remaining analysis, we focus on the case when each researcher places the same weight on pecuniary compensation and peer effect, i.e.  $\beta = 1$  for all researchers.<sup>7</sup> For this case, there are a number of different possible equilibria. As shown in Theorem 1, none of these have overlap between the outside and academic sectors. The possible equilibria differ in whether they have no, partial, or full overlap of the support sets of the universities, whether university B succeeds in hiring researchers, and whether or not the academic sector is constrained by the outside sector. Here and henceforth we use the term “no overlap” when the overlap consists of only a threshold type, which has measure zero.

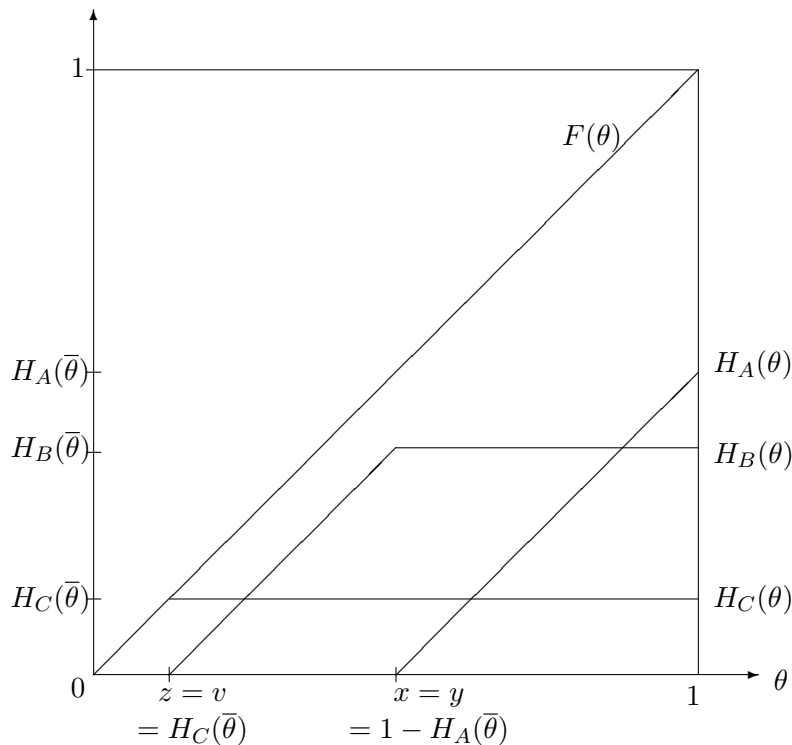
Prior to characterizing the different possible equilibria, we make the following assumption about the type-distribution of researchers, which makes a tractable closed-form solution possible.

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<sup>6</sup>We thank a referee for suggesting the connection between heterogeneity in  $\beta$  and the overlap between the academic and outside sectors.

<sup>7</sup>When the weight researchers assign to pecuniary compensation relative to peer effect is homogenous, then researchers only differ in their ability, so we henceforth use the term “type” to refer to ability type.

Figure 2: Distribution of researchers across universities when there is full segregation



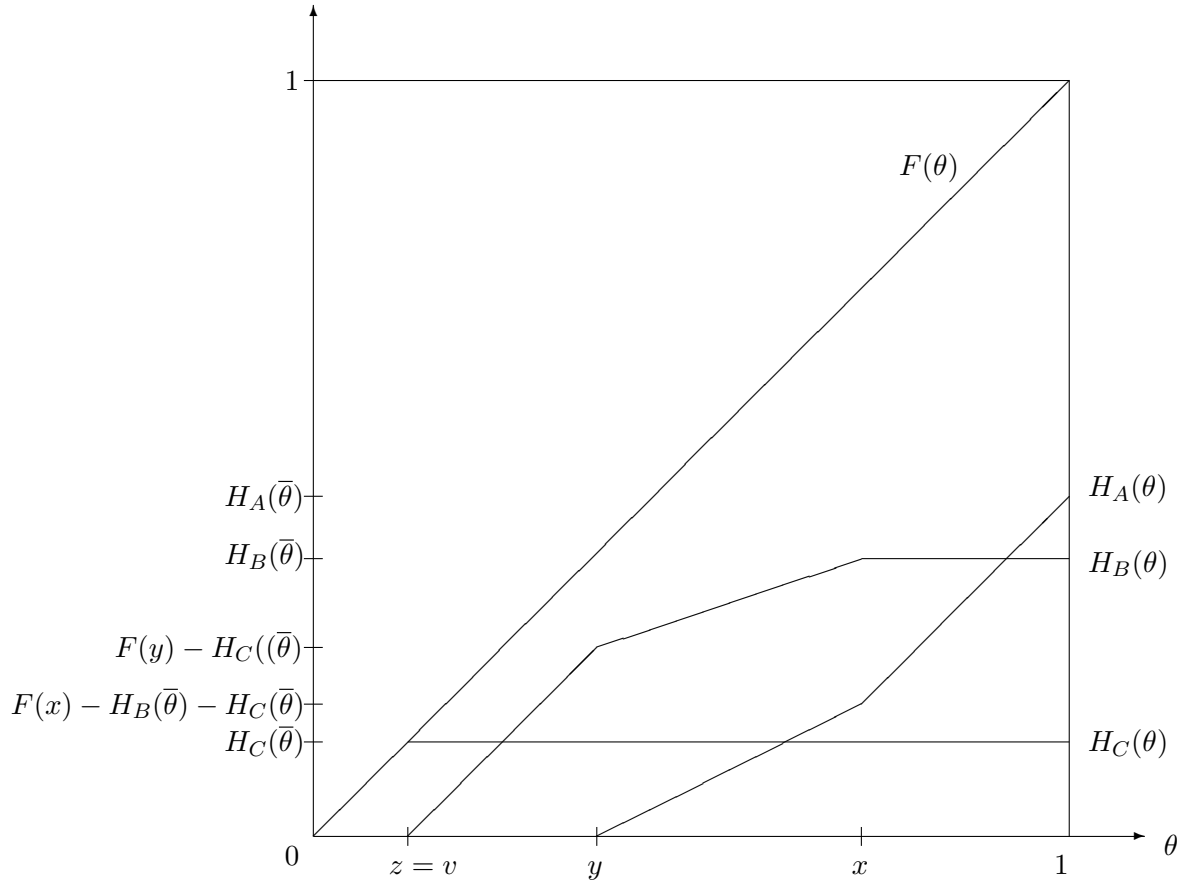
**Assumption 1.** *The distribution of  $\theta$  is uniform on  $[\underline{\theta}, \bar{\theta}] = [0, 1]$ , i.e.  $F(\theta) = \theta$  on  $[0, 1]$ .*

There are eight possible scenarios in which the academic sector is active, as well as one scenario with an empty academic sector. We now proceed to describe the different possible equilibrium scenarios.

**Unconstrained Full Segregation (UFS).** In UFS, the support sets of the universities do not overlap, and the wage difference between the outside and academic sectors is sufficiently low that the outside option does not affect the academic sector. Both universities are able to satisfy their demands. The interval at the top of the ability distribution works for the better university,  $A$ , and the interval immediately below that works for the less good university,  $B$ . Figure 2 illustrates a situation with full segregation.

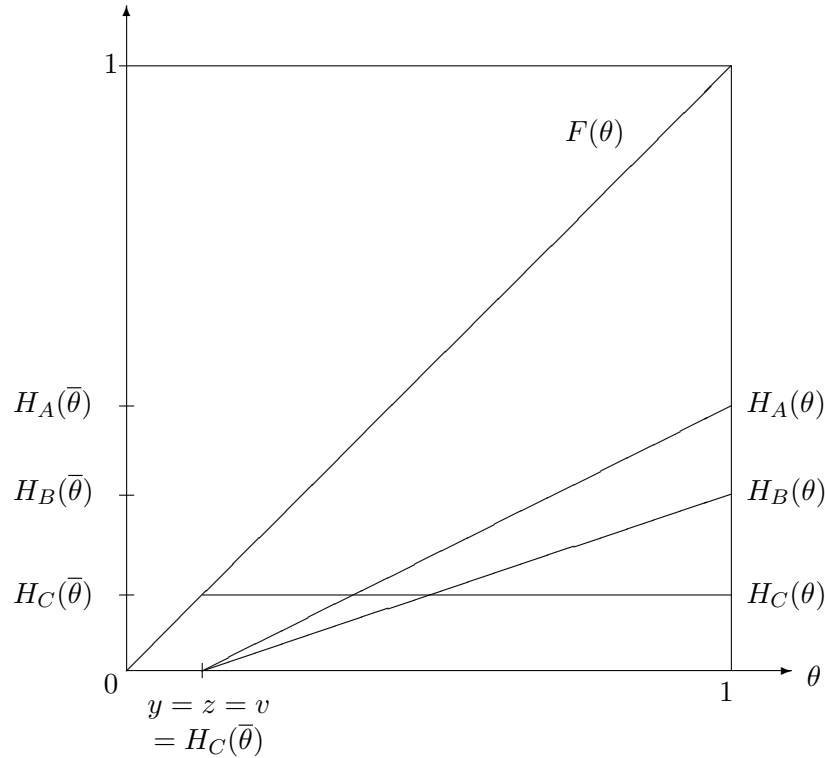
**Unconstrained Partial Overlap (UPO).** In UPO, there is partial overlap of positive measure between the support sets of the universities. Also, the outside option is low

Figure 3: Distribution of researchers across universities when there is partial overlap



enough that it does not affect the academic sector, and hence the universities are still able to satisfy their demands for researchers. The top of the ability distribution works for the better university,  $A$ , but there are intermediate types for which some researchers work for the better university and others work for the less good university. Thus, moving from the right of the ability distribution to the left, the interval at the very top works exclusively for  $A$ , all types in an interval immediately left of that are split between  $A$  and  $B$ , while the next interval works exclusively for  $B$ . Finally, all sufficiently less able researchers work in the outside sector. Figure 3 illustrates a situation with partial overlap.

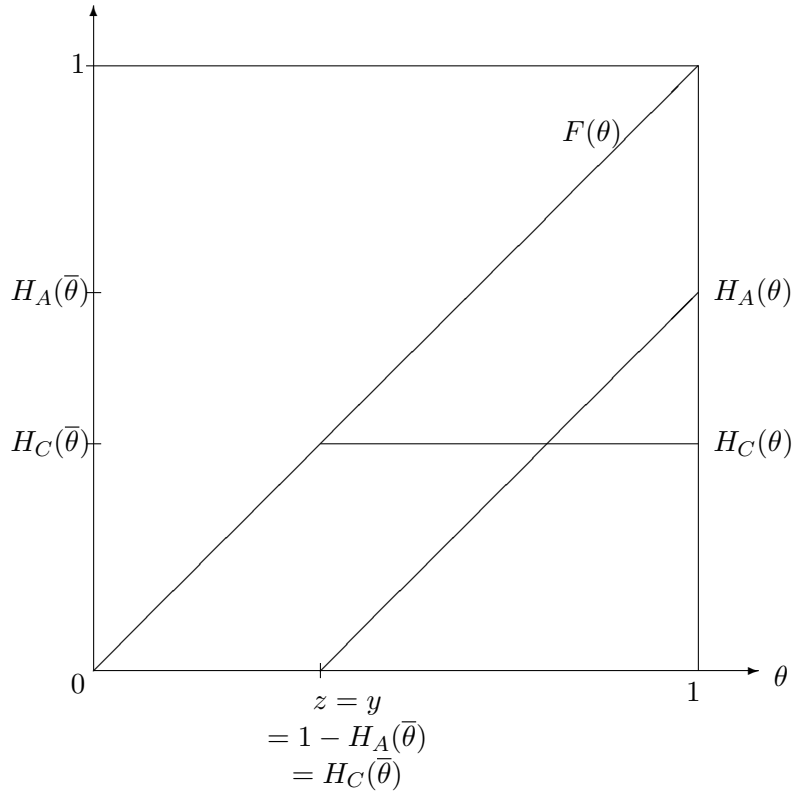
Figure 4: Distribution of researchers across universities when there is full overlap



**Unconstrained Full Overlap (UFO).** In UFO, there is full overlap between the support sets of the universities. All types who work in academia have a certain fraction of researchers working for  $A$  and the remainder for  $B$ . As a consequence, in UFO the institutions are of the same average quality. Similarly to UFS and UPO, the outside option is low enough that it does not constrain the academic sector, and thus the universities' demands for researchers are satisfied. Figure 4 illustrates a situation with full overlap.

**Constrained Equilibria.** When the outside option is sufficiently attractive it affects the academic sector because more researchers are attracted to the outside sector. The universities are then unable to fully satisfy their demands for researchers. For each type of unconstrained equilibrium described above, there is a corresponding constrained equilibrium. We denote these **Constrained Full Segregation (CFS)**, **Constrained Partial Overlap (CPO)**, and **Constrained Full Overlap (CFO)**. In those equilibria, the size of the academic sector is constrained by the outside option. Figures 2, 3, and 4 illustrate CFS, CPO, and CFO equilibria, respectively.

Figure 5: Distribution of researchers across universities when there is a single university



**Unconstrained Single University (USU).** In USU, the top of the ability distribution works exclusively for university  $A$ , and the remaining researchers work for the outside sector. Thus, university  $B$  is not active because it does not succeed in hiring any researchers. Although the academic sector as a whole is constrained, the outside option is low enough that it does not constrain university  $A$ , so university  $A$ 's demand for researchers is satisfied. Thus, this equilibrium differs from the other unconstrained equilibria because although university  $A$  is unconstrained the total demand for researchers is not satisfied. The corresponding constrained equilibrium is **Constrained Single University (CSU)**, in which the size of university  $A$  is also constrained by the outside sector, and university  $B$  remains empty. Figure 5 illustrates the situation with a single university.

**Empty Academic sector (EAS).** The final scenario is one of an empty academic sector. This occurs if the outside option is sufficiently attractive that all researchers want to work only in the outside sector.



Figures 2 through 5 are also representative when Assumption 1 is not satisfied, that is, when the distribution of  $\theta$  is not uniform. In that case, the curves are non-linear, but the different types of equilibria are qualitatively unchanged. Assumption 1 makes a tractable closed-form solution possible.

For equilibria in which at least one university is empty, the equilibrium also depends on researchers' beliefs about what their ranking would be if they moved to an empty university. We specify researchers' beliefs with the following assumption about rank at an empty university:

**Assumption 2.** *If a researcher moves to an empty university, then their rank is zero. That is,  $r_i(\theta) = 0 \forall \theta$  if  $H_i(1) = 0$ .*

Given the universities' demands  $\tilde{H}_A$  and  $\tilde{H}_B$ , the equilibrium that prevails is determined by the wage functions  $C(\theta)$  and  $W(\theta)$  for the outside and academic sectors and by the weight  $\alpha$  the researchers give to ranking compared with peer-effect in their utility function. Theorem 2 presents the existence, uniqueness, and comparative statics results, and Figure 6 shows how the equilibrium varies in  $(c_0 - w_0, \alpha)$ -space.

**Theorem 2.** *There exists a unique (up to the relative size of the universities in CFO equilibria)<sup>8</sup> stable sorting equilibrium under Assumptions 1 and 2.*

*For low values of the base salary difference  $c_0 - w_0$  between the outside and academic sectors the academic sector is unconstrained by the outside sector, but for larger values of  $c_0 - w_0$  the academic sector is constrained. In any constrained equilibrium, an increase in  $c_0 - w_0$*

1. *strictly decreases the size of the academic sector,*
2. *does not increase the quality difference  $m_A - m_B$  between the universities, and*
3. *strictly decreases the quality difference  $m_A - m_B$  in a CFS or CPO equilibrium.*

*The value of  $c_0 - w_0$  for which the equilibrium becomes constrained is strictly increasing in  $\alpha$  for some values of  $\alpha$  and non-decreasing in  $\alpha$  for all values of  $\alpha$ .*

*When the weight  $\alpha$  researchers assign to rank relative to peer effect is low, there is full segregation between the universities. For intermediate values of  $\alpha$  there is partial overlap. For high values of  $\alpha$  there is full overlap.<sup>9</sup> The values of  $\alpha$  for which the overlap becomes partial and full are strictly decreasing in  $c_0 - w_0$  for some values of  $c_0 - w_0$  and non-increasing in  $c_0 - w_0$  for all values of  $c_0 - w_0$  for which both universities are active.*

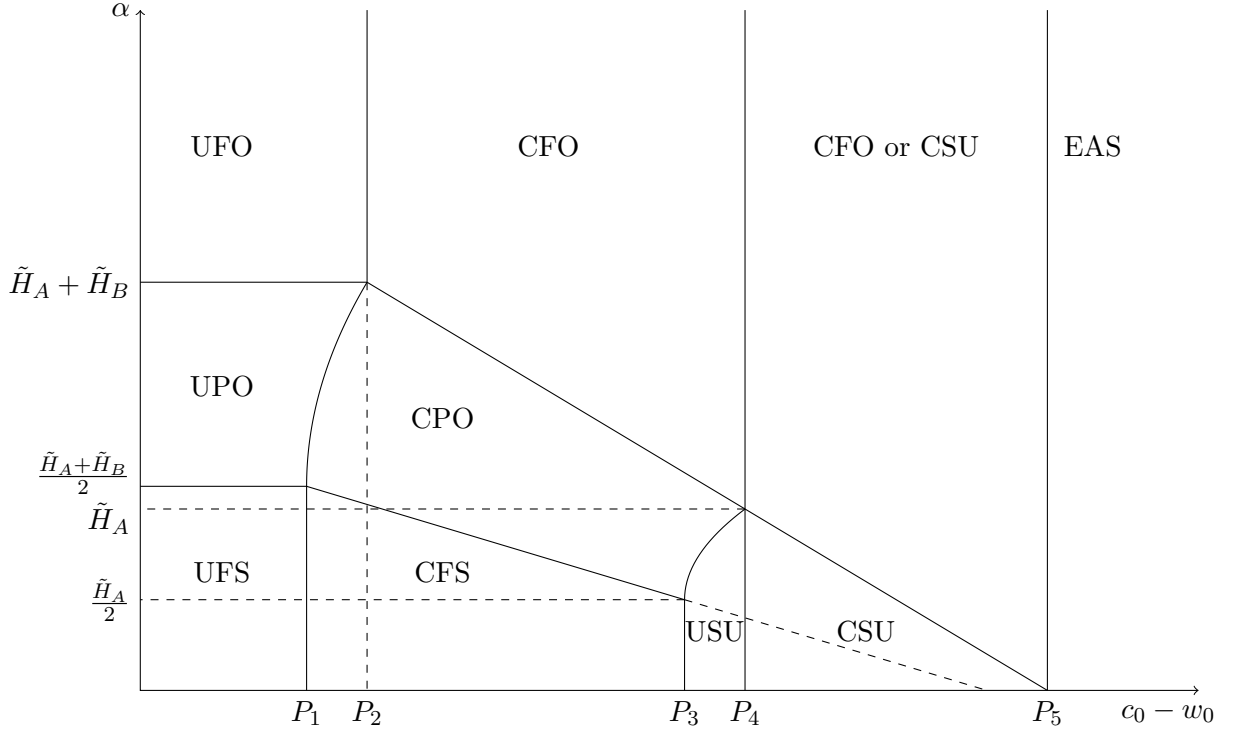
*In both CFS and CPO equilibria, university A's demand for researchers is always satisfied, i.e.  $H_A(1) = \tilde{H}_A$ .*

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<sup>8</sup>Because the relative size of the universities is undetermined in CFO equilibria, there is a set of parameter values for which the size of university B in the CFO equilibrium may in fact be zero, in which case the equilibrium is CSU. This indeterminacy does not affect welfare.

<sup>9</sup>When  $c_0 - w_0$  exceeds a threshold, at least one university may be inactive for all values of  $\alpha$ .

Figure 6: Sorting equilibrium with affine salary functions



Definitions for the  $c_0 - w_0$  intercepts and equations for the boundaries between the different categories of equilibria are in Appendix B.

Let the salary functions for both the academic and outside sectors be affine, with  $C(\theta) = c_0 + \delta_c \theta$  and  $W(\theta) = w_0 + \delta_w \theta$ . In any constrained equilibrium, the effects of a decrease in the difference in salary progression  $\delta_w - \delta_c$  between the academic and outside sectors are as stated in items 1 through 3 above.<sup>10</sup>

**Proof:** Please see Appendix B for the exact boundaries between the different categories of equilibria and for the proof of Theorem 2.

Figure 6 illustrates the result in Theorem 2 when the salary functions are affine with  $C(\theta) = c_0 + \delta_c \theta$  and  $W(\theta) = w_0 + \delta_w \theta$ .<sup>11</sup> For general salary functions, the category to which the equilibrium belongs varies across  $(c_0 - w_0, \alpha)$ -space in a similar fashion. In particular,

<sup>10</sup>We have written the difference in salary progression as that for the academic sector minus that for the outside sector (opposite to how we wrote the difference in base salary) in order for it to be positive and for “decreasing” meaning decreasing in absolute value. A decrease thus implies a better outside option.

<sup>11</sup>Figure 6 illustrates the sorting equilibrium for the case of university A having a greater demand for

the UFS/UPO and UPO/UFO boundaries are horizontal and the UFS/CFS, UFO/CFO, CFS/USU, USU/CSU, CFO/CSU, and CFO/EAS boundaries are vertical. Furthermore, the CFS/CPO and CPO/CFO boundaries are both downward sloping in general, but they need not be linear. Also in general, the UPO/CPO and CPO/USU boundaries are upward sloping curves.

Our model allows for an important dimension that is absent in Damiano et al. (2010), namely the value of the outside option. We analyze what happens to equilibrium outcomes as the value of the outside option varies, both in terms of base salary  $c_0$  and, for affine salary functions, wage progression  $\delta_c$ . Varying the outside option endogenously induces both voluntary and involuntary exit of researchers from academia and also affects the universities' qualities as well as the quality difference between them. Moreover, as seen in Theorem 2 and Figure 6 and discussed subsequently, there is an interesting interaction between the preference parameter  $\alpha$  and the value of the outside option, which naturally cannot be analyzed in the absence of an outside sector.

For low values of  $\alpha$  and  $c_0 - w_0$ , the equilibrium is unconstrained full segregation (UFS). Since  $\alpha$  is low, the researchers care little about their ranking within their institution. Instead they care a lot about the peer-effect. Therefore, the researchers sort themselves into the two institutions in non-overlapping intervals with all types in  $[1 - \tilde{H}_A, 1]$  working for  $A$  and all types in  $[1 - \tilde{H}_A - \tilde{H}_B, 1 - \tilde{H}_A]$  working for  $B$ . The remaining types work in the outside sector. The academic sector is unaffected by the outside sector and the universities can fully satisfy their demands.

If  $\alpha$  increases, holding  $c_0 - w_0$  constant, the lowest types at university  $A$  eventually become indifferent between  $A$  and  $B$ , in which case the equilibrium becomes unconstrained with partial overlap (UPO). Researchers now care enough about their ranking that when  $c_0 - w_0$  increases further, some bottom types from university  $A$  find it worthwhile to move to university  $B$ . The peer-effect is smaller at  $B$ , but their ranking is higher. Hence, the equilibrium takes an overlapping interval form, with  $y < x$ . All types in  $[1 - \tilde{H}_A - \tilde{H}_B, 1]$  work in the academic sector, while the remaining types work in the outside sector. Thus, the universities' demands are again fully satisfied and the academic sector is unaffected by the outside sector. As  $\alpha$  increases, the overlap between the universities grows and the quality difference between them decreases. More researchers will find it worthwhile to give up the higher peer effect at university  $A$  and move to university  $B$  in order to get a higher relative ranking among their peers.

If  $\alpha$  continues to increase (still holding  $c_0 - w_0$  constant), then all types become indifferent between working at  $A$  and  $B$ , in which case the equilibrium is unconstrained full overlap (UFO). Then there is no quality difference between the universities. The researchers care enough about ranking that even the slightest quality difference would warrant the lowest ranked researcher in the better institution to move to the less good institution. The aca-

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researchers than university  $B$ . Figure 7 illustrates this for the case of university  $B$  having a greater demand for researchers than university  $A$  and is presented in Appendix B.

demographic sector remains unaffected by the outside sector, and the universities' demands are fully satisfied.

In these three unconstrained equilibria, there is involuntary exit from academia. The involuntary exit occurs exactly because the universities are unconstrained which happens when the outside option is not sufficiently attractive. That is, once the universities' demands for researchers are satisfied, there are no more jobs in the academic sector, but the outside option is sufficiently unattractive that some of the remaining types, who each has measure zero and thus individually would not affect the average quality of the universities, would prefer to work in academia. In many fields, the relevant equilibrium will be one of the unconstrained equilibria.

The intuition that the equilibrium changes from full segregation to partial overlap to full overlap as  $\alpha$  increases through the unconstrained equilibria can also be found in Damiano et al. (2010). However, they have no outside sector, and with no outside sector, Figure 6 collapses to the vertical axis. In Damiano et al., the universities' demand is always for the full set of researchers, and with no outside sector, all researchers always work in the academic sector. There is no exit, either voluntary or involuntary. Naturally, their model cannot be used to analyze what happens when the outside option becomes more attractive.

In our model, the outside sector becomes more attractive from either an increase in the base salary difference between the outside and academic sectors,  $c_0 - w_0$ , or a decrease in the salary progression difference between the academic and outside sectors,  $\delta_w - \delta_c$ .

For low values of  $c_0 - w_0$  and  $\alpha$ , the equilibrium is UFS. If the base salary difference,  $c_0 - w_0$ , is sufficiently small, then all researchers prefer the academic sector to the outside sector because the peer effect compensates even the lowest type for the difference in base salaries. Increasing  $c_0 - w_0$ , that is, salaries in the outside sector relative to salaries in the academic sector (holding  $\alpha$  constant), makes the outside sector more attractive. If  $c_0 - w_0$  increases enough, then the equilibrium becomes constrained full segregation (CFS). In CFS, the lowest ranked researcher in B is indifferent between staying at B and moving to the outside sector. If  $c_0 - w_0$  increases further, this researcher strictly prefers the outside sector to B. Since researchers can always switch to the outside sector, this researcher will move to the outside sector, causing the size of university B to decrease, raising the average ability at university B. Since the concern for relative ranking  $\alpha$  is low, we still have full segregation. Therefore, in CFS the researchers sort themselves into the two institutions and the outside sector in non-overlapping intervals with all types in  $[1 - \tilde{H}_A, 1]$  working for A, all types in  $[1 - \tilde{H}_A - H_B(\bar{\theta}), 1 - \tilde{H}_A]$  working for B, and the remaining types working in the outside sector. Thus, university B can no longer satisfy its demand. For further increases in  $c_0 - w_0$ , one of two things can happen. One possibility is that the quality difference between the universities becomes small enough that the utility from ranking starts to dominate that from peer effect for some researchers who consequently decide to move to B. The equilibrium then becomes constrained partial overlap (CPO). The other possibility is that  $\alpha$  is too low for the ranking component to become dominating. In that case, university B eventually becomes empty (USU). Increasing  $c_0 - w_0$  further, the lowest

type at university  $A$  is eventually indifferent between  $A$  and the outside sector, and the equilibrium becomes constrained single university (CSU).

Suppose next that we start in unconstrained partial overlap (UPO) and increase  $c_0 - w_0$ , holding  $\alpha$  constant. Initially, the lowest ranked researcher at  $B$  derives higher utility from working at  $B$  than he would from working in the outside sector, and the academic sector is unaffected by the increase in  $c_0 - w_0$ . Once  $c_0 - w_0$  reaches the UPO/CPO boundary, the lowest ranked researcher at  $B$  is indifferent between the outside sector and university  $B$ . Once  $c_0 - w_0$  increases further, this researcher strictly prefers the outside sector to university  $B$ . Since researchers can always switch to the outside sector, as with full segregation, he moves to the outside sector, causing the size of university  $B$  to decrease and the average ability of university  $B$  to rise. As  $c_0 - w_0$  increases further, the quality difference between the universities decreases until it is zero and the equilibrium becomes constrained full overlap (CFO). The less researchers care about their ranking within a university, the higher is the difference in base salaries for which the equilibrium becomes CFO. This is because low  $\alpha$  means that researchers derive little utility from ranking, so the difference in peer effect must be smaller before ranking dominates for a given type, who then moves to  $B$ . For sufficiently low  $\alpha$ , the ranking component never dominates.

Finally, if we start in unconstrained full overlap (UFO) and increase  $c_0 - w_0$ , the academic sector eventually becomes constrained as researchers find it worthwhile to move to the outside sector, and the equilibrium becomes CFO. As  $c_0 - w_0$  increases further, the total size of the academic sector falls. In CFO, the size of the academic sector is pinned down, but the size of each university is not determined. However, the relative size of universities does not affect researchers' utility because each type enjoys the same ranking and peer effect independent of the relative size of each university in CFO. Also, once  $c_0 - w_0$  reaches a threshold level ( $P_4$  in Figure 6), the equilibrium can be either CFO or CSU, depending on whether the sizes of the universities have decreased simultaneously or university  $B$  has been depleted first. If  $c_0 - w_0$  continues to increase, the academic sector is eventually empty (EAS).

The concern for ranking interacts with the outside option beyond what has already been described. A higher value of  $\alpha$  implies that a (weakly) higher value of  $c_0 - w_0$  is needed before the outside sector starts to constrain the academic sector. The reason is that a higher  $\alpha$  is associated with a higher weight on the utility derived from working in academia relative to that stemming from salary. In any constrained equilibrium, the researcher who is indifferent between academia and the outside sector has a rank of zero, so concern for ranking does not directly affect his utility. However, if there is partial overlap between universities, then increasing the concern for ranking increases the overlap and increases the peer effect at the less good university. So increasing the concern for ranking affects the lowest ranked researcher in academia by increasing their peer effect. Therefore, there are certain points in CPO starting from which increases in  $\alpha$  eventually make the equilibrium unconstrained, first with partial overlap and for even higher values of  $\alpha$  with full overlap.

A decrease in the salary progression difference  $\delta_w - \delta_c$  between the academic and outside sectors makes the academic sector less attractive, similar to how an increase in the base salary difference  $c_0 - w_0$  materializes. In particular, the academic salary of the lowest types in the academic sector decreases relative to their outside sector salary. These types now prefer to work in the outside sector. Hence, the size of the academic sector decreases. In CPO and CFS, as more researchers move to the outside sector, they leave university  $B$ . This increases the quality of university  $B$ , since those who left are of lower ability than those remaining at university  $B$ . As a result, the quality difference between the universities decreases.

In each of the constrained equilibria, all exit from academia is voluntary because the outside option is sufficiently attractive that types in the nonacademic sector prefer that sector to either university. Also, to summarize the intuition from Theorem 2, in CFS and CPO, when the outside option becomes better, it not only increases competition between academia and the outside sector, but also increases competition within the academic sector, since the quality difference between the universities decreases. An increase in the outside option also reduces the amount of university  $B$ 's demand that is satisfied (and later reduces that of university  $A$ 's).

In many fields, for example English and history, the outside sector does not constrain universities. In these fields, our results suggest that the allocation of researchers across universities is not affected by the outside sector or salary changes in the outside sector, and that exit from academia is involuntary. In contrast, the outside sector constrains universities in fields with a high outside option, possibly accounting, engineering, or law. In these fields, our results suggest that an increase in the outside sector salary reduces the quality difference across universities and the size of lower ranked departments, and that we observe voluntary exit from academia. If the institutions are interpreted as country clubs rather than universities, a reasonable parameterization would have the equilibrium fall in one of the unconstrained categories, since a major reason to join a country club is exactly to enjoy the peer effect. Our results then suggest that a marginal change in the price of playing golf on public golf courses would not affect the clientele of the country clubs.

A final result has to do with the mean salaries in the two universities:

**Corollary 1** (Endogenous salary difference). *Under the conditions of Theorem 2, the following hold: If there is wage progression in the academic sector (i.e.  $\delta_w > 0$ ), then the difference in average salary between the universities is proportional to the quality difference  $m_A - m_B$  between the universities. The difference in average salary is strictly positive in any equilibrium that does not have full overlap.*

Corollary 1 states that even though the two universities use the same salary function, and thus pay the same salary to any given type, if  $\delta_w > 0$  such that there is salary progression in the academic sector, then there will be a higher mean salary in university  $A$  than  $B$ . The difference in mean salaries is due to the different demographics at the two universities: on average researchers at  $A$  are more able than those at  $B$ , and as a

result those at  $A$  receive a higher salary on average. The model hence predicts that we should observe higher mean salaries at better universities, but that salary differences would disappear if we could control for differences in ability of those working there.

A related point is that the model predicts higher average academic salaries in fields with a high outside option. This is also due to differences in the demographics. Thus, in the context of our model, observed salary differences across fields are due to the better outside option making it a more selective group of researchers working in academia, rather than due to the better outside option driving up the entire academic wage function for the field. In the real world, we probably observe a mix of the two effects.

There is a number of interesting questions that could potentially be investigated using the present paper as a stepping stone. One generalization would be to allow for more than two institutions. With more than two institutions, increases in the outside option may compress the quality difference between all institutions, or they may compress quality differences only between lesser institutions.

In the present paper, each university hires the best researchers it attracts with the exogenous academic salary function. Another generalization could be to make the salary function discretionary for each institution. If salary compensation for a given type were endogenous, and potentially also a function of relative rank, then the institutions would have an explicit way of attracting high ability researchers. A lower ranked institution may then compete more aggressively in salary for intermediate abilities, because these provide positive externalities at the less good institution and negative externalities at the better institution. By overpaying (relative to the better school) for intermediate abilities, the lesser school can attract these intermediate abilities, which helps it raise its profile and in turn attracts other solid researchers.

One could also change how the peer effect is modelled so that top researchers in a department are given more weight than low ranked researchers. Finally, our equilibrium concept implicitly assumes that the institutions can perfectly observe a researcher's ability. If they instead could only observe ability with some noise, a modified equilibrium concept would be needed. While interesting, these issues are beyond the scope of the present paper and are left for future research.

## 4 Concluding remarks

We develop a sorting model of researchers, differing in ability, who are employed across universities and an outside sector. Each researcher derives utility from pecuniary compensation, and, in the academic sector, their rank and the peer effect. There is a unique stable equilibrium allocation of researchers in which the top of the ability distribution is allocated to the academic sector and the bottom of the ability distribution is allocated to the outside sector. If the academic sector is constrained by the outside sector, we find that an increase in competition from the outside sector decreases the quality difference between the higher

and lower ranked universities. This decrease in quality difference happens whether the increase in competition is from a higher general salary level or a stronger salary progression in the outside sector. These comparative statics suggest that in fields where the outside option is very attractive, one would expect the difference in research quality between top ranked and lower ranked departments to be smaller than in fields where the outside option is less attractive. Increased competition from the outside sector also decreases the difference in average salaries between the two universities. This difference suggests that in fields where the outside option is more attractive, we would expect the differences in average salaries across the academic sector to be smaller than in fields where the outside option is less attractive.

## A Proof of Theorem 1

Since  $T_i$ ,  $i = A, B, C$ , is defined as the closure of the set of types at which  $H_i$  is strictly increasing,  $\inf T_i = \min T_i$  and  $\sup T_i = \max T_i$ . We first prove that if  $T_A \neq \emptyset$ ,  $T_B \neq \emptyset$ , and  $m_A \geq m_B$  then  $\max T_A \geq \max T_B$  and  $\min T_A \geq \min T_B$ . Suppose to the contrary that  $\max T_A < \max T_B$ . Then there exists  $\hat{\theta} \in T_B$  such that  $r_B(\hat{\theta}) < 1$  and  $r_A(\hat{\theta}) = 1$ . Since  $m_A \geq m_B$ , agents of ability type  $\hat{\theta}$  strictly prefer  $A$ . This is a contradiction since ability type  $\hat{\theta}$  is higher than the lowest ability type working for  $A$  and thus has the option of moving to  $A$ . Suppose now, to reach another contradiction, that  $\min T_A < \min T_B$ . Then there exists  $\hat{\theta} \in T_B$  such that  $r_B(\hat{\theta}) = 0$  and  $r_A(\hat{\theta}) > 0$ . Since  $m_A \geq m_B$ , agents of ability type  $\hat{\theta}$  strictly prefer  $A$ . This is again a contradiction since type  $\hat{\theta}$  is higher than the lowest ability type working for  $A$  and thus has the option of moving to  $A$ .

We now show that if  $T_B \neq \emptyset$  and  $T_C \neq \emptyset$ , then  $\max T_B \geq \max T_C$  and  $\min T_B \geq \min T_C$ . To see this, note that  $T_B \neq \emptyset$  implies that  $m_B \in [\min T_B, \max T_B]$ . Also, note that for  $\theta = \min T_B$ , we have that  $r_B(\min T_B) = 0$ , so  $V_B(\beta, \min T_B) = m_B + \beta W(\min T_B)$ .

Suppose now, in order to reach a contradiction, that  $\min T_C > \min T_B$ . The support set  $T_i$  is defined as the closure of the set of types at which  $H_i$  is strictly increasing, so  $\min T_B$  must be a limit point of some neighborhood that belongs to  $T_B$ , and this neighborhood must necessarily be to the right of the minimum. Then for  $\theta = \min T_C$ , we have that

$$r_B(\min T_C) > 0. \quad (2)$$

Since researchers always have the possibility of switching to the outside sector, it must hold that

$$V_B(\beta, \min T_B) = m_B + \beta W(\min T_B) \geq \beta C(\min T_B) = V_C(\beta, \min T_B) \quad \forall \beta, \quad (3)$$

because otherwise some type  $(\beta, \theta)$  with  $\theta \in T_B$  close to  $\min T_B$  would move to the outside sector. Inequality (3) implies that

$$m_B \geq \beta [C(\min T_B) - W(\min T_B)] \quad \forall \beta. \quad (4)$$



Since  $W'(\theta) \geq C'(\theta)$  for all  $\theta$  and  $\min T_C > \min T_B$ , it must be the case that

$$C(\min T_B) - W(\min T_B) \geq C(\min T_C) - W(\min T_C). \quad (5)$$

But (2), (4) and (5) imply that  $m_B + \alpha r_B(\min T_C) > \beta [C(\min T_C) - W(\min T_C)] \forall \beta$ , which is equivalent to  $V_B(\beta, \min T_C) > V_C(\beta, \min T_C) \forall \beta$ . Hence, a researcher in  $T_C$  close to  $\min T_C$  would be better off switching to  $B$ , which contradicts that  $\min T_C > \min T_B$ . Thus,  $\min T_B \geq \min T_C$ .

Suppose instead, to reach another contradiction, that  $\max T_C > \max T_B$ . Then it must hold that

$$V_C(\beta, \max T_C) = \beta C(\max T_C) > m_B + \alpha + \beta W(\max T_C) = V_B(\beta, \max T_C) \forall \beta, \quad (6)$$

since researchers of ability type  $\theta = \max T_C$  have the option of moving to  $B$ , but are choosing  $C$  over  $B$ . Inequality (6) is equivalent to

$$\beta [C(\max T_C) - W(\max T_C)] > m_B + \alpha \quad \forall \beta. \quad (7)$$

Since  $C(\theta) - W(\theta)$  is non-increasing in  $\theta$ , and  $\max T_C > \max T_B$ , it must be the case that

$$C(\theta) - W(\theta) \geq C(\max T_C) - W(\max T_C) \quad (8)$$

for all  $\theta \in T_B$ . Also,  $r_B(\theta) \leq 1$  for all  $\theta \in T_B$ . This, together with (7) and (8) implies that

$$\beta [C(\theta) - W(\theta)] > m_B + \alpha r_B(\theta) \Leftrightarrow V_C(\beta, \theta) > V_B(\beta, \theta) \quad (9)$$

for all  $\theta \in T_B$  and for all  $\beta$ . Because researchers can always switch to the outside sector, this implies that  $T_B = \emptyset$ . Hence, assuming that  $\max T_C > \max T_B$  has led to a contradiction because it was assumed that  $T_B \neq \emptyset$ . Thus,  $\max T_C \leq \max T_B$ . An analogous argument shows that if  $T_A \neq \emptyset$  and  $T_C \neq \emptyset$ , then  $\max T_A \geq \max T_C$  and  $\min T_A \geq \min T_C$ .

We next show that  $T_A$ ,  $T_B$ , and  $T_C$  are intervals. To see this, suppose that  $T_A$  is non-empty and not an interval. Then there exist types  $\hat{\theta}$  and  $\tilde{\theta}$ , with  $\hat{\theta} < \tilde{\theta}$  such that all ability types on  $(\hat{\theta}, \tilde{\theta})$  choose  $B$  or  $C$ , while  $\hat{\theta} \in T_A$  and  $\tilde{\theta} \in T_A$ . Thus it must be that  $H_A(\hat{\theta}) = H_A(\tilde{\theta})$  and either  $H_B(\hat{\theta}) < H_B(\tilde{\theta})$  and/or  $H_C(\hat{\theta}) < H_C(\tilde{\theta})$ . Furthermore, since  $T_A$  is the closure of the set of types for which  $H_A$  is strictly increasing, types in small neighborhoods below  $\hat{\theta}$  and above  $\tilde{\theta}$  are in the support set of  $A$ .

If  $H_B(\hat{\theta}) < H_B(\tilde{\theta})$  then, since types  $\hat{\theta}$  and  $\tilde{\theta}$  have the option of switching between  $A$  and  $B$ , both ability types must be indifferent between  $A$  and  $B$ :

$$\alpha r_A(\hat{\theta}) + m_A = \alpha r_B(\hat{\theta}) + m_B \quad (10)$$

and

$$\alpha r_A(\tilde{\theta}) + m_A = \alpha r_B(\tilde{\theta}) + m_B. \quad (11)$$

Since  $r_A(\hat{\theta}) = r_A(\tilde{\theta})$ , equations (10) and (11) imply that  $r_B(\hat{\theta}) = r_B(\tilde{\theta})$ , which contradicts that  $H_B(\hat{\theta}) < H_B(\tilde{\theta})$ .

If instead  $H_C(\hat{\theta}) < H_C(\tilde{\theta})$  then, since all ability types  $\check{\theta} \in (\hat{\theta}, \tilde{\theta})$  have the option of switching between  $A$  and  $C$ , it must be the case that

$$V_A(\beta, \check{\theta}) = m_A + \alpha r_A(\check{\theta}) + \beta W(\check{\theta}) \leq \beta C(\check{\theta}) = V_C(\beta, \check{\theta}) \quad (12)$$

for all  $\beta$  and for all  $\check{\theta} \in (\hat{\theta}, \tilde{\theta})$ . Since  $\hat{\theta} \in T_A$ , ability types  $\check{\theta}$  in a small neighborhood below  $\hat{\theta}$  are also in  $T_A$ . For these  $\check{\theta}$ ,  $C(\check{\theta}) - W(\check{\theta}) \geq C(\hat{\theta}) - W(\hat{\theta})$  because  $C(\theta) - W(\theta)$  is non-increasing in  $\theta$ , and  $r_A(\check{\theta}) < r_A(\hat{\theta})$ . This together with (12), implies that  $m_A + \alpha r_A(\check{\theta}) < \beta [C(\check{\theta}) - W(\check{\theta})] \forall \beta \Leftrightarrow V_A(\beta, \check{\theta}) < V_C(\beta, \check{\theta})$  for all  $\beta$ , so these ability types would prefer to switch to the outside sector, which contradicts that they belong to the support set of  $A$ .

Hence, assuming that there exist types  $\hat{\theta}$  and  $\tilde{\theta}$ , with  $\hat{\theta} < \tilde{\theta}$  such that all types on  $(\hat{\theta}, \tilde{\theta})$  choose  $B$  or  $C$ , while  $\hat{\theta} \in T_A$  and  $\tilde{\theta} \in T_A$  has lead to a contradiction. It follows that  $T_A$  is an interval. A similar argument proves that  $T_B$  is an interval.

Suppose finally that  $T_C$  is non-empty and not an interval. Then there exist ability types  $\hat{\theta}$  and  $\tilde{\theta}$ , with  $\hat{\theta} < \tilde{\theta}$  such that all ability types on  $(\hat{\theta}, \tilde{\theta})$  choose  $A$  or  $B$ , while  $\hat{\theta} \in T_C$  and  $\tilde{\theta} \in T_C$ . Suppose, without loss of generality, that  $H_B(\hat{\theta}) < H_B(\tilde{\theta})$ .

Since ability types  $\check{\theta} \in (\hat{\theta}, \tilde{\theta})$  have the option of switching between  $B$  and  $C$ , it must be the case that

$$V_B(\beta, \check{\theta}) = m_B + \alpha r_B(\check{\theta}) + \beta W(\check{\theta}) \geq \beta C(\check{\theta}) \forall \beta \quad (13)$$

which gives

$$m_B + \alpha r_B(\check{\theta}) \geq \beta [C(\check{\theta}) - W(\check{\theta})] \forall \beta. \quad (14)$$

For  $\acute{\theta}$  in neighborhood above  $\tilde{\theta}$ ,  $C(\acute{\theta}) - W(\acute{\theta}) \geq C(\tilde{\theta}) - W(\tilde{\theta})$  because  $C(\theta) - W(\theta)$  is non-increasing in  $\theta$ , and  $r_B(\acute{\theta}) > r_B(\tilde{\theta})$ . With (14), this implies that

$$m_B + \alpha r_B(\acute{\theta}) > \beta [C(\acute{\theta}) - W(\acute{\theta})] \forall \beta \Rightarrow V_B(\beta, \acute{\theta}) > V_C(\beta, \acute{\theta}) \forall \beta. \quad (15)$$

This contradicts that  $\acute{\theta} \in T_C$ , so  $T_C$  must be an interval.

Next we establish that if  $T_B \neq \emptyset$  and  $T_C \neq \emptyset$ , then  $\max T_C \geq \min T_B$  for heterogenous  $\beta$ s and  $\max T_C = \min T_B$  for a homogenous  $\beta$ . A similar argument applies to show these inequalities with  $T_A$  replacing  $T_B$  when  $T_B = \emptyset$ ,  $T_A \neq \emptyset$  and  $T_C \neq \emptyset$ .

Since researchers always have the option of switching to the outside sector, we must have that

$$V_B(\beta, \min T_B) = m_B + \beta W(\min T_B) \geq \beta C(\min T_B) = V_C(\beta, \min T_B) \text{ for some } \beta. \quad (16)$$

Which implies that  $m_B \geq \beta [C(\min T_B) - W(\min T_B)]$  for some  $\beta$ .

If some researchers of ability type  $\theta = \min T_B$  prefer the outside sector to university B, i.e.  $m_B < \hat{\beta} [C(\min T_B) - W(\min T_B)]$  for some  $\hat{\beta}$ , then  $\max T_C > \min T_B$ . Suppose

instead, for a contradiction, that  $\max T_C \leq \min T_B$ . Since researchers always have the option of switching to the outside sector, we must have that  $V_B(\beta, \min T_B) \geq V_C(\beta, \min T_B)$  for all  $\beta$ , which implies  $m_B \geq \beta [C(\min T_B) - W(\min T_B)]$  for all  $\beta$ . This contradicts that  $m_B < \hat{\beta} [C(\min T_B) - W(\min T_B)]$  for some  $\hat{\beta}$ . Therefore,  $\max T_C > \min T_B$  if  $m_B < \hat{\beta} [C(\min T_B) - W(\min T_B)]$  for some  $\hat{\beta}$ .

If  $W(\min T_B) \geq C(\min T_B)$ , then (16) holds for all  $\beta$  and  $\min T_B = \max T_C$ . To see this, suppose that  $\max T_C > \min T_B$ . Then we have  $r_B(\max T_C) > 0$ . Under the supposition we must have  $V_C(\beta, \max T_C) \geq V_B(\beta, \max T_C)$  for some  $\beta$ , which implies

$$0 \geq m_B + \alpha r_B(\max T_C) + \beta [W(\max T_C) - C(\max T_C)]. \quad (17)$$

Since  $W(\theta) - C(\theta)$  is non-decreasing in  $\theta$  and  $W(\min T_B) - C(\min T_B) \geq 0$ , we have that  $W(\max T_C) - C(\max T_C) \geq 0$ . This contradicts (17) because  $r_B(\max T_C) > 0$  and  $m_B \geq 0$ . Therefore, if  $W(\min T_B) \geq C(\min T_B)$ , then  $\max T_C \leq \min T_B$ . Finally, by definition an equilibrium allocation is a feasible allocation and thus has  $H_A(\theta) + H_B(\theta) + H_C(\theta) = F(\theta)$  for all  $\theta \in [\underline{\theta}, \bar{\theta}]$ , i.e. in any equilibrium every researcher works in either  $A$ ,  $B$ , or  $C$ . Therefore, and because  $T_A$ ,  $T_B$ , and  $T_C$  are intervals,  $\min T_B \geq \max T_C \Rightarrow \min T_B = \max T_C$ . Then, again by feasibility of the equilibrium, it must also be the case that  $\max T_B \geq \min T_A$ .

To establish that  $\max T_C = \min T_B$  with homogenous  $\beta$ , suppose that  $\max T_C > \min T_B$ . A researcher of ability type  $\min T_B$  is the lowest ability researcher who works for  $B$  and therefore has rank  $r_B(\min T_B) = 0$ . Since it is already established that  $\min T_B \geq \min T_C$ , it must hence be that

$$V_B(\min T_B) \geq V_C(\min T_B) \Leftrightarrow m_B + \beta W(\min T_B) \geq \beta C(\min T_B) \Leftrightarrow m_B \geq \beta [C(\min T_B) - W(\min T_B)]. \quad (18)$$

Researchers with  $\theta \in (\min T_B, \max T_C)$  must be indifferent between  $B$  and the outside option. Furthermore,  $C(\theta) - W(\theta) \leq C(\min T_B) - W(\min T_B)$  for  $\theta \in (\min T_B, \max T_C)$ , since  $W'(\theta) \geq C'(\theta)$  for all  $\theta$ . We thus have that for  $\theta \in (\min T_B, \max T_C)$ ,

$$V_B(\theta) = V_C(\theta) \Leftrightarrow \alpha r_B(\theta) + m_B = \beta [C(\theta) - W(\theta)] \leq \beta [C(\min T_B) - W(\min T_B)] \leq m_B,$$

using the result in (18). This implies that  $r_B(\theta) \leq 0$ , a contradiction with  $\max T_C > \min T_B$ . Therefore,  $(\min T_B, \max T_C) = \emptyset$ , which establishes that  $\min T_B \geq \max T_C$ . Following the same argument as the case with heterogenous  $\beta$ s,  $\min T_B \geq \max T_C \Rightarrow \min T_B = \max T_C$  and  $\max T_B \geq \min T_A$ . Hence, we have established that there exists  $v \in [\underline{\theta}, \bar{\theta}]$  such that  $\theta \in [\underline{\theta}, v]$  work in the outside sector, while  $\theta \in [v, \bar{\theta}]$  work in the academic sector. ■

## B Complete characterization for, and proof of, Theorem 2:

**Complete characterization for Theorem 2:** Under Assumptions 1 and 2 there exists a unique stable sorting equilibrium.

Defining the following constants

- $P_1 \equiv \frac{2-2\tilde{H}_A-\tilde{H}_B}{2} - \Delta(1 - \tilde{H}_A - \tilde{H}_B)$
- $P_2 \equiv \frac{2-\tilde{H}_A-\tilde{H}_B}{2} - \Delta(1 - \tilde{H}_A - \tilde{H}_B)$
- $P_3 \equiv (1 - \tilde{H}_A) - \Delta(1 - \tilde{H}_A)$
- $P_4 \equiv \frac{2-\tilde{H}_A}{2} - \Delta(1 - \tilde{H}_A)$
- $P_5 \equiv 1 - \Delta(1),$

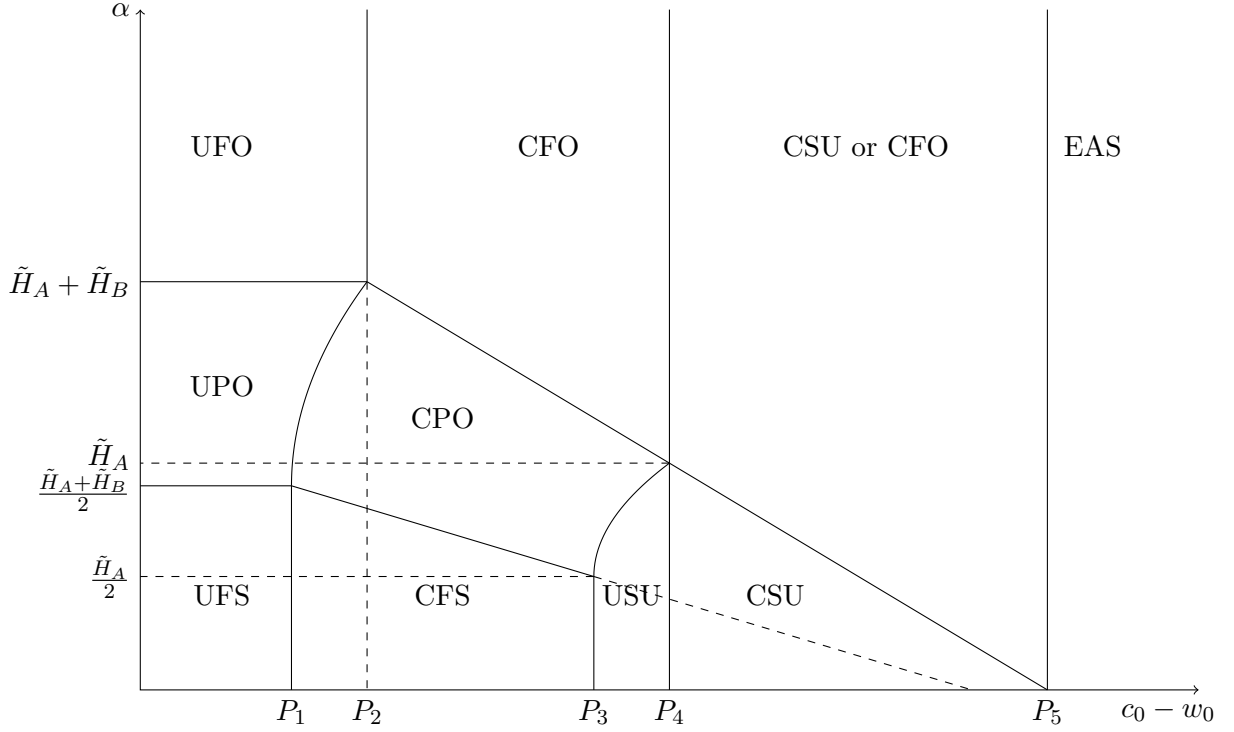
and the following functions

- $\rho_1(\alpha) = \frac{2(1-\alpha)-\tilde{H}_A}{2} - \Delta(1 - 2\alpha)$
- $\rho_2(\alpha) = \frac{2-\tilde{H}_A}{2} + 2\alpha \left( \frac{\alpha}{\tilde{H}_A} - 1 \right) - \Delta(1 - \tilde{H}_A)$
- $\rho_3(\alpha) = \frac{2-\tilde{H}_A-\tilde{H}_B}{2} + \frac{2\alpha\tilde{H}_A}{\tilde{H}_A+\tilde{H}_B} \left( \frac{\alpha}{\tilde{H}_A+\tilde{H}_B} - 1 \right) - \Delta(1 - \tilde{H}_A - \tilde{H}_B)$
- $\rho_4(\alpha) = \frac{2-\alpha}{2} - \Delta(1 - \alpha),$

the equilibrium is

- UFS if  $c_0 - w_0 < P_1$ , and  $\alpha \leq \frac{\tilde{H}_A+\tilde{H}_B}{2}$
- CFS if  $c_0 - w_0 \geq P_1$ ,  $c_0 - w_0 \leq \rho_1(\alpha)$ , and  $c_0 - w_0 < P_3$
- USU if  $c_0 - w_0 \geq P_3$ ,  $c_0 - w_0 \geq \rho_2(\alpha)$  for  $\alpha \geq \frac{\tilde{H}_A}{2}$ , and  $c_0 - w_0 < P_4$
- UPO if  $c_0 - w_0 < \rho_3(\alpha)$ , and  $\frac{\tilde{H}_A+\tilde{H}_B}{2} < \alpha < \tilde{H}_A + \tilde{H}_B$
- CPO if  $c_0 - w_0 > \rho_1(\alpha)$ ,  $c_0 - w_0 \geq \rho_3(\alpha)$  for  $\alpha \geq \frac{\tilde{H}_A+\tilde{H}_B}{2}$ ,  $c_0 - w_0 < \rho_2(\alpha)$ , and  $c_0 - w_0 < \rho_4(\alpha)$
- UFO if  $c_0 - w_0 < P_2$ , and  $\alpha \geq \tilde{H}_A + \tilde{H}_B$
- CFO if  $c_0 - w_0 \geq P_2$ ,  $c_0 - w_0 \geq \rho_4(\alpha)$ , and  $c_0 - w_0 < P_4$
- CSU if  $P_4 \leq c_0 - w_0$ , and  $c_0 - w_0 < \rho_4(\alpha)$
- CFO or CSU if  $P_4 \leq c_0 - w_0 < P_5$ , and  $c_0 - w_0 \geq \rho_4(\alpha)$
- EAS if  $P_5 \leq c_0 - w_0$ .

Figure 7: Sorting equilibrium with affine salary functions for  $\tilde{H}_A > \tilde{H}_B$



**Proof of Theorem 2:** To prove Theorem 2, we first solve for the cut-offs  $v$ ,  $z$ ,  $y$ , and  $x$ , as well as for the slopes of the non-normalized type distributions  $H_A$ ,  $H_B$ , and  $H_C$  across  $(c_0 - w_0, \alpha)$ -space. We can then solve for the regions of  $(c_0 - w_0, \alpha)$ -space in which the different equilibria exist, for the universities' qualities, and for the influence of the outside option on the size of the academic sector in the constrained equilibria. We proceed to show that the equilibria described in Theorem 2 are stable and that there is a unique stable equilibrium across  $(c_0 - w_0, \alpha)$ -space. Finally, we prove the comparative statics results.

Recall that according to Assumption 1,  $\bar{\theta} = 1$ . Also, note that equation (1) gives that

$$r_i(\theta) = \frac{H_i(\theta)}{H_i(1)}.$$

A researcher of type  $z$  is the highest ability researcher who works in the outside sector. Hence,  $H_C(\theta)$  is constant and equal to  $H_C(1)$  on  $[z, 1]$ . As established in Theorem 1,  $z = v$  and thus  $H_B(z) = H_A(z) = 0$ , and it follows that  $H_C(z) = F(z) - H_A(z) - H_B(z) = F(z)$ . Since  $F(z) = z$ , it follows that  $z = H_C(1)$ . On  $[0, z)$  everyone works in the outside sector.

We conclude that the non-normalized distribution of types in the outside sector is

$$H_C(\theta) = \begin{cases} F(\theta) & \text{if } \theta \in [0, z) \\ F(z) & \text{if } \theta \in [z, 1]. \end{cases}$$

Consider first full segregation. With full segregation,  $x = y$ . The non-normalized distributions of types,  $H_A$ ,  $H_B$ , and  $H_C$ , under full segregation are illustrated in Figure 2. By Theorem 1, it is the top end of the ability distribution that will choose to work in academia. With full segregation, all of the highest ability researchers choose to work for the better university  $A$ , while the best of the remaining researchers choose to work for  $B$ . Only a researcher with  $\theta = x$  is indifferent between  $A$  and  $B$ . It follows easily, in addition to the values for  $H_C(\theta)$  found above, that under full segregation

$$H_B(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, z] \\ F(\theta) - H_C(1) & \text{if } \theta \in (z, x) \\ F(x) - H_C(1) & \text{if } \theta \in [x, 1]. \end{cases}$$

and

$$H_A(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, x) \\ F(\theta) - H_B(1) - H_C(1) & \text{if } \theta \in [x, 1] \end{cases}$$

In particular, we have that  $z = v = H_C(1)$  and  $x = y = 1 - H_A(1) = H_B(1) + H_C(1)$ .

The peer effect at university  $A$  is then given by

$$m_A = \frac{2 - H_A(1)}{2}$$

while the peer effect at university  $B$  is given by

$$m_B = \frac{2 - 2H_A(1) - H_B(1)}{2}.$$

The full segregation equilibrium is unconstrained as long as  $\frac{2 - 2\tilde{H}_A - \tilde{H}_B}{2} \geq c_0 - w_0 + \Delta(1 - \tilde{H}_A - \tilde{H}_B)$ , that is, as long as the peer effect in  $B$  when both universities' demands are satisfied is at least as high as the wage difference between the outside and academic sectors for the threshold type  $1 - \tilde{H}_A - \tilde{H}_B$ . When this is satisfied, the lowest ranked researcher in  $B$  has higher utility from working at  $B$  than from working in the outside sector. Therefore, the boundary between UFS and CFS is given by

$$c_0 - w_0 = \frac{2 - 2\tilde{H}_A - \tilde{H}_B}{2} - \Delta(1 - \tilde{H}_A - \tilde{H}_B).$$

Full segregation also requires that the difference in peer effect is large enough that no researcher wishes to move from  $A$  to  $B$ . Under UFS, this condition is given by

$$m_A - m_B = \frac{\tilde{H}_A + \tilde{H}_B}{2} \geq \alpha. \tag{19}$$

As long as (19) is satisfied, the lowest ranked researcher at  $A$  has at least as high utility from staying at  $A$  as from moving to  $B$ . Thus, the North boundary of UFS is given by

$$\alpha = \frac{\tilde{H}_A + \tilde{H}_B}{2}.$$

When instead  $\frac{2-2\tilde{H}_A-\tilde{H}_B}{2} < c_0 - w_0 + \Delta(1 - \tilde{H}_A - \tilde{H}_B)$ , the full segregation equilibrium is constrained and the size of the academic sector is determined by the equation

$$\frac{2 - 2H_A(1) - H_B(1)}{2} = c_0 - w_0 + \Delta(1 - H_A(1) - H_B(1)). \quad (20)$$

That is, the size of the academic sector is pinned down by  $m_B = C(1 - H_A(1) - H_B(1)) - W(1 - H_A(1) - H_B(1))$  such that the lowest ranked researcher in  $B$  gets exactly the same utility from working at  $B$  as he would from working in the outside sector.

The lowest ranked researcher at  $B$ ,  $\min T_B$ , strictly prefers  $A$  to  $B$  in CFS, since his ranking is zero at either university and  $m_A > m_B$  in CFS. Suppose, in order to reach a contradiction, that  $H_A(1) < \tilde{H}_A$ . Since  $\min T_B \in T_B$ , condition (ii) of Definition 2 implies that  $V_B(\min T_B) \geq V_A(\min T_B)$ , which contradicts that  $\min T_B$  strictly prefers  $A$  to  $B$ . We therefore have that  $H_A(1) = \tilde{H}_A$  in CFS. This together with equation (20) implies that under CFS the size of university B,  $H_B(1)$ , is determined implicitly by the equation

$$c_0 - w_0 = \frac{2 - 2\tilde{H}_A - H_B(1)}{2} - \Delta(1 - \tilde{H}_A - H_B(1)). \quad (21)$$

In CFS, university B hires some researchers,  $H_B(1) > 0$ . Setting  $H_B(1) = 0$  in equation (21) gives the CFS/USU boundary:

$$c_0 - w_0 = 1 - \tilde{H}_A - \Delta(1 - \tilde{H}_A). \quad (22)$$

At the CFS/CPO boundary, the lowest type at university A is indifferent between university A and university B:  $\frac{\tilde{H}_A + H_B(1)}{2} = \alpha$ . Rearranging, we have  $H_B(1) = 2\alpha - \tilde{H}_A$ . Therefore, we need  $\alpha \geq \frac{\tilde{H}_A}{2}$  for  $H_B(1) \geq 0$ .

Substituting  $H_B(1) = 2\alpha - \tilde{H}_A$  into equation (21) gives the CFS/CPO boundary:

$$c_0 - w_0 = \frac{2(1 - \alpha) - \tilde{H}_A}{2} - \Delta(1 - 2\alpha) \quad \text{for } \alpha \geq \frac{\tilde{H}_A}{2}. \quad (23)$$

Totally differentiating equation (23) gives that on the CFS/CPO boundary,

$$\frac{d\alpha}{d(c_0 - w_0)} = - [1 - 2\Delta'(1 - 2\alpha)]^{-1} < 0,$$

so the CFS/CPO boundary is downward sloping.

Consider now partial overlap. With partial overlap,  $y < x < 1$ . The non-normalized distributions of types,  $H_A$ ,  $H_B$ , and  $H_C$ , under partial overlap are illustrated in Figure 3. Again, by Theorem 1, it is the top end of the ability distribution that will choose to work in academia. When there is overlap, all researchers with ability  $\theta$  in the interval  $[y, x]$  are indifferent between the two universities, thus both  $H_A(\theta)$  and  $H_B(\theta)$  are increasing on  $[y, x]$  as illustrated in Figure 3.

To find the non-normalized type distributions  $H_A$  and  $H_B$  under partial overlap, we first find their values at the critical points  $z$ ,  $y$ , and  $x$ . With  $z = v$ , we see that  $H_B(y) = F(y) - H_C(1)$ , since researchers in  $(z, y)$  are allocated exclusively to  $B$ . A researcher of type  $x$  is the highest ability researcher who works for  $B$ , hence  $H_B(x) = H_B(1)$  and  $H_B(\theta)$  is constant thereafter. Since  $y$  is the lowest type who works for  $A$ ,  $H_A(y) = 0$ . Finally, researchers in  $(x, 1]$  are allocated exclusively to  $A$ , hence  $H_A(x) = F(x) - H_B(1) - H_C(1)$ .

Since a researcher of type  $y$  is indifferent between  $A$  and  $B$ , we must have that

$$\begin{aligned} V_B(y) = V_A(y) &\Leftrightarrow \alpha \frac{F(y) - H_C(1)}{H_B(1)} + m_B + W(y) = \alpha \frac{H_A(y)}{H_A(1)} + m_A + W(y) \\ &\Leftrightarrow \alpha \frac{y - H_C(1)}{H_B(1)} + m_B = m_A. \end{aligned} \quad (24)$$

Since a researcher of type  $x$  is also indifferent between  $A$  and  $B$ , we have that

$$\begin{aligned} V_B(x) = V_A(x) &\Leftrightarrow \alpha \frac{H_B(1)}{H_B(1)} + m_B + W(x) = \alpha \frac{F(x) - H_B(1) - H_C(1)}{H_A(1)} + m_A + W(x) \\ &\Leftrightarrow m_B = \alpha \frac{x - 1}{H_A(1)} + m_A. \end{aligned} \quad (25)$$

Equations (24) and (25) imply that

$$\frac{y - H_C(1)}{H_B(1)} = \frac{1 - x}{H_A(1)}. \quad (26)$$

Furthermore, researchers with  $\theta \in (y, x)$  are also indifferent between  $A$  and  $B$ . Hence, for  $\theta \in (y, x)$ ,

$$\begin{aligned} V_B(\theta) = V_A(\theta) &\Leftrightarrow \alpha \frac{H_B(\theta)}{H_B(1)} + m_B + W(\theta) = \alpha \frac{H_A(\theta)}{H_A(1)} + m_A + W(\theta) \\ &\Leftrightarrow H_B(\theta) = \frac{H_B(1)}{H_A(1)} H_A(\theta) + \frac{H_B(1)}{\alpha} (m_A - m_B). \end{aligned} \quad (27)$$

It follows from (27) and the fact that on this interval  $H_A(\theta) + H_B(\theta) = F(\theta) - H_C(1) = \theta - H_C(1)$ , that  $H_A(\theta)$  and  $H_B(\theta)$  must be linear on  $(y, x)$ , and if  $H_A(1) > H_B(1)$  then  $H_B(\theta)$  is flatter than  $H_A(\theta)$ , and vice versa.



We can find the slopes of  $H_A$  and  $H_B$  on  $[y, x]$  from the critical points above and conclude that under partial overlap the non-normalized distribution of types at university  $A$  is

$$H_A(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, y) \\ \frac{x+H_A(1)-1}{x-y}\theta - y\frac{x+H_A(1)-1}{x-y} & \text{if } \theta \in [y, x] \\ F(\theta) - H_B(1) - H_C(1) & \text{if } \theta \in (x, 1] \end{cases}$$

while the non-normalized distribution of types at university  $B$  is

$$H_B(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, z] \\ F(\theta) - H_C(1) & \text{if } \theta \in (z, y) \\ \frac{1-y-H_A(1)}{x-y}\theta - x\frac{1-y-H_A(1)}{x-y} + H_B(1) & \text{if } \theta \in [y, x] \\ H_B(1) & \text{if } \theta \in (x, 1]. \end{cases}$$

The peer effect at university  $B$  is then given by

$$m_B = \int_z^y \frac{1}{H_B(1)} \theta d\theta + \int_y^x \frac{1 - F(y) - H_A(1)}{H_B(1)(x-y)} \theta d\theta = \frac{1}{2} \frac{y^2 - z^2}{H_B(1)} + \frac{1}{2} \frac{1 - y - H_A(1)}{H_B(1)} (x + y), \quad (28)$$

while the peer effect at university  $A$  is

$$m_A = \int_y^x \theta \frac{F(x) + H_A(1) - 1}{H_A(1)(x-y)} d\theta + \int_x^1 \frac{1}{H_A(1)} \theta d\theta = \frac{1}{2} \left( \frac{x-1}{H_A(1)} + 1 \right) (x + y) + \frac{1}{2} \frac{1 - x^2}{H_A(1)}. \quad (29)$$

Using (28) and (29), the difference in peer effect between  $A$  and  $B$  is given by

$$m_A - m_B = \frac{1}{2} (x + y) \left( 1 + \frac{x-1}{H_A(1)} - \frac{1-y-H_A(1)}{H_B(1)} \right) + \frac{1}{2} \frac{1-x^2}{H_A(1)} - \frac{1}{2} \frac{y^2 - z^2}{H_B(1)}.$$

This and equation (26) imply that

$$m_A - m_B = \frac{H_A(1) + H_B(1)}{2H_A(1)^2} (1-x)(2H_A(1) - 1 + x),$$

which together with equation (25) can be used to find that

$$x = 1 - H_A(1) + H_A(1) \left( \frac{2\alpha}{H_A(1) + H_B(1)} - 1 \right). \quad (30)$$

Plugging this back into (26) then gives that

$$y = 1 - H_A(1) - H_B(1) \left( \frac{2\alpha}{H_A(1) + H_B(1)} - 1 \right). \quad (31)$$

Using (29), (30), and (31), we now have that

$$m_A = \frac{2 - H_A(1) - H_B(1)}{2} + \frac{2\alpha H_B(1)}{H_A(1) + H_B(1)} \left( 1 - \frac{\alpha}{H_A(1) + H_B(1)} \right) \quad (32)$$

and

$$m_B = \frac{2 - H_A(1) - H_B(1)}{2} + \frac{2\alpha H_A(1)}{H_A(1) + H_B(1)} \left( \frac{\alpha}{H_A(1) + H_B(1)} - 1 \right). \quad (33)$$

From equations (30) and (31) it follows that  $x \geq y$  is equivalent to

$$\alpha \geq \frac{H_A(1) + H_B(1)}{2}, \quad (34)$$

which is consistent with the North boundaries of UFS and CFS. From equation (30) it also follows that  $x \leq 1$  if and only if

$$\alpha \leq H_A(1) + H_B(1), \quad (35)$$

and  $x = 1$  if and only if (35) holds with equality. When the latter is true it is also the case that  $y = 1 - H_A(1) - H_B(1)$ , c.f. (31), and the overlap is full rather than partial.

The partial overlap equilibrium is unconstrained as long as the peer effect at university  $B$ , which is given in (33), calculated with the universities actual demands  $\tilde{H}_A$  and  $\tilde{H}_B$  is at least as high as the wage difference between the outside and academic sectors. That is, the equilibrium is UPO as long as

$$\frac{2 - \tilde{H}_A - \tilde{H}_B}{2} + \frac{2\alpha\tilde{H}_A}{\tilde{H}_A + \tilde{H}_B} \left( \frac{\alpha}{\tilde{H}_A + \tilde{H}_B} - 1 \right) \geq c_0 - w_0 + \Delta(1 - \tilde{H}_A - \tilde{H}_B). \quad (36)$$

Equality in (36) defines a curve in  $(c_0 - w_0, \alpha)$ -space, which is the boundary between UPO and CPO. Solving for  $c_0 - w_0$  in (36), the boundary is given by

$$c_0 - w_0 = \frac{2 - \tilde{H}_A - \tilde{H}_B}{2} + \frac{2\alpha\tilde{H}_A}{\tilde{H}_A + \tilde{H}_B} \left( \frac{\alpha}{\tilde{H}_A + \tilde{H}_B} - 1 \right) - \Delta(1 - \tilde{H}_A - \tilde{H}_B). \quad (37)$$

It follows from (34) and (35) that the South boundary of UPO is given by

$$\alpha = \frac{\tilde{H}_A + \tilde{H}_B}{2} \quad (38)$$

and the North boundary of UPO is given by

$$\alpha = \tilde{H}_A + \tilde{H}_B.$$

Totally differentiating (37) gives that the UPO/CPO boundary is upward sloping,

$$\frac{d\alpha}{d(c_0 - w_0)} = \frac{(\tilde{H}_A + \tilde{H}_B)^2}{2\tilde{H}_A} \left[ 2\alpha - \tilde{H}_A - \tilde{H}_B \right]^{-1} > 0$$

because from (38)  $\alpha \geq \frac{\tilde{H}_A + \tilde{H}_B}{2}$  in UPO.

When (36) does not hold, the partial overlap equilibrium is constrained and the size of the academic sector is determined by the equation

$$\frac{2 - H_A(1) - H_B(1)}{2} + \frac{2\alpha H_A(1)}{H_A(1) + H_B(1)} \left( \frac{\alpha}{H_A(1) + H_B(1)} - 1 \right) = c_0 - w_0 + \Delta(1 - H_A(1) - H_B(1)). \quad (39)$$

That is, the size of the academic sector is pinned down by  $m_B = C(1 - H_A(1) - H_B(1)) - W(1 - H_A(1) - H_B(1))$  such that the lowest ranked researcher in  $B$  gets exactly the same utility from working at  $B$  as he would from working in the outside sector.

The lowest ranked researcher at  $B$ ,  $\min T_B$ , strictly prefers  $A$  to  $B$  in CPO, since his ranking is zero at either university and  $m_A > m_B$  in CPO. Suppose, in order to reach a contradiction, that  $H_A(1) < \tilde{H}_A$ . Since  $\min T_B \in T_B$ , condition (ii) of Definition 2 implies that  $V_B(\min T_B) \geq V_A(\min T_B)$ , which contradicts that  $\min T_B$  strictly prefers  $A$  to  $B$ . We therefore have that  $H_A(1) = \tilde{H}_A$  in CPO. Thus, the size of university B in CPO is determined implicitly by the equation

$$c_0 - w_0 = \frac{2 - \tilde{H}_A - H_B(1)}{2} + \frac{2\alpha \tilde{H}_A}{\tilde{H}_A + H_B(1)} \left( \frac{\alpha}{\tilde{H}_A + H_B(1)} - 1 \right) - \Delta(1 - \tilde{H}_A - H_B(1)). \quad (40)$$

We need  $H_B(1) \geq 0$  in CPO. Setting  $H_B(1) = 0$  in equation (40) give the CPO/USU boundary:

$$c_0 - w_0 = \frac{2 - \tilde{H}_A}{2} + 2\alpha \left( \frac{\alpha}{\tilde{H}_A} - 1 \right) - \Delta(1 - \tilde{H}_A). \quad (41)$$

Totally differentiating (41) gives that the boundary between CPO and USU is upward sloping,

$$\frac{d\alpha}{d(c_0 - w_0)} = \frac{1}{2} \left[ \frac{2\alpha}{\tilde{H}_A} - 1 \right]^{-1} \geq 0$$

because with  $H_A(1) = \tilde{H}_A$  and  $H_B(1) = 0$  equation (34) gives that  $\alpha \geq \frac{\tilde{H}_A}{2}$ .

At the CPO/CFO boundary, university B hires researchers of the highest type,  $x = 1$ . Setting  $x = 1$  and  $H_A(1) = \tilde{H}_A$  in equation (30) gives  $H_B(1) = \alpha - \tilde{H}_A$ . Thus,  $H_B(1) \geq 0$  if and only if  $\alpha \geq \tilde{H}_A$  at the CPO/CFO boundary. Substituting  $\alpha = \tilde{H}_A + H_B(1)$  into (40) gives CPO/CFO boundary:

$$c_0 - w_0 = \frac{2 - \alpha}{2} - \Delta(1 - \alpha) \text{ for } \alpha \geq \tilde{H}_A. \quad (42)$$

Totally differentiating equation (42) gives that the CPO/CFO boundary is downward sloping:

$$\frac{d\alpha}{d(c_0 - w_0)} = - \left[ \frac{1}{2} - \Delta'(1 - \alpha) \right]^{-1} < 0.$$

Consider now full overlap. With full overlap,  $x = 1$  and  $y = H_C(1)$ . In this case,  $A$  and  $B$  share all types in academia, i.e. all types in academia are indifferent between  $A$  and  $B$ . Then

$$m_A = m_B = \frac{2 - H_A(1) - H_B(1)}{2}. \quad (43)$$

This scenario is illustrated in Figure 4.

The full overlap equilibrium is unconstrained as long as

$$\frac{2 - \tilde{H}_A - \tilde{H}_B}{2} \geq c_0 - w_0 + \Delta(1 - \tilde{H}_A - \tilde{H}_B). \quad (44)$$

If (44) is not satisfied, the full overlap equilibrium is constrained, that is, CFO. Then the size of the academic sector is determined by

$$\frac{2 - H_A(1) - H_B(1)}{2} = c_0 - w_0 + \Delta(1 - H_A(1) - H_B(1)). \quad (45)$$

In CFO, all types in academia are indifferent between  $A$  and  $B$ . Thus, the size of each university is undetermined. However, the relative university sizes are irrelevant for the researchers' utilities.

Setting  $H_A(1) + H_B(1) = 0$  in (45) gives that the CFO/EAS boundary is

$$c_0 - w_0 = 1 - \Delta(1). \quad (46)$$

Setting  $H_A(1) = \tilde{H}_A$  and  $H_B(1) = 0$  in (45) gives that for  $c_0 - w_0 \geq P_4$ , the CFO equilibrium may in fact be a CSU equilibrium.

As we show later, CFO is only stable if  $\alpha \geq H_A(1) + H_B(1)$ . Plugging  $\alpha = H_A(1) + H_B(1)$  into (45) gives that the South CFO boundary is

$$c_0 - w_0 = \frac{2 - \alpha}{2} - \Delta(1 - \alpha). \quad (47)$$

Consider finally single university. The non-normalized distributions of types,  $H_A$ ,  $H_B$ , and  $H_C$ , under single university are illustrated in Figure 5. By Theorem 1, the top end of the distribution will choose to work for the better university  $A$ , and the remaining researchers will work for the outside sector. It follow that under single university

$$H_C(\theta) = \begin{cases} F(\theta) & \text{if } \theta \in [0, z) \\ F(z) & \text{if } \theta \in [z, 1], \end{cases}$$

$$H_B(\theta) = 0 \quad \forall \theta \in [0, 1],$$

and

$$H_A(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, z) \\ F(\theta) - H_C(1) & \text{if } \theta \in [z, 1]. \end{cases}$$

In particular, we have that  $z = y = H_C(1)$ .

The peer effect at university  $A$  is then given by

$$m_A = \frac{2 - H_A(1)}{2}$$

and the peer effect at university  $B$  is  $m_B = 0$ .

Single university requires that no researcher would like to move from university  $A$  to university  $B$ . Under Assumption 2, a researcher will have a rank of zero if he moves to an empty university. The lowest type in  $A$  weakly prefers university  $A$  to university  $B$  if and only if  $m_A \geq \alpha r_B(1 - H_A(1)) + m_B$ . Under Assumption 2, this condition becomes  $m_A \geq m_B$ . This condition always holds because  $m_A = \frac{2 - H_A(1)}{2}$  and the peer effect at university  $B$  is  $m_B = 0$  because there are no peers at the empty university  $B$ . Note that all other types in  $A$  also prefer to stay in  $A$ , since for those types  $m_A + \alpha r_A(\theta) > 0 \Leftrightarrow V_A(\theta) > V_B(\theta)$ .

The lowest type in  $A$  weakly prefers university  $A$  to the outside sector as long as  $\frac{2 - H_A(1)}{2} \geq c_0 - w_0 + \Delta(1 - H_A(1))$ . The size of university  $A$  is unconstrained if  $\frac{2 - \tilde{H}_A}{2} \geq c_0 - w_0 + \Delta(1 - \tilde{H}_A)$ . That is, if the peer effect in  $A$  is at least as high as the wage difference between the outside and academic sectors for the lowest type in  $A$ ,  $1 - \tilde{H}_A$ . Therefore, the USU/CSU boundary is given by

$$c_0 - w_0 = \frac{2 - \tilde{H}_A}{2} - \Delta(1 - \tilde{H}_A).$$

If instead  $\frac{2 - \tilde{H}_A}{2} < c_0 - w_0 + \Delta(1 - \tilde{H}_A)$ , then the single university equilibrium is constrained and the size of university  $A$  is implicitly determined by

$$c_0 - w_0 = \frac{2 - H_A(1)}{2} - \Delta(1 - H_A(1)). \quad (48)$$

Setting  $H_A(1) = 0$  gives the CSU/EAS boundary,

$$c_0 - w_0 = 1 - \Delta(1). \quad (49)$$

When this is satisfied, the highest type is indifferent between university  $A$  and the outside sector.

Note, for  $c_0 - w_0 \in \left[ \frac{2 - \tilde{H}_A}{2} - \Delta(1 - \tilde{H}_A), 1 - \Delta(1) \right]$  and  $c_0 - w_0 \geq \frac{2 - \alpha}{2} - \Delta(1 - \alpha)$ , both CFO and CSU equilibria are possible.

In full overlap equilibria, since all types in academia are indifferent between  $A$  and  $B$ , equation (10) is satisfied for all types in academia. Therefore, full overlap equilibria are in principle possible across  $(c_0 - w_0, \alpha)$ -space. However, as we will show next, they are only stable when  $\alpha \geq H_A + H_B$ , resulting in the unique stable equilibrium being as depicted in Figures 6 and 7.

We show stability by applying Theorem 6.5 in Stokey and Lucas (1989). Let  $\mu$  denote the mean type in academia. That is,

$$\mu = \frac{2 - H_A(1) - H_B(1)}{2}.$$

Note that

$$\frac{H_A(1)}{H_A(1) + H_B(1)} m_A + \frac{H_B(1)}{H_A(1) + H_B(1)} m_B = \mu. \quad (50)$$

Let  $t$  denote the difference in average types between the universities, i.e.

$$t = m_A - m_B.$$

Under full overlap,  $t = 0$ , while under full segregation,  $t = \frac{H_A(1) + H_B(1)}{2}$ . Under partial overlap,

$$t \in \left( 0, \frac{H_A(1) + H_B(1)}{2} \right).$$

Define also

$$D(t) = m_A(t) - m_B(t).$$

By (50),

$$D(t) = \frac{H_A(1) + H_B(1)}{H_B(1)} (m_A(t) - \mu). \quad (51)$$

A stationary point has  $D(t) = t$ . By Theorem 6.5 in Stokey and Lucas (1989), a stationary point is stable if the derivative  $\frac{dD(t)}{dt}$  is less than one in absolute value.

Taking the derivative of  $D(t)$  in (51) gives

$$\frac{dD(t)}{dt} = \frac{H_A(1) + H_B(1)}{H_B(1)} \frac{dm_A}{dt}. \quad (52)$$

Taking the derivative w.r.t.  $t$  in (29) yields

$$\frac{dm_A}{dt} = \frac{1}{2} \left( \frac{x-1}{H_A(1)} + 1 \right) \frac{dy}{dt} + \frac{1}{2} \left( \frac{x-1}{H_A(1)} + 1 \right) \frac{dx}{dt} + \frac{1}{2} \frac{x+y}{H_A(1)} \frac{dx}{dt} - \frac{x}{H_A(1)} \frac{dx}{dt}. \quad (53)$$

By (24),

$$t = \alpha \frac{y-1 + H_A(1) + H_B(1)}{H_B(1)},$$

which gives that

$$\frac{dy}{dt} = \frac{H_B(1)}{\alpha}. \quad (54)$$

Similarly, by (25),

$$t = \alpha \frac{1-x}{H_A(1)},$$

yielding that

$$\frac{dx}{dt} = -\frac{H_A(1)}{\alpha}. \quad (55)$$

Combining (52), (53), (54), and (55), we now have that

$$\frac{dD(t)}{dt} = \frac{H_A(1) + H_B(1)}{\alpha} \left( \frac{x - 1 + H_A(1)}{2H_A(1)} - \frac{y - 1 + H_A(1)}{2H_B(1)} \right). \quad (56)$$

Now, consider first full overlap allocations. For these,  $t = 0$  and  $m_A = m_B$ . Hence,  $D(0) = 0$ , so full overlap allocations are stationary points. Plugging  $x = 1$  and  $y = 1 - H_A(1) - H_B(1)$  into (56) gives that

$$\frac{dD(0)}{dt} = \frac{H_A(1) + H_B(1)}{\alpha}.$$

Thus,  $|\frac{dD(0)}{dt}| \leq 1$  if and only if  $\alpha \geq H_A(1) + H_B(1)$ . This implies that UFO allocations are stable for  $\alpha \geq \tilde{H}_A + \tilde{H}_B$ . Under CFO on the other hand,  $\alpha \geq H_A(1) + H_B(1)$  only North-East of the CPO/CFO boundary given by equation (42). Therefore, CFO allocations are stable for

$$c_0 - w_0 \geq 1 - \frac{\alpha}{2} - \Delta(1 - \alpha). \quad (57)$$

For EAS,  $m_A = 0$  and  $\mu = 0$  from (52). It follows that  $D(t) = 0$ . Therefore,  $t = 0$  is a stationary point. The EAS equilibrium is stable because  $\frac{dD(t)}{dt} = 0$  for all  $t$ .

Consider next full segregation allocations. For these  $t = \frac{H_A(1) + H_B(1)}{2}$  and  $m_A = \frac{2 - H_A(1)}{2}$ . By inserting this and the expression for  $\mu$  into (51), it follows that  $D(t) = \frac{H_A(1) + H_B(1)}{2} = t$ . Therefore,  $t = \frac{H_A(1) + H_B(1)}{2}$  is a stationary point. Under full segregations  $x = y = 1 - H_A(1)$ . Plugging this into (56) shows that  $\frac{dD(t)}{dt} = 0$  for all  $t$ . It follows that the full segregation equilibria are stable.

Next consider partial overlap allocations. By (32) and (33),  $t = 2\alpha \left(1 - \frac{\alpha}{H_A(1) + H_B(1)}\right)$ . Inserting (29) and the expression for  $\mu$  into (51) it follows that  $D(t) = 2\alpha \left(1 - \frac{\alpha}{H_A(1) + H_B(1)}\right) = t$ . Therefore,  $t = 2\alpha \left(1 - \frac{\alpha}{H_A(1) + H_B(1)}\right)$  is a stationary point. By (24),  $y = \frac{tH_B(1)}{\alpha} + 1 - H_A(1) - H_B(1)$ , while by (25),  $x = 1 - \frac{tH_A(1)}{\alpha}$ . Plugging these expressions into (56) shows that  $\frac{dD(t)}{dt} = \frac{H_A(1) + H_B(1)}{\alpha} \left(1 - \frac{t}{\alpha}\right)$ . It follows that

$$\frac{dD \left(2\alpha \left(1 - \frac{\alpha}{H_A(1) + H_B(1)}\right)\right)}{dt} = 2 - \frac{H_A(1) + H_B(1)}{\alpha}.$$

Thus,

$$\left| \frac{dD \left(2\alpha \left(1 - \frac{\alpha}{H_A(1) + H_B(1)}\right)\right)}{dt} \right| \leq 1$$

if and only if  $\alpha \leq H_A(1) + H_B(1)$ .

This implies that UPO allocations are stable for  $\alpha \leq \tilde{H}_A + \tilde{H}_B$ . Under CPO, on the other hand,  $\alpha \leq H_A(1) + H_B(1)$  only South-West of the CPO/CFO boundary given by equation (42). Therefore, CPO allocations are stable for

$$c_0 - w_0 \leq 1 - \frac{\alpha}{2} - \Delta(1 - \alpha).$$

Finally, in SU equilibria, the peer effect at university B,  $m_B$ , is zero. Therefore,  $m_B$  does not depend on  $m_A$  or the mean type in academia,  $\mu$ . Also, by Assumption 2 the rank at B of any researcher who moves to the empty university B is zero. Therefore, any researcher in university A or the outside sector will prefer their location to university B.

**Proof of base salary comparative statics:** (i) That  $H_B(1)$  is decreasing in  $c_0 - w_0$  for CFS and CPO equilibria follows from taking the total derivative in (21) and (40) and solving for  $\frac{dH_B(1)}{d(c_0 - w_0)}$ . In CFS,  $\frac{dH_B(1)}{d(c_0 - w_0)} = \frac{-1}{0.5 - \Delta'(1 - \tilde{H}_A - H_B(1))} < 0$ . In CPO,

$$\frac{dH_B(1)}{d(c_0 - w_0)} = \frac{1}{-0.5 - \frac{2\alpha\tilde{H}_A}{(\tilde{H}_A + H_B(1))^2} \left( \frac{2\alpha}{\tilde{H}_A + H_B(1)} - 1 \right) + \Delta'(1 - \tilde{H}_A - H_B(1))} < 0$$

because  $\frac{2\alpha}{\tilde{H}_A + H_B(1)} - 1 \geq 0$  in CPO. That  $H_A(1) + H_B(1)$  is decreasing in  $c_0 - w_0$  for CFO follows from taking the total derivative of (45) and finding  $\frac{d(H_A(1) + H_B(1))}{d(c_0 - w_0)} = \frac{-1}{0.5 - \Delta'(1 - H_A(1) - H_B(1))} < 0$ .

(ii) Totally differentiating (48) gives  $\frac{dH_A(1)}{d(c_0 - w_0)} = \left[ -\frac{1}{2} + \Delta'(1 - H_A(1)) \right]^{-1} < 0$ .

(iv) In CFS,  $m_A - m_B = \frac{\tilde{H}_A + H_B(1)}{2}$  and it follows that  $\frac{d(m_A - m_B)}{d(c_0 - w_0)} < 0$  since  $\frac{dH_B(1)}{d(c_0 - w_0)} < 0$ .

In CPO,  $m_A - m_B = 2\alpha \left( 1 - \frac{\alpha}{\tilde{H}_A + H_B(1)} \right)$ . Taking the derivative gives that  $\frac{d(m_A - m_B)}{d(c_0 - w_0)} = \frac{2\alpha^2}{(\tilde{H}_A + H_B(1))^2} \frac{dH_B(1)}{d(c_0 - w_0)}$ , which is negative since  $\frac{dH_B(1)}{d(c_0 - w_0)} < 0$ .

(iii) The result for CPO and CFS follows from (iv). For CFO, the quality difference is zero and does not change with  $c_0 - w_0$ . In the unconstrained equilibria, the academic sector is unaffected by changes in the outside option, and in particular the universities' qualities do not depend on  $c_0 - w_0$ .

**Proof of salary progression comparative statics:** (i) That  $H_B(1)$  is increasing in  $\delta_w - \delta_c$  for CFS and CPO equilibria follows from plugging  $\Delta(\theta) = -(\delta_w - \delta_c)\theta$  into (21) and (40), taking the total derivatives, and solving for  $\frac{dH_B(1)}{d(\delta_w - \delta_c)}$ . In CFS,  $\frac{dH_B(1)}{d(\delta_w - \delta_c)} = \frac{1 - \tilde{H}_A - H_B(1)}{0.5 + \delta_w - \delta_c} > 0$ . In CPO,

$$\frac{dH_B(1)}{d(\delta_w - \delta_c)} = \frac{1 - \tilde{H}_A - H_B(1)}{0.5 + \frac{2\alpha\tilde{H}_A}{(\tilde{H}_A + H_B(1))^2} \left( \frac{2\alpha}{\tilde{H}_A + H_B(1)} - 1 \right) + \delta_w - \delta_c} > 0.$$



That  $H_A(1)+H_B(1)$  is increasing in  $\delta_w-\delta_c$  for CFO follow from plugging  $\Delta(\theta) = -(\delta_w-\delta_c)\theta$  into (45), taking the total derivative, and solving for  $\frac{d(H_A(1)+H_B(1))}{d(\delta_w-\delta_c)} = \frac{1-H_A(1)-H_B(1)}{0.5+\delta_w-\delta_c} > 0$ .

(iii) Following the proof of the base salary comparative statics,  $\frac{d(m_A-m_B)}{d(\delta_w-\delta_c)} > 0$  in CFS and CPO since  $\frac{dH_B(1)}{d(\delta_w-\delta_c)} > 0$ .

(ii) The result for CPO and CFS follows from (iii). For CFO, the quality difference is zero and does not change with  $\delta_w - \delta_c$ . In the unconstrained equilibria, the academic sector is unaffected by changes in the outside option, and in particular the universities' qualities do not depend on  $\delta_w - \delta_c$ .

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